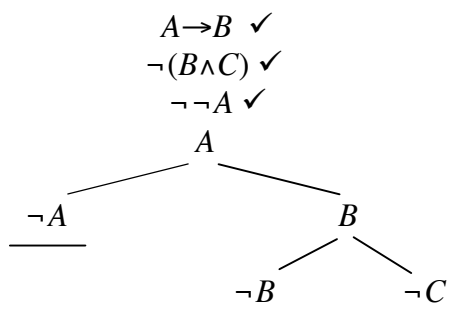


PY1003 Exam 2005 – Model Solutions

1. i. $A \rightarrow B, \neg(B \wedge C) \vdash \neg A$



invalid

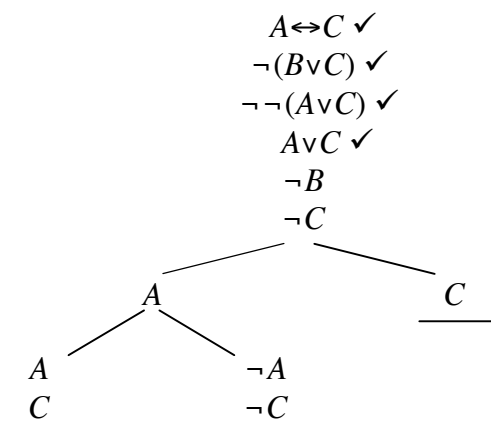
Countermodel:

$I(A) = T$

$I(B) = T$

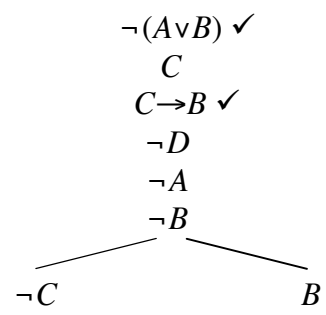
$I(C) = F$

ii. $A \leftrightarrow C, \neg(B \vee C) \vdash \neg(A \vee C)$



valid

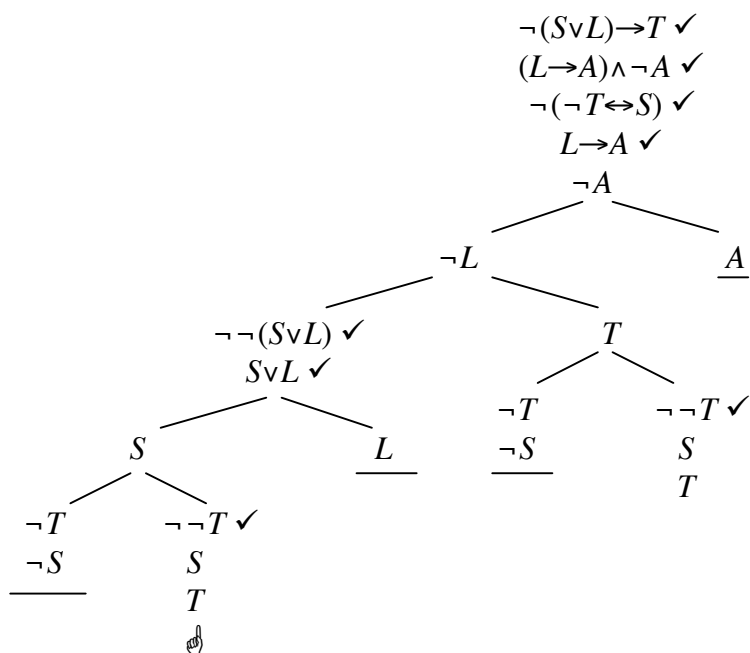
iii. $\neg(A \vee B), C, C \rightarrow B \vdash D$



valid

2. Key: T : The train will be on time.
 S : There is a signal failure.
 L : There are leaves on the track.
 A : It is autumn.

$$\neg(S \vee L) \rightarrow T, (L \rightarrow A) \wedge \neg A \vdash \neg T \leftrightarrow S$$

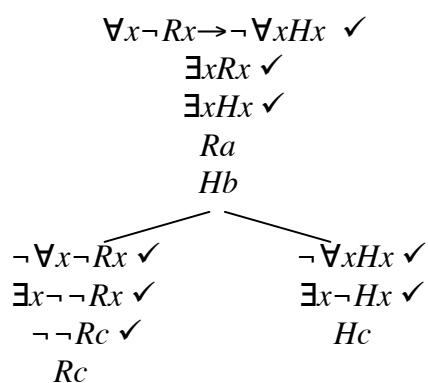


invalid

Countermodel:

$$\begin{aligned} I(T) &= T \\ I(S) &= T \\ I(L) &= F \\ I(A) &= F \end{aligned}$$

