

PY1003 Resit 2005 – Model Solutions

1. (a) $\neg(A \vee B) \vdash A \rightarrow \neg B$

$$\begin{array}{l}
 \neg(A \vee B) \checkmark \\
 \neg(A \rightarrow \neg B) \checkmark \\
 A \\
 \neg \neg B \\
 \neg A \\
 \hline
 \neg B
 \end{array}$$

valid

1. (b) $(A \rightarrow B) \rightarrow C, \neg C \vdash \neg A$

$$\begin{array}{c}
 (A \rightarrow B) \rightarrow C \checkmark \\
 \neg C \checkmark \\
 \neg \neg A \checkmark \\
 A \\
 \swarrow \quad \searrow \\
 \neg(A \rightarrow B) \quad \underline{C} \\
 \begin{array}{l}
 A \\
 \neg B
 \end{array}
 \end{array}$$

invalid

Countermodel:

$$I(A) = T$$

$$I(B) = F$$

$$I(C) = F$$

1. (c) $(A \vee B) \vee (A \leftrightarrow C), \neg C \vdash \neg A$

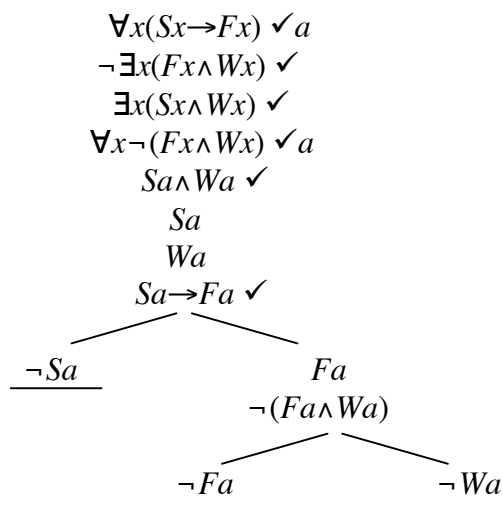
$$\begin{array}{c}
 (A \vee B) \vee (A \leftrightarrow C) \checkmark \\
 \neg C \\
 \neg \neg A \checkmark \\
 A \\
 \swarrow \quad \searrow \\
 A \vee B \quad A \leftrightarrow C \\
 \swarrow \quad \searrow \quad \swarrow \quad \searrow \\
 A \quad B \quad A \quad \neg A \\
 \quad \quad \quad \underline{C} \quad \underline{\neg C}
 \end{array}$$

Model: $I(A) = T; I(B) = T; I(C) = F$

invalid

- 2./3. (a) Key: Sx : x is a salmon
 Fx : x is a fish
 Wx : x has wings

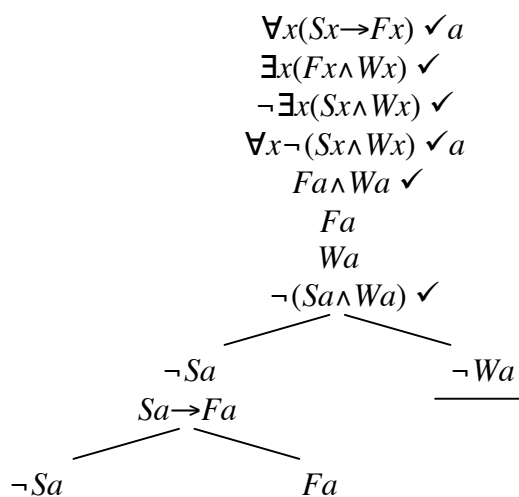
$$\{\forall x(Sx \rightarrow Fx), \neg \exists x(Fx \wedge Wx), \exists x(Sx \wedge Wx)\}$$



inconsistent

- 2./3. (b) Key: Sx : x is a salmon
 Fx : x is a fish
 Wx : x has wings

$$\{\forall x(Sx \rightarrow Fx), \exists x(Fx \wedge Wx), \neg \exists x(Sx \wedge Wx)\}$$



