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Notes, Comments, and Letters to the Editor

Alliances and negotiations

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Abstract

Negotiations often occur not between individuals but among groups (companies, trade unions, political parties, etc.) each representing composite interests, whose bargaining behaviour is regulated by some collective decision mechanism. In this paper we provide a natural model of such circumstances. We formalise the notion of ‘alliance’ and show how different preference aggregation procedures within the alliance affect the bargaining outcome. In particular, we find that unanimity procedures lead to ‘more aggressive’ negotiating tactics than majority procedures.

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1. Introduction

Negotiations occur not only between individuals but also—and perhaps typically—between organisations. Unlike individuals, organisations do not have preferences that can be treated as primitives. Organisations are conglomerations of composite interests, whose bargaining stance is determined by some collective decision mechanism: consider for instance negotiations involving trading blocks, federations, trade unions, political parties, public companies, business partnerships, and so on. In this paper, we introduce a framework to study how the collective

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mechanisms (e.g. majority versus unanimity voting) used to determine ‘aggregate’ behaviour in a specific type of organisation (which we call an ‘alliance’) are related to bargaining outcomes, and we derive some interesting principles.

For a vivid example, in the diplomatic negotiations leading Nato to take military action against Yugoslavia in 1999, the alliance was often accused of taking an intransigent stance. In particular, the perception was that the ‘hawks’ in the alliance (mainly the British and American governments) were calling the shots despite the strong preference of a number of other members for a more accommodating stance. When faced with this accusation, Nato representatives invariably replied that Nato is an organisation based on unanimity, and that any position taken through complete consensus by so many countries could hardly be blamed for being extremist. This argument may sound convincing at first. But does it stand up to scrutiny? Motivated by such type of questions, we introduce a formal definition of a (bargaining) *alliance* and analyse its behaviour. An alliance is a group of ‘similarly but not identically motivated members’. More precisely, there are two features that distinguish an alliance. It is a group of individual decision makers who: (i) share a common interest, yet also have heterogeneous preferences; (ii) must take a common stance in negotiations.

Our proposal is to model the two seemingly contradictory features in (i) as follows. We assume that members of an alliance share the same preference ordering over the feasible alternatives; yet, their preferences differ in ‘intensity’. In other words, members exhibit different degrees of keenness on a common good. This is formalised by assuming that members have von Neumann–Morgenstern utility functions which, while not coinciding, are all monotonic increasing when alternatives are listed in an appropriate order, which is then the common preference ordering of the members. In thinking about this definition, one may be helped by considering a ‘shareholder model’: presumably all shareholders of a company agree that higher profits are better than lower ones; however, usually any two distinct shareholders will differ in the evaluation of risky prospects of future profits. In the NATO example, all countries agree on some basic security aims; but they disagree on the precise evaluation of these aims and of the outcome in case of breakdown. We use the specific term ‘alliance’ to distinguish it, for example from the notion of a committee, where no restrictions are made on the heterogeneity of preferences of the members. For instance, a union bargaining over wages is an alliance rather than a committee. Although clearly this distinction will often be blurred in reality (e.g. in our NATO example), we think it is clear at the abstract level and a driving force of many negotiations between organisations.¹ As for (ii), we consider several internal decision procedures by which a collective stance is reached by the members of an alliance. We contrast procedures based on *unanimity* with procedures based on *majority*. This allows us to test the seemingly common view that unanimity procedures should yield ‘less extreme’ negotiating stances. We find that, to the contrary, unanimity rules

¹Here we are concerned with negotiation rather than production problems, while in the classical analysis of military alliances of [9] the main objective is the provision of a public good. More recent literature in this vein has emphasised the aspects of impure public goods and joint production (see [13] for an excellent survey).

provide a fertile ground for more aggressive behaviour. The rough intuition for this result is simple. With unanimity, for an alliance to ‘concede’ (namely, to accept a proposal by the opponent) a consensus must be reached; but this means that each individual member will be able to block proposals by the opponent. This has the consequence that any proposal that is accepted in equilibrium must satisfy even the ‘most aggressive’ members. Obviously some finer details are lacking in this informal explanation. For example, which member qualifies as ‘most aggressive’ is not determined by the parameters characterising the preferences of the members of the alliance alone, but also depends on the preferences of the opponent. In particular, we stress that ‘aggressiveness’ cannot be identified with lack of risk aversion: members can be differently ‘aggressive’ even in the particular case when they share the same degree of risk aversion. This is due to the fact that, because the breakdown point is evaluated differently across members, they face different risky prospects when deciding whether to accept a proposal.² Majority voting, conversely, leads by the same logic to a ‘median’ position among those held by the members.

