PY1003: Introduction to Logic

Lecture 2

Sentential logic revisited I

1. Logical form

The logical form of an argument is the structure, or pattern, that various arguments can share. For instance, all of the following arguments have the same logical form:

1. Ross is in his office or Ross is in the library
2. It is not the case that Ross is in his office.
3. Therefore, Ross is in the library.

1. The sky is yellow or the sky is pink
2. It is not the case that the sky is yellow
3. Therefore, the sky is pink

1. Quarks have flavour or quarks have spin
2. It is not the case that quarks have flavour
3. Therefore, quarks have spin.

The validity of an argument depends upon its logical form. For instance, all of the arguments above are valid. One of the aims of logic is to enable us to detect the logical form of an argument. We do this by formalising the argument – that is, translating it into a language of logic. The language we will be looking at in the next three lectures is called sentential logic. It should be familiar from the Reason and Argument module last semester – but we'll be looking at it in a bit more detail.

2. Atomic sentences and compound sentences

Consider the following claim:

(1) If Jackson is convicted, then either he is guilty or the jury is biased

(1) is a complete, grammatical sentence of English. But when we look closely, we see that it contains several smaller sentences. These smaller sentence are 'Jackson is convicted', 'he is innocent', and 'the jury is biased.' These smaller sentences are connected to form (1) using the phrases 'If… then…' and 'either… or'

At this point, it is useful to introduce some terminology. An *atomic sentence* is a sentence that contains no other sentences. A *complex sentence* is a sentence such as (1) which contains one or more smaller sentences. Finally, a *sentential connective* is a phrase such as "if… then" which is used to turn atomic sentences into compound sentences.

3. Sentential letters

By translating an argument into sentential logic, we aim to display the relation between the atomic sentences of the argument. The first step in this process is to replace each atomic sentence of the argument with a capital letter, such as 'A', 'B' or 'C'. These are sometimes called *sentential letters.* To see how this works, return to the argument about Ross:

1. Ross is in his office or Ross is in the library
2. It is not the case that Ross is in his office
3. Therefore, Ross is in the library.

We can replace the sentence 'Ross is in his office' with the letter 'A' and the sentence 'Ross is in the library' with the letter 'B'. Then we can write out the argument as follows:

1. A or B
2. It is not the case that A
3. Therefore, B

4. Sentential connectives

The second stage is to replace the sentential connectives with symbols that represent them. We'll be using the following symbols.

It is not the case that A	$\neg A$	negation
A and B	$A \wedge B$	conjunction
A or B	$A \vee B$	disjunction
If A then B	$A \rightarrow B$	conditional
A if and only if B	$A \leftrightarrow B$	biconditional

These symbols are sometimes called *logical constants* because (unlike the sentential letters) they always have the same meaning. Using these symbols, we can complete our translation of the argument about Ross into the language of sentential logic:

1. $A \vee B$
2. $\neg A$
3. B

Notice that we have eliminated the word "therefore". This is because it is a word of English, which does not belong in the language of sentential logic. Instead, we rely on the line under the second premiss to distinguish between premisses and conclusion. Sometimes, however, it is useful to have a symbol which plays the role of the word 'therefore'. The symbol that we use is called a turnstile, and it looks like this: |-. Using the turnstile, we can write out the argument above as follows:

A v B, ¬A |- B

This way of presenting an argument is sometimes called *sequent form.* And a sentence of sentential logic, such as 'A v B', is sometimes called a *formula.*

5. Brackets and scope

Consider the compound sentence:

(2) Either I will stay in and study tonight, or I will go to the cinema.

This sentence contains three atomic sentences. We'll translate them as follows:

A: I'll stay in tonight
B: I'll study tonight
C: I'll go to the cinema

As a first attempt to translate (2) into the language of sentential logic, we might come up with the following:

(3) A ∧ B v C

But (3) is ambiguous. On the first reading, it means the same as (2). On the second reading, however, it means that I will stay in tonight, and *in addition* I will either study or go to the cinema. The difference between the two sentences is a difference in the way in which the sentential connectives join the sentences together. On the first reading, the connective '∧' joins together A and B. On the second reading, it joins together the compound sentence 'A ∧ B' with the atomic sentence C. We call this sort of difference, a difference in scope.

In logic, we capture differences in scope by using brackets. If we wish to indicate that a connective joins two sentences together, we enclose these two sentences, together with the connective itself, within brackets. Thus we can distinguish the two different readings of (2) as follows:

(3') (A ∧ B) v C
(3'') A ∧ (B v C)

There is one further complication. Consider the formula

(4) ¬A v B

At first glance, we might suspect that (4) is ambiguous too. It might be read in either of the following ways:

(4') (¬A) v B

(4′′) ¬(A v B)

But in fact this appearance is incorrect. There is a special convention governing negation. The convention is that the negation symbol, '¬', always takes the smallest possible scope. Thus the scope of the negation in (4) is 'A', and we do not need to use brackets to indicate that this is the case. If we wish the scope to be the compound sentence 'A v B', we need to use brackets – as in (4′′) above.

6. A rule for determining scope

Many people find it quite easy see the scope of two-place connectives. However, if you find yourself confused about the issue, you may find the following rule useful (adapted from Forbes, *Modern Logic*, p.15):

> The scope of a two-place connective in a sentence is that part of the sentence enclosed in the closest pair of matching brackets within which the connective lies, if there is such a pair, or else the entire formula, if there is not.

Remember, though, that this rule does not apply to negation.

7. Summary and further reading

The aim of this lecture has been to remind ourselves of the language of sentential logic. This language consists of the sentential letters, the logical constants, pairs of brackets, and the turnstile. By translating arguments into the language of sentential logic, we aim to make their logical form apparent.

If you want to read more about these topics, please try:

- Colin Howson, *Logic with Trees,* ch. 1, section 2
- Graeme Forbes, *Modern Logic*, ch. 2, sections 1-4