

Neo-Fregeanism and Quantifier Variance

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Abstract

Sider argues that, of maximalism and quantifier variance, the latter promises to let us make better sense of neo-Fregeanism. I argue that neo-Fregeans should, and seemingly do, reject quantifier variance. If they must choose between these two options, they should choose maximalism.

1: Introduction

Benacerraf's problem lies at the heart of philosophy of mathematics: the truth of mathematical statements appears to require the existence of infinitely many abstract mathematical objects, yet we seem to know these truths without having perceptual access to abstracta. What grounds our knowledge of mathematics? Philosophical responses typically take one of two paths, attempting either to dispense with abstract objects, or else to show how we can have knowledge of them.

Neo-Fregeans take the second path, arguing that our knowledge of logic together with our knowledge of a definitional fact about the word 'number' secures our knowledge of abstracta and arithmetic. The definitional fact is Hume's Principle:

$\forall F \forall G$ [the number of Fs = the number of Gs iff the Fs stand in one-one correspondence with the Gs]

We know that there are concepts which stand in one-one correspondence with each other (e.g. the concept *being non-self-identical* stands in one-one correspondence with itself); via the biconditional, we can thus establish that there are numbers. Moreover, we can derive the axioms of Peano Arithmetic from this starting point using logic alone.

To those of us who make a living out of the ‘philosophical science of ontology’¹, this looks like audacious sleight of hand. Of course we can stipulate that ‘the number of the Fs = the number of the Gs’ is true iff the Fs stand in one-one correspondence with the Gs, but can we really then treat that phrase as referring to entities called ‘numbers’? There is apparently no commitment to the existence of numbers on the right-hand side of Hume’s Principle: can stipulation create such a commitment on the left?

Ted Sider believes that our disagreement with neo-Fregeans can be traced to broader disagreements about the nature of language and reality, and the proper constraints upon metaphysics; I agree. He discusses two rival metaontologies—quantifier variance and maximalism—and tries out each as an underpinning for neo-Fregeanism. He doesn’t attribute either view to actual neo-Fregeans, though he does argue that quantifier variance at least supplies all that neo-Fregeans can reasonably hope for.

In response, I will attempt the following. First, I will argue that one of Sider’s attempts to simplify the debate—by shifting attention away from the ‘syntactic priority’ thesis and the role of singular terms—manages instead to distort the debate. Second, I will argue that maximalism is a closer match to actual neo-Fregeanism than is quantifier variance, and that if neo-Fregeans must choose between these two options, maximalism is the right choice to make. One apparent difficulty for my argument is that it is not clear how maximalism can make sense of neo-Fregean talk of ‘reconceptualization’ or ‘recarving content’; I address this difficulty by drawing upon the work of David Armstrong and David Lewis.

2: The Importance of Singular Terms

Taken in isolation the stipulation of Hume’s Principle would seem unobjectionable: we could read ‘the number of Fs is identical to the number of Gs’ as a semantically-simple synonym for ‘the Fs are in one-one correspondence with the Gs’. Moreover, taken in isolation, the neo-Fregean claim that ‘the number of Fs is identical to the number of Gs’ really does have the logical form of an identity statement would also

¹ Crispin Wright, *Frege’s Conception of Numbers as Objects*, Aberdeen: Aberdeen University Press, p. 52.

seem unobjectionable: we could understand Hume's Principle as a bold hypothesis about numbers.

The power and the mystery of neo-Fregeanism derives from combining these two claims: Hume's Principle may be stipulated true, *and* its apparent logical form is genuine, so that if there are any concepts in one-one-correspondence, then there are numbers. Consequently, much effort has been devoted to the question whether 'the number of Fs' and 'the number of Gs' really function as singular terms on the left-hand side of Hume's Principle.²

Sider, however, suggests that this particular debate is a mere distraction. What's crucial is whether sentences like 'the number of Fs is identical to the number of Gs' are susceptible to the usual quantificational laws. For example, suppose that we establish via the biconditional that 'the number of non-self-identical things is identical to the number of non-self-identical things' is true. Neo-Fregeans want to infer from this that there is something which is identical to the number of non-self-identical things. Sider recommends that they simply stipulate that 'the number of Fs' and similar terms are liable to such quantificational treatment and, moreover, that the components of such terms are semantically significant, so that we can infer from the existence of something which is identical to the number of non-self-identical things to the existence of a number.