Technically, ours is a particular model of multiperson bargaining. In view of the fact that such models are often known to exhibit a great multiplicity of equilibria, one might initially be skeptical about the value of this enterprise. But it turns out that focussing on alliances pays off by yielding sharp predictions and by generating, in most cases (although not always), a *unique* equilibrium—*without* imposing stationarity as is often done in multiperson models of bargaining.

2. The model

In order to simplify the exposition we assume in the basic model that a single agent negotiates with an alliance. In Section 3.3, we show that the results we obtain under this simplification essentially carry over to the case of two alliances. $N + 1$ agents negotiate over alternatives $s \in S \equiv [0, 1]$,³ where for notational simplicity we assume that N is odd. The breakdown event (distinct from any alternative) is denoted b . Each agent i has preferences over the set of lotteries on $S \cup \{b\}$, $\mathcal{L}(S \cup \{b\})$, which are representable by a von Neumann–Morgenstern (henceforth vNM) utility function $u_i: \mathcal{L}(S \cup \{b\}) \rightarrow \mathcal{R}$. Abusing notation, we use the same symbol u_i to represent preferences over riskless alternatives. The agents $i = 2, \dots, N + 1$ form an alliance in the sense explained in the introduction. Let A denote the set of these agents. So, given $s, s' \in S$, $u_i(s) \geq u_i(s')$ for some $i \in A$ implies $u_j(s) \geq u_j(s')$ for all $j \in A$. Agent 1 has opposite preferences to A over riskless alternatives, that is, given $s, s' \in S$, $u_1(s) \geq u_1(s')$ for some $i \in A$ implies $u_1(s) \leq u_1(s')$. For all $i \in \{1\} \cup A$ there exists $s \in A$

² Moreover, the intuition provided does not explain why it is credible for the hawks to block unsatisfactory proposals in *all* equilibria.

³ This domain restriction is not necessary for our results. Qualitatively similar results can be obtained for a generic set of alternatives S that generates a bargaining frontier which is a continuous strictly decreasing and concave function. Details are available from the authors upon request.

such that $u_i(s) > u_i(b)$. For $i \in A$, u_i is increasing and concave on $[0, 1]$, while u_1 is decreasing and concave on $[0, 1]$.⁴

Note that we do not assume that, given a probability p , for all $i \in \{1\} \cup A$, for all $s \in S$ there exists $s' \in S$ such that $u_i(s') = pu_i(s) + (1 - p)u_i(b)$. This means that, in the bargaining game described below, for some player not all simple lotteries constructed using alternatives in S have a certainty equivalent in S . This way of proceeding seems appropriate in the absence of an explicit discussion of the mechanism by which the ‘agenda’ S is arrived at. It may well be the case, for example, that for some member of A the breakdown event is so bad that the prospect of risking a breakdown with some positive probability or otherwise getting some ‘poor’ alternative is worse than any alternative in S . Therefore we wish not to impose any restrictions as to the relationship between $u_i(b)$ and the values taken on by $u_i(s)$, except that there exists some s such that $u_i(s) > u_i(b)$ for each i . On the contrary, in models of *two-person* bargaining with risk it is usually assumed (e.g. [1,2,7,8,10]), that $u_i(b) = \min_{s \in S} u_i(s)$ or $u_i(b) \geq \min_{s \in S} u_i(s)$. In this way we are able to introduce an additional source of heterogeneity (beside different degrees of risk aversion) among members of A , namely the extent to which breakdown is disliked.⁵

2.1. Bargaining

In the bargaining process specified below, agents in A must make *collective* proposals to agent 1. Similarly, agents in A will have to *collectively* either accept or reject proposals by agent 1. We call the mechanism by which agents in A make such collective proposals, acceptances or rejections an *internal procedure*, or simply a *procedure*. Once an internal procedure \mathcal{P} has been fixed, we use expressions such as ‘proposal by A ’, ‘acceptance by A ’ and ‘rejection by A ’ treating A as a single player. Given any internal procedure \mathcal{P} , negotiations between 1 and A proceed in an alternating offers fashion. That is, A , using the procedure \mathcal{P} , proposes an alternative $s \in [0, 1]$, which 1 can either accept or reject. In the former case, negotiations end with agreement on s . Otherwise, negotiations continue with probability p ; 1 makes a proposal $s' \in [0, 1]$ to A , who can accept or reject, using the procedure \mathcal{P} . In the former case, negotiations end with agreement on s' . Otherwise, negotiations continue with probability p , with A making a proposal. Negotiations proceed in this way over an unbounded number of rounds (numbered $r = 0, 1, 2, \dots$) until agreement is reached, with agent A proposing in even rounds and 1 proposing in odd rounds. After a rejection by A or 1, with probability $1 - p \in (0, 1)$ negotiations break down

