Both Sides of the Story: Skill-biased Technological Change, Labour Market Frictions, and Endogenous Two-Sided Heterogeneity

Fabio R. Aricó†
University of St Andrews

SEPTEMBER 2009

ABSTRACT

This paper presents a stylised framework to examine how skill-biased technological change and labour market frictions affect the relationship between economic expansion and unskilled unemployment. The first part of the analysis focuses on the investment decisions in skill-acquisition and technology adoption activities faced by workers and firms in response to the introduction of an innovative technology. The second part examines how endogenous two-sided heterogeneity in the labour market affects the macroeconomic outcomes in terms of unemployment, technological diffusion, and economic expansion. To conclude, the framework is used to discuss the effects of alternative forms of policy intervention on agents' investment decisions and on the macroeconomic outcomes.

JEL Classification: C78, J24, J64, O33.
Keywords: skill-biased technological change, market frictions, two-sided heterogeneity.

* The author is grateful to Robin Naylor, Charles Nolan, Margaret Stevens, Jonathan Temple, David Ulph, Myrna Wooders, and Lei Zhang for their useful comments. Financial support from the University of Warwick and the ESRC (PTA-030-2002-00458) is gratefully acknowledged.
† School of Economics and Finance, Castlecliff, The Scores, St Andrews, Fife KY16 9AL, Scotland, UK. Tel: +44 (0) 1334 461953. E-mail: Fabio.Arico@st-andrews.ac.uk.

CASTLECLIFFE, SCHOOL OF ECONOMICS & FINANCE, UNIVERSITY OF ST ANDREWS, KY16 9AL
TEL: +44 (0)1334 462445 FAX: +44 (0)1334 462444 EMAIL: cdma@st-and.ac.uk

www.st-and.ac.uk/cdma
1 Introduction

In this paper we consider an economy experiencing a wave of skill-biased technological change in presence of labour market frictions. The first part of our analysis considers the investment decisions in skill-acquisition and technology adoption activities faced by workers and firms in response to the introduction of an innovative technology. The second part examines how endogenous two-sided heterogeneity in the labour market affects the macroeconomic outcomes in terms of unemployment, technological diffusion, and economic expansion. In our model the policy-maker aims to promote the diffusion of an innovative skill-biased technology avoiding an increase in unskilled unemployment. We show that, when skill-acquisition and technology adoption activities are both endogenous, this result can be achieved by introducing either subsidies to education or to innovation. Nevertheless, the choice of the most appropriate form of intervention depends on the fundamental parameters of the economy.

Our framework aims to provide an analysis of the interaction between skill-biased technological change and labour market frictions in the determination of unskilled unemployment and economic expansion. According to the Solow-Phelps paradigm, technological change and unemployment have been traditionally considered as two separate economic phenomena. Nevertheless, recent contributions have challenged this view.\(^1\) Pissarides (1990) suggests that, when the labour market is affected by search and matching frictions, firms planning to invest in new technology hire workers today to save on hiring costs in the future –the capitalisation effect– generating negative correlation between technological change and unemployment. Aghion and Howitt (1994) reply that, when technological change is skill-biased, the introduction of new technologies causes a process of reallocation of workers from old plants to technologies of more recent vintage. This creative destruction effect may induce positive correlation between technological change and unemployment. Acemoglu (1997) remarks that the Neo-Schumpeterian approach to skill-biased technological change has enriched our understanding about the relationship between economic growth and unemployment, but it neglects the role played by human capital and skill-formation. In his model, Acemoglu (1997) shows that when innovating firms promote training activities for workers, they generate further incentives for other firms to innovate by increasing the number of skilled workers in the labour force. Thus, subsidies to innovation incentivise both technology adoption and skill-acquisition activities at the same time. This process creates multiplier effects leading to an equilibrium with complete diffusion of the innovative technology and low unemployment.