'The focus on the debate is then placed where it should be: on whether the joint stipulation of [Hume's Principle] and [the logical form claim] is coherent.'³

This account of neo-Fregeanism as founded upon a joint stipulation does make sense of some critical reactions to the view. Reductionists accept the stipulation of Hume's Principle, but deny that the left-hand side reveals any ontological commitment to numbers; rejectionists accept that the left-hand side has its apparent logical form, but reject the stipulation of Hume's Principle.

² See Fraser MacBride, 'Speaking with Shadows: A Study of Neo-Logicism', *British Journal for the Philosophy of Science*, 54 (2003): 103-163, for discussion and references.

³ Theodore Sider, 'NeoFregeanism and Quantifier Variance', *Proceedings of the Aristotelian Society Supplementary Volume #* (2007): ##-##, p.6 in manuscript.

But there are two problems with Sider's attempt to re-frame neo-Fregeanism in this way. First, the issues at stake in the about whether 'the number of Fs' is a singular term will simply reappear in debate about whether the left-hand side of instances of the biconditional satisfy semantic compositionality and penetrability by quantifiers. For neo-Fregeans, all it takes for something to be a singular term is for it to behave like a singular term (for example, to stand on one side of a genuine identity sign), and all it takes for such a term to refer is for it to feature in a true atomic statement. The fact that we can consistently stipulate Hume's Principle to be true reveals that we were already committed to the existence of numbers by our acceptance of one-one correspondences. Second, taking the logical form claim to be a matter of stipulation obscures the fact that it flows directly from neo-Fregeans' metaontological views about syntax, semantics, extra-linguistic reality, and the connections amongst them.

Perhaps my difference with Sider here is just a difference of emphasis: I say that the neo-Fregeans' claim about the logical form of Hume's Principle is a consequence of their metaontology, whereas Sider takes it to be a stipulation whose joint-stipulability with the truth of Hume's Principle is supposed to be licensed by the metaontology. Still, I think this difference of emphasis is significant, for two reasons. First, it shows that the debate about singular terms which Sider recommends sidestepping is integral to the debate about neo-Fregean metaontology; this in turn reminds us that neo-Fregeans do not separate language and world in the neat fashion we scientific ontologists prefer. Second, the reduction of neo-Fregeanism to a joint stipulation naturally prompts the hostile thought that neo-Fregeans are trying to stipulate numbers into existence, rather than, as they see it themselves, revealing our existing ontological commitments.

3: Quantifier Variance or Maximalism?

Sider suggests two possible metaontologies for neo-Fregeanism—quantifier variance and maximalism—and while he doesn't attribute either of these to actual neo-Fregeans, he does recommend that they adopt quantifier variance. In contrast, I will argue both that maximalism is closer to the view of actual neo-Fregeans, and that it provides them with a stronger position. In the first of these, at least, I side with Matti Eklund, who has provided more detailed arguments for assimilating neo-Fregeanism

to maximalism, as well as highlighting some pitfalls for this view.⁴ I will outline the two metaontologies then explain why neo-Fregeans should (and perhaps do) favour maximalism.

The core claim of quantifier variance is that there is a range of equally good meanings for the existential quantifier, none of which is simply a restriction of the others. Central to Sider's paper is the welcome development of a formal framework for understanding this 'neo-Carnapian' idea, which is derived from Eli Hirsch.⁵ Quantifier variance is intended to make sense of 'no-fault' disagreements about ontology where neither party appears to be quantifying over a domain which is simply a restriction of that intended by the other. Hirsch's focus is the metaphysics of material objects, and in particular the question of whether arbitrary sums of familiar objects exist: in one sense of 'exist' they do, in another sense they don't, and there's no deeper meaning to the question whether they *really* exist. But Sider shows how the same framework may cohere with neo-Fregeanism.