⁴Another conceivable way to model an alliance would be to imagine that there are k issues to be discussed. The feasible set is $[0, 1]^k$, the members of the alliance have increasing preferences over each issue, while player 1 has decreasing preferences over each issue. The problem with this method is that one can in general find a pair of issues, say s and s' , such that some members of the alliance prefer s to s' , while the rest of the members *and* player 1 prefer s' to s . Therefore this modelling choice would not reflect our idea of the alliance as a group of members who are always like-minded.

⁵The possible lack of a feasible certainty equivalent does not create particular problem when $S = [0, 1]$ but requires some analysis in the general case. See [5] for details.

irreversibly and the breakdown event occurs. Given a procedure \mathcal{P} , we denote by $\Gamma(\mathcal{P})$ the bargaining game in which procedure \mathcal{P} is used.

2.2. Internal q -quotas procedures

Under a q -quota procedure, the eventual proposal by A as well as the decision to reject or accept a proposal by 1, are arrived at through non-cooperative negotiation games. Regarding A 's responses, given a proposal s by 1, each $i \in A$ either accepts or rejects, in an ordering to be specified later. Then s is accepted and the game ends if at least a quota q of members of A accept. Otherwise, the game moves, with probability $p_A \in (0, 1)$, to the next round, in which a proposal by A must be made in the following way. Each $i \in A$ proposes an alternative s_i . If, and only if, $s_i = s$ for at least a quota q of members of A for some $s \in S$, s constitutes A 's proposal. Failure to reach the quota, both when accepting and when proposing, results with probability $1 - p_A \in (0, 1)$ in irreversible breakdown of internal negotiations, in which case the outcome is for simplicity identified with b . If internal negotiations continue (with probability p_A), each $i \in A$ makes a new proposal, and so on until a quota q of members agrees or breakdown ensues. Regarding the order of moves, in the main text we focus on a *simultaneous* q -quota procedure, denoted \mathcal{P}^q , whereby all $i \in A$ move simultaneously both when accepting and when proposing. It is possible to show that the same results can be obtained within a sequential procedure (see [6]). We study the two cases of majority ($q = \frac{1}{2}$) and unanimity ($q = 1$). As will be apparent from the analysis the extension to the intermediate cases is obvious.

2.3. Equilibrium notion

The basic notion of equilibrium is that of subgame perfect equilibrium (s.p.e.). However, we also need to model the situation in which agents $i \in A$ act non-cooperatively but, being members of an alliance, openly negotiate and sign binding agreements to *coordinate* their strategies if needed. We thus require the actions in the internal negotiation games to be stable not only with respect to *individual* deviations of each $i \in A$ but also with respect to *joint* deviations by *groups* of agents in A . That is, we consider s.p.e. in which at no information set can a subset of agents $A' \subset A$ improve the payoff of each agent in A' by jointly changing their actions, given the equilibrium continuation. We define a subgame perfect equilibrium with this property a *jointly stable* subgame perfect equilibrium (j.s.s.p.e.). The requirement of joint stability merely serves the function of excluding artificial equilibria which are created by the use of a simultaneous move internal negotiation game.⁶ For example, without this restriction, there would be (stationary) s.p.e. where A always proposes an alternative s such that $u_1(s) \geq u_1(s')$ for all $s' \in S$ even if the continuation payoff for player 1 by rejecting and counterproposing is strictly less than $pu_1(s)$; this would

⁶Hence, we do not regard this requirement as sufficiently interesting to be part of a specific procedure. Also, note that a j.s.s.p.e. differs from the notion of Strong Equilibrium because it applies only to the members of the alliance, *not* to set of all players.

happen as no $i \in A$ could individually deviate to a better proposal. It is possible to show that in the case of a sequential protocol for the internal negotiation phase, subgame perfection yields the same outcome as jointly stable subgame perfection (see [6]).

