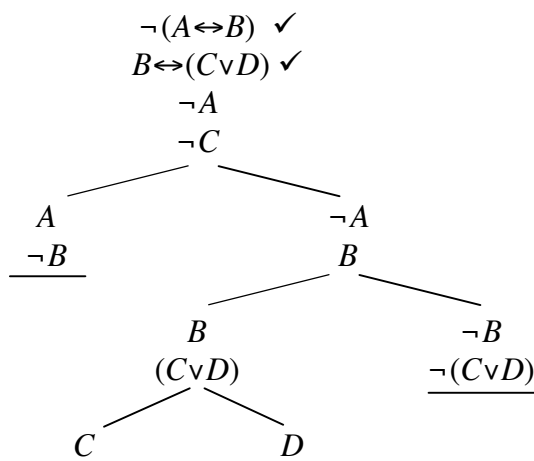


PY1003 Exam 2004 – Model Solutions

1. a. $\neg(A \leftrightarrow B), B \leftrightarrow (C \vee D), \neg A \vdash C$



invalid

Countermodel:

$I(A) = F$

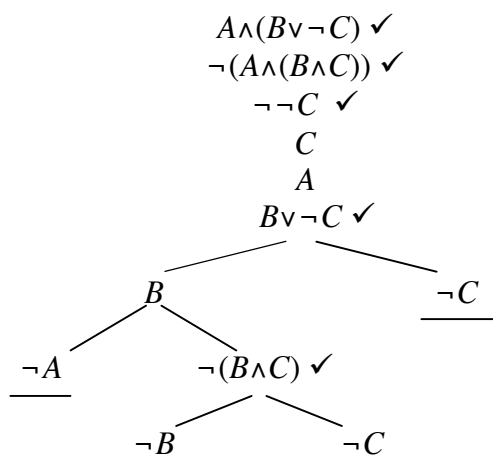
$I(B) = T$

$I(C) = F$

$I(D) = T$

1. b. $A \wedge (B \vee \neg C), \neg(A \wedge B \wedge C) \vdash \neg C$

NB: ' $\neg(A \wedge B \wedge C)$ ' has to be replaced by ' $\neg((A \wedge B) \wedge C)$ ' or ' $\neg(A \wedge (B \wedge C))$ '.



valid

2. a. (i) Key: Dx : x is a dog
 Cx : x is a cat
 Mx : x is a mammal

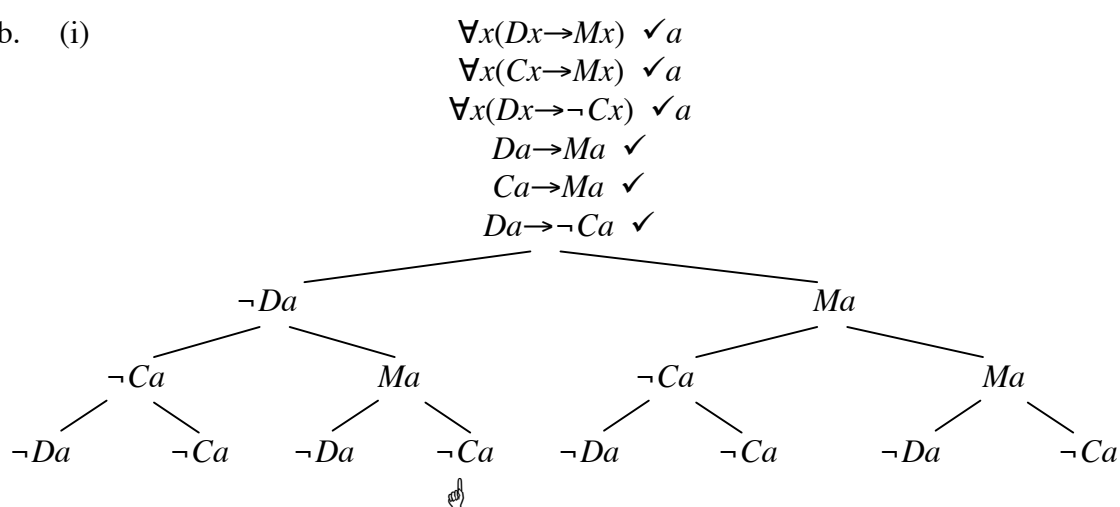
$$\{\forall x(Dx \rightarrow Mx), \forall x(Cx \rightarrow Mx), \forall x(Dx \rightarrow \neg Cx)\}$$

2. a. (ii) Key: Px : x is pompous Domain: people
 Lxy : x likes y

$$\{\exists xPx, \neg \exists x \exists y(Lxy \wedge Py), \forall x \exists y Lxy\}$$

NB: ' $\neg \exists x \exists y(Lxy \wedge Py)$ ' could also be formalised as ' $\forall x \forall y(Lxy \rightarrow \neg Py)$ '.

