PY1003: Introduction to Logic
Lecture 9
Predicate Logic II

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Office Hours: Monday 3-4 and Friday 10-11

NO EXAM THIS FRIDAY!!!!

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Summary from the last lecture:

- **Names and predicates to represent the content of sentences.**
- **Existential quantifier and individual variables.**
- **Existential statements involving logical connectives**
- **Negative existential statements**
- **Pitfalls:**
  - **$\exists x\ (Gx \wedge Fx)$**
  - **$\exists x\ Gx \wedge \exists x\ Yx$**

Structure of the lecture:

I. More shortcomings of sentential logic

II. Universal quantifier

III. Universal quantifier and compound sentences

IV. Universal quantifier and existential quantifier

V. Domains of quantification

VI. Arguments using quantifiers

1. More limitations of sentential logic

George is a banker (Ba)
No banker is generous ($\neg \exists x (Bx \wedge Gx)$)
Therefore, George is not generous. ($\neg Ga$)

Everyone is mortal
Therefore, David is mortal

A: Everyone is mortal
B: David is mortal
A |- B

All men are mortal
All mortals fear dying
Therefore, all men fear dying.

The aim of this lecture:

To learn how to translate the argument above into the language of predicate logic.

2. The universal quantifier

(1) Everyone is mortal

Every person is such that he/she is mortal

For all x, x is mortal

For all x, Mx

(1'') ∀x Mx

<u>Some terminology</u>

***Universal quantifiers*:**

All…
Everybody
Everyone…
Everything….

∀: the *universal quantifier symbol*

The universal quantifier *binds* the variable x: ∀x Mx

The universal quantifier – some examples

Everything is green

$\forall x\ Gx$

If everything is green then everything is coloured

$\forall x\ Gx \rightarrow \forall x\ Cx$

If everyone is rich, then David is happy

$\forall x\ Rx \rightarrow Ha$

3. Universal quantifiers and compound sentences

REMEMBER:

Someone is happy and someone is rich
$\exists x\ Hx \wedge \exists x\ Rx$

Contrast this with:

Someone is happy and rich
$\exists x\ (Hx \wedge Rx)$

Everyone is happy and everyone is rich
$\forall x\ Hx \wedge \forall x\ Rx$

Compare this with:

Everyone is happy and rich
$\forall x\ (Hx \wedge Rx)$

-> There is no difference in the latter case.

Universal quantifiers and compound sentences (cont)

(5) Everyone who is rich is happy

Every person is such that if he/she is rich, he/she is happy.

For all x, if x is rich then x is happy

(5') $\forall x\ (Rx \rightarrow Hx)$

while:

Everyone is happy and rich
$\forall x\ (Hx \wedge Rx)$

entails that everyone has two properties (being happy and rich)

(5) says that *if* a person is rich then it is wise.

-> Doesn't say anything about poor (not-rich) people. Their existence (of not-rich people) is left open.

Some more examples:

- **All philosophers are good at logic:**
- **$\forall x\ (Px \rightarrow Lx)$**

- **All healthy people sleep for at least eight hours a night:**
- **$\forall x\ (Hx \rightarrow Sx)$**

- **All Scottish philosophers are good at logic**
- **$\forall x\ ((Sx \wedge Px) \rightarrow Lx)$**

- **Whales are mammals:**
- **$\forall x\ (Wx \rightarrow Mx)$**

Compound sentences (contd.)

It is not the case that all philosophers are good at logic:

(Px: x is a philosopher, Lx: x is good at logic)

Everyone who does yoga is relaxed

(Yx: x does yoga, Rx: x is relaxed)

Everyone is happy and relaxed

(Hx: x is happy, Rx: x is relaxed.)

Only the rich are happy

(Rx: x is rich, Hx: x is happy)

4. Quantifier interdefinability

(5) It is not the case that everyone is happy

(5') $\neg \forall x\, Hx$

Compare this with:

(6) There is someone who is not happy

(6') $\exists x \neg Hx$

(5') and (6') are logically equivalent!

<u>Quantifier interdefinability:</u>

$\exists x\, Px$ iff $\neg \forall x \neg Px$

$\forall x\, Px$ iff $\neg \exists x \neg Px$

Some other examples:

No banker is generous

$\neg \exists x (Bx \wedge Gx)$

$\forall x (Bx \rightarrow \neg Gx)$

No one who is unhappy is wise

Paraphrase 1:

It is not the case that there is someone who is unhappy and wise

$\neg \exists x (\neg Hx \wedge Wx)$

Paraphrase 2:

Everyone who is wise is happy

$\forall x (Wx \rightarrow Hx)$

5. Domains of discourse

Everyone is happy

$$\forall x\, Hx$$

We here made an implicit assumption about the *domain of discourse*!

We assumed that the universal quantifier *ranges* over people and not beer cans (e.g.)

<u>Terminology</u>

The collection of people or objects to whom the quantifiers are relativised is the *domain of discourse.*

If a quantifier is relativised towards a certain domain of discourse we say that it *ranges over* that domain.

Domain of discourse

Convention

State explicitly the domain of discourse when providing a translation.

Everyone is happy

Domain: people

Hx: x is happy

$\forall$x Hx

Other *domains of discourse:*

- **Places**
- **Times**

It is sunny everywhere

Domain: Places

$\forall$x Sx

It is loud all the time
Domain: Times

$\forall$x Lx

Alternatively, the following is also acceptable:

Everyone is happy

Hx: x is happy

Px : x is a person

∀x (Px -> Hx)

It is sunny everywhere

Px: x is a place

Sx: x is sunny

∀x (Px -> Sx)

It is loud all the time
Tx: x is a time
Lx: x is loud

∀x (Tx -> Lx)

VI. Arguments with universal quantifiers

<u>Everyone is mortal</u>

David is mortal

Translation key:

Mx: x is mortal, a: David

$\forall$x Mx |- Ma

All men are mortal ($\forall$x (Px -> Mx))

All mortals fear dying ($\forall$x (Mx -> Fx))

Therefore, all men fear dying. ($\forall$x (Px -> Fx))

We don't yet have a means of *proving* that these arguments are valid. But you will start developing such a method in the next lecture.

Limitations in our expressive power:

“Many things” or “Most things”

“Usually”

“Seldom”

“Often”

These can’t be properly represented but are there any convincing arguments that involve these expressions?