3. Results

The space of alternatives is the unit interval, $S = [0, 1]$. Any $s \in S$ could measure for instance the level of a tariff, or more abstractly a ‘stance’ between two extremes. This restriction of the space of alternatives does not essentially alter the nature of the results.⁷ However, the analysis carried out in this context allows one to use and extend the technique introduced in [12]. For $i \in A$ and for all $s \in [0, 1]$ define the ‘certainty equivalent’ functions $d_i : [0, 1] \times \{0, 1, \dots, \infty\} \rightarrow [0, 1]$ as $d_i(s, r) = s' \in S$ such that $u_i(s') = p^r u_i(s) + (1 - p^r) u_i(b)$ if such an s' exists and 0 otherwise. Similarly, $d_1(s, r) = s' \in S$ such that $u_1(s') = p^r u_1(s) + (1 - p^r) u_1(b)$ if such an s' exists and 0 otherwise. To simplify notation we write throughout $d_i(s)$ instead of $d_i(s, 1)$ for all $i \in \{1\} \cup A$.

3.1. Unanimity

We consider first the unanimity game $\Gamma(\mathcal{P}^1)$. Let \bar{s}_{Ai} (resp. \underline{s}_{Ai}) be the supremum (resp. infimum) of the non-empty (see below) set of certainty equivalents for player $i \in A$ of the j.s.s.p.e. outcomes (alternative-round pairs) in subgames where A makes a proposal. Similarly, let \bar{s}_1 (resp. \underline{s}_1) be the supremum (resp. infimum) of the set of certainty equivalents of j.s.s.p.e. outcomes in subgames where agent 1 makes a proposal. Note that $u_i(\bar{s}_{Ai}) \geq u_i(\underline{s}_{Ai})$ for all $i \in A$, whereas $u_1(\bar{s}_1) \leq u_1(\underline{s}_1)$. Let G^A denote subgames where A is the proposer and let G^1 denote subgames where 1 is the proposer.

Theorem 1. *The game $\Gamma(\mathcal{P}^1)$ has a unique j.s.s.p.e., which is stationary and with immediate agreement on the alternative s_A^* characterised as follows. Let s_i^* be the unique equilibrium alternative which would be agreed upon in a two-person bargaining game between agent i alone and agent 1. Then s_A^* is preferred by all agents in A to all s_i^* , or $s_A^* \geq s_i^*$ for all $i \in A$.*

Proof. The values \bar{s}_{Ai} , \underline{s}_{Ai} , \bar{s}_1 and \underline{s}_1 can be bounded by several inequalities. For any proposal s by agent 1 such that $s < d_i(\underline{s}_{Ai})$ for some $i \in A$, it is optimal, if feasible, for this agent i to induce a subgame of type G^A . With the procedure \mathcal{P}^1 it is feasible for agent i to induce G^A by rejecting s . Thus the certainty equivalent of any equilibrium outcome in subgames where agent 1 acts as proposer must not be smaller than the

⁷Details are available from the authors upon request.

certainty equivalent of \underline{s}_{Ai} for each $i \in A$:

$$\underline{s}_1 \geq d_i(\underline{s}_{Ai}) \text{ for all } i \in A. \tag{1}$$

Conversely, any proposal s by agent 1 such that $s > d_i(\bar{s}_{Ai})$ for each agent $i \in A$ must be accepted by each agent i in equilibrium, and therefore by A as a whole. This bounds from above the certainty equivalent for agent 1 of equilibrium outcomes when he proposes, as follows⁸

$$\bar{s}_1 \leq \max_{i \in A} d_i(\bar{s}_{Ai}). \tag{2}$$

Next, observe that in any s.p.e. agent 1 must accept any proposal s by A such that $s < d_1(\underline{s}_1)$. Now consider the situation where agents in A either never reach an agreement on which proposal to make, or do so with delay. This cannot be part of a j.s.s.p.e., since agents in A can Pareto improve (by avoiding delays in the internal proposal game and by ensuring immediate acceptance by agent 1) on any set of actions yielding any member $i \in A$ less than $u_i(d_1(\underline{s}_1))$. Thus the requirement of joint stability ensures⁹

$$\underline{s}_{Ai} \geq d_1(\underline{s}_1) \text{ for all } i \in A. \tag{3}$$