2. b. (i)



consistent

Model:

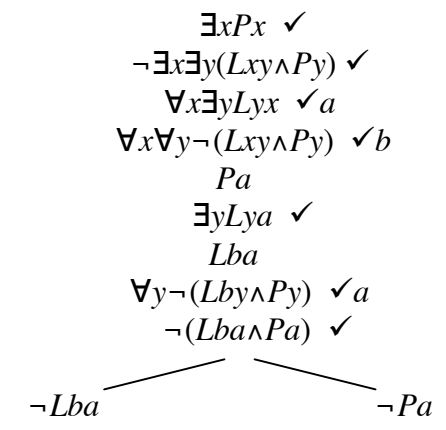
$$D = \{a\}$$

$$I(C) = \emptyset$$

$$I(D) = \emptyset$$

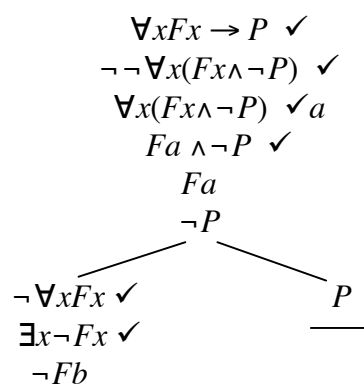
$$I(M) = \{a\}$$

2. b. (ii)



inconsistent

3. a. $\forall xFx \rightarrow P \vdash \neg \forall x(Fx \wedge \neg P)$



invalid

Countermodel:

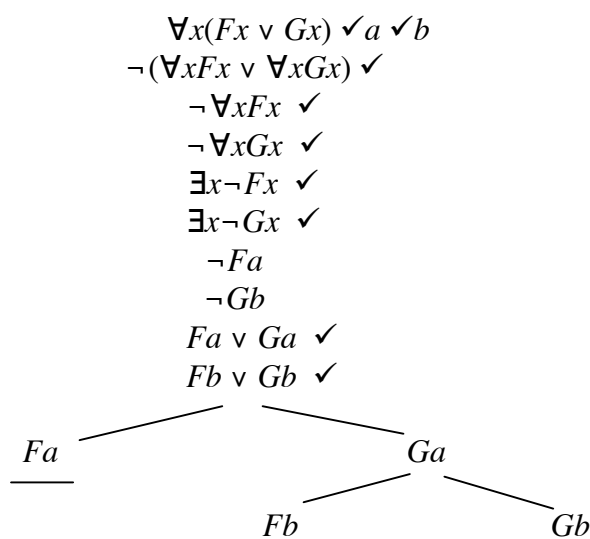
$$D = \{a, b\}$$

$$I(P) = F$$

$$I(F) = \{a\}$$

NB: This year, we have not introduced such “mixed cases” – they will not be on the exam.

3. b. $\forall x(Fx \vee Gx) \vdash \forall xFx \vee \forall xGx$



invalid

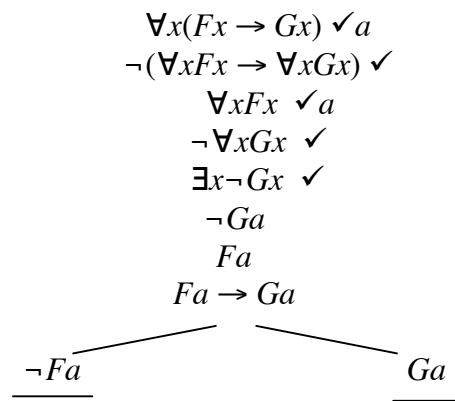
Countermodel:

$$D = \{a, b\}$$

$$I(F) = \{b\}$$

$$I(G) = \{a\}$$

3. c. $\forall x(Fx \rightarrow Gx) \vdash \forall xFx \rightarrow \forall xGx$



valid