JOURNAL OF MODERN OPTICS, 1998, VOL. 45, NO. 6, 1231–1237

The production of multiringed Laguerre–Gaussian modes by computer-generated holograms

J. ARLT, K. DHOLAKIA, L. ALLEN and M. J. PADGETT

J. F. Allen Physics Research Laboratories, School of Physics and Astronomy, University of St Andrews, North Haugh, St Andrews, Fife KY16 9SS, Scotland

(*Received 15 September 1997*)

Abstract. A computer-generated hologram is used for the production of high-order multiringed Laguerre–Gaussian modes. These holograms differ from those previously reported in that they have an additional circular discontinuity. The holograms are used in transmission and are designed to convert the fundamental Hermite–Gaussian laser mode into a Laguerre–Gaussian mode with specific azimuthal and radial indices. The optical efficiency exceeds 40% with a radial mode purity of approximately 80%

In recent years, considerable interest has been shown in the generation of laser beams containing phase singularities [1–4]. These beams are best described in terms of Laguerre–Gaussian modes which have an azimuthal phase term of $\exp(i\ell\phi)$ and possess a well defined orbital angular momentum of $\ell\hbar$ per photon [5]. The identification of light beams with a well defined orbital angular momentum, which is distinct from the spin component, has led to a number of exciting studies. These include the transfer of the orbital angular momentum to macroscopic objects [6, 7], the mode transformation upon nonlinear frequency doubling [8], and the interaction of these beams with atomic systems [9, 10].

For a Laguerre–Gaussian (LG) mode, the amplitude u_p^ℓ is given by

$$u_p^\ell(r,\phi,z) \propto \exp\left(-\frac{ikr^2}{2R}\right)\exp\left(-\frac{r^2}{w^2}\right)\exp(-i(2p+\ell+1)\psi)\exp(-i\ell\phi)(-1)^p \times\left(2\frac{r^2}{w^2}\right)^{\ell/2} L_p^\ell\left(\frac{2r^2}{w^2}\right), \quad (1)$$

where R is the wave-front radius of curvature, w is the radius for which the Gaussian term falls to $1/e$ of its on-axis value, ψ is the Guoy phase and $L_p^\ell(x)$ is a generalized Laguerre polynomial. LG modes with $p = 0$ are single annular rings with a well defined azimuthal phase given by ℓ. For $p > 0$, the modes are multiringed with $p + 1$ equal to the number of radial nodes.

LG beams may be produced directly from a laser [4] or, more usually, by the conversion of Hermite–Gaussian (HG) laser modes. The Gouy phase shift introduced by a pair of cylindrical lenses can be used to convert a high-order HG mode of indices m and n into a pure LG mode with $\ell = m - n$ and $p = \min(m, n)$ [2]. However, this relies on the ability to generate high-order HG modes, which most commercial lasers are designed to avoid. Consequently, if a commercial laser is to be used as the source, it is desirable to generate the LG mode directly from the fundamental zero-order Gaussian mode. Spiral phase plates [11, 12] and computer-generated holograms [13] have both been used. However, to date, efficient conversion has only been demonstrated for single-ringed LG modes, that is those with radial mode index $p = 0$.

In this paper we extend the use of computer generated holograms to the generation of LG modes with $p > 0$. The holograms comprise a forked diffraction grating with ℓ dislocations. The resulting screw phase dislocation produced on the axis of the diffracted beams gives the $\exp(\mathrm{i}\ell\phi)$ phase structure and the characteristic annular intensity pattern in the far field. When the forked-grating holograms are illuminated with a fundamental Gaussian mode, the resulting beam is a superposition of an infinite number of LG modes of the same ℓ index but with a range of values of p. The decomposition of the resulting beam can be performed analytically and the $p = 0$ mode is found to contribute to 78.5% to the intensity of the first-order diffracted beam [11, 13]. Other hologram designs which allow one to generate pure LG modes have also been demonstrated but their design inevitably leads to a low efficiency [14].