The next inequality simply follows from the fact that agent 1 in any s.p.e. must reject any proposal s by A such that $s > d_1(\bar{s}_1)$

$$\bar{s}_{Ai} \leq d_1(\bar{s}_1) \text{ for all } i \in A. \tag{4}$$

The above $3N + 1$ inequalities characterising a j.s.s.p.e. can now be considerably simplified. Clearly \underline{s}_{Ai} can be attained in a j.s.s.p.e. with immediate agreement: given $d_1(\underline{s}_1)$, choose the stationary j.s.s.p.e. in which all $i \in A$ always propose $d_1(\underline{s}_1)$; since $d_1(\underline{s}_1)$ is the certainty equivalent corresponding to the supremum of the j.s.s.p.e. payoffs for agent 1, he must accept. Therefore, since the agreement $d_1(\underline{s}_1)$ is obtained with no delay in subgames G^A , the certainty equivalent of $d_1(\underline{s}_1)$ for all $i \in A$ is simply $d_1(\underline{s}_1)$ itself. So $\underline{s}_{Ai} = d_1(\underline{s}_1)$ for all $i \in A$ by inequality 3. Let us denote by \underline{s}_A this common value of the \underline{s}_{Ai} .

A similar reasoning establishes that for all $i \in A$ it must be $d_1(\bar{s}_1) = \bar{s}_{Ai} \equiv \bar{s}_A$. Then the following system is implied:

$$\underline{s}_1 \geq d_i(\underline{s}_A) \text{ for all } i \in A, \tag{5}$$

$$\bar{s}_1 \leq \max_{i \in A} d_i(\bar{s}_A), \tag{6}$$

$$\underline{s}_A = d_1(\underline{s}_1), \tag{7}$$

$$\bar{s}_A = d_1(\bar{s}_1). \tag{8}$$

Since 5 can be written as $\underline{s}_1 \geq \max_{i \in A} d_i(\underline{s}_A)$, relations (5)–(8) also characterise the stationary s.p.e. of a fictitious two player alternating offers bargaining game, Γ^{1R} , in

⁸ Recall that this corresponds to a lower bound on the expected utility agent 1 can get when a proposer.

⁹ Of course, there may be s.p.e. (not jointly stable) in which agents coordinate in \mathcal{P}^1 on a proposal strictly lower than $d_1(\underline{s}_1)$. See Remark 1 below.

which the players are agent 1 and a ‘representative’ agent R whose utility function u_R is defined stepwise as follows. Let the function $k : [0, 1] \rightarrow \{2, \dots, N + 1\}$ be defined by $k(s) = \min\{\arg \max_{i \in A} d_i(s)\}$. Then $u_R(s) = u_{k(s)}$ for all $s \in [0, 1]$, so that by definition R ’s certainty equivalent function d_R is given by $d_R(s) = \max_{i \in A} d_i(s)$. That is, d_R corresponds to the upper envelope of the d_i functions (see example in Fig. 1, where the graph of d_R is the thick line originating in $(0, 0)$). Although u_R is typically not concave or even continuous, some crucial properties of the d_i are inherited by d_R . Firstly, the continuity of the d_i implies that d_R is continuous.

Moreover, a straightforward extension of a standard result (see e.g. [10, pp. 74–75]) shows that concavity of the functions u_i implies *increasing loss to delay* of an additional round: that is, $\phi_i(s) = s - d_i(s)$ is an increasing function. This implies immediately that $\phi_R(s) = s - d_R(s)$ is also an increasing function. This fact, by the analysis in [12], guarantees existence and uniqueness of the s.p.e. agreements of the fictitious game Γ^{1R} , and hence of the original game $\Gamma(\mathcal{P}^1)$. Note for future reference that these values are geometrically characterised by the unique intersection (s_A^*, s_1^*) between the graphs of the functions d_1 and d_R , that $d_1(d_R(s_A^*)) = s_A^*$ and that $d_1(d_R(s)) < s$ for $s > s_A^*$.

Finally, we can characterise the equilibrium of Γ^{1R} in terms of pairwise bargaining games between agent 1 and each agent $i \in A$ individually. All such bargaining games have a unique s.p.e. (which is stationary). Denote by s_i^* the equilibrium proposals by agent $i \in A$ in the game with agent 1. These values again correspond to the (unique) intersections of the graph of the function d_1 with that of each function d_i , and $d_1(d_i(s_i^*)) = s_i^*$ for all $i \in A$.