We embrace Acemoglu’s critique, and we share the view that further attention should be devoted to the role of human capital formation in the process of diffusion of new technologies. However, we argue that Acemoglu’s approach is still exclusively focused on firms’ side of the labour market because, in his model, technology adoption

\(^1\)See Aricò (2003) for a survey of recent contributions on the relationship between economic growth and unemployment.
and training decisions are both led by firms. In contrast, we develop a framework where both sides of the labour market face a specific investment decision: firms decide whether to invest in the innovative technology, and workers decide whether to acquire the skills to operate such technology. We show that, in presence of skill and technology complementarities, and labour market frictions, the analysis of the strategic interaction between workers and firms provides new insights to the understanding of the relationship between economic expansion and unemployment. In our framework, forward-looking workers form expectations on the number of firms adopting the new technology and on the number of other workers investing in education. On the opposite side of the labour market, firms develop a similar strategic behaviour. Workers know that, if more firms invest in the new technology, they will have a higher probability to be matched with a vacancy demanding a high level of skills, and have further incentive to invest in education. At the same time, we assume that the education technology displays congestion externalities, and that when more workers access education, the cost of upgrading skills becomes higher. In a similar fashion, firms’ incentives to innovate depend positively on the expected number of skilled workers in the labour market, as well as negatively on the number of other innovators due to the presence of bottle-neck effects in the process of technology adoption. Thus, in our framework, strategic complementarities do not arise within the firms’ side of the labour market but across the two sides of the labour market. Additionally, congestion externalities in the process skill-acquisition and technology adoption generate strategic substitutabilities within each side of the labour market.

There is an increasing consensus on the fact that the most recent waves of technological change have been of a skill-biased nature. To match this stylised fact, recent research has focused on models displaying heterogeneity in the distribution of skills across workers and heterogeneity in the distribution of technologies across firms. Nevertheless, many of these contributions endogenise either the distribution of skills or the distribution of technologies, taking the heterogeneity affecting the opposite side of the labour market as exogenously given. For instance, Acemoglu (1999) and Violante (2002) focus on endogenous technology adoption in presence of exogenous skill heterogeneity. Albrecht and Vroman (2002) and Caselli (1999), instead, develop models of endogenous skill-formation with exogenous technological change. Very few contributions –such as Chari and Hopenhayn (1991), Young (1993), Redding (1996), Gautier (2002), Lloyd-Ellis and Roberts (2002), and Navarro (2009)– are able to characterise a fully endogenous bilateral interaction between the processes of skill-acquisition and technology adoption. However, these contributions are not part of the

---

2 As a matter of fact, the skills provided by firms through training are often of a specific nature, and they are unlikely to be completely transferable across different jobs. To this extent, we regard skills gained through education as a form of human capital that is easier to be transferred over the process of job turnover.

3 Recent empirical evidence in favour of this hypothesis has been provided by Autor et. al. (1998), Berman et. al. (1998), Hollanders and Ter Weel (2002), Kahn and Lim (1998), and Machin and Van Reenen (1998).
same literature. In some of these models the labour market is affected by search and matching frictions. In others, the labour market is assumed to be competitive, failing to offer an explanation for the persistence of unskilled unemployment. Moreover, the majority of these models focus their analysis on extreme results (e.g. no diffusion vs. complete diffusion of the innovative technology), and miss to provide practical prescriptions for policy-making.

Our aim is to address these issues within a unique coherent framework. Thus, we develop a model able to (i) identify the role of skill-biased technological change and search and matching frictions in the determination of endogenous two-sided heterogeneity in the labour market, (ii) describe the strategic interaction between workers and firms, and (iii) inform the policy-maker about the effects of alternative forms of intervention on the rate of unemployment, and on the degree of technological diffusion and economic expansion. We find that subsidies to either education or innovation both foster economic expansion, but generate ambiguous effects on unemployment. For this reason, the social planner must evaluate with attention the opportunity cost of these two policies, as different economies might benefit from different forms of intervention.

