

Truth, Demonstration, and Knowledge: A Solution to the Paradox of Knowability

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Semantic anti-realism (henceforth simply ‘anti-realism’) is the doctrine that *there is a conceptual connection between truth and our recognition of it*. As qualifiedly applied to a particular discourse d , anti-realism is the doctrine that there is a conceptual connection between the truth of sentences belonging to d and our recognition of it.¹ Assuming henceforth English to be the language used by the discourse in question, this conceptual connection has quite naturally been supposed to be captured by the formulation of an *epistemic* constraint on the notion of truth operating over the discourse:

(EC⁻) ‘ A ’ is true only if it is feasibly knowable that A ,

whereby ‘ A ’ is henceforth schematic for whichever sentence is supposed to fall within the scope of the anti-realist claim.²

¹Call the unqualified form of anti-realism, meant to apply to sentences belonging to *whichever* discourse, ‘*global* anti-realism’. Call a qualified form of anti-realism, meant to apply just to sentences belonging to a *particular* discourse d , ‘*local* anti-realism with respect to d ’. The discussion of this paper is insensitive to this distinction.

²Note that, throughout, expressions will be individuated in such a way as to make redundant the usual relativization to languages of the truth predicate. Note also that, if we want to preserve their disquotational character, (EC⁻) and its subsequent relatives must be restricted to *unambiguous and context-independent* sentences; alternatively, ‘true’ must be substituted with ‘true-in-the-present-context’ (assuming context also to resolve ambi-

Three features of the *modal epistemic* operator ‘it is feasibly knowable that’ are worth remarking upon right at the outset. Firstly, the operator is intended to be *factive* in the following sense: its being feasibly knowable that A implies ‘ A ’s being true (call this ‘the factivity constraint’). (EC^-) can therefore be strengthened to:

(EC) ‘ A ’ is true iff it is feasibly knowable that A .

Secondly, that the knowability in question is a feasible one means that the relevant possible situations witness to a claim of feasible knowability are situations concerning beings endowed *with our actual cognitive powers* or, at most, with some finite extensions thereof (call this ‘the extendability constraint’). Thirdly, that the knowability in question is a feasible one means that the relevant possible situations witness to a claim of feasible knowability are situations in which the available evidence is *at least nomologically constrained by the present state of the actual world*—that is, by the state at which the claim of feasible knowability is to be evaluated (call this ‘the cosmological constraint’).

Section 1 introduces a particular form of anti-realism and a problem posed to it by a well-known reasoning. Section 2 presents a version of an influential argument for anti-realism which applies only to discourses using a particular kind of language, yielding, for such discourses, the (EC)-version of the form of anti-realism in question. Section 3 makes explicit the distinction between two different kinds of language, showing that the problematic reasoning does not even get off the ground wrt the form of anti-realism in question as applied to a discourse using the first kind of language; it also starts to develop an anti-realist approach to the second and more interesting kind of language. Section 4 completes this task, yielding a non-(EC)-version of the form of anti-realism in question applying to discourses using the second kind of language, and shows how the problematic reasoning breaks down wrt it.

1 The Paradox of Knowability

Unrestricted anti-realism is the doctrine that, whichever schema (such as (EC)) is to constrain the truth of sentences belonging to a discourse, *every* substitution-instance of it holds.³ Under some natural assumptions, (EC)-

guity). For ease of discussion, I will henceforth ignore ambiguity and context-dependence.

³Note that this distinction between unrestricted and restricted anti-realism is orthogonal to the distinction between global and local anti-realism. Note also that unrestricted anti-realism licences no liar- or knower-like paradox without assumptions that are by

unrestricted anti-realism is refuted by the following simple reasoning, originally published in Fitch [1963], pp. 138–9 and known as ‘the paradox of knowability’.

Start with the assumption that it is known⁴ that $[A$ and it is not known that $A]$.⁵ Then, by distribution of knowledge over conjunction, it is known that A and it is known that it is not known that A . By conjunction elimination on the first conjunct, it is known that A . By conjunction elimination on the second conjunct, it is known that it is not known that A . By factivity of knowledge, it is not known that A . Contradiction. By *reductio*, it is not known that $[A$ and it is not known that $A]$. By necessitation, necessarily, it is not known that $[A$ and it is not known that $A]$. Given that metaphysical necessity (of ignorance) implies, as it were, “feasible” necessity (of ignorance), and therefore the negation of feasible possibility of knowledge, this result, together with (contraposed) (EC)-unrestricted anti-realism, implies that ‘ A and it is not known that A ’ is untrue;⁶ together with the *disquotational* schema for *truth*:

(DT) ‘ A ’ is true iff A ,

this yields in turn the result (*substitutionally* quantifying in sentence position and using as quotation device a quotation function creating an autonomous context) that there are no unknown truths.⁷

themselves already sufficient to generate the standard semantic and epistemic paradoxes. Together with the possible gappiness of some sentences, they will henceforth be ignored.

⁴Throughout, read ‘it is known that’ and its relatives as implicitly existentially quantifying over subjects and times.

⁵Throughout, I use square brackets to disambiguate scope in English.

⁶I take for granted, in this paper, the derivability of this conclusion from (EC)-unrestricted anti-realism.

⁷Use the (contraposed) right-to-left direction of (DT) to go from the untruth of ‘ A and it is not known that A ’ to its negation; use the left-to-right direction of (DT) to go from there to the negation of ‘‘ A ’ is true and it is not known that A ’.