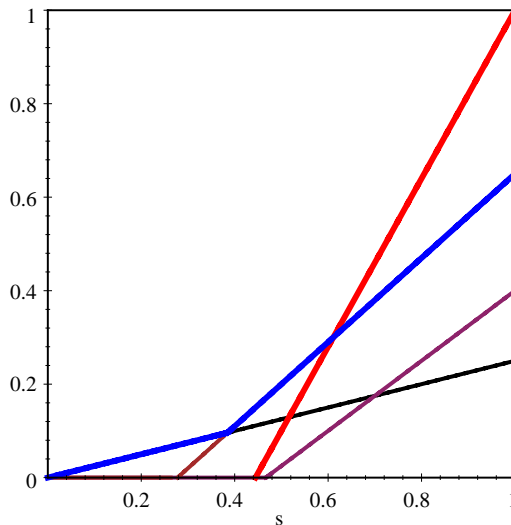


Fig. 1. Equilibrium in the $\Gamma(\mathcal{P}^1)$ game ($A = \{2, 3, 4\}$). The $d_{i \in A}$ functions originate in $(0, 0)$ and d_1 in $(1, 1)$.

The point of intersection between the graph of d_1 and the graph of d_R is a point of intersection between the graph of d_1 and the graph of some d_i , $i \in A$. Since the intersection (x^i, y^i) between the graph of d_1 and the graph of each d_i is unique and has $x^i = s_i^*$, it follows that $s_A^* = s_j^*$ for some $i \in A$. Suppose now that there exists $j \in A$ for which $s_A^* < s_j^*$. Because the graph of d_1 intersects the graph of d_R only once, it cannot be the case that $d_R(s_j^*) = d_j(s_j^*)$; hence from $d_R(s) \geq d_i(s)$ for all $s \in [0, 1]$ we have $d_R(s_j^*) > d_j(s_j^*)$. By the fact that d_1 is nondecreasing this implies $d_1(d_R(s_j^*)) \geq d_1(d_j(s_j^*))$. But, since $d_1(d_j(s_j^*)) = s_j^*$, this contradicts the fact that $d_1(d_R(s)) < s$ for $s > s_A^*$. We can conclude that $s_A^* \geq s_i^*$ for all $i \in A$.

A stationary j.s.s.p.e. of $\Gamma(\mathcal{P}^1)$ can now be constructed in the obvious way. \square

The unanimity procedure leads an alliance to adopt a negotiating stance which coincides with that of the agent which is, in terms of the outcome achievable in a hypothetical pairwise bargaining with agent 1, the ‘most aggressive’ in the alliance. In effect, the unanimity rule has the same effect as a form of ‘optimal delegation’: it is as if the ‘best’ member of the alliance was delegated to negotiate with the opponent. Note that the identity of this member depends crucially on the preferences of the members of the alliance and cannot be identified on the basis of the preferences of the members of the alliance alone. For instance, in terms of Fig. 1, if the d_1 function were flatter (intersecting the horizontal axis at, say, 0.2), the best member of the alliance would be the one whose d_i function starts from the origin. Furthermore, the solution is highly insensitive to the preferences of the less aggressive members. Any change in such preferences which does not make one of the less aggressive members the most aggressive will not have any impact on the equilibrium agreement. Also, while the less aggressive members clearly benefit from their membership in the alliance (in the sense that they obtain a higher payoff than they would in individual bargaining), still *within* the game they may be forced at some (out of equilibrium) nodes by the most aggressive member to take actions which they consider suboptimal: in particular, there will be out of equilibrium nodes in which the most aggressive member forces a rejection of offers that seem perfectly acceptable to other members. Finally, the proof makes clear that the internal procedure could be simplified without changing the result. In particular, the internal procedure could consist simply of a single stage where if there is no agreement breakdown occurs.

The fact that there is no equilibrium other than the one ‘most unfavourable’ to agent 1 is quite remarkable. For instance, one might have expected agent 1 to condition his actions on the identity of rejectors to prevent such an outcome to obtain. Namely, following a rejection by the most aggressive member of the alliance, agent 1 could threaten to make a very unfavourable counteroffer (from the point of view of A). Our result shows that such tactics can never be credible.

Remark 1. The requirement of joint stability has considerable bite. In its absence, many other s.p.e. exist. For example, consider the following strategies: all $i \in A$ always propose 0 and accept any proposal by agent 1; agent 1 always proposes 0 and accepts only proposals not greater than $d_1(0)$. These strategies constitute an s.p.e. of

$\Gamma(\mathcal{P}^1)$ because although each individual member of A has the power to disrupt the proposal game within A , this would only delay by one round the attainment of the same outcome.

3.2. Majority

The analysis of majority procedures follows closely that of unanimity procedures, so it is only sketched. Consider the game $\Gamma(\mathcal{P}^{\frac{1}{2}})$ first. Analogously to the previous section, we can define bounds on the infimum and supremum of the set of certainty equivalents of j.s.s.p.e. outcomes in subgames of type G^1 and G^A . Inequalities (3) and (4) must hold as before, since their justification does not depend on the internal procedure used by A , nor does the reasoning establishing the existence of a common \underline{s}_A (and similarly a common \bar{s}_A). What is affected is instead the proposing behaviour of agent 1, who must now satisfy just the majority of members of A for any of his proposals to be accepted.

