PY1003: Introduction to Logic
Lecture 17 (corrected)
Identity, Number and Definite Descriptions

1. 'Only'

Suppose that we wish to express the claim:

(1) John only loves himself

Let 'a' stand for John, and 'Lxy' to mean 'x loves y'. Then we can translate (1) as follows:

(1*) $Laa \wedge \forall x\,(Lax \rightarrow x = a)$

This formula tells us that John loves himself, and that for any x, if John loves x, then x is identical to John. We use a similar strategy to express claims such as

(2) There is only one philosopher who is good as football.

Let 'Px' mean 'x is a philosopher' and 'Fx' mean 'x is good at football'. Then our translation will be:

(2*) $\exists x\,((Px \wedge Fx) \wedge \forall y\,((Px \wedge Fy) \rightarrow x = y))$

2. 'At least'

Claims such as

(3) There is at least one god

are easily expressed using the existential quantifier. Using 'Gx' to mean 'x is a god', we write:

(3*) $\exists x\, Gx$

But now consider the assertion that:

(4) There are at least two gods

How are we to express this in the language of predicate logic? As follows:

(4*) $\exists x\, \exists y\, ((Gx \wedge Gy) \wedge \neg\, x=y)$

In theory, this strategy can be used to express the claim that there are at least *n* Fs, when *n* is any natural number. In practice, however, the formulae rapidly become unwieldy!

3. Number

Suppose, now, that we wish to maintain that there are exactly *n* gods. How are to do this? (Remembering that the language of predicate logic does not contain numbers.) Our strategy is as follows. We note that the claim:

(5) There are exactly *n* gods

is equivalent to the following conjunction:

(5*) There are at least *n* gods and there are no more than *n* gods

We then use the resources above in order to express this conjunction. Starting with the simplest case, consider the claim that

(6) There is exactly one god

We express this as follows:

(6*) $\exists x\ (Gx \wedge \forall y\ Gy\ (y = x))$

The first conjunct tells us that there is at least one god. The second tells us that any god that any god that exists will be identical to this first god. Now consider a more complicated claim:

(7) There are exactly two gods

Here is how we translate this claim:

(7*) $\exists x\ \exists y\ (((Gx \wedge Gy) \wedge \neg\ x=y) \wedge \forall z\ (Gz \rightarrow (z=x \vee z=y)))$

The first conjunct tells us that there are at least two gods. The second tells us that if there is a further god, then this further god must, in fact, be identical to one of the previous two. In theory, we can use the same strategy to express any claim about natural numbers – although, again, the formulae rapidly become unmanageable.

4. Some arguments

(i) If there are at least two gods, there must be at least one god. We can demonstrate that this is the case using a truth-tree:

$\exists x\ \exists y\ ((Gx \wedge Gy) \wedge \neg\ x=y)\ \surd$

$\neg\ \exists x\ Gx\ \surd$

|

$\forall x\ \neg\ Gx\ \ \surd a$

|

$\exists y\ ((Ga \wedge Gy) \wedge \neg\ a=y)\ \surd$

|

$(Ga \wedge Gb) \wedge \neg\ a=b\ \surd$

|

$Ga \wedge Gb\ \surd$

$\neg\ a=b$

|

Ga

Gb

|

$\underline{\neg\ Ga}$

(ii) Consider the following argument:

Fred is the only philosopher in the room
Mary is in the room
<u>Fred is not identical to Mary</u>
Therefore, Mary is not a philosopher

Let ‘Px’ mean ‘x is a philosopher’ and ‘Rx’ mean ‘x is in the room’. Let ‘a’ stand for Fred and ‘b’ for Mary. Then we translate the argument as follows:

$(Pa \wedge Ra) \wedge \forall x\ ((Px \wedge Rx \rightarrow x{=}a),\ Rb\ \neg a{=}b \mid\!-\ \neg Pb$

We can now show that the argument is valid using a truth tree:

$(Pa \wedge Ra) \wedge \forall x\ ((Px \wedge Rx) \rightarrow x{=}a)\ \surd$

Rb

$\neg a{=}b$

$\neg\neg Pb\ \surd$

|

Pb

|

$Pa \wedge Ra\ \surd$

$\forall x\ ((Px \wedge Rx) \rightarrow x{=}a)\ \surd\ b$

|

Pa

Ra

|

$(Pb \wedge Rb) \rightarrow b = a\ \surd$

/ \

$\neg\ (Pb \wedge Rb)\ \surd$ $b = a$

/ \ |

$\neg Pb$ $\neg Rb$ $\neg\ b{=}b$ (First Identity Rule)

|

<u>$b{=}b$</u> (Second Identity Rule)

5. Definite Descriptions

Definite descriptions are noun phrases which identify a unique person or object by reference to its properties. They usually begin with the word ‘the’. Examples of definite descriptions include:

The Queen of England
The man in the corner
The American student
The largest city in the world
Henry’s sister

How should we translate sentences that involve definite descriptions into the language of predicate logic? In order to answer this question, compare the following two sentences:

(8) The Queen of England is rich
(9) A Queen of England is rich

The difference between these two sentences is that the former implies *uniqueness.* If 8 is true, then there is only one man in the corner. In contrast, the second sentence is compatible with the existence of a whole bunch of men in the corner. We take this into account when we translate sentences involving definite descriptions. For instance, we translate (8) as:

(8*) $\exists x\ ((Qx \wedge Rx) \wedge \forall y\ (Qy \rightarrow y=x))$

Here are some more examples:

The American student is cheeky: $\exists x\ (((Ax \wedge Sx) \wedge Cx) \wedge \forall y\ (Ax \wedge Sx \rightarrow x=y))$
Someone loves the Queen of England: $\exists x\ \exists y\ ((Qx \wedge Lxy) \wedge \forall z\ (Qz \rightarrow z=x))$

PHILOSOPHCICAL NOTE: This treatment of definite descriptions enables us to solve what was once seen as a pressing philosophical problem. Consider the following sentence

(10) The King of France is bald

Some philosophers argued that this sentence is false, since it is not the case that the King of France is bald. Others, however, argued that it is meaningless, since the phrase 'the King of France' lacks a referent. If we translate 10 into the language of predicate logic, however, it becomes:

(10') $\exists x\ ((Kx \wedge Bx) \wedge \forall y\ (Ky \rightarrow y=x))$

It can be quickly seen that using a truth tree that (10') implies the following claim:

(11) $\exists x\ Kx$

That is, (10') implies that there exists a King of France. Since this claim is false, (10') is false too. And since (10') is false, so is the claim that the King of France is bald.

6. Summary and further reading

In this lecture, we have learnt how to use the identity symbol to express numerical phases, including 'only' and 'at least', and to express sentences involving definite descriptions. Further reading includes:

- Howson, *Logic with Trees,* ch9
- Jeffey, *Formal Logic*, ch.5
- Forbes, *Modern Logic*, ch.7., sec 2.