The form of the forked holograms is calculated from a simple general formula. For a binary grating the boundaries between the transparent and opaque areas are given, in polar coordinates, by

$$\frac{\ell\phi}{\pi} = n + \frac{2r}{\Lambda}\cos\phi, \tag{2}$$

where $n = 0, \pm 1, \pm 2, \ldots$ and Λ is the period of the grating at large distances away from the fork. The efficiency of the hologram can be increased by making it a phase hologram and blazing it to maximize the power in the chosen diffracted order. The transmittance function $T(r, \phi)$ of the hologram can then be expressed as [15]

$$T(r,\phi) = \exp\left[\mathrm{i}\delta H(r,\phi)\right], \tag{3}$$

where δ is the amplitude of the phase modulation. The pattern of the hologram $H(r,\phi)$ is given by

$$H(r,\phi) = \frac{1}{2\pi}\mathrm{mod}\left(\ell\phi - \frac{2\pi}{\Lambda}r\cos\phi,\ 2\pi\right), \tag{4}$$

where $\mathrm{mod}(a, b) = a - b\,\mathrm{int}(a/b)$. For $p = 0$ modes, such phase holograms have been reported [15] to have optical efficiencies higher than 50%

To generate LG modes with $p > 0$ we introduce a circular discontinuity, of radius w_{holo}, into the blazed hologram about which the phase of the grating is advanced by π (figure 1). Although the holograms still do not produce pure LG modes, the resulting diffracted beams can again be expressed as a superposition of LG modes. The two free Gaussian beam parameters, that is the waist size w_0 and the position of the waist, give an infinite number of LG basis sets. Only after choice of these two parameters is the decomposition unique. The complex expansion coefficients of the decomposition of the mth diffraction order can then be calculated by means of a scalar product [11]:

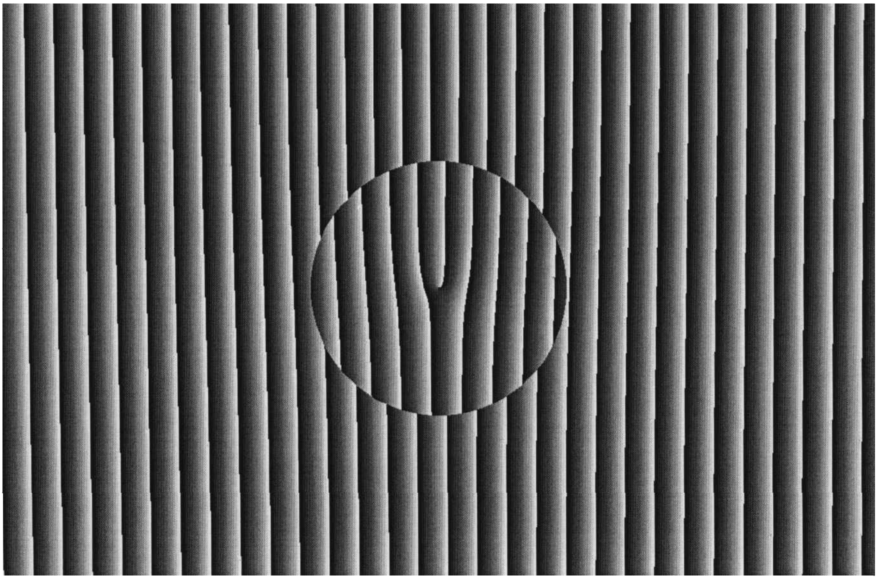


Figure 1. Blazed hologram to generate a higher-order LG beam with $p = 1$, $\ell = 1$.

$$a_{p\ell} = \left\langle T(r,\phi)E_{\mathrm{in}}(r), u_p^{\ell}(r,\phi,0)\exp\left(-im\frac{2\pi}{\Lambda}r\cos\phi\right)\right\rangle$$
$$= \iint T(r,\phi)E_{\mathrm{in}}(r)\left[u_p^{\ell}(r,\phi,0)\exp\left(-im\frac{2\pi}{\Lambda}r\cos\phi\right)\right]^* r\,\mathrm{d}r\,\mathrm{d}\phi, \qquad (5)$$

where

$$E_{\mathrm{in}}(r) = \left(\frac{2}{\pi}\right)^{1/2}\frac{1}{w_{\mathrm{beam}}}\exp\left(-\frac{r^2}{w_{\mathrm{beam}}^2}\right) \qquad (6)$$