Let $m : [0, 1] \rightarrow \{2, \dots, N + 1\}$ be defined, for any $s \in [0, 1]$, as the lowest number for which $\#\{i \in A | d_i(s) \leq d_{m(s)}\} = \frac{N+1}{2}$. That is, $m(s)$ is that agent $j \in A$ for whom, given $s \in [0, 1]$, his d_j function assumes the median value in the set of values assumed at s by all d_i (if there is more than one agent with this property, take the one with the lowest index). The j.s.s.p.e. is characterised by the following conditions (where the requirement of joint stability is now needed for the inequalities corresponding to (1)):

$$\underline{s}_1 \geq d_{m(\underline{s}_A)}(\underline{s}_A), \tag{9}$$

$$\bar{s}_1 \leq d_{m(\bar{s}_A)}(\bar{s}_A), \tag{10}$$

$$\underline{s}_A = d_1(\underline{s}_1), \tag{11}$$

$$\bar{s}_A = d_1(\bar{s}_1). \tag{12}$$

Inequality (9) states that the ‘best’ agreement that agent 1 can obtain is bounded by what the *majority* of members of A are prepared to accept. For any proposal less than $d_{m(\underline{s}_A)}(\underline{s}_A)$ a majority would find it profitable to reject and obtain at worst \underline{s}_A in the next round, whose certainty equivalent is $d_i(\underline{s}_A)$ for agent i . So at least the agent who is pivotal at \underline{s}_A , $m(\underline{s}_A)$, must be satisfied by player 1’s proposal. Symmetrically for condition (10). The analysis of the previous section can now be carried through with the function $d_M : [0, 1] \rightarrow [0, 1]$ defined by $d_M(s) = d_{m(s)}$ (which inherits from the d_i exactly the same properties as d_R) in place of d_R , yielding:

Theorem 2. *The game $\Gamma(\mathcal{P}^{\frac{1}{2}})$ has a unique j.s.s.p.e., which is stationary and with immediate agreement on the alternative s_A^* characterised as follows. Let s_i^* be the unique equilibrium alternative which would be agreed upon in a two-person bargaining game between agent i alone and agent 1. Then s_A^* is the median of the set $\{s_i^*\}_{i \in A}$.*

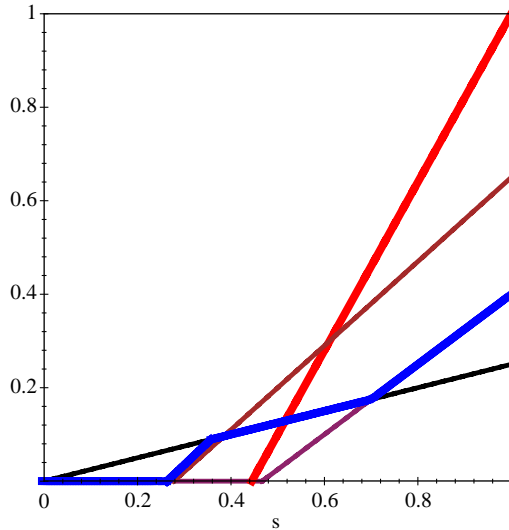


Fig. 2. Equilibrium in the $\Gamma(\mathcal{P}^{\frac{1}{2}})$ game ($A = \{2, 3, 4\}$).

The equilibrium can be visualised as the intersection of the d_1 function with the ‘median envelope’ of the d_i functions, d_M (see example in Fig. 2). The majority procedure leads an alliance to adopt a negotiating stance which coincides with that of the agent which is median in terms of the outcome achieved in hypothetical pairwise bargaining with 1.