The reminder of this paper is organised in five sections. Section 2 outlines the set-up of our framework. Section 3 describes the matching algorithm operating in the labour market, and the process of production and distribution of output between wages and profits. Section 4 focuses on the investment decisions in skill-acquisition and technology adoption faced by workers and firms. Section 5 characterises the macroeconomic outcomes in terms of unemployment, technological diffusion, and economic expansion. Section 6 provides a discussion of our findings and concludes.

2 The model

We consider an economy composed of a continuum of workers $i \in I$ and a continuum of firms $j \in J$ of the same size: $I \equiv J \equiv [0, 1]$. In the beginning, all firms produce an identical final good using the same technology. All workers are endowed with the same skills, until a new and more productive technology is exogenously made available in the economy. The introduction and the diffusion of the new technology is skill-biased, as the new technology can be operated only by workers who are endowed with a sufficient amount of skills.

Firms decide whether to innovate and adopt the new ‘high’ technology by paying a fixed investment cost, or to retain the low-technology profile. The investment decision of each firm is denoted by $t_j \in \{t^L, t^H\}$, where $t^L$ and $t^H$ represent the two available strategies: retaining the low-technology or investing in the high-technology.

---

4We assume that the number of workers is identical to the number of firms. This constitutes a benchmark case where, potentially, each worker could be matched with a firm, and each firm could be matched with a worker.
On the opposite side of the labour market, workers decide whether to invest in a costly skill-acquisition process, and gain the knowledge to operate the high-technology, or to remain unskilled. The available strategies for each worker are represented by $h_i \in \{h_{us}, h^s\}$, where $h_{us}$ reflects the decision to remain unskilled and $h^s$ represents the decision of investing in skill-acquisition. At the end of the investment stage of the model, a fraction of the labour force, $\phi^*$, emerges as having invested in skill-acquisition and a proportion, $\mu^*$, of firms has having acquired the high-technology. Thus, the equilibrium distributions of skills and technologies $(\phi^*, \mu^*)$, are endogenously determined in this economy. The proportion of workers investing in skill-acquisition is defined as:

$$\phi^* = \int_{i \in I} [h^s (i) = h^s] \, di$$  \hspace{1cm} (1)$$

and the proportion of firms investing in technology adoption is defined as:

$$\mu^* = \int_{j \in J} [t^* (j) = t^H] \, dj$$  \hspace{1cm} (2)$$

where $h^s (i)$ and $t^* (j)$ denote the equilibrium investment decision of worker $i$ and firm $j$, respectively.

Each worker faces a specific cost of investment in skill-acquisition denoted by $c_i$. At the same time, each firm faces a specific cost of investment in technology adoption, denoted by $d_j$. Investment costs are uniformly distributed across agents, such that: $d_j \sim U[0, D]$ and $c_i \sim U[0, C]$. This means that every worker (every firm) can acquire the same level of skills (the same innovative technology), but the investment costs differ between workers (and between firms).

We assume that the education technology –not explicitly modelled in this framework– generates congestion costs for workers investing in skill-acquisition activities. The higher the number of workers investing in skill-acquisition, the higher the congestion cost incurred by each worker. In particular, we assume that the congestion cost of skill-acquisition for each worker is a quadratic function of the proportion of skilled workers, such that: $\gamma \phi^2$, with $\gamma > 0$. At the same time, on the opposite side of the labour market, also firms experience congestion costs in the process of technology adoption as the price of the high-technology equipment increases with the demand for innovation. Even in this case, we assume that the congestion cost of technology adoption for each firm is a quadratic function of the number of innovating firms: $\rho \mu^2$, with $\rho > 0$.

Investment decisions having been taken, workers and firms access the labour market. The labour market is uncoordinated and a process of matching allows the two sides of the labour market to meet, randomly allocating workers to firms. We assume that each firm opens a single vacancy and employs a worker if s/he is able to operate its technological equipment. In particular, we assume