is the field amplitude of the normalized incident beam with waist size w_{beam} and $T(r,\phi)$ the transmittance of the hologram. The relative weight of the modes is given by

$$I_{p\ell} = |a_{p\ell}|^2. \qquad (7)$$

For an ideal blazed grating all intensity would be diffracted in one order with $m = 1$. Integration of equation (5) over ϕ shows that all the modes present in the beam diffracted by the hologram have the same azimuthal mode index ℓ. The weighting of the different p indices, however, depends on the ratio of the waist size w_{beam} to the radius w_{holo} of the discontinuity. The decomposition may readily be performed numerically using routines written within the software package Mathematica [16]. For the numerical evaluation the hologram was assumed to be placed exactly at the beam waist of the incident beam. The LG modes used for the decomposition have their waist at the same position but their waist size is defined as w_{holo}. Figure 2 shows the calculated decomposition of the beam produced by a hologram with a single discontinuity. As can be seen from the figure, when $w_{\mathrm{beam}}/w_{\mathrm{holo}} \approx 2$, the contribution of the $p = 1$ mode peaks at approximately 80%

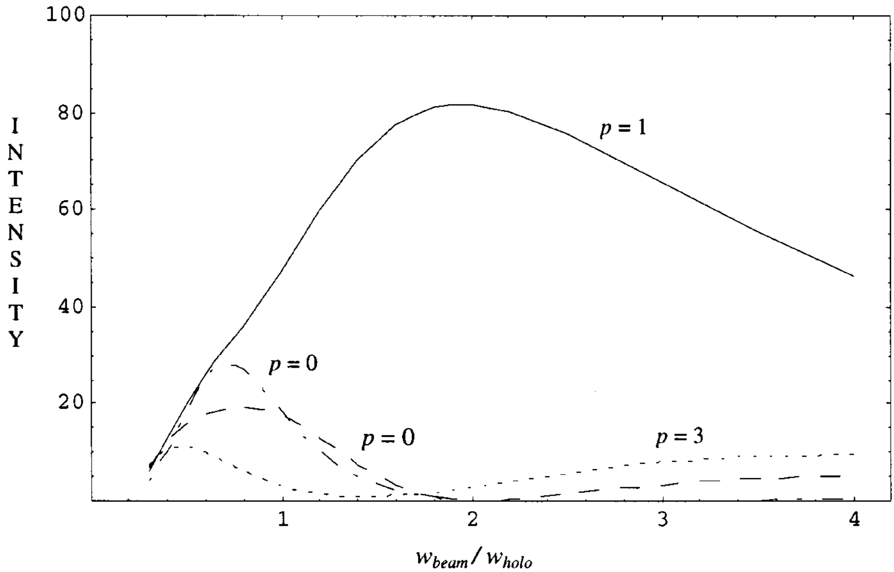
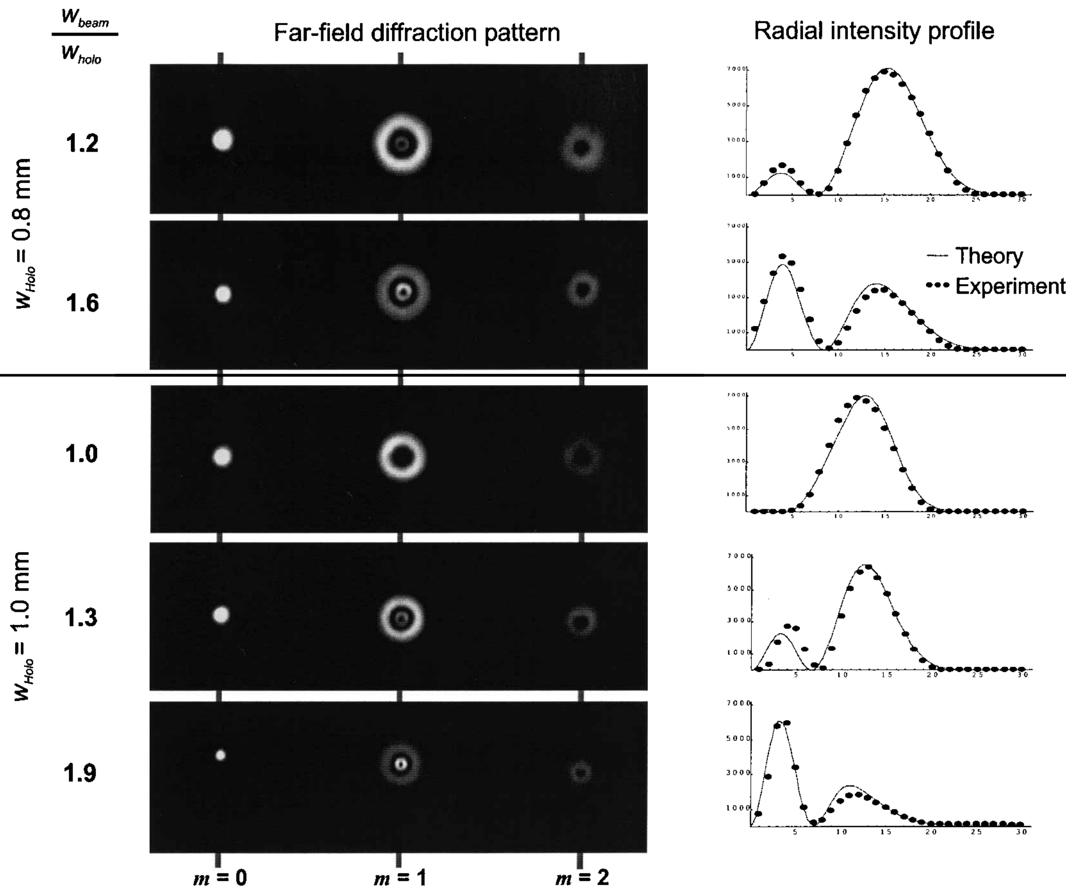


