Good Politicians' Distorted Incentives

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Abstract

I construct a political agency model that provides a new explanation for sub-optimal policy making decisions by incumbents. I show that electoral incentives can induce politicians to address less relevant issues, disregarding more important ones. Issue importance is defined in terms of the utility voters would receive if the issue was solved. Contrary to existing literature, sub-optimal policy making occurs even when voters are perfectly informed about issues’ characteristics and politicians are policy oriented. I provide an explanation that relies on the negative correlation between issue importance and probability of solving it: for a given effort exerted by incumbents, less relevant issues guarantee higher probability of success. In equilibrium, voters cannot commit to re-elect the incumbent if and only if the most important issue was solved. This is because solving the easy issue also constitutes a positive signal about incumbents’ type. Whenever re-election is sufficiently valuable, then, politicians will choose to address less relevant and easier issues.

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1 Introduction

A key element for the correct functioning of representative democracies is the performance of elected politicians. In this respect, elections provide the main tool to correct the moral hazard and adverse selection components intrinsic in politician-voter relationship. An important question to ask then is whether elections achieve this goal. The answer provided by the literature is “not always” (see Ashworth (2012) for a review): the interplay between asymmetric information and the personal gains from holding office can distort politician’s incentive and induce them to take decisions that go against the best interest of voters.

This paper contributes to the political agency literature by providing a new channel through which elections can distort politician’s incentives. The simple model I analyse delivers the following clear and important message: elections can induce politicians to address issues that are easy to tackle, instead of focusing on important – but more complicated – ones. Most importantly, this happens even if voters are perfectly informed about issues’ characteristics.

Consider a country affected by two types of issues: an important one, that would deliver high utility to voters if solved, but is characterized by a consistent risk of failure; and another issue that is relatively easier to solve, but is not the priority for voters’ interests. Voters know the characteristics of the issues but are uncertain about the type of incumbent in power. In particular, they do not know whether the politician is only driven by a private desire for holding office or whether she is (also) intrinsically interested in solving the issues. Furthermore, they are not able to perfectly monitor the effort exerted by the politician, but can only verify if an issue was solved or not. Suppose they observe that only the easy issue was solved. Does this observation increase the likelihood of having a good politician in power? The answer is yes: because of the utility she derives from solving the issues, a good politician always exerts higher effort than a bad one, and is therefore more likely to solve any issue. Now turn to a good incumbent’s choice: on the one hand, solving the more important issue would deliver higher utility; on the other hand, this comes at a high risk of failure and the easier issue might be a safer choice. The second consideration prevails whenever being in power in the second period is important enough.

Political agency literature has focused on different sources of asymmetric information between voters and politicians. In some cases, voters cannot ob-
serve politicians’ competence level (among others, Rogoff (1990), Besley and Case (1995) and Banks and Sundaram (1998)). When asymmetric information is combined with voters’ uncertainty about the optimal action to take, politicians have incentives to pander by choosing the action that is ex ante more popular rather than following their private signal (Canes-Wrone et al. (2001), Ashworth and Shotts (2010)). In Maskin and Tirole (2004), pandering arises from voters uncertainty about politician’s preference ranking over two possible actions. In Coate and Morris (1995), voters cannot observe politicians’ attitude towards pleasing a special interest group. Politicians must choose whether to implement a public project or transfer resources to the group. The authors show that whenever voters are unsure about the quality of the project, inefficient transfers will be implemented.

In these papers, if voters were perfectly informed about project characteristics and optimality of actions, no distortion would arise. This is the main difference with respect to my work. In my model, voters know exactly what the right choice is and how problematic the issues are, but are unable to force the politician to act in their interest. In order to do so, they would need to commit to re-elect the incumbent only if the more important issue was solved. Assume however that, with some probability, external factors might prevent the resolution of the important issue. These could be, for example, the absence of cooperation by other countries on international matters or the presence of a highly inefficient bureaucracy in the country (Gratton et al. (2017)). Voters are aware of the possibility but do not know the exact state of the world. In this situation, committing not to re-elect the incumbent when she solved the easy and less important issue would be sub-optimal: if external factors prevent the resolution of the important issue and given voters’ re-election strategy, only a good incumbent would have incentives to exert effort on the less important issue.