Using an appropriate natural-deduction system, the whole reasoning can be formalized as follows:

1	(1)	$\exists st\mathcal{K}_{s,t}(A \wedge \neg \exists st\mathcal{K}_{s,t}A)$	A
1	(2)	$\exists st\mathcal{K}_{s,t}A \wedge \exists st\mathcal{K}_{s,t} \neg \exists st\mathcal{K}_{s,t}A$	1 \mathcal{K} -D
1	(3)	$\exists st\mathcal{K}_{s,t}A$	2 \wedge -E
1	(4)	$\exists st\mathcal{K}_{s,t} \neg \exists st\mathcal{K}_{s,t}A$	2 \wedge -E
1	(5)	$\neg \exists st\mathcal{K}_{s,t}A$	4 \mathcal{K} -E
1	(6)	\perp	3,5 \neg -E
	(7)	$\neg \exists st\mathcal{K}_{s,t}(A \wedge \neg \exists st\mathcal{K}_{s,t}A)$	1,6 \neg -I
	(8)	$\Box \neg \exists st\mathcal{K}_{s,t}(A \wedge \neg \exists st\mathcal{K}_{s,t}A)$	7 \Box -I
	(9)	$\neg \Diamond \exists st\mathcal{K}_{s,t}(A \wedge \neg \exists st\mathcal{K}_{s,t}A)$	8 $\Box \neg \varphi(\vec{\xi}) \Leftrightarrow \neg \Diamond \varphi(\vec{\xi})$

Classically, this result is logically equivalent to every truth's being known. If you accept classical logic and think that it is not the case that every truth is known, you had better reject (EC)-unrestricted anti-realism. If you accept intuitionistic logic and think that there are unknown truths, you too had better reject (EC)-unrestricted anti-realism (and, by the way, you had better reject it even if you only accept the intuitionistically weaker claim that it is not the case that there are no unknown truths).

Should we take the foregoing reasoning to give us reasons for disbelieving unrestricted anti-realism *in general*? No, for there arguably is at least one way of capturing what the advertised connection between truth and our recognition of it consists in which yields a schema different from (EC) and which, contrary to (EC)-unrestricted anti-realism, does not fall prey to the paradox of knowability, or so I shall argue.

2 Manifestation and Anti-Realism

One major line of argument for anti-realism runs as follows. What does *understanding* an expression ultimately consist in? In some capacity or other in *using* the expression,⁸ otherwise understanding would be an utterly mysterious capacity for beings like us to possess. For what else could its *basis* be, since there clearly must be one? And how would it be possible to *assess* whether it is present or not, as we clearly do? And how would it be possible for it to be *transmissible*, as it clearly is? It is therefore reasonable to place the following *manifestation constraint* on understanding: whatever understanding of an expression a speaker possesses, she must ultimately be able to manifest it in her use of that or other related expressions.

$$\begin{array}{ll}
(10) & \neg\blacklozenge\exists st\mathcal{K}_{s,t}(A \wedge \neg\exists st\mathcal{K}_{s,t}A) & 9 \neg\blacklozenge\varphi(\vec{\xi}) \Rightarrow \neg\blacklozenge\varphi(\vec{\xi}) \\
(11) & \neg T^i A \wedge \neg\exists st\mathcal{K}_{s,t}A & 10 \text{ (EC)} \\
(12) & \neg(A \wedge \neg\exists st\mathcal{K}_{s,t}A) & 11 \text{ (DT)} \subset \\
(13) & \neg(TQ(A) \wedge \neg\exists st\mathcal{K}_{s,t}A) & 12 \text{ (DT)} \supset \\
(14) & \Pi P\neg(TQ(P) \wedge \neg\exists st\mathcal{K}_{s,t}P) & 13 \text{ } \Pi\text{-I} \\
(15) & \neg\Sigma P(TQ(P) \wedge \neg\exists st\mathcal{K}_{s,t}P) & 14 \Pi\nu_0\neg\varphi(\vec{\nu}_1) \Rightarrow \neg\Sigma\nu_0\varphi(\vec{\nu}_1),
\end{array}$$

whereby $\ulcorner\mathcal{K}_{\tau_0,\tau_1}\varphi(\vec{\xi})\urcorner$ formalizes $\ulcorner\tau_0$ knows at τ_1 that $\varphi(\vec{\xi})\urcorner$, \blacklozenge is a feasible-possibility operator and $\ulcorner Q\urcorner$ a suitable quotation functor.

⁸The important and widespread phenomenon of *semantic deference* can safely be ignored in this context, as we can restrict our attention to the non-deferring speakers competent with the expression. Clearly, whatever conclusion we reach concerning the relevant fragment of the language will also hold for the fragment of the language spoken by the deferring speakers, as the “fragments” in question are one and the same, at least under a semantic deference thesis strong enough to call in the first place for a restriction of the argument to follow.

Now, *one* use linguistic expressions can be put to is *assertion* (whatever that is): so we may ask what practical capacities can be manifested by a participant in such a use.⁹ On the one hand, we should all agree that assertion is a *norm-governed* practice: acts of assertion are *warranted by* and in turn *warrant* certain things. Henceforth, focus on the input side of assertion.¹⁰ A speaker manifests her taking the world's being *W* to warrant any assertion of a sentence *s* by and only by being disposed (*under some idealization*)¹¹ to assert *s if presented*¹² with the world's being *W*. Conversely, a speaker manifests her taking any assertion of *s* to be warranted only by the world's being *W* by and only by being disposed to assert *s only if* presented with the world's being *W*.¹³ This much she *can* do. But, crucially, this much is also *all* she can do: the practice of assertion is so constituted that nothing over and above a speaker's disposition to take the world's being in a certain way to be a necessary or sufficient condition for an assertion of a certain sentence

⁹It may be objected that the practical capacities involved in understanding an expression need not have anything to do with its use in assertion, even when the expression is sometimes so used. That may be so (and certainly is so if there is no extant practice of assertion). But the point is hardly of any help *for the realist*, as a parallel argument for anti-realism will still go through whatever the privileged understanding-constituting linguistic practices turn out to be. Anyhow, assertion will certainly be such a practice in a community whose *only* linguistic practice is assertion, and we can restrict our claims to such a community, as results will be interesting enough.

¹⁰It is essential to distinguish two different kinds of warrant for an assertion: *objective* warrant—that which makes an assertion at least partially correct *independently of the speaker's situation*—and *subjective* warrant—that which makes an assertion at least partially correct and *depends on the speaker's situation*. For instance, on the one hand, the world's being such that snow is white is an objective warrant for an assertion of 'Snow is white': such an assertion is at least partially correct in a way that it would not be if snow were not white, and it is so no matter what the speaker's situation is. On the other hand, the world's being such that the speaker believes that snow is white is a subjective warrant for an assertion of 'Snow is white': such an assertion is at least partially correct in a way that it would not be if the speaker did not believe that snow is white, and it is so because the speaker is in a particular situation (namely, the one of believing that snow is white). Henceforth, focus on objective warrant.

¹¹The notion of idealization in question is the same as the one explicated by the extendability constraint.