Figure 2. Intensity of different LG ($p, \ell = 1$) modes as a function of the ratio $w_{\rm beam}/w_{\rm holo}$.

In this work we used blazed holograms with a period $\Lambda = 0.175$ mm and radii $w_{\rm holo}$ typically between 0.8 and 1.5 mm. The computer-generated pattern was written directly onto colour film (Kodak Ektachrome Professional 100) using an Agfa slide writer film recorder (4096×2732 pixels on a slide of $35\,\rm mm \times 23\,\rm mm$ size). Contact prints were then made onto holographic film. The film was developed and bleached using a rehalogenation bleach [17]. The combination of blazing and bleaching gave a typical optical efficiency into the first diffracted order of approximately 40% This is not as high as the earlier work on modes with $p = 0$, which used specialist holographic glass plates. However, we felt that the simplicity gained by using the combination of standard photographic film and the slide writer justified the loss in overall optical efficiency. By changing to better holographic material and by compensating for the nonlinearities of the film by adjusting the computer-generated grey scale, further improvement in efficiency should be readily achievable.

A number of holograms have been fabricated and the mode decomposition of the resulting beams analysed as a function of $w_{\rm beam}/w_{\rm holo}$. Figure 3 shows the far-field diffraction patterns and the resulting mode decomposition for a hologram designed to give $\ell = 1$ in the first order. As expected, the $p = 1$ component peaks at just over 80%when $w_{\rm beam}/w_{\rm holo} \approx 2$.

This approach can be extended to the generation of modes with $p > 1$ by the inclusion of additional circular discontinuities at larger radii. The discontinuities are canonically spaced according to the radii of the zeros of the desired LG mode. For example to generate a LG mode with $p = 2$, $\ell = 1$, two discontinuities with radii $r_1 = \left[(3 - 3^{1/2})/2\right]^{1/2} w_{\rm holo}$ and $r_2 = \left[(3 + 3^{1/2})/2\right]^{1/2} w_{\rm holo}$ are used. Figure 4 shows the far-field diffraction pattern for such a hologram with two circular discontinuities illuminated with a fundamental mode with $w_{\rm beam}/w_{\rm holo} \approx 2.5$ to give a beam with three rings. Decomposition of the resulting beam shows that more than 75%is associated with the $p = 2$ mode.