3. i. $\{\forall x \neg Rx \rightarrow \neg \forall x Hx, \exists x Rx, \exists x Hx\}$

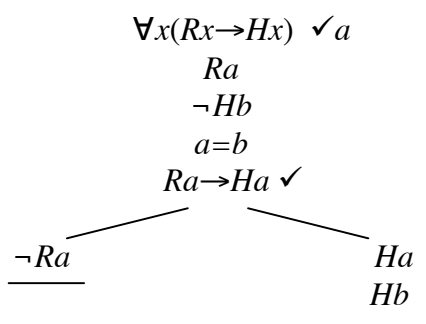


Model:

$$\begin{aligned} D &= \{a, b, c\} & I(R) &= \{a, c\} \\ & & I(H) &= \{b\} \end{aligned}$$

consistent

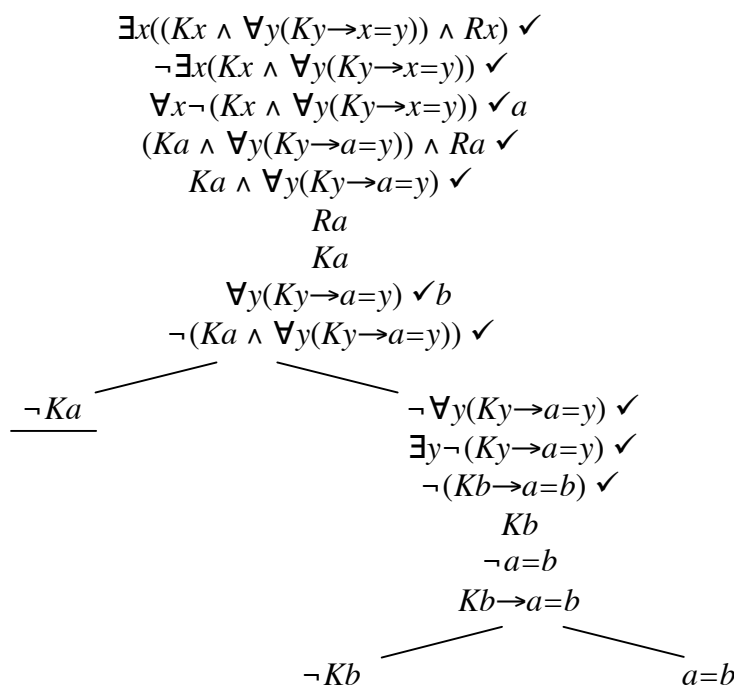
3. ii. $\{\forall x(Rx \rightarrow Hx), Ra, \neg Hb, a=b\}$



inconsistent

4. i. Key: Kx : x is a King of England
 Rx : x is rich

$\exists x((Kx \wedge \forall y(Ky \rightarrow x=y)) \wedge Rx) \vdash \exists x(Kx \wedge \forall y(Ky \rightarrow x=y))$

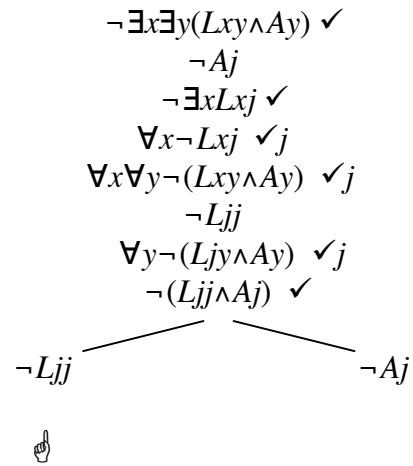


valid

4. ii. Key: Ax : x is arrogant domain : people
 Lxy : x likes y
 j : Juan

$\neg \exists x \exists y (Lxy \wedge Ay), \neg Aj \vdash \exists x Lxj$

NB: ' $\neg \exists x \exists y (Lxy \wedge Ay)$ ' could also be formalised as ' $\forall x \forall y (Lxy \rightarrow \neg Ay)$ '.



Countermodel:

$D = \{j\}$

$I(L) = \emptyset$

$I(A) = \emptyset$