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# PY1003 Additional Exercises

1. Formalise into the language of first-order logic (i.e. predicate logic) the following sets of sentences. Clearly exhibit your key.

   a. {No dolphins are mammals; All mammals are fish; Some dolphins are fish}

   b. {Everyone loves logicians; No logician is not cool; Nobody loves anyone who is cool}

   c. {No philosopher detest anyone who detests Derrida; Richard is a philosopher; Richard detests Derrida; Derrida detest himself}

2. Decide using the tree method the consistency of the set of sentences in (1.) above. If they are consistent, exhibit a model.

3. Decide the validity of the following sequents using the tree method. If they are invalid exhibit a counterexample.

   a. $\forall x(Px \rightarrow Mx), \neg\exists x(Sx \wedge Mx) \vdash \neg\exists x(Sx \wedge Px)$

   b. $\neg\forall x(Fx \leftrightarrow Gx) \vdash \exists x(\neg Fx \leftrightarrow Gx)$

   c. $\forall x\forall y(Rxy \rightarrow Rxx), \forall yRay \vdash \exists x\neg Rxx$

And for those who can’t get enough:

4. Translate the following argument into first-order logic. Use a tree to determine whether the argument is valid. If it is invalid, exhibit a counterexample.

   Marcus is in his favourite pub; Marcus’ favourite pub is the Cellar. Therefore, Marcus is in the Cellar.

# Model Solutions

1. a. Key: $Dx$ : $x$ is a dolphin
$Mx$ : $x$ is a mammal
$Fx$ : $x$ is a fish

$\{\neg\exists x(Dx\wedge Mx);\ \forall x(Mx\rightarrow Fx);\ \exists x(Dx\wedge Fx)\}$

b. Key: $Lx$ : $x$ is a logician domain: people
$Cx$ : $x$ is cool
$Rxy$ : $x$ loves $y$

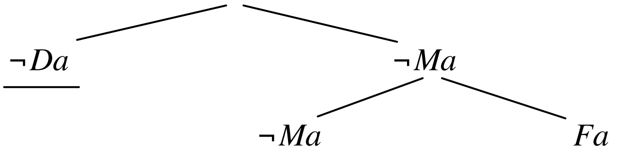
$\{\forall x\forall y(Ly\rightarrow Rxy);\ \neg\exists x(Lx\wedge\neg Cx);\ \neg\exists x\exists y(Rxy\wedge Cx)\}$

c. Key: $Px$ : $x$ is a philosopher
$Dxy$ : $x$ detests $y$
$r$ : Richard
$d$ : Derrida

$\{\neg\exists x(Px\wedge\exists y(Dxy\wedge Dyd));\ Pr;\ Drd;\ Ddd\}$

2. a.

$\neg\exists x(Dx\wedge Mx)$ ✓
$\forall x(Mx\rightarrow Fx)$ ✓$a$
$\exists x(Dx\wedge Fx)$ ✓
$\forall x\neg(Dx\wedge Mx)$ ✓$a$
$Da\wedge Fa$ ✓
$Da$
$Fa$
$\neg(Da\wedge Ma)$ ✓
$Ma\rightarrow Fa$ ✓



**consistent**

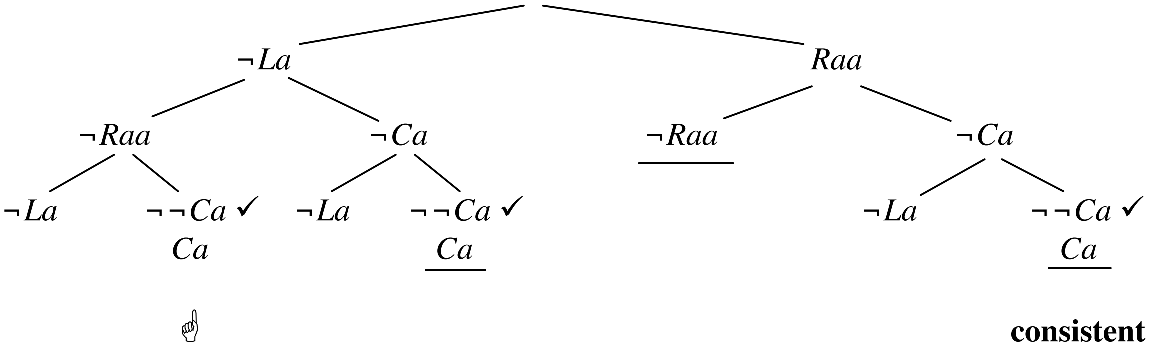
Model:

D = $\{a\}$

I(D) = $\{a\}$
I(F) = $\{a\}$
I(M) = $\varnothing$

2. b.

