

PY1003: Introduction to Logic
Lecture 9
Predicate Logic I

Structure of the lecture:

- I. The limitations of sentential logic**
- II. Names and predicates**
- III. Quantifiers and individual variables**
- IV. Existential quantifiers and conjunctions**
- V. Negated existential statements**
- VI. Arguments in predicate logic**

1. The limitations of sentential logic

David is rich

Therefore, someone is rich

A: David is rich

B: Someone is rich

A |- B

But this argument form is invalid!

I (A) = T

I (B) = F

→ We need more expressive resources to represent arguments.

Our aim: to learn a new language of logic.

We'll call this language *predicate logic*.

2. Names and predicates

(1) David is rich

This consists of two components:

A name - David

A predicate – is rich

Names: a, b, c...

Predicates: P, Q, R...

Translation key:

a: David

Rx: x is rich

(1) David is rich

(1') Ra

Names and predicates (contd.)

Translation key:

Rx: **x is rich**

Hx: **x is happy**

a: **David**

b: **Victoria**

David is happy: Ha

Victoria is happy: Hb

We can combine sentences of predicate logic using the *logical constants*...

David is happy and Victoria is happy:

Ha \wedge Hb

If David is not rich then Victoria is not happy:

$\neg Ra \rightarrow \neg Hb$

Names and Predicates (contd.): Examples

London is large but St Andrews is not.

a: London, b: St Andrews, Lx: x is large

Either Helen or Sarah stole the money

a: Helen, b: Sarah, Sx: x stole the money

Helen stole the money unless Sarah did.

Features of names/predicates:

- **Names stand for/refer/denote *specific* persons or things.**
- **A predicate is something that applies to an object, is true of an object or which an object satisfies.**
- **A predicate are sometimes said to stand for a property. (x is wise)**

→ What about ‘I’, ‘You’, ‘tomorrow’?

Some terminology:

a, b, c - *Individual letters.*

x, y, z - *Individual variables*

P, Q, R - *Predicate letters*

3. Quantifiers and individual variables

(2) Someone is rich

Some person is such that he/she is rich

For some x , x is rich

For some x , Rx

(2') $\exists xRx$

Some terminology

Existential quantifiers:

someone

something

there exists a person who is...

At least one thing is...

\exists - the *existential quantifier symbol*

Individual variables and quantifiers (contd.)

Translation key:

Rx: x is rich

Hx: x is happy

a: David

At least one person is happy:

$\exists x Hx$

If someone is rich then someone is happy

$\exists x Rx \rightarrow \exists x Hx$

There is someone such that if she/he is rich then she/he is happy.

$\exists x (Rx \rightarrow Hx)$

If someone is rich then David is happy

$\exists x Rx \rightarrow Ha$

Individual variables and quantifiers (contd.)**There is an omnipotent being****Ox: x is an omnipotent being****Something is large****Lx: x is large****If something is red then something is coloured****Rx: x is red, Cx: x is coloured****If something is red then it is coloured****Rx: x is red, Cx: x is coloured**

4. Existential quantifiers and conjunctions

(3) Someone is happy and someone is rich

(3') $\exists x Hx \wedge \exists x Rx$

Contrast this with:

(4) Someone is happy and rich

(4') $\exists x (Hx \wedge Rx)$

In (3'), the main connective is the conjunction.

In (4'), the main connective is the existential quantifier.

Another example

Something is green and something is yellow

$\exists x Gx \wedge \exists x Yx$

Something is green and yellow

$\exists x (Gx \wedge Yx)$

5. Negated existential statements

Nobody is happy

It is not the case that somebody is happy

$\neg \exists x Hx$

Contrast this with:

There is someone who is not happy

$\exists x \neg Hx$

Another example:

There is nothing green: $\neg \exists x Gx$

There is something that is not green: $\exists x \neg Gx$

More complex examples:

There is nothing that is green and there is nothing that is red.

$$\neg \exists x Gx \wedge \neg \exists x Rx$$

There is nothing that is green and red.

$$\neg \exists x (Gx \wedge Rx)$$

Nothing is both red and green.

$$\neg \exists x (Gx \wedge Rx)$$

If nothing is red then nothing is coloured.

$$\neg \exists x Rx \rightarrow \neg \exists x Cx$$

6. Arguments in predicate logic

1. David is rich

2. Therefore, someone is rich

Rx: x is rich

a: David

Ra |- $\exists x Rx$

1. Grass is green

2. Therefore, something is green

1. London is in England

2. Therefore, something is in England