¹²The notion of a presentation in question is to be understood as follows: an object *o* is presented to a subject *s* as being *F* iff there is no better objective epistemic position *s* could be in in order to find out whether *o* is *F* (whereby the range of candidate epistemic positions is subject to the cosmological constraint).

¹³The foregoing assumes that sentences of the language have their own necessary and sufficient warrant conditions. The assumption may however be challenged on holistic grounds, at least for some languages. Only a body of sentences of the language would then be such as to have its own necessary and sufficient warrant conditions. The argument will nevertheless still go through as applied to the relevant body of sentences of the language.

to be warranted can be manifested in it.

To take the world's being W to be the necessary and sufficient condition for an assertion of a sentence s to be warranted is thus to have a disposition to assert (under some idealization) s iff presented with the world's being W . Now, the implicit *modality* in the notion of a *disposition* is such as at least to *require* the world's being *recognizably* W (by a competent speaker for s , under some idealization, if presented to her) if it is W at all. For reflect that any epistemic status falling short of knowledge would either imply the possibility of a false belief in similar circumstances—thereby preventing the disposition in question from being a disposition to assert (under some idealization) s *only if* presented with the world's being W —or the representational inaccessibility of the world's being W —thereby preventing the disposition in question from manifesting the speaker's *taking* the world's being W to be the necessary and sufficient condition for an assertion of s to be warranted.

Moreover, if the world is recognizably W (by a competent speaker for s , under some idealization, if presented to her) if it is W at all, then it is *feasibly knowably* W (by a competent speaker for s), given that the idealization and accessibility in question are accommodated, respectively, by the extendability and cosmological constraints. But, certainly, the world's being W is the necessary and sufficient condition for an assertion of s to be warranted iff speakers take it to be so. We thus conclude that the necessary and sufficient condition for an assertion of a certain sentence to be warranted is such that it is feasibly knowably satisfied if it is satisfied at all.¹⁴

¹⁴The argument can be represented as follows:

- (i) A speaker manifests her taking the world's being W to be a sufficient (necessary) condition for an assertion of a sentence s to be warranted by and only by being disposed (under some idealization) to assert s if (only if) presented with the world's being W . (Required by the manifestation constraint on understanding.)
- (ii) The implicit modality in the notion of a disposition is such as at least to require the world's being recognizably W (by a competent speaker for s , under some idealization, if presented to her) if it is W at all. (Knowledgeability requirement on the manifesting disposition.)
- (iii) Hence, the world must be *feasibly knowably* W (by a competent speaker for s) if it is W at all. (From (i) and (ii) by the extendability and cosmological constraints.)
- (iv) But: the world's being W is a sufficient (necessary) condition for an assertion of s to be warranted iff speakers take it to be so. (Conventionality of sentences' warrant conditions.)
- (v) Therefore, the necessary and sufficient condition for an assertion of a certain sentence to be warranted is such that it is feasibly knowably satisfied if it is satisfied at all. (From (iii) and (iv).)

On the other hand, we should all also agree that assertion is a *world-directed* practice: acts of assertion *represent* the world as being in a certain way and are evaluated accordingly. For instance, whether snow is white or not is in this sense always relevant to the evaluation of an assertion of ‘Snow is white’. And it seems that this is so because *that* snow is white is what ‘Snow is white’ *says*. If ‘Snow is white’ did not say that snow is white, it would be hard to see how whether snow is white or not could still be in this sense always relevant for the evaluation of an assertion of ‘Snow is white’. The dimension of evaluation in question, connecting the status of an asserted sentence with the way the world is and the way the sentence says it is, is best identified with the one of *truth*, for it is part of the meaning of the truth predicate that the following principle connecting *truth* and *saying*:

(TS) For every sentence φ , φ is true iff the world is the way φ says it is

holds.

But what an English sentence ‘*A*’ *says* cannot outrun what a competent speaker for English *understands* it to say. And, by the manifestation constraint on understanding, what a competent speaker for English understands ‘*A*’ to say is something the speaker can manifest in the practice of assertion. Since, as we have seen, all a speaker can manifest in the practice of assertion is what counts as warranting the assertion of ‘*A*’ (say, the world’s being *W*), the world is the way ‘*A*’ says it is iff it is *W*.¹⁵ But, by the previous argu-

¹⁵Equivalence in ‘say that’-contexts is notoriously a highly controversial matter. But even if it turns out that ‘*A*’ does not say that the world is *W*, what it says is something which is in some sense *analytically equivalent* with the world’s being *W* (and that is sufficient to validate the claim in the text that the world is the way ‘*A*’ says it is iff it is *W*). Think of the equivalence between something’s being a vixen and its being a female fox. The equivalence in question between its being the case that *A* and its being the case that *B* may then be cashed out as the fact that its being the case that *A* iff *B* is known by a speaker simply by dint of her competence for a language, insofar as this implies her taking that its being the case that *B* is the necessary and sufficient condition for its being the case that *A*.

With this notion of analytical equivalence, we can also extract a notion of a *defeasible* warrant from the notion of an *undefeasible* warrant with which we have implicitly been working. Say that a speaker takes the world’s being *D* defeasibly to warrant any assertion of a sentence *s*, iff, for some *U* and *C*, the speaker takes the world’s being *U* (undefeasibly) to warrant any assertion of *s* and the world’s being *U* is analytically equivalent with the world’s being *D* and *C*. (This ensures that the speaker’s belief that the world is *C* will carry with it her believing the world’s being *D* to be a sufficient condition for an assertion of *s* to be warranted—a sufficient condition unless the speaker’s belief that the world is *C* is defeated.)

ment, the world is W iff it is feasibly knowably so (by a competent speaker for ‘ A ’), and, given a highly plausible principle of *closure of feasible knowability under analytic equivalence*, if it is feasibly knowably W , then what ‘ A ’ says is feasibly knowably the case. Therefore, what ‘ A ’ says is something that is the case iff it is feasibly knowably the case (the right-to-left direction being guaranteed in turn by the factivity constraint). Given (DT) and the *disquotational* schema for *sentences’ saying*:

(DSS) ‘ A ’ says that A ,

we finally have that ‘ A ’ is true iff it is feasibly knowable that A —that is, (EC)-unrestricted anti-realism!¹⁶

3 Compositionality and Demonstration

You may think that, given what we have seen in section 1, something must have gone terribly wrong somewhere in section 2. Not so. The argument run in section 2 is fine as far as it goes, for it assumes the language used by the discourse in question to be quite an unusual one—namely, to be a language whose sentences do not have *semantically relevant parts*. Sentences of such a language cannot be understood *compositionally*: they must be understood, as it were, from scratch wrt any other sentence, for there can be no relevant semantic *relation* between any distinct sentences of the language. Of course, this is not the case for English nor for actual natural languages in general: for instance, for every sentences φ and ψ , if one understands φ , ψ and ‘and’, then one also understands $\ulcorner\varphi$ and $\varphi\urcorner$, $\ulcorner\varphi$ and $\psi\urcorner$, $\ulcorner\psi$ and $\varphi\urcorner$, $\ulcorner[\varphi$ and $\psi]$ and $\varphi\urcorner$, and so forth.