Decomposition into LG modes ($\ell = 1$)

p	Amplitude	Intensity [%]
0	0.32	10.1
1	0.77	59.8
2	0.36	13.0
3	0.12	1.5

p	Amplitude	Intensity [%]
0	0.15	2.1
1	0.88	77.3
2	0.18	3.1
3	0.11	1.2

p	Amplitude	Intensity [%]
0	0.43	18.3
1	0.69	47.7
2	0.42	17.7
3	0.17	3.0

p	Amplitude	Intensity [%]
0	0.27	7.1
1	0.81	65.3
2	0.32	10.2
3	0.11	1.1

p	Amplitude	Intensity [%]
0	0.07	0.5
1	0.9	81.7
2	0.05	0.2
3	0.15	2.2

Figure 3. Comparison between experimental results and numerically predicted radial intensity profiles and mode decompositions.

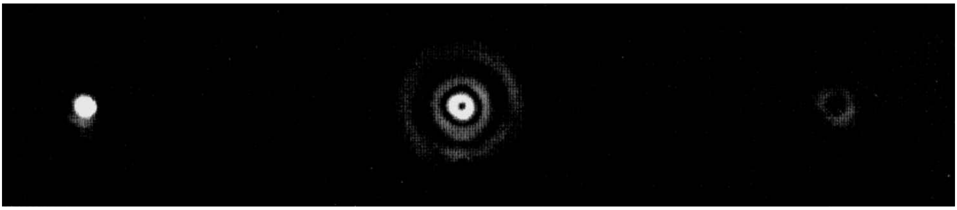


Figure 4. LG mode with $\ell = 1$ and $p = 2$ generated by a hologram with two annular discontinuities.

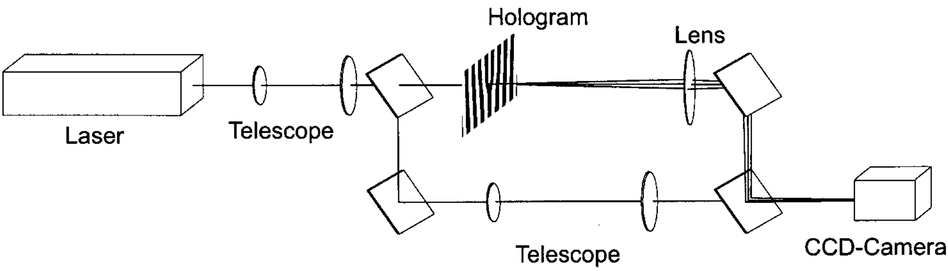


Figure 5. Experimental set-up to measure the phase structure of the generated beams. CCD, charge-coupled device.

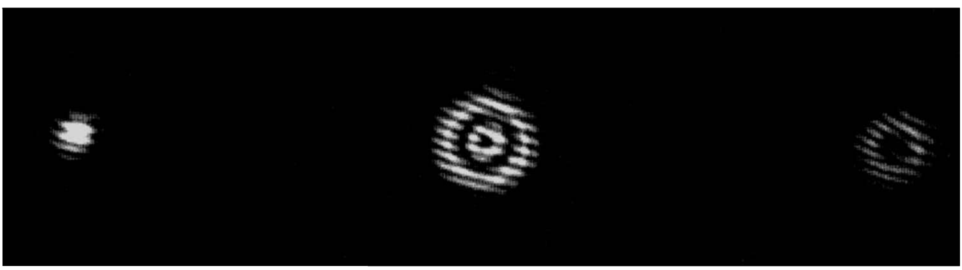


Figure 6. Interference experiment revealing the phase structure of the generated beams.

The intensity distribution is in good agreement with that predicted, but a question remains as to the phase structure of the beams produced. The phase structure of LG beams has previously been confirmed by observing the interference pattern between the LG mode and a plane wave [18]. Figure 5 shows a similar experimental configuration to observe the phase structure of the holographically generated beams. Figure 6 shows the resulting interferogram obtained using a hologram designed to give $\ell = 1$ in the first order. The zero-order non-diffracted beam exhibits straight fringes showing a plane-wave phase structure. The first-order beam exhibits a single fork in the fringes, indicating an $\exp(i\phi)$ phase structure. Note also that the inner and outer ring are out of phase. The second-order beam has a double fork, indicating an $\exp(i2\phi)$ phase structure. However, the second-order beam has only a single ring. This is because the phase advance of π for the first order produces an advance of 2π for the second order; this gives no discontinuity in phase in the far field. The third-order beam, not shown here, has two rings with three forks indicating an $\exp(i3\phi)$ phase structure.