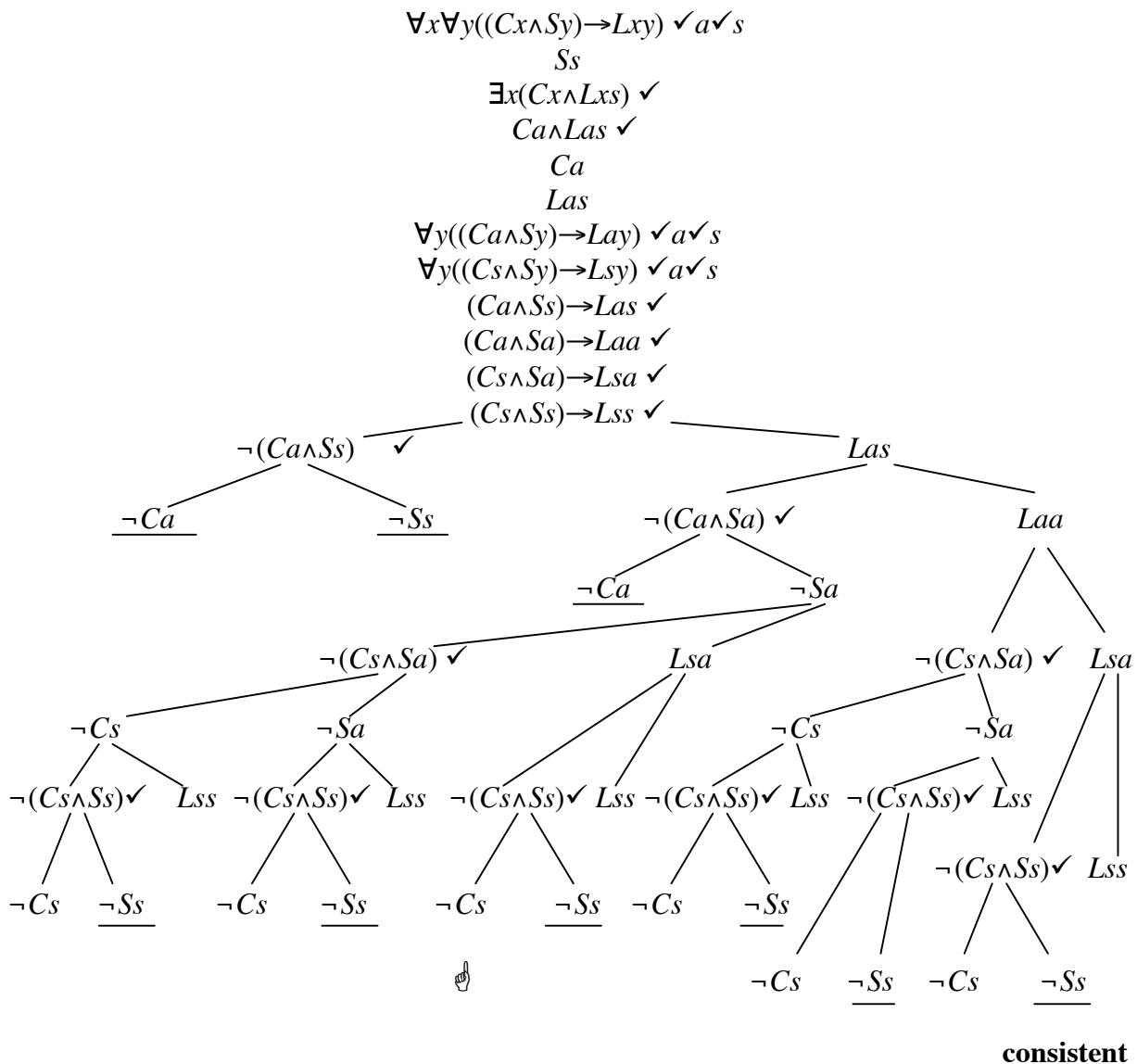
consistent

Model:

$$\begin{array}{ll}
 D = \{a\} & I(S) = \emptyset \\
 & I(F) = \{a\} \\
 & I(W) = \{a\}
 \end{array}$$

2./3. (c) Key: Sx : x is a salmon
 Cx : x is a cat
 Lxy : x loves y
 s : Sally

$\{\forall x\forall y((Cx\wedge Sy)\rightarrow Lxy), Ss, \exists x(Cx\wedge Lxs)\}$



Model:

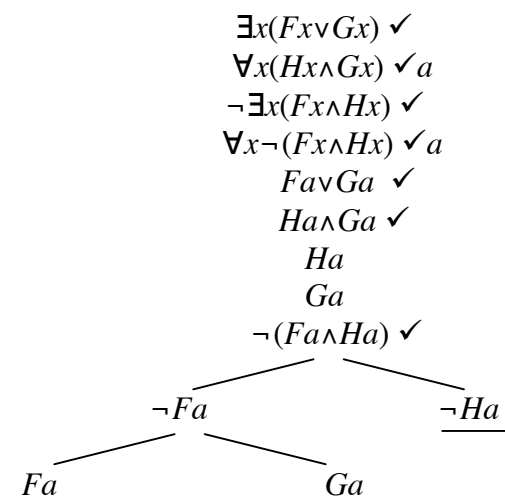
$D = \{a, s\}$

$I(C) = \{a\}$

$I(S) = \{s\}$

$I(L) = \{<s, a>, <a, s>\}$

4. (a) $\exists x(Fx \vee Gx), \forall x(Hx \wedge Gx) \vdash \exists x(Fx \wedge Hx)$



invalid

Countermodel:

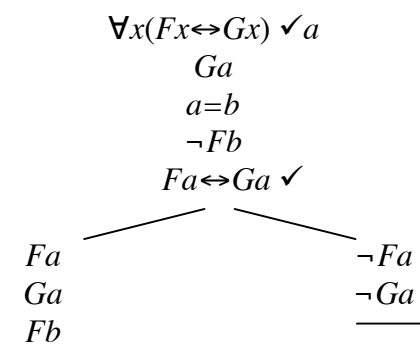
$$D = \{a\}$$

$$I(F) = \emptyset$$

$$I(G) = \{a\}$$

$$I(H) = \{a\}$$

4. (b) $\forall x(Fx \leftrightarrow Gx), Ga, a=b \vdash Fb$



valid