A standard objection to neo-Fregeanism is that there just might not be any objects which satisfy the definition of 'number' laid down by the stipulation of Hume's Principle. Surely the most we can stipulate is that *if* there are numbers, then they satisfy Hume's Principle?⁶ On the quantifier variance interpretation, neo-Fregeans take the stipulation of Hume's Principle to shift the meaning of the quantifiers such that it becomes true to say that numbers exist. Roughly speaking, pre-stipulation, 'numbers exist' is false but post-stipulation 'numbers exist' is true, because the meaning of 'exist' shifts at stipulation. Crucially, neither of these meanings of 'exist' is objectively better or more natural than the other; there's no deeper meaning to the question whether numbers *really* exist.

This is only a rough formulation, since strictly speaking 'numbers exist' is meaningless, rather than false, before the stipulation of Hume's Principle. A little more carefully: anyone who said pre-stipulation 'there are infinitely many objects'

⁴ Matti Eklund, 'Neo-Fregean Ontology', *Philosophical Perspectives* 20 (2006): 95-121.

⁵ Eli Hirsch, 'Quantifier Variance and Realism', *Philosophical Issues*, 12 (2002): 51-73.

⁶ Hartry Field, 'Critical Notice of Wright's *Frege's Conception of Numbers as Objects*', *Canadian Journal of Philosophy*, 14 (1984): 637-62.

would have spoken falsely; anyone who made the homophonic claim post-stipulation would speak truly.

The second metaontology, maximalism, is the luxurious view that everything which can exist does exist. Neo-Fregeans apparently only need maximalism about the abstract; on this interpretation, the consistency of Hume's Principle shows that numbers can exist, then maximalism guarantees that they do.⁷ (Maximalism about the abstract is much like 'full-blooded Platonism': this is the view, as formulated by Mark Balaguer (1998), that every mathematical object which could possibly exist does exist.)

If maximalism is to be straightforwardly incompatible with quantifier variance, it must include the rejection of the quantifier variance thesis, as well as the positive claim about what objects there are. Without this, debate about the truth of maximalism would look like just the sort of ontological 'disagreement' which Hirsch's quantifier variance is intended to deflate, and it would not be clear that the views were genuine rivals.

Suitably understood, maximalism and quantifier variance are competing answers to at least three different questions. First, which metaontology is true? Second, what is the intended metaontology of actual neo-Fregeans? Third, which metaontology best serves the goals of neo-Fregeanism? The first question is explicitly sidelined in Sider's paper, except for passing remarks about the falsity of both views; I too will ignore it. Sider also sets aside the second question; I will give it a little more attention. The third question gets most airtime, and Sider's conclusion is that 'NeoFregeans should embrace quantifier variance'.⁸ I will disagree.

My method is the following: I will examine several different aspects of neo-Fregeanism, including some positive features and some difficulties for the view; in each case I will discuss which of maximalism and quantifier variance would be most advantageous to neo-Fregeans, and which, if either, seems close to the views of actual

⁷ Eklund (2006) argues, however, that it will prove difficult for neo-Fregeans to restrict their maximalism to the abstract: the views about language and reality which underwrite the method of abstraction also apply to the concrete realm.

⁸ Sider (2007), p.1 in MS.

neo-Fregeans. I do not claim that this procedure is conclusive, but it does tend to favour maximalism.

4: Mutually Incompatible Abstraction Principles

I will confront a key difficulty for maximalism first. As Sider points out in his sections 4 and 5.2, individually consistent but pairwise inconsistent abstraction principles are a problem for neo-Fregeanism under the maximalist interpretation but not under the quantifier variance interpretation. An extended version of this argument is central to Eklund's criticism of maximalist neo-Fregeanism.⁹ I concede that this problem does provide a good reason for neo-Fregeans to adopt quantifier variance rather than maximalism, if they cannot solve the problem otherwise.