When comparing voters’ expected utility when the incumbent can run for re-election or not, I obtain conclusions that are in line with existing literature (e.g. Barro (1973), Ferejohn (1986), Besley and Smart (2007)). Elections increase voters expected utility by allowing them to screen politicians and by partially re-aligning their incentives to those of the electorate. At the same time, however, the distortion created in politician’s incentives decreases voters’ utility. This reduces the positive effects of allowing incumbents to run for a second term.

In the paper, I make the simplifying assumption that the politician can only exert effort on one of the two issues. This is an ideal starting point
towards the analysis of the same problem under multitasking. A more recent research on political agency has developed in this direction. For example, Ash et al. (2016) consider incumbents’ effort allocation over a divisive and a common value issue. They show that incumbents can posture by over-providing effort on that issue. This happens because of their desire to signal that their policy preferences over the issue are aligned with those of the majority. Fox and Van Weelden (2015) focus on the choice between preparing for a potential crisis and working to make a normal situation even better. When voters are unsure about politicians ability, the incumbent will sub-optimally exert too little effort in preparation for the crisis.

The reminder of the paper is organized as follows. Section 2 introduces the model and the main assumptions. Section 3 presents the results. An alternative specification of the model in which no external friction on issue resolution exists is discussed in Section 4. Section 5 concludes.

2 The Model

An incumbent politician is confronted with two issues, $a$ and $b$, and must choose which one to address before the end of the term. I assume she cannot address both. The probability of solving an issue depends on two factors. First, exogenous constraints might prevent its resolution. I assume there exist three states of the world, $\sigma_a$, $\sigma_b$ and $\sigma_{ab}$, depending on which issue(s) can be solved: only $a$, only $b$ or both $a$ and $b$, respectively. The second factor determining the probability of solving an issue is incumbent’s effort. Let $e \in [0, \infty)$ denote the amount of effort she exerts on an issue, if she decides to address it. If no exogenous constraint prevents the resolution of issue $i$, this is solved with probability $\pi_i(e) \in [0, 1)$. I assume $\pi_i(\cdot)$ increasing and concave and such that $\pi_i(0) = 0$ and $\lim_{e \to \infty} \pi_i(e) = 1$. Combining the two factors, the probability of solving issue $i$ for a given level of effort $e$ exerted by the incumbent is

$$P_i(e|\sigma) = \begin{cases} \pi_i(e) & \text{if } \sigma \in \{\sigma_i, \sigma_{ab}\} \\ 0 & \text{otherwise} \end{cases}$$

if the incumbent has decided to address the issue, it is always zero otherwise. Voters do not know the state of the world and are unable to monitor politician’s effort decisions. They only observe which issue, if any, was solved. I
denote this outcome by $s \in \{a, b, \emptyset\}$, with $s = a$ ($s = b$) if issue $a$ ($b$) was solved and $s = \emptyset$ otherwise. Voters have identical preferences over the issues and receive utility $u_i$ from the resolution of issue $i \in \{a, b\}$. I make the following additional assumptions on the characteristics of the two issues

**Assumption 1.** For all $e \in [0, \infty)$,

i) Issue $b$ is easier to solve: $\pi_b(e) > \pi_a(e)$;

ii) Issue $a$ is more important: $\pi_a(e)u_a > \pi_b(e)u_b$;

For any level of effort $e$ exerted, the incumbent suffers a cost $c(e)$, with $c(\cdot)$ increasing and convex. There are two types of incumbent: a *bad* ($B$) type only cares about being re-elected and obtains rent $R$ from being in office in the second period. A *good* ($G$) type cares about being re-elected as well as solving the issues$^1$. In this respect, her preferences are identical to voters’. An incumbent is good with probability $\gamma$. Voters’ goal is to have a good incumbent in power in the next period.$^2$ Let $\hat{\gamma}(s)$ be voters’ updated beliefs about the type of incumbent, conditional on observing outcome $s$. Re-electing the incumbent is optimal if an only if $\hat{\gamma}(s) \geq \gamma$.

