Lecture 12

PY1003

Quantifiers and Trees

ANNOUNCMENT:

NO TUTORIALS THIS WEEK!!!!

Sorry about the confusion….

Office Hours: Monday 3-4 and Fridays 10-11
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Summary of last lecture

- **I. Introduced the tree-rule for existential quantifier**
- **II. Introduced the tree-rule for universal quantifier**
- **III. Examples involving the quantifier**
- **IV. Briefly discussed the tree-rules for the negated existential/universal quantifier**

Plan for this lecture

I. Reminder for the tree rules of the existential and universal quantifier.
II. Some examples using both quantifiers.
III. Introduce in the negated universal/ existential quantifier.
IV. Discuss some examples using all types of quantified statements and some more “rule’s of thumb”.

1. Reminder for the tree rules of the existential and universal quantifier.

Last week we introduced the following rule for the existential quantifier:

$$\exists x Rx \quad \surd$$

$$|$$

$$R\tau$$

for some new constant τ *not* occurring already on the branch.

To motivate the rule that a *new* constant *not* occurring on the branch has to be used, I put forth the following example:

(1) David is rich

(2) There is someone who is both rich and happy

(3) David is happy

Ra

∃ x (Rx ∧ Hx)

¬ Ha

|

Ra ∧ Ha

|

Ha

----- (closed)

Ra

∃ x (Rx ∧ Hx) √

¬ Ha

|

Rb ∧ Hb

|

Hb

|

Rb

Issue:

Using the latter tree do we thereby say that David is not the person who is rich?

Remember: The issue is always whether, as a *matter of logic* the conclusion follows. The point here is just to show that it is not a matter of logic that David is happy (assuming the two premises).

The fact that we attributed 'being happy' to some constant 'b', just highlights that there is a model in which the premises are true but the conclusion is false.

Thus, it's not (logically) necessary that David is happy if the two premises are true.

The Universal quantifier reconsidered:

The rule:

For *every* name τ we already have in the branch, we must write 'G(τ)' in the branch.

If no term yet occur in the branch, then we can enter 'G(a)' in the branch.

The universal quantifier is not ticked off since it if a new term is introduced later in the tree, the quantifier will always apply again to this new term.

Problem:

When we show that an argument is invalid/valid we can only "finish" the tree provided all rules have been applied. Yet, how do we know when to stop if we can't tick off the universal quantifier?

Solution:

- **Consider a universal quantified statement as "finished" if you applied all constants occurring on the tree. (Keep stock of all constants)**

- **If the universally quantified is used to introduce a *new constant*, then only introduce a new constant *once*. If after introducing a new term the tree remains open, then further application will not close the tree.**

2. Some examples using both quantifiers

$$\exists x\, Fx$$
$$\forall x\, \neg (Fx \lor Gx)$$

Rule of Thumb

If possible, always apply the rule for the existential quantifier before applying any rule for the universal quantifier.

Another example:

$$\forall x\ (Fx \lor Gx)$$
$$\exists x\ \neg\ (Fx)$$
$$\exists x\ \neg\ (Gx)$$

3. Introduce (again) the negated universal/existential quantifier.

It is not the case that someone is happy

$\neg \exists x\, Hx$

Everyone is unhappy

$\forall x \neg Hx$

they are logically equivalent!

➔ **From this we can motivate the tree-rule for ¬ ∃**

Tree-rule for ¬ ∃

$\neg \exists x Fx$ √

|

$\forall x \neg Fx$

The same idea applies to $\neg\forall xFx$

It is not the case that everyone is happy

$$\neg \forall x\, Hx$$

There is someone who is not happy

$$\exists x \neg Hx$$

So, the tree rule will be

Tree-rule for $\neg \forall x$

$$\neg \forall x\, Fx \;\checkmark$$

$$|$$

$$\exists x \neg Hx$$

4. Some examples

There is someone who is rich
Therefore there is someone who is rich or happy

$$\exists x\ Rx$$
$$\neg(\exists x(Rx \lor Hx))$$

Some more rule’s of thumb:

(1) Apply the rule for the negated existential/ universal quantifier as early as possible.

(2) Once there are no negated quantifiers left – always first get rid of the existential quantifier before applying the rule for the universal quantifier.

(3) Don’t introduce new constants/names by means of the universal quantifier unless you have to.

(4) Always introduce a new term when using the rule for the existential quantifier.

(5) Keep count of all terms that have been introduced, so you know when to “finish off” the universal quantifier.

All men are mortal (∀x (Px -> Mx))
All mortals fear dying (∀x (Mx -> Fx))
Therefore, all men fear dying (∀x (Px -> Fx))

∀x (Px -> Mx)
∀x (Mx -> Fx)
¬ (∀x (Px -> Fx))

No banker is generous

$\forall x\,(Bx \rightarrow \neg Gx)$

$\neg \exists x\,(Bx \wedge Gx)$

$$\neg(\forall x\,(Bx \rightarrow \neg Gx) \leftrightarrow \neg \exists x\,(Bx \wedge Gx))$$

More examples (some logically equivalent statements)

Only the rich are happy

(Rx: x is rich, Hx: x is happy)

(1) $\forall x\ (Rx \rightarrow Hx)$
(2) $\forall x\ (\neg Hx \rightarrow \neg Rx)$
(3) $\neg \exists x\ (Hx \wedge \neg Rx)$

We can show that they are all logically equivalent. To do this we show that (1) $\Rightarrow$ (2) $\Rightarrow$ (3) $\Rightarrow$ (1)