¹⁶The argument can be represented as follows:

- (I) What an English sentence ‘ A ’ says cannot outrun what a competent speaker for English understands it to say. (Connection between saying und understanding.)
- (II) Hence, since all a speaker can manifest in the practice of assertion is what counts as warranting the assertion of ‘ A ’ (say, the world’s being W), the world is the way ‘ A ’ says it is iff it is W . (From (I) and the manifestation constraint on understanding.)
- (III) The world is W iff it is feasibly knowably so. (From (v).)
- (IV) Hence, the world is the way ‘ A ’ says it is iff it is feasibly knowably so. (From (II) and (III) by closure of feasible knowability under analytic equivalence (left-to-right) and the factivity constraint (right-to-left).)
- (V) Therefore, ‘ A ’ is true iff it is feasibly knowable that A . (From (IV), (TS) and (SS).)

Once the argument for (EC)-unrestricted anti-realism wrt a *non-compositional* language n has been laid out in detail, it is clear why the paradox of knowability does not tell against it. For, to recall, understanding of a sentence φ belonging to n requires a speaker to be disposed (under some idealization) to assert φ iff she is presented with what φ says, whereby nothing short of knowledge that what φ says is the case suffices for being so disposed. But if what φ says is, for some P , that P and it is not known that P , then no one can ever be so disposed, exactly because of the validity of the reasoning (1)-(7) exploited by the paradox of knowability. So, by the manifestation constraint on understanding, no one can ever so much as *understand* φ . So φ is crucially *not* a sentence of n and thus falls outside of the scope of unrestricted (EC) for n . No paradox arises: for every true sentence φ of n , it is feasibly knowable that what φ says is the case, and this does not collapse on every true sentence φ of n being such that it is known that what φ says is the case. For languages for which the anti-realist argument from the theory of understanding does indeed licence unrestricted (EC) (namely, non-compositional languages), the paradox of knowability does not even get off the ground: it arises from substituting in (EC) a sentence saying, for some P , that P and it is not known that P , but any such sentence is not a sentence of any non-compositional language and thus cannot be substituted in (EC) for any such language.

On the other hand, we should not expect the anti-realist argument from the theory of understanding to licence unrestricted (EC) when applied to a *compositional* language. If a language has a compositional semantics, we only require, for every sentence belonging to it, that understanding of the sentence consist in understanding its ultimate semantically relevant parts and their modes of composition. For our purposes, we can focus on a simple interpreted standard first-order epistemic language without identity and with only general terms of arbitrary arity as its non-logical constants.

From the point of view which sees assertion as a *norm-governed* practice, the basic notion will now be the notion of an act of *application* of a *formula* to certain *objects* being warranted by certain things. The practice of assertion is so constituted that nothing over and above a speaker's disposition to take certain objects' being in a certain way to be the necessary and sufficient condition for an application of a certain formula to them to be warranted can be manifested in it.

Start with the *atomic formulae*. A speaker manifests her taking a sequence of objects¹⁷ being W to warrant application of an atomic formula

¹⁷The set-theoretic machinery is adopted only for convenience of exposition, but is ultimately dispensable.

$\varphi(\vec{\xi})$ to it by and only by being disposed (under some idealization) to apply $\varphi(\vec{\xi})$ to a sequence presented to her *if* the sequence is W . Conversely, a speaker manifests her taking any application of $\varphi(\vec{\xi})$ to be warranted only by a sequence's being W by and only by being disposed (under some idealization) to apply $\varphi(\vec{\xi})$ to a sequence presented to her *only if* the sequence is W . Say that a sequence *permonstrates* a formula $\varphi(\vec{\xi})$ iff the application of $\varphi(\vec{\xi})$ to it is warranted. The previous considerations linking taking something as warrant for something, dispositions and knowledge require that a sequence be *recognizably* W (by a competent speaker for $\varphi(\vec{\xi})$, under some idealization, if presented to her) if it is W at all.

Moreover, if a sequence is recognizably W (by a competent speaker for $\varphi(\vec{\xi})$, under some idealization, if presented to her) if it is W at all, then it is *feasibly knowably* W (by a competent speaker for $\varphi(\vec{\xi})$), given that the idealization and accessibility in question are accommodated, respectively, by the extendability and cosmological constraints. But, certainly, a sequence's being W is the necessary and sufficient condition for an application of $\varphi(\vec{\xi})$ to it to be warranted iff speakers take it to be so. We thus conclude that the necessary and sufficient condition for a sequence to permonstrate an atomic formula is such that it is feasibly knowably satisfied if it is satisfied at all.¹⁸

¹⁸The argument can be represented as follows:

- (i') A speaker manifests her taking a sequence's being W to be a sufficient (necessary) condition for an application of an atomic formula $\varphi(\vec{\xi})$ to be warranted by and only by being disposed (under some idealization) to apply $\varphi(\vec{\xi})$ to a sequence presented to her if (only if) the sequence is W . (Required by the manifestation constraint on understanding.)
- (ii') The implicit modality in the notion of a disposition is such as at least to require the sequence's being recognizably W (by a competent speaker for $\varphi(\vec{\xi})$, under some idealization, if presented to her) if it is W at all. (Knowledgeability requirement on the manifesting disposition.)
- (iii') Hence, the sequence must be feasibly knowably W (by a competent speaker for $\varphi(\vec{\xi})$) if it is W at all. (From (i') and (ii') by the extendability and cosmological constraints.)
- (iv') But: the sequence's being W is a sufficient (necessary) condition for an application of $\varphi(\vec{\xi})$ to be warranted iff speakers take it to be so. (Conventionality of atomic formulae's warrant conditions.)
- (v') Therefore, the necessary and sufficient condition for a sequence to permonstrate an atomic formula is such that it is feasibly knowably satisfied if it is satisfied at all. (From (iii') and (iv').)