## 3 Results

The main goal of the paper is to show how the pressure of re-election can change politician’s incentives against voters’ interests. In order to do so, set $R = 0$ first. This corresponds to a situation where the politician cannot run for re-election (for example, in presence of term limits). A $B$ type would always shirk and exert no effort. The optimal amount of effort that a good incumbent would exert on issue $i$ would instead solve

$$\max_e \pi_i(e)u_i - c(e) \quad (1)$$

Let $\hat{e}_i$ denote the solution for issue $i$. By Assumption 1, point ii),

$$\pi_a(\hat{e}_a)u_a - c(\hat{e}_a) \geq \pi_a(\hat{e}_b)u_a - c(\hat{e}_b)$$

$$> \pi_b(\hat{e}_b)u_b - c(\hat{e}_b)$$

$^1$For a good politician, the parameter $R$ could be interpreted as the expected utility of being able to solve future issues in the country if in power.

$^2$This is equivalent to imposing a two-term limit for the incumbent and assuming that new issues appear in the second period. In the second period, a good politician’s preferences are perfectly aligned with voters’, while a bad politician will exert no effort.
so that a good incumbent would always try to solve issue \(a\), whenever this is possible.

Now let \(R > 0\). Let \(v = (v_s)_{s \in \{a,b,\emptyset\}} \in \{0,1\}^3\) represent a generic re-election strategy for voters, with \(v_s = 1\) if they decide to re-elect the incumbent after observing outcome \(s \in \{a,b,\emptyset\}\), \(v_s = 0\) otherwise. In particular, denote by \(v^* = \{1,1,0\}\) the strategy prescribing to re-elect the incumbent if any of the issues was solved. The strategy is optimal if and only if \(\tilde{\gamma}(i) \geq \gamma\), for both \(i \in \{a,b\}\). Now assume the incumbent expects voters to play \(v^*\) and consider effort provision by the two types. In state \(\sigma_i\), the maximization problems for a bad and a good type are

\[
\max_e \pi_i(e) R - c(e) \tag{2}
\]

\[
\max_e \pi_i(e) [u_i + R] - c(e) \tag{3}
\]

respectively. Let \(e^*_{iB}\) and \(e^*_{iG}\) denote their solutions, for each issue \(i\). From the first order conditions,

\[
\frac{c'(e^*_i)}{\pi'_i(e^*_i)} = R < u_i + R = \frac{c'(e^*_G)}{\pi'_i(e^*_G)}
\]

and since \(c'()/\pi'_i()\) is an increasing function of effort, \(e^*_i > e^*_{iB}\). In state \(\sigma_{ab}\), a bad incumbent always addresses issue \(b\) as

\[
\pi_b(e^*_{bB}) R - c(e^*_{bB}) \geq \pi_b(e^*_{aB}) R - c(e^*_{aB}) > \pi_a(e^*_{aB}) R - c(e^*_{aB})
\]

The first inequality directly follows from \(e^*_{bB}\) being the solution of problem (2), the second from Assumption 1, point i). A good type chooses issue \(a\) in state \(\sigma_{ab}\) only if

\[
\pi_i(e^*_{aG}) [u_a + R] - c(e^*_{aG}) \geq \pi_b(e^*_{bG}) [u_b + R] - c(e^*_{bG}) \tag{4}
\]

This choice depends on \(R\). Proposition 1 shows that there exists a lower bound \(\bar{R}\) on the rents from re-election such that, for all values of \(R > \bar{R}\), a good politician will address \(b\) in state \(\sigma_{ab}\).

**Proposition 1.** There exists \(\bar{R} \in [0,\infty)\) such that, whenever \(R > \bar{R}\), in equilibrium

- Voters re-elect if any of the two issues was solved
In state $\sigma_a$, both politicians exert effort on $a$

In states $\sigma_b$ and $\sigma_{ab}$, both politicians exert effort on $b$

The proof of the proposition (in the appendix) consists of two steps. First, conditional on voters adopting strategy $v^*$, I show that such $R$ exists. Secondly, I show that $v^*$ is optimal for voters given the behavior it induces on politicians. Intuitively, since a good politician always exerts higher effort than a bad one, the resolution of any issue increases the belief of having a good incumbent in power.