$\forall x \forall y(Ly \rightarrow Rxy)\ \checkmark a$
$\neg\exists x(Lx \wedge \neg Cx)\ \checkmark$
$\neg\exists x \exists y(Rxy \wedge Cx)\ \checkmark$
$\forall x \neg(Lx \wedge \neg Cx)\ \checkmark a$
$\forall x \neg\exists y(Rxy \wedge Cx)\ \checkmark a$
$\neg\exists y(Lay \wedge Ca)\ \checkmark$
$\forall y \neg(Ray \wedge Ca)\ \checkmark a$
$\neg(Raa \wedge Ca)\ \checkmark$
$\neg(La \wedge \neg Ca)\ \checkmark$
$\forall y(Ly \rightarrow Ray)\ \checkmark a$
$(La \rightarrow Raa)\ \checkmark$



**consistent**

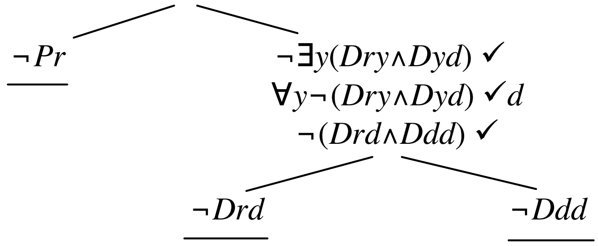
Model:

D = {*a*}

I(*C*) = {*a*}
I(*L*) = ∅
I(*R*) = ∅

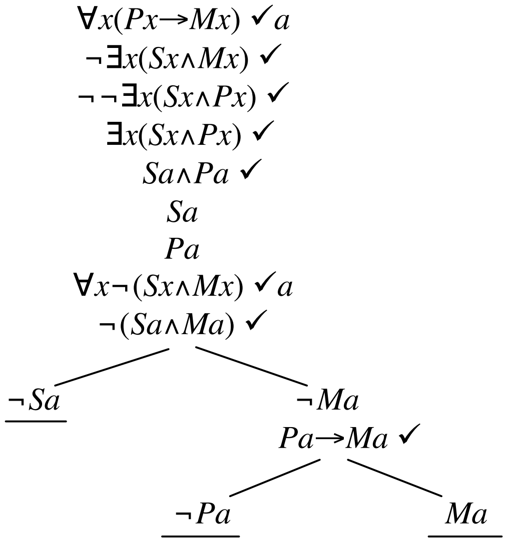
2. c.

$\neg\exists x(Px \wedge \exists y(Dxy \wedge Dyd))\ \checkmark$
$Pr$
$Drd$
$Ddd$
$\forall x \neg(Px \wedge \exists y(Dxy \wedge Dyd))\ \checkmark r$
$\neg(Pr \wedge \exists y(Dry \wedge Dyd))\ \checkmark$



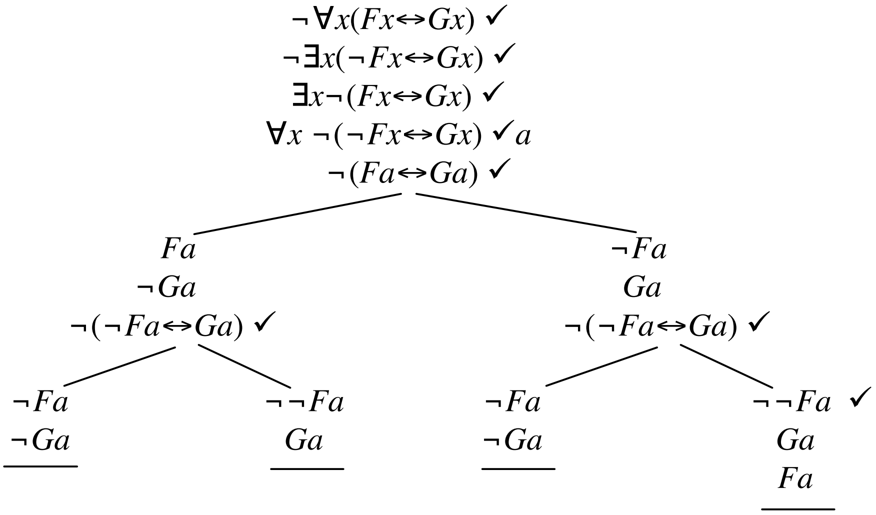
**inconsistent**

3. a.



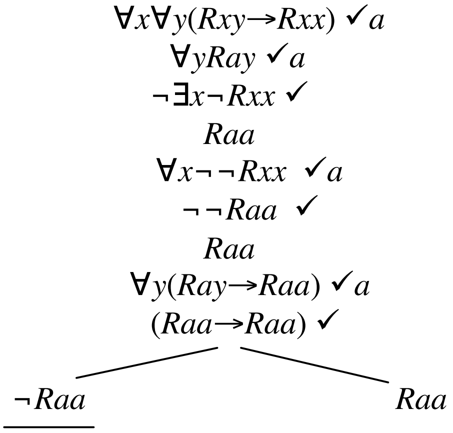
**valid**

3. b.



**valid**

3. c.



**invalid**

Countermodel:
D = {$a$}　　　　I($R$) = {<$a$, $a$>}

4. E-mail me your solution, and I let you know whether you got it right! ☺