Enter *logical operators*. They form complex expressions out of simpler ones, the necessary and sufficient permonstration conditions of the compound being a *function* of the necessary and sufficient permonstration conditions of its components. Since, in general, a speaker's believing that a sequence permonstrates a formula $\varphi(\vec{\xi})$ is displayed by the speaker's disposition to apply $\varphi(\vec{\xi})$ to it, a speaker's taking that the mode of composition characteristic of an n -ary logical operator Ω is a certain function will be manifested by the speaker's disposition (under some idealization) to apply $\Omega\varphi_0(\vec{\xi}_0), \varphi_1(\vec{\xi}_1), \dots, \varphi_{n-1}(\vec{\xi}_{n-1})$ to a sequence presented to her being an *isomorphic* function of the speaker's dispositions to apply $\varphi_0(\vec{\xi}_0), \varphi_1(\vec{\xi}_1), \dots, \varphi_{n-1}(\vec{\xi}_{n-1})$ to it.

That is the master thought in this dialectic: compositionality requires nothing more than the Ω -relevant isomorphism. Once this is in place, nothing more is required for understanding of $\Omega\varphi_0(\vec{\xi}_0), \varphi_1(\vec{\xi}_1), \dots, \varphi_{n-1}(\vec{\xi}_{n-1})$ (that is, for one's taking that the necessary and sufficient permonstration condition of $\Omega\varphi_0(\vec{\xi}_0), \varphi_1(\vec{\xi}_1), \dots, \varphi_{n-1}(\vec{\xi}_{n-1})$ is the value of the function associated with Ω when applied to the necessary and sufficient permonstration conditions of $\varphi_0(\vec{\xi}_0), \varphi_1(\vec{\xi}_1), \dots, \varphi_{n-1}(\vec{\xi}_{n-1})$), for every formulae $\varphi_0(\vec{\xi}_0), \varphi_1(\vec{\xi}_1), \dots, \varphi_{n-1}(\vec{\xi}_{n-1})$ (assuming understanding of $\varphi_0(\vec{\xi}_0), \varphi_1(\vec{\xi}_1), \dots, \varphi_{n-1}(\vec{\xi}_{n-1})$). In particular, given compositionality, it is *not* required that a speaker's taking that the operation characteristic of Ω is a certain function be manifested by nothing less than the speaker's disposition (under some idealization) to apply $\Omega\varphi_0(\vec{\xi}_0), \varphi_1(\vec{\xi}_1), \dots, \varphi_{n-1}(\vec{\xi}_{n-1})$ to a sequence presented to her iff things *are* in a certain way as regards the permonstration conditions of $\varphi_0(\vec{\xi}_0), \varphi_1(\vec{\xi}_1), \dots, \varphi_{n-1}(\vec{\xi}_{n-1})$, as such a requirement would amount, in effect, to treating $\Omega\varphi_0(\vec{\xi}_0), \varphi_1(\vec{\xi}_1), \dots, \varphi_{n-1}(\vec{\xi}_{n-1})$ as a linguistic expression without semantically relevant parts, expression whose understanding is unrelated to understanding of any other expression of the language.

A speaker may be disposed (under some idealization) to apply $\varphi(\vec{\xi}_0) \wedge \psi(\vec{\xi}_1)$ to a sequence presented to her iff she is disposed to apply $\varphi(\vec{\xi}_0)$ to it *and* she is disposed to apply $\psi(\vec{\xi}_1)$ to it. She thereby manifests her taking a sequence permonstrating $\varphi(\vec{\xi}_0)$ and permonstrating $\psi(\vec{\xi}_1)$ to be the necessary and sufficient condition for the sequence to permonstrate $\varphi(\vec{\xi}_0) \wedge \psi(\vec{\xi}_1)$.

A speaker may be disposed (under some idealization) to apply $\varphi(\vec{\xi}_0) \vee \psi(\vec{\xi}_1)$ to a sequence presented to her iff she is *either* disposed to apply $\varphi(\vec{\xi}_0)$ to it *or* disposed to apply $\psi(\vec{\xi}_1)$ to it. She thereby manifests her taking a sequence permonstrating $\varphi(\vec{\xi}_0)$ or permonstrating $\psi(\vec{\xi}_1)$ to be the necessary

and sufficient condition for the sequence to permonstrate $\varphi(\vec{\xi}_0) \vee \psi(\vec{\xi}_1)$.

A speaker may be disposed (under some idealization) to apply $\varphi(\vec{\xi}_0) \supset \psi(\vec{\xi}_1)$ to a sequence presented to her iff, *if* she is disposed to apply $\varphi(\vec{\xi}_0)$ to it, *then* she is disposed to apply $\psi(\vec{\xi}_1)$ to it. She thereby manifests her taking a sequence permonstrating $\psi(\vec{\xi}_1)$ if it permonstrates $\varphi(\vec{\xi}_0)$ to be the necessary and sufficient condition for the sequence to permonstrate $\varphi(\vec{\xi}_0) \supset \psi(\vec{\xi}_1)$.

A speaker may be disposed (under some idealization) to apply $\neg\varphi(\vec{\xi}_0)$ to a sequence presented to her iff, if she is disposed to apply $\varphi(\vec{\xi}_0)$ to it, then, for some formula $\psi(\vec{\xi}_1)$ whose application to it she *rejects*,¹⁹ she is disposed to apply $\psi(\vec{\xi}_1)$ to it. She thereby manifests her taking a sequence permonstrating, for some absurd formula $\psi(\vec{\xi}_1)$, $\psi(\vec{\xi}_1)$ if it permonstrates $\varphi(\vec{\xi}_0)$ to be the necessary and sufficient condition for the sequence to permonstrate $\neg\varphi(\vec{\xi}_0)$.