That said, the responses of actual neo-Fregeans to the problem do suggest that their intended metaontology is closer to maximalism than to quantifier variance. In their 2001 prospectus Hale and Wright consider (i) rejecting all abstraction principles which require the universe to have a determinate cardinality; (ii) holding "that *either* (though not of course both) of such a pair of [mutually inconsistent] abstractions is acceptable...there is simply no fact of the matter with which an otherwise acceptable abstraction might conflict and we are in effect merely presented with choices about in which of two incompatible directions we should develop the notion of set"; and (iii) "A third, as it seems to us attractive, line of response, would be to argue that [an abstraction is unacceptable] unless there is no other abstraction incompatible with it which has exactly the same other virtues."¹⁰ It's hard to be certain, but (ii) does sound like quantifier variance, with its various distinct domains of existents, whilst the "attractive" (iii) suggests a single domain, as entailed by maximalism.

5: Duds

Richard Heck points out that first-order equivalence relations are easily gerrymandered: if we do not place restrictions on which relations are suitable for abstraction, we will be lumbered with all sorts of peculiar abstracta.¹¹ Consider an arbitrary equivalence relation, Q , one of whose equivalence classes contains just

⁹ Eklund (2006), section VI.

¹⁰ Bob Hale and Crispin Wright, *The Reason's Proper Study*, Oxford: Oxford University Press (2001), p.426.

¹¹ Richard Heck, 'Syntactic Reductionism', *Philosophia Mathematica* 3, 8 (2000): 124-49.

Heck's shoes, Queen Victoria, and the blackboard in 104 Emerson Hall. Now $Dud(a) = Dud(b)$ iff aQb .

Hale and Wright are rather sanguine about Duds:

Why, after all, shouldn't Fregean abstracts be just as 'proliferated' as sets? This not to deny that if any well-defined first-order equivalence relation is allowed to sustain an abstraction, then some of the resulting abstracts—like Duds—will rightly impress as frivolous. But there seems no good reason why that impression should issue in scepticism about such entities, or why tolerance of them should somehow trivialize the abstractionist account in more serious cases. As an analogy: someone whose notion of set is initially grounded in predication will be inclined to find sets to which no naturally conceived property or relation corresponds equally bizarre on first encounter. Hale and Wright (2001): p.424, fn.8.

Again this isn't decisive, but both the lack of concern and the analogy with sets are a poor match with quantifier variance. Sets are out there and quantified over, regardless of whether and when we define predicates to match them; systematicity pushes us to recognise the full set-theoretical universe, moving beyond our initially restricted view of what exists. Similarly, according to maximalist neo-Fregeanism, abstracta are out there, and quantified over, regardless of whether and when we implicitly define concepts to match them; Heck and systematicity push us to move beyond our initially restricted view of what exists. But according to quantifier variance doesn't fit so well here: at any time, it's true to say that only those abstracta exist which are required by abstraction principles already stipulated true. So the discussion of Duds provides good reason for thinking neo-Fregeans do not intend to endorse quantifier variance.

Is this a wise choice? The quantifier variance interpretation might seem preferable to maximalism on this count, since it would in some sense reduce the number of Dud-like entities in our ontology: before the relevant abstraction principle is rustled up, the peculiar entities do not fall within the scope of our quantifiers, and it's true then to say that they don't exist. This is a meagre advantage, however, given that merely stipulating a gerrymandered abstraction principle is enough to bring the relevant objects into the domain of our quantifiers, making it true to say that they exist.