Whenever $R$ is sufficiently large, then, issue $a$ will be addressed only in state $\sigma_a$, while both types of politician will tackle issue $b$ in the other two states. Notice that Proposition 1 holds for any arbitrarily small (but positive) probability of states $\sigma_a$ and $\sigma_b$. Given this, it is worth comparing voters’ expected utility when a politician is or is not subject to re-election, for the limit case where the probabilities of those states tend to zero. Let $\rho(\sigma)$ represent the (ex ante) probability of state $\sigma$ and $V$ the expected utility of having a good politician in power in the second period (see footnote 2). I denote voters’ expected utilities when the politician is subject to re-election or not as $EU_R$ and $EU$, respectively. Then

$$\lim_{\rho(\sigma_a) \to 0, \rho(\sigma_b) \to 0} EU_R = \gamma \{ \pi_b(e_{bG}^*)(u_b + V) + [1 - \pi_b(e_{bG}^*)]\gamma V \} + (1 - \gamma) \{ \pi_b(e_{bB}^*)u_b + [1 - \pi_b(e_{bB}^*)]\gamma V \}$$

$$\lim_{\rho(\sigma_a) \to 0, \rho(\sigma_b) \to 0} EU = \gamma \pi_a(\hat{e}_{aG})u_a + \gamma V$$

Taking the difference between the two,

$$\Delta EU = \gamma [\pi_b(e_{bG}^*)u_b - \pi_a(\hat{e}_{aG})u_a] + (1 - \gamma) [\pi_b(e_{bG}^*) - \pi_b(e_{bB}^*)]\gamma V + (1 - \gamma) \pi_b(e_{bB}^*)u_b \quad (5)$$

The second and third term in (5) represent the known positive effects of allowing the incumbent to run for re-election. First, this allows voters to screen good from bad politicians and increase expected second period utility (second term). Secondly, elections reduce moral hazard by providing incentives to exert effort for a bad incumbent (third term). The first term is negative whenever $R$ and the difference between $u_a$ and $u_b$ are sufficiently large. It represents the utility loss from the distortion of incentives for the incumbent.
3.1 Other equilibria

I now briefly discuss the existence of other possible equilibria by considering alternative re-election strategies by voters. Let $R > R$. Start by assuming that voters re-elect if and only if issue $i$ was solved ($v \in \{\{1, 0, 0\}, \{0, 1, 0\}\}$). This is optimal only if, after observing that issue $j \neq i$ was solved, voters believe to have a bad incumbent in power, i.e. $\tilde{\gamma}(j) < \gamma$. Given voters’ re-election rule, however, a bad incumbent would have no incentives to exert effort on issue $j$: this would only increase costs, without affecting the probability of being re-elected. More precisely, a bad incumbent will exert effort $e_{jB}^*$ in states $\sigma_i$ and $\sigma_{ab}$ and no effort in state $\sigma_j$.

On the contrary, a good incumbent would always exert a positive amount of effort on issue $j$ in (at least) state $\sigma_j$, as this would still give her a positive expected utility from solving the issue. More precisely, optimal effort by a good incumbent is $e_{jG}^*$ in state $\sigma_i$, $\hat{e}_j$ in state $\sigma_j$ (where $\hat{e}_j$ solves (1) above) and either $e_{jG}^*$ or $\hat{e}_j$ in state $\sigma_{ab}$, depending on whether

$$
\pi_i(e_{jG}^*)[u_i + R] - c(e_{jG}^*) \geq \pi_j(\hat{e}_j)u_j - c(\hat{e}_j)
$$

holds. Given the optimal effort by the two types in each state, we have

$$
\tilde{\gamma}(j) = \rho(\sigma_j|j) + \rho(\sigma_{ab}|j)Prob(G|j, \sigma_{ab})
$$

where $\rho(\sigma|j)$ denotes the probability of state $\sigma$ conditional on issue $j$ being solved. Since

$$
Prob(G|j, \sigma_{ab}) = \begin{cases} 
\frac{\gamma \pi_i(e_{jG}^*)}{\gamma \pi_i(e_{jG}^*) + (1-\gamma)\pi_i(e_{jB}^*)} > \gamma & \text{if (6) holds} \\
1 & \text{otherwise}
\end{cases}
$$

we must have $\tilde{\gamma}(j) > \gamma$.