A speaker may be disposed (under some idealization) to apply $\forall\xi_0\varphi(\vec{\xi}_1)$ to a sequence presented to her iff she is disposed to apply $\varphi(\vec{\xi}_1)$ to *every* sequence differing from it at most at its ξ_0 -relevant place. She thereby manifests her taking a sequence to be such that every sequence differing from it at most at its ξ_0 -relevant place permonstrates $\varphi(\vec{\xi}_1)$ to be the necessary and sufficient condition for the sequence to permonstrate $\forall\xi_0\varphi(\vec{\xi}_1)$.

A speaker may be disposed (under some idealization) to apply $\exists\xi_0\varphi(\vec{\xi}_1)$ to a sequence presented to her iff she is disposed to apply $\varphi(\vec{\xi}_1)$ to *some* sequence differing from it at most at its ξ_0 -relevant place. She thereby manifests her taking a sequence to be such that some sequence differing from it at most at its ξ_0 -relevant place permonstrates $\varphi(\vec{\xi}_1)$ to be the necessary and sufficient condition for the sequence to permonstrate $\exists\xi_0\varphi(\vec{\xi}_1)$.

A speaker may be disposed (under some idealization) to apply $\mathcal{K}_{\xi_0, \xi_1}\varphi(\vec{\xi}_2)$ to a sequence presented to her iff she is disposed to apply $\varphi(\vec{\xi}_2)$ to it and a permonstration of $\varphi(\vec{\xi}_2)$ by it is *implemented in a knowledge-conferring way* by the value of ξ_0 at the value of ξ_1 . She thereby manifests her taking a knowledge-conferring implementation by the value of ξ_0 at the value of ξ_1 of a permonstration of $\varphi(\vec{\xi}_2)$ by a sequence to be the necessary and sufficient condition for the sequence to permonstrate $\mathcal{K}_{\xi_0, \xi_1}\varphi(\vec{\xi}_2)$.

Finally, say that a formula is a *sentence* iff no free variable occurs in it, and that there is a *demonstration* of a sentence φ iff φ is permonstrated by some (every) sequence. It follows that there is a demonstration of $\varphi \wedge \psi$ iff there is a

¹⁹Rejection is taken here to be a primitive attitude constitutive of the overall practice of assertion.

demonstration of φ and a demonstration of ψ ; that there is a demonstration of $\varphi \vee \psi$ iff there is either a demonstration of φ or a demonstration of ψ ; that there is a demonstration of $\varphi \supset \psi$ iff, if there is a demonstration of φ , then there is a demonstration of ψ ; that there is a demonstration of $\neg\varphi$ iff, if there is a demonstration of φ , then there is a demonstration, for some absurd sentence ψ , of ψ ; that there is a demonstration of $\forall\xi\varphi(\xi)$ iff there is a permonstration of $\varphi(\xi)$ by every sequence; that there is a demonstration of $\exists\xi\varphi(\xi)$ iff there is a permonstration of $\varphi(\xi)$ by some sequence;²⁰ that there is a demonstration of $\exists\xi_0\xi_1\mathcal{K}_{\xi_0,\xi_1}\varphi(\xi_0,\xi_1)$ iff, for some sequence, there is a permonstration of $\varphi(\xi_0,\xi_1)$ by the sequence and this permonstration is implemented in a knowledge-conferring way by the value of ξ_0 at the value of ξ_1 .

4 Truth, Demonstration, and Knowledge

From the point of view which sees assertion as a *world-directed* practice, acts of application *represent* their objects as being in a certain way and are evaluated accordingly. For instance, whether snow is white or not is in this sense always relevant to the evaluation of an application of ‘ x is white’ to it. And it seems that this is so because *that* it is white is what ‘ x is white’ *says* of snow. If the formula did not say of snow that it is white, it would be hard to see how whether snow is white or not could still be in this sense always relevant for the evaluation of an application of ‘ x is white’ to it. The dimension of evaluation in question, connecting the status of an applied expression with the way the objects it is applied to are and the way the expression says they are, is best identified with that of *satisfaction*,²¹ for it is part of the meaning of the satisfaction predicate that the following principle connecting *satisfaction* and *saying*:

²⁰If our language is enriched with constants for every object of the domain and the definition of demonstration is expanded in a suitable way so as to cover the new set of atomic sentences, the *demonstration functionality* of the quantifiers and of the knowledge operator can be restated as follows: there is a demonstration of $\forall\xi\varphi(\xi)$ iff, for every constant κ , there is a demonstration of $\varphi(\kappa/\xi)$; there is a demonstration of $\exists\xi\varphi(\xi)$ iff, for some constant κ , there is a demonstration of $\varphi(\kappa/\xi)$; there is a demonstration of $\exists\xi_0\xi_1\mathcal{K}_{\xi_0,\xi_1}\varphi(\xi_0,\xi_1)$ iff, for some constants κ_0 and κ_1 , there is a demonstration of $\varphi(\kappa_0/\xi_0,\kappa_1/\xi_1)$ and this demonstration is implemented in a knowledge-conferring way by the denotation of κ_0 at the denotation of κ_1 .

²¹The intuitive semantic notion here is the one of *being true of*, whose arity is variable. The notion of satisfaction can then be extracted by first forming a 2-ary predicate $\ulcorner\xi_0$ is true* of $\xi_1\urcorner$ true of formulae and sequences and then taking its converse.

- (SS) For every formula $\varphi(\vec{\xi})$, a sequence $\langle o_0, o_1, \dots \rangle$ satisfies $\varphi(\vec{\xi})$ iff o_0, o_1, \dots are the way $\varphi(\vec{\xi})$ says they are

holds.