6: Impredicativity

Neo-Fregeans want to use Hume's Principle to establish the existence of infinitely many numbers, and to do so without relying on the assumption that there are infinitely many non-numerical objects. This means that Hume's Principle must be interpreted in such a way that numbers are amongst the objects falling under the concepts whose one-one correspondence is discussed on the right-hand side. So Hume's Principle is impredicative: 'its first-order quantifiers must be construed as ranging over, *inter alia*, objects of that very kind whose concept it is intended to introduce.'¹²

A more explicit, second-order formulation of Hume's Principle makes this clear:

$$\forall F \forall G [(\text{the number of } Fs = \text{the number of } Gs) \text{ iff} \\ \exists R \forall x ((Fx \rightarrow \exists ! y (Gy \ \& \ Rxy)) \ \& \ (Gx \rightarrow \exists ! y (Fy \ \& \ Ryx)))]$$

If impredicativity is a problem, it is because impredicative definitions are viciously circular. The apparent circularity here is not metaphysical: Hume's Principle is not supposed to show how numbers themselves are somehow composed out of non-numerical objects. Any ban on impredicativity must instead arise from a requirement for epistemic noncircularity. Hume's Principle is supposed to function as an introduction to, or explanation of, the concept 'number' for those who are already familiar with the sorts of facts described on the right-hand side of the biconditional. Thus it is crucial to the success of neo-Fregeanism that it be possible to grasp the content of the right-hand-side without already having a grasp of 'number', even though numbers fall under the concepts whose equinumerosity is discussed on the right-hand side. Recognising the importance of the issue, Hale and Wright argue that one can understand a concept, understand it well enough to know whether the objects falling under it stand in a one-one correspondence with the objects falling under some other concept, without knowing whether numbers fall under that concept.¹³

¹² Hale and Wright (2001), p.21.

¹³ Bob Hale, 'Dummett's Critique of Wright's Attempt to Resuscitate Frege', *Philosophia Mathematica* (3) 2 (1994): 122-47; Crispin Wright, 'On the Harmless Impredicativity of $N^=$ (Hume's Principle)', in *Philosophy of Mathematics Today*, edited by Matthias Schirn (1998), Oxford: Clarendon Press, 339-68.

If maximalism is true, the neo-Fregeans must be right about what it takes to grasp a concept. Given maximalism, all sorts of things exist, things which no-one has ever explicitly considered (including lots of Dud-like entities). Presumably many of these things fall under concepts which we all understand (like *being self-identical*), so it must be possible to understand a concept without having a firm grasp on all the things which fall under it. Maximalism makes it compulsory to think of our understanding of concepts roughly as the neo-Fregeans do, on pain of our not understanding even very familiar concepts. So if we grant maximalism to the neo-Fregeans, the problem of impredicativity dissolves.

But the quantifier variance interpretation makes impredicativity look more worrying. Recall the central idea: by stipulating the truth of Hume's Principle, we shift the meanings of the quantifiers in a way which makes the biconditional true. So the stipulation of Hume's Principle changes the meaning of the one-one correspondence claim on the right-hand side of the biconditional:

The view is *not* that, after introducing Hume's Principle, the quantifiers in mathematical sentences mean something different from the quantifiers in nonmathematical sentences. The idea is rather that all quantifiers throughout the language have changed. Sider (2007), section 5.1

Given this quantifier variance interpretation, it is less clear whether someone could be introduced to the concept of number as the neo-Fregeans say. Before the stipulation of Hume's Principle, one-one correspondence claims do *not* quantify over numbers; after stipulation they do. We lose the cosy picture of our Hero¹⁴, extending his language by correlating claims involving new words (like 'number' or 'direction') with claims involving old words (like 'one-one correspondence' or 'parallel'), whose meaning he already grasps. Instead, Hero must somehow first grasp the *shifted* meaning of the old words, then use this to grasp the meaning of the new words. Moreover we lose the cosy picture of a Hero who already knows that the concept *being-non-self-identical* is equinumerous with itself, and can use this knowledge, via the stipulation of Hume's Principle, to acquire knowledge of numbers. Instead, Hero must somehow work out

¹⁴ Wright (1998).

that ‘being-non-self-identical is equinumerous with itself’ expresses a truth in the new language, before he can use this knowledge to acquire knowledge of numbers.