A similar reasoning holds for the strategies prescribing to never re-elect or always re-elect the incumbent ($v \in \{\{0, 0, 0\}, \{1, 1, 1\}\}$). The equilibrium considered in Proposition 1 is however not unique. For sufficiently large $R$, other equilibria could exist where no politician exerts positive effort and voters re-elect even if no issue was solved ($v \in \{\{0, 0, 1\}, \{0, 1, 1\}, \{1, 0, 1\}\}$). I disregard this type of equilibria in the analysis.\(^3\)

\(^3\)Note that the existence of these equilibria only strengthens the conclusions on the negative effect of re-election on politicians’ incentives.
4 Unique state of the world

As mentioned already, the results discussed in the previous section hold for infinitesimally small probabilities of states $\sigma_a$ and $\sigma_b$. For the limit case in which these probabilities are exactly zero (so that both issues can always be solved with positive probability if the incumbent exerts effort on them), two other equilibria arise where voters re-elect the incumbent only if one specific issue was solved. Let $v^a = \{1, 0, 0\}$ be the re-election strategy that retains the incumbent if and only if issue $a$ was solved. Under $v^a$, both politicians will address issue $a$ and exert an amount of effort $e_{aB}^*$ and $e_{aG}^*$ that solve (2) and (3) (with $i = a$), respectively. As shown before, $e_{aG}^* > e_{aB}^*$, which in turn implies $\hat{\gamma}(a) > \gamma$. Finally, not re-electing the incumbent if $b$ was solved can be supported by out-of-equilibrium beliefs $\hat{\gamma}(b) < \gamma$.

Under $v^a$, a good incumbent does not face any tradeoff, as the only issue that guarantees re-election is also the one that maximizes her expected utility. The tradeoff re-emerges if voters re-elect if and only if issue $b$ was solved. Let $v^b = \{0, 1, 0\}$ denote this re-election strategy. Under $v^b$, a bad politician will exert an amount of effort $e_{bB}^*$ that solves (2) (with $i = b$). A good politician will have the choice between trying to solve $a$ and not being re-elected for sure or trying to solve $b$ to maximize her chances of being in power in the second period. She will choose $b$ if and only if

$$\pi_b(e_{bG}^*)[u_b + R] - c(e_{bG}^*) \geq \pi_a(\hat{e}_a)u_a - c(\hat{e}_a)$$

where $\hat{e}_a$ is the solution of problem (1) above. A sufficient condition for this to hold is $R > R$. Indeed, for all $R$ above the threshold,

$$\pi_b(e_{bG}^*)[u_b + R] - c(e_{bG}^*) \geq \pi_a(e_{aG}^*)[u_a + R] - c(e_{aG}^*) \\
\geq \pi_a(\hat{e}_a)[u_a + R] - c(\hat{e}_a) \\
> \pi_a(\hat{e}_a)u_a - c(\hat{e}_a)$$

The first inequality holds because (4) is violated whenever $R > R$. The second is a direct consequence of $e_{aG}^*$ being the solution of (3). The discussion proves the following proposition:

**Proposition 2.** Whenever $\sigma_{ab}$ is the only possible state of the world, there exists an equilibrium where both incumbents address issue $a$ and voters re-elect if and only if that issue is solved. Furthermore, if $R \geq R$, there exists other two equilibria where both incumbents choose issue $b$ and voters re-elect if issue $b$ is solved.
5 Conclusions

The paper considered a political agency model where an incumbent politician must decide which issue to address. Issues differ in their importance for voters and in how easily they can be solved. More precisely, the more important issue is also more difficult to solve. Voters are perfectly aware of the characteristics of the two issues, but cannot observe whether the incumbent is purely office motivated or also cares about solving the issues. Furthermore, they cannot monitor her effort.

The key conclusion of the paper is that, even if voters know what would be the right action to be taken by the incumbent, they are unable to commit not to re-elect her if she chooses to solve the less important issue. This creates a tradeoff for a good politician between solving the issue that would maximize the (first period) expected utility and addressing the one that would maximize the chances of re-election. When re-election is sufficiently important, the incumbent will choose the second.

The distortion of politicians’ incentives created by the possibility to run for a second term partially offsets the positive effects produced by elections on voters expected utility (through screening and the correction of moral hazard).