In this work we have extended the production of LG modes using computer-generated holograms to include modes with $p > 0$. The beams produced have a uniquely defined azimuthal index ℓ, and high radial mode purity. Holographic converters use the fundamental Gaussian mode. This is particularly useful if a commercial laser is to be used as the source. In addition, removing the need for cylindrical lenses eliminates any risk of introducing astigmatism into the beam; even a small amount of astigmatism can lead to a break up of the LG mode in anisotropic media [19]. Our specific interest in these modes is their interaction with nonlinear media where the orbital angular momentum results in additional constraints. Our earlier investigation of frequency doubling has shown that LG modes with $p > 0$ behave differently from the $p = 0$ modes. Frequency-doubled $p = 0$ modes are themselves LG modes, whereas higher radial modes produce a new class of mode, which we call Gegenbauer–Gaussian modes [20]. We aim to extend this work to investigate the process of parametric down-conversion with modes possessing orbital angular momentum.

References

[1] Heckenberg, N. R., McDuff, R., Smith, C. P., and White, A. G., 1992, *Optics Lett.*, **17,** 221.

[2] Beijersbergen, M. W., Allen, L., van der Veen, H. E. L. O., and Woerdman, J. P., 1993, *Optics Commun.*, **96,** 123.

[3] Basistiy, I. V., Bazhenov, V. Y., Soskin, M. S., and Vasnetsov, M. V., 1993, *Optics Commun.*, **103,** 422.

[4] Harris, M., Hill, C. A., and Vaughan, J. M., 1994, *Optics Commun.*, **106,** 161.

[5] Allen, L., Beijersbergen, M. W., Spreeuw, R. J. C., and Woerdman, J. P., 1992, *Phys. Rev.* A, **45,** 8185.

[6] He, H., Friese, M. E. J., Heckenberg, N. R., and Rubinsztein-Dunlop, H., 1995, *Phys. Rev. Lett.*, **75,** 826.

[7] Simpson, N. B., Dholakia, K., Allen, L., and Padgett, M. J., 1997, *Optics Lett.*, **22,** 52.

[8] Dholakia, K., Simpson, N. B., Padgett, M. J., and Allen, L., 1996, *Phys. Rev.* A, **54,** R3742.

[9] Power, W. L., Allen, L., Babiker, M., and Lembessis, V. E., 1995, *Phys. Rev.* A, **52,** 479.

[10] Lai, W. K., Babiker, M., and Allen, L., 1997, *Optics Commun.*, **133,** 487.

[11] Beijersbergen, M. W., Coerwinkel, R. P. C., Kristensen, M., and Woerdman, J. P., 1994, *Optics Commun.*, **112,** 321.

[12] Turnbull, G. A., Robertson, D. A., Smith, G. M., Allen, L., and Padgett, M. J., 1996, *Optics Commun.*, **127,** 183.

[13] Heckenberg, N. R., McDuff, R., Smith, C. P., Rubinsztein-Dunlop, H., and Wegener, M. J., 1992, *Opt. quant. Electron.*, **24,** S951.

[14] Freude, W., Grau, G. K., Lieber, W., and Wupperman, B., 1988, *Appl. Optics*, **27,** 138.

[15] He, H., Heckenberg, N. R., and Rubinsztein-Dunlop, H., 1995, *J. mod. Optics*, **41,** 217.

[16] Mathematica 3.0 1997 (Wolfram Research)

[17] Kim, N., 1994, *Optics Commun.*, **105,** 1.

[18] Padgett, M. J., Arlt, J., Simpson, N., and Allen, L., 1996, *Am. J. Phys.*, **64,** 77.

[19] Mamaev, A. V., Saman, M., and Zozula, A. A., 1997, *Technical Digest*, QTuK7. Quantum Electronics and Laser Science Conference '97.

[20] Courtial, J., Dholakia, K., Allen, L., and Padgett, M. J., 1997, *Phys. Rev.* A, **56,** 4193.