Now, this difficulty should not be over-stated, since the change in meaning induced by quantifier variance should not be over-stated. As Sider points out, if quantifier variance is not to collapse into the uninteresting claim that the string of letters ‘exists’ could have meant something else, then the various available meanings for the existential quantifier must all in some sense be *existential* meanings, related in some significant way (perhaps their shared inferential rôle is key). So Hero doesn’t face the difficult task of translating his knowledge into an entirely novel language. But recall that, for the purposes of neo-Fregeanism, Hero’s epistemic position must be extremely secure. Adopting a quantifier variance interpretation makes this more difficult to demonstrate; adopting maximalism makes it rather easy.

7: Pre- and Post-Stipulation Asymmetry

As Sider shows, a crucial feature of the quantifier variance approach is that it treats the various meanings for the quantifiers as equally good, and thereby dissolves many apparent disagreements about what exists. This feature distinguishes quantifier variance from certain other attempts to resolve ontological disagreements, those which invoke quantification over a restricted domain. Everyday folk deny that there is any such thing as Noel’s-nose-plus-Liam’s-fist; rather than class these everyday beliefs false, advocates of unrestricted composition may argue that everyday quantifiers range over the restricted domain of ordinary objects, not the universal domain.

Here is Hirsch on such attempts to defend unrestricted composition:

This suggestion is misguided, I think. Ordinary people must have a concept of ‘existence *simpliciter*’. They must understand how to use the quantifier unrestrictedly; otherwise no such use could be part of the English language...if we explain to ordinary people that the [arbitrary sum] in question need not be any kind of familiar thing, it need not be an interesting thing or the sort of thing one would normally talk about, they still regard the sentence [which claims that it exists] as insanely false. Hirsch (2002), p.65

According to Hirsch and his quantifier variance view, philosophical advocates of unrestricted composition use quantifiers which are simply *different* from those used in everyday speech; these quantifiers are not better, less restricted, more encompassing versions of those used by ordinary people.

So quantifier variance encapsulates a kind of symmetry amongst the different meanings for the quantifiers. Maximalism is at odds with this: the speaker who denies the existence of objects which *could* exist either speaks falsely (though perhaps forgivably), or else speaks truly only by virtue of using a quantifier which fails to range over the entire universal domain.

How does this disagreement play out for neo-Fregeanism? Suppose that, before the stipulation of Hume's Principle, Hero denies that there are infinitely many objects (he's not equipped to deny that there are numbers, as such). According to the quantifier variance interpretation, Hero speaks the truth, as indeed he would if he added 'and I mean that absolutely, not even if we include all the things we don't normally talk about'. Maximalists, in contrast, count his denial as false, at least if it is intended to have entirely general scope; Hero's quantifiers already range over numbers (and much else), even if he can be forgiven for not yet realising this.

According to neo-Fregeans, however, abstraction merely reveals the ontological commitments which are already present in claims about one-one-correspondence between concepts (or parallelism between lines, as the case may be).

The Fregean platonist holds...that the ontological commitments of the right-hand sides are just what are displayed in the surface grammar of the left-hand sides—an ontology of both directions and lines. But that seems to imply that even if we speak exclusively in the vocabulary of the right-hand sides, we nevertheless refer, willy-nilly, to directions as well. Indeed, this would be so even if we had no inkling of the concept of direction and never introduced direction terminology. Wright 1990, p. 164 as reprinted in Hale and Wright (2001)

(It's only 'seems to imply' because Wright goes on to argue that although claims about parallel lines ontologically commit us to directions—even if we have no inkling of this—they do not thereby involve *reference* to directions.)

On this view, concept-formulation via stipulation reveals aspects of the world which might otherwise have remained hidden. We improve our epistemic position, our knowledge of what (already) exists, by stipulating Hume's Principle, thereby acquiring a new concept, and by employing logic; post-stipulation we know more about the world. There seems to be an asymmetry between pre- and post-stipulation positions which cannot be captured by the quantifier variance interpretation, according to which no particular meaning of the existential quantifier is the most natural, or objectively best.

As Sider points out, a great advantage of quantifier variance is that it provides neo-Fregeans with a guarantee that existence claims about numbers are true:

If [contrary to quantifier variance] there were a single distinguished quantificational meaning, then it would be an open possibility that numbers, directions and other abstracta are simply missing from existence in the distinguished sense of 'existence' even though we speak in a perfectly consistent way about them...But given quantifier variance, this is not an open possibility. Sider (2007) final page.