Gratton et al. (2017) study the dynamic interaction between law-making activities and the quality of bureaucracy. When the latter is highly inefficient, less competent politicians engage in the production of useless reforms. In a situation of political instability, this allows them to be recognized as active reformers, while the slow wheels of bureaucracy prevent voters to observe the true quality of the reform. In turn, the over-production of unnecessary reforms fuels the inefficiency of the bureaucratic process, leading to a steady-state characterized by a jammed bureaucracy and a high production of useless reforms. Combining insights of this paper with the intuition behind my model could produce interesting results. Assume reforms are not just good or bad, but also differ in how easily they can be implemented. When a bad reform is also hard to implement, the incentives to promote it are even stronger for a less competent incumbent, as voters are less likely to observe its quality. This reinforces the conclusions in Gratton et al. (2017). Some novel insight could appear when looking at the behavior of a competent incumbent. In Gratton et al. (2017), this type always implements a good reform. If implementing an easy reform can send a negative signal to voters (either because they do not want politicians to choose irrelevant tasks or because they believe
any politician can implement an easy reform), a competent incumbent might refrain from promoting a reform even if it is good. In equilibrium, then, the total number or reforms could be lower, but their average quality would be worse.

6 Appendix

Proof of Proposition 1. Assume voters play strategy \( v^* = \{1, 1, 0\} \) and define the two functions

\[
f_a(e) = \pi_a(e)[u_a + R]
\]

and

\[
f_b(e) = \pi_b(e)[u_b + R]
\]

By the assumptions on \( \pi_i(\cdot) \), \( f_a(0) = f_b(0) = 0 \), and \( f'_b(0) > f'_a(0) \) if and only if

\[
R > \frac{\pi'_b(0)u_b - \pi'_a(0)u_a}{\pi'_b(0) - \pi'_a(0)} \equiv R_0
\]

Thus, for all \( R > R_0 \) and \( e \) sufficiently small, it must be that \( f_b(e) > f_a(e) \). Since \( \lim_{e \to \infty} f_a(e) > \lim_{e \to \infty} f_b(e) \), there must exist a point \( \bar{e} > 0 \) such that \( f_b(\bar{e}) = f_a(\bar{e}) \) and \( f_b(e) > f_a(e) \) for all \( e \in (0, \bar{e}) \). More precisely, \( \bar{e} = \bar{e}(R) \) and, by the implicit function theorem

\[
\frac{\partial \bar{e}(R)}{\partial R} = \frac{\pi_b(\bar{e}) - \pi_a(\bar{e})}{f'_b(\bar{e}) - f'_a(\bar{e})} > 0
\]

Denote by \( R \) the value of \( R \) such that

\[
f_a(\bar{e}(R)) = f_b(\bar{e}(R)) = c(\bar{e}(R))
\]

Then, for all \( R > R \geq R_0 \),

\[
f_b(e) - c(e) > f_a(e) - c(e)
\]

for all \( e \leq \bar{e}(R) \). Since \( e^*_i \) for both \( i \), we have that

\[
\pi_b(e^*_iG)[u_b + R] - c(e^*_iG) \geq \pi_b(e^*_aG)[u_b + R] - c(e^*_aG) > \pi_a(e^*_aG)[u_a + R] - c(e^*_aG)
\]

To conclude the proof, one must show that \( v^* \) is optimal. Let \( R > R \) and denote by \( \rho(\sigma|s) \) voters’ belief about the probability of state \( \sigma \in \{\sigma_a, \sigma_b, \sigma_{ab}\} \)
occurring, given the outcome $s \in \{a, b, \emptyset\}$. The updated beliefs about the quality of the incumbent are

\[
\tilde{\gamma}(a) = \frac{\gamma \pi_a(e^*_a)}{\gamma \pi_a(e^*_a) + (1 - \gamma) \pi_a(e^*_B)} > \gamma
\]

\[
\tilde{\gamma}(b) = \frac{\gamma \pi_b(e^*_b)}{\gamma \pi_b(e^*_b) + (1 - \gamma) \pi_b(e^*_B)} > \gamma
\]

\[
\tilde{\gamma}(\emptyset) = \rho(|\sigma_a|) \frac{\gamma[1 - \pi_a(e^*_a)]}{\gamma[1 - \pi_a(e^*_a)] + (1 - \gamma)[1 - \pi_a(e^*_B)]} + [1 - \rho(|\sigma_a|)] \frac{\gamma[1 - \pi_b(e^*_b)]}{\gamma[1 - \pi_b(e^*_b)] + (1 - \gamma)[1 - \pi_b(e^*_B)]} < \gamma
\]

where the inequalities follow from the fact that $e^*_i > e^*_i$ for both $i$. \qed

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