But what an English formula ‘ $A(\vec{v})$ ’ *says* cannot outrun what a competent speaker for English *understands* it to say. And, by the manifestation constraint on understanding, what a competent speaker for English understands ‘ $A(\vec{v})$ ’ to say is something the speaker can manifest in the practice of assertion. Since, as we have seen, all a speaker can manifest in the practice of assertion is what counts as a permonstration of ‘ $A(\vec{v})$ ’ by a sequence, o_0, o_1, \dots are the way ‘ $A(\vec{v})$ ’ says they are iff $\langle o_0, o_1, \dots \rangle$ permonstrates ‘ $A(\vec{v})$ ’. By (SS), a sequence satisfies ‘ $A(\vec{v})$ ’ iff it permonstrates ‘ $A(\vec{v})$ ’.²²

Now, a formula $\varphi(\vec{\xi})$ is true iff it is a sentence satisfied by some (every) sequence. By the previous argument, we can then conclude that an expression $\varphi(\vec{\xi})$ is true iff it is a sentence permonstrated by some (every) sequence—that is, iff there is a demonstration of it.²³ What the anti-realist argument from the theory of understanding yields in the case of a compositional language is thus the following *demonstration* constraint on the notion of truth operating over a certain discourse:

²²And so, given the *disquotational* schema for *formulae’s saying*:

- (DFS) ‘ $A(\vec{v})$ ’ says of o_0, o_1, \dots that they are A ,

the corresponding *activity* principles for permonstration and demonstration follow.

²³The argument can be represented as follows:

- (I’) What an English formula ‘ $A(\vec{v})$ ’ says cannot outrun what a competent speaker for English understands it to say. (Connection between saying and understanding.)
- (II’) Hence, since all a speaker can manifest in the practice of assertion is what counts as a permonstration of ‘ $A(\vec{v})$ ’ by a sequence, o_0, o_1, \dots are the way ‘ $A(\vec{v})$ ’ says they are iff $\langle o_0, o_1, \dots \rangle$ permonstrates ‘ $A(\vec{v})$ ’. (From (I’) and the manifestation constraint on understanding.)
- (III’) Hence, a sequence satisfies ‘ $A(\vec{v})$ ’ iff it permonstrates ‘ $A(\vec{v})$ ’. (From (II’) and (SS).)
- (IV’) A formula is true iff it is a sentence satisfied by some (every) sequence. (Connection between truth and satisfaction.)
- (V’) Therefore, a formula is true iff it is a sentence permonstrated by some (every) sequence—that is, iff there is a demonstration of it. (From (III’) and (IV’).)

(DC) ‘ A ’ is true iff there is a demonstration of ‘ A ’.

Crucially, the argument does not discriminate between different kinds of sentences, and so (DC) holds *unrestrictedly*. Is (DC)-unrestricted anti-realism refuted by the paradox of knowability?

To see that this is not the case, return first to the basic notion of a permonstration. A speaker manifests her taking a W sequence to permonstrate an atomic formula $\varphi(\vec{\xi})$ by and only by being disposed (under some idealization) to apply $\varphi(\vec{\xi})$ to a sequence presented to her if the sequence is W . As already remarked, W sequences permonstrate $\varphi(\vec{\xi})$ iff speakers take them to do so. Therefore, $\varphi(\vec{\xi})$ is permonstrated by a sequence if the sequence *is* W —whereby being W need not imply anything about a speaker’s situation. Nothing more is required from a permonstration of $\varphi(\vec{\xi})$ by a sequence. In particular, nothing more is required in terms of a speaker’s *epistemic* position.

Analogously, a speaker manifests her taking the sufficient permonstration conditions of a compound $\Omega\varphi_0(\vec{\xi}_0), \varphi_1(\vec{\xi}_1), \dots, \varphi_{n-1}(\vec{\xi}_{n-1})$ (with Ω ’s being an n -ary logical operator) to be a certain function of the sufficient permonstration conditions of $\varphi_0(\vec{\xi}_0), \varphi_1(\vec{\xi}_1), \dots, \varphi_{n-1}(\vec{\xi}_{n-1})$ (so that, for instance, things’ being W as regards the sufficient permonstration conditions of $\varphi_0(\vec{\xi}_0), \varphi_1(\vec{\xi}_1), \dots, \varphi_{n-1}(\vec{\xi}_{n-1})$ with respect to a sequence is sufficient for the sequence’s permonstrating $\Omega\varphi_0(\vec{\xi}_0), \varphi_1(\vec{\xi}_1), \dots, \varphi_{n-1}(\vec{\xi}_{n-1})$) by and only by being such that her disposition (under some idealization) to apply $\Omega\varphi_0(\vec{\xi}_0), \varphi_1(\vec{\xi}_1), \dots, \varphi_{n-1}(\vec{\xi}_{n-1})$ to a sequence presented to her is a certain function of her dispositions to apply $\varphi_0(\vec{\xi}_0), \varphi_1(\vec{\xi}_1), \dots, \varphi_{n-1}(\vec{\xi}_{n-1})$ to it. As already remarked, a sequence permonstrates $\Omega\varphi_0(\vec{\xi}_0), \varphi_1(\vec{\xi}_1), \dots, \varphi_{n-1}(\vec{\xi}_{n-1})$ iff speakers take it to do so. Therefore, $\Omega\varphi_0(\vec{\xi}_0), \varphi_1(\vec{\xi}_1), \dots, \varphi_{n-1}(\vec{\xi}_{n-1})$ is permonstrated by a sequence if things *are* W as regards the sufficient permonstration conditions of $\varphi_0(\vec{\xi}_0), \varphi_1(\vec{\xi}_1), \dots, \varphi_{n-1}(\vec{\xi}_{n-1})$ with respect to it—whereby being W need not imply anything about a speaker’s situation. Again, nothing more is required from a permonstration of $\Omega\varphi_0(\vec{\xi}_0), \varphi_1(\vec{\xi}_1), \dots, \varphi_{n-1}(\vec{\xi}_{n-1})$ by a sequence. Again, in particular, nothing more is required in terms of a speaker’s *epistemic* position.

I submit this is a basic fact about the *logic* of manifesting one’s understanding of a rule in general. Consider the manifestation of a player’s understanding of chess rules. All a player can do is to accept moves made in accordance with those rules and reject any other move, but this is crucially *not* taken to manifest the player’s understanding that two chess situations s_0

and s_1 are such that s_1 is a permissible development of s_0 *only if* the player judges them to be so; what the player’s behaviour is taken to manifest is, rather, her understanding that s_0 and s_1 are such that s_1 *simply is* a permissible development of s_0 , no matter whether one judges it to be so or not. The logic of manifesting one’s understanding is thus peculiarly *opaque*: what is understood is what is manifested, but what is manifested does not include the (unavoidable) fact that it is manifested.