3.3. Two alliances

Although for simplicity we have limited ourselves to analysing negotiations between an alliance and a single opponent, it is possible, by using the same techniques, to extend all results to the case of negotiations between two alliances. Let $A = \{1, \dots, N\}$ and $B = \{N + 1, \dots, N + M\}$ be two alliances with opposite preferences over alternatives in S . Denote by $\Gamma(\mathcal{P}_A, \mathcal{P}_B)$ the game between two alliances in which A uses the internal procedure \mathcal{P}_A and B uses the internal procedure \mathcal{P}_B , and where A is the first proposer. Then we can state the following result (for which we omit the now obvious proof):

Theorem 3. In games $\Gamma(\mathcal{P}_A^1, \mathcal{P}_B^1)$ and $\Gamma(\mathcal{P}_A^{\frac{1}{2}}, \mathcal{P}_B^{\frac{1}{2}})$ let s_{iX}^* , with $X = A, B$ and $i \in \{A \cup B\} \setminus X$, be the unique¹⁰ equilibrium alternative which would be agreed upon in a game between agent i alone and alliance X . Then: (i) the game $\Gamma(\mathcal{P}_A^1, \mathcal{P}_B^1)$ has a unique j.s.s.p.e., which is stationary and with immediate agreement on an alternative s_{AB}^* such that s_{AB}^* is preferred by all agents in $\{A \cup B\} \setminus X$ to all s_{iX}^* ; (ii) the game

¹⁰From Theorems 1 and 2.

$\Gamma(\mathcal{P}_A^{\frac{1}{2}}, \mathcal{P}_B^{\frac{1}{2}})$ has a unique j.s.s.p.e., which is stationary and with immediate agreement on the alternative s_{AB}^* which is the median of the set $\{s_{iX}^*\}_{i \in A}$.

4. Concluding remarks

The distinguishing feature of an alliance in this paper is that its members share the same ordinal preferences over alternatives, but with different intensities. This is admittedly a ‘pure’ case, which however captures the relevant aspects of a number of real life situations.¹¹ It is in the very nature of an alliance that its members *must* share some general objectives. The more broadly defined the objectives, the more likely it is that ordinal agreement holds.

Our main findings can be summarised as follows: (i) Unanimity rules are ‘better’ for an alliance than majority rules, in the sense that the equilibrium outcome for an alliance under unanimity dominates the one under majority. More specifically, unanimity rules lead the alliance to behave as if the ‘most aggressive’ member had been delegated to bargain on its behalf. But the identity of such a member is determined not only by the preferences of the members of the alliance, but also by those of the opponents;¹² (ii) ‘Weaker’ members benefit from the presence of ‘stronger members’ in an alliance, in the sense that, for any procedure, acquiring a more aggressive member (one who would obtain a better result in individual bargaining) does not worsen, and possibly strictly improves, the equilibrium outcome for an alliance. However, weaker members may be forced, in virtue of their being bound by the internal procedural rules, to take actions they regard as suboptimal at out of equilibrium nodes.

Two models which are closely related to ours at a conceptual level are [11,4]. [11] consider a multiperson model of war of attrition with two alternatives and incomplete information. Players can be classified into two groups according to which of the two alternatives they prefer. Although the focus, scope and setting of their paper is very different from ours, one of the main points we make—that unanimity leads to more aggressive behaviour—is confirmed in a natural interpretation of their model.¹³ Haller and Holden [4] study a three-stage game situation where large groups (continuum) of agents negotiate by appointing a representative, with the proviso that any agreement he strikes (based on the Nash bargaining solution) has to be ratified by the members, who have to choose the size of the majority required for ratification. Different ratification requirements therefore work as constraints on the

¹¹For example, in a defence alliance presumably all members prefer higher levels of security to lower ones, but their willingness to contribute to it differs. Furthermore, the ordinal unanimity of preferences can be obtained by a careful definition of the basic alternatives.

¹²More extreme outcomes when unanimity is required have been shown to occur in different settings. For example [3] contrast unanimity and majority procedures in a jury setting and show that the unanimity rule increases the probability of making mistakes.

¹³In which the expected concession dates are interpreted as a measure of aggressiveness.

representative's actions. They show that a larger majority ratification rule yields a more favourable outcome, although this entails the risk of rejection by a minority of the group.

Clearly, the choice of a procedure for an alliance has many other facets which we have not considered in this paper. A major example of these, often debated in practice, is the extent to which the majority rule would yield more *timely* decisions. A second major issue of interest is how differential information between members would affect the alliance behaviour (this is one of the main elements in [11]). A full blown bargaining model with incomplete information is beyond the scope of the present analysis and it would be significantly more complex.¹⁴ Thirdly, we have eschewed the analysis of membership rules. All these issues deserve further research.

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¹⁴However, we have a simple two period example with one sided offers by the uninformed party. The intuition that the unanimity rule confers an advantage to the alliance is confirmed, in the sense that the best perfect Bayesian equilibrium for the alliance under unanimity rule cannot be achieved in a p.b.e. under majority rule. See [6].

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