This is true; my claim is that maximalism also closes off this worrying possibility, as Sider himself shows, whilst staying closer to the spirit of neo-Fregeanism.

8: Conceptual Recarving

Finally, Sider argues that quantifier variance makes best sense of the neo-Fregean 're-carving' or 'reconceptualization' idea. According to Hale and Wright this is:

Frege's idea at *Grundlagen* 64 that the two halves of an instance of an abstraction principle may be seen as 're-carvings' of a single content:

specifically, that we are free to reconceive the obtaining of an equivalence relation on objects of a certain kind as the identification of objects of a new kind. Hale and Wright (2001) p.421.

In concluding his paper, Sider writes:

Most importantly, the quantifier variance interpretation is a way—the *only* way, as far as I can see—of making sense of the idea that abstraction principles ‘reconceptualize’ facts about, e.g., parallelism and equinumerosity. The core of quantifier variance is that the facts do not demand a unique description in the language of quantifiers. The facts about parallelism can be described by saying ‘there are only lines’ or they can be described by saying ‘there are lines and directions’. Sider (2007) last page

The mapping of quantifier variance onto recarving is not especially smooth. The Fregean claim is that, for any *instance* of the biconditional, the description on the left is equivalent to the description on the right: in some sense, the fact that these two particular lines are parallel just is the fact that they have the same direction. Recarving is something that can be done to parallelism facts one at a time, though it is the abstraction principle which gives us the general recipe for doing so.

In contrast, Sider’s quantifier variance explicitly turns recarving into something that is done just once for the whole realm, by stipulating the abstraction principle: ‘the totality of states of affairs about equinumerosity can be reconceptualized as states of affairs involving the existence of a domain of numbers.’ (section 1.4) This is no accident: no-one should think that directions enter the scope of our existential quantifier piecemeal as we consider various pairs of parallel lines.

So there’s some awkwardness here: quantifier variance doesn’t quite capture reconceptualization as neo-Fregeans intend it. But does it do better than maximalism, at least? You might very well think so. If maximalism is true, then Hume’s Principle merely introduces a term for those entities (numbers) which, whilst being entirely distinct from concepts, are correlated with them in a certain way. Sure, maximalism guarantees that there are such entities, but how could this process be described as

‘reconceptualization’ or ‘recarving’? I think there are two questions here. First, how can commitment to a fact about entities of one sort involve commitment to a fact about entities of another, distinct sort? Second, how can such a relation between distinct entities have anything to do with ‘reconceptualization’?

In considering the first question, we may consult two noble laureates of the philosophical science of ontology: David Armstrong and David Lewis. Armstrong writes, repeatedly and notoriously, that supervenient entities are an ‘ontological free lunch’, and that ‘What supervenes is no addition of being’.¹⁵ Lewis wrote, not so often, but notoriously nevertheless, that once we are committed to the existence of some objects, commitment to their fusion is no further commitment. ‘It just *is* them. They just *are* it.’¹⁶

Consider your ontological commitments. Now ‘add’ an object which is identical (numerically identical) to one of the objects you already accept; clearly this is no further commitment, no addition to being. Indeed, you could stipulate that ‘Robert’ be another name for Lucy, an object you’re already committed to; then a commitment to the existence of Robert is no further commitment, it’s just another way of expressing your existing commitments.

For Lewis, mereological fusion is ontologically innocent because ‘it commits us only to things that are identical, so to speak, to what we were committed to before’.¹⁷ That ‘so to speak’ must do some delicate work, as Sider himself has shown elsewhere.¹⁸ Lewis doesn’t accept the strong view that many-one composition and one-one identity are the very same relation; rather, he accepts an ‘analogical’ version of the thesis, according to which one-one identity is one kind of identity, while composition, and indeed parthood, are other kinds of identity.