A permonstration must thus be accurately distinguished from its *knowledge-conferring implementation*. It suffices for a sequence to permonstrate a formula $\varphi(\vec{\xi})$ that it be W —whereby being W need not imply anything about a speaker’s situation; but for a speaker to acquire knowledge that the objects of the sequence are the way $\varphi(\vec{\xi})$ says they are the sequence must be presented to her and appropriate computational processes must occur. There is no general guarantee that these conditions can always be satisfied, since the way $\varphi(\vec{\xi})$ says objects are may exactly be the way they are only if these conditions are not satisfied (think of the following values for ‘ $\varphi(\vec{\xi})$ ’ as respective counterexamples: ‘No one is in a good epistemic position’, ‘Everyone is drunk’, . . . and, of course, ‘There are 1,963 trees in St Andrews and it is not known that there 1,963 trees in St Andrews’).

This distinction between a permonstration and its knowledge-conferring implementation is essential also to avoid a too easy validation of the object-language conditional $\varphi(\vec{\xi}_0) \supset \exists \xi_1 \xi_2 \mathcal{K}_{\xi_1, \xi_2} \varphi(\vec{\xi}_0)$ by an anti-realist semantics, validation which does not even go through the detour of the paradox of knowability (s. Hart [1979], p. 165, n. 3; the insightful reply in Williamson [1982], pp. 206–7—hinging upon the distinction between a proof *type* and a proof *token*—has been a major source of inspiration for this paper). If the basic notion of an anti-realist semantics were the one of a knowledge-conferring implementation of a permonstration, there would be a knowledge-conferring implementation of a permonstration of $\varphi(\vec{\xi}_0) \supset \exists \xi_1 \xi_2 \mathcal{K}_{\xi_1, \xi_2} \varphi(\vec{\xi}_0)$ by a sequence iff, if there is a knowledge-conferring implementation of a permonstration of $\varphi(\vec{\xi}_0)$ by the sequence, then there is a knowledge-conferring implementation of a permonstration of $\exists \xi_1 \xi_2 \mathcal{K}_{\xi_1, \xi_2} \varphi(\vec{\xi}_0)$ by the sequence. And there would be a knowledge-conferring implementation of a permonstration of $\exists \xi_1 \xi_2 \mathcal{K}_{\xi_1, \xi_2} \varphi(\vec{\xi}_0)$ by a sequence s iff there is a knowledge-conferring implementation of a permonstration by s of ‘ \top There is a knowledge-conferring implementation of a permonstration of ‘ $\varphi(\vec{\xi}_0)$ ’ by s^{-1} . The absurd $\varphi(\vec{\xi}_0) \supset \exists \xi_1 \xi_2 \mathcal{K}_{\xi_1, \xi_2} \varphi(\vec{\xi}_0)$ would then be validated by the not absurd and indeed traditional epistemological view that knowledge is such that, for every formula $\varphi(\vec{\xi}_0)$, there cannot be a knowledge-conferring implementation of a permon-

stration of $\varphi(\vec{\xi}_0)$ by a sequence s without there being a knowledge-conferring implementation of a permonstration by s of \ulcorner There is a knowledge-conferring implementation of a permonstration of $\varphi(\vec{\xi}_0)$ by $s \urcorner$.

The absurd $\varphi(\vec{\xi}_0) \supset \exists \xi_1 \xi_2 \mathcal{K}_{\xi_1, \xi_2} \varphi(\vec{\xi}_0)$ is *not* so validated if the basic notion of an anti-realist semantics is the one of a permonstration rather than the one of a knowledge-conferring implementation of a permonstration: for it is consistent, for some non- \mathcal{K} -initial formula $\varphi(\vec{\xi}_0)$, that there is a permonstration of $\varphi(\vec{\xi}_0)$ by a sequence without there being a knowledge-conferring implementation of it, even if knowledge is such that, for every formula $\varphi(\vec{\xi}_0)$, there cannot be a knowledge-conferring implementation of a permonstration of $\varphi(\vec{\xi}_0)$ by a sequence s without there being a knowledge-conferring implementation of a permonstration by s of \ulcorner There is a knowledge-conferring implementation of a permonstration of $\varphi(\vec{\xi}_0)$ by $s \urcorner$.

The distinction between a permonstration and its knowledge-conferring implementation implies an analogous distinction between a demonstration and its knowledge-conferring implementation. We can then finally see how, contrary to (EC)-unrestricted anti-realism, (DC)-unrestricted anti-realism—the version of unrestricted anti-realism which *is* the conclusion of the anti-realist argument from the theory of understanding (if the language is compositional)—does not fall prey to the paradox of knowability. Mimicking Fitch’s original reasoning, we can go from ‘There is a demonstration of ‘ A ’ and it is not known that A ’ to ‘There is a demonstration of ‘ A ’ and there is a demonstration of ‘It is not known that A ’ (by demonstration functionality of conjunction). But this is *not* absurd (as, on the contrary, ‘It is known that A and it is known that it is not known that A ’ is), since, as we have seen, there being a demonstration of ‘ A ’ does not imply the existence of a knowledge-conferring implementation of such a demonstration (even though it does imply the feasible knowability that A , if ‘ A ’ is atomic). Fitch’s reasoning thus breaks down: it is *consistent* that there is both a demonstration of ‘ A ’ and a demonstration of ‘It is not known that A ’, and therefore consistent (by demonstration functionality of conjunction) that there is a demonstration of ‘ A and it is not known that A ’.²⁴

Two lines of inquiry immediately present themselves at this point:

- (a) As we have seen, there being a demonstration of ‘ A ’ does not *generally* imply knowledge that A , nor feasible knowability that A , nor metaphys-

²⁴Of course, given demonstration functionality of conjunction and factivity of demonstration, one can successfully mimic Fitch’s original reasoning wrt ‘ A and there is no demonstration of ‘ A ’, but this only yields the not absurd, left-to-right direction of the new anti-realist constraint (DC).

ically possible knowledge that A . Can the logical relations among these notions be made more precise, and can interesting conditions be then specified under which some implication *locally* holds?

- (b) Can (DC) play the role traditionally played by (EC) in various anti-realist arguments (for example, in the anti-realist argument for revision of classical logic)? In this respect, does its *unrestricted* validity give it any advantage against versions of anti-realism based on a *restriction* of the specific constraint (such as (EC)) meant to capture the connection—so central for anti-realism—between truth and our recognition of it?²⁵

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²⁵[Acknowledgements.]