Armstrong is bolder. A commitment to supervenient entities is not anything additional to a commitment to the subvening entities, whether or not the superveners are identical to the subveners. Moreover, in the special case of composition, the whole is

¹⁵ D.M. Armstrong, *A World of States of Affairs*, Cambridge: Cambridge University Press (1997), p.12.

¹⁶ David Lewis, *Parts of Classes*, Oxford: Blackwell (1991), p.81.

¹⁷ Lewis (1991) p.82.

¹⁸ Theodore Sider, ‘Parthood’, *Philosophical Review*, forthcoming.

nothing in addition to its parts, and the parts are nothing in addition to the whole, so ‘mereological wholes are identical with all their parts taken together. Symmetrical supervenience yields identity’.¹⁹ A paragraph later, Armstrong writes ‘Like other free lunches, this one gives and takes away at the same time. You get the supervenient for free, but you do not really get an extra entity’. Again, this is a delicate matter: it seems that supervening entities are identical to subveners only where supervenience is symmetric, yet somehow superveners are not really extra entities, even where they fail to be identical to subveners.

Lewis and Armstrong’s views are hardly pellucid, but they do have the sociological advantage of being familiar to many contemporary metaphysicians. And they provide a model for the idea that facts about distinct objects can in some sense be the same fact. The sum is not, for Lewis, literally identical with its many parts, but commitment to the parts already involves commitment to the whole. For Armstrong, supervening entities are real, and are not in general identical to subveners, but a commitment to them is somehow no additional commitment.

This is strongly reminiscent of the neo-Fregean idea that we are already ontologically committed to numbers by our commitment to equinumerous concepts. Recall Wright’s efforts to establish that, despite this, equinumerosity claims do not involve *reference* to numbers; likewise, Lewis and Armstrong would presumably accept that I do not *refer* to a complex object when I refer to the objects which are its parts, even though I thereby commit myself to its existence.

The Australian free lunch resembles the Scottish generosity about ontology, but neither Lewis or Armstrong suggest that supervenient entities are in any sense constructed by our ‘conceptualization’ of them. But nor do neo-Fregeans endorse such an anti-realism. Their picture involves a single fact (or content, or state of affairs) which includes both a pair of parallel lines and a ‘pair’ of identical directions. Which of these entities we ‘see’ depends upon the perspective from which we consider the fact; it is the shift between perspectives which involves recarving or reconceptualizing the fact.

¹⁹ Armstrong (1997), p.12.

Consider an analogy. Suppose that a cat is ontologically more basic than any of its parts. What does this mean? Hard to say exactly, but perhaps the cat is ‘more than the sum of its parts’, or the parts are ontologically dependent upon the cat, or it is essential to the parts that they are parts of the cat, yet inessential to the cat that it have those very parts. Perhaps it’s the converse of whatever philosophers have in mind when they think that macroscopic entities exist, but are less basic than the microscopic entities which compose them.

Now, think of the cat as divided into parts. Which parts? You might consider: head, torso, limbs, tail. Then: cells. Then: left half, right half. Then: front half, back half. No particular decomposition is the most fundamental; the most complete account is the one which includes every part, though this may not be the most useful account; there is a good sense in which commitment to the parts involved in one decomposition just is commitment to the parts involved in every other, as indeed is commitment to the cat. All of the parts exist before you think of them: to say that the parts are secondary to or dependent upon the cat is not to say either that they fail to exist, nor that their existence depends upon our recognising them.

My suggestion, of course, is that on this picture the parts stand to the cat as do the components of the fact to the fact itself. Are you recarving the cat as you idly contemplate it? You’re not physically chopping it up, merely shifting your attention between its various parts; indeed I can think of nothing—concept, thought, whatever—which is literally segmented during this process. Yet the metaphor seems reasonable.

So maximalism offers neo-Fregeans the opportunity to get (more or less) what they want, and, provided we adopt certain other views about ontological free lunches and the primacy of facts, then maximalism even promises to make sense of recarving. But do neo-Fregeans feel the need for *any* explicit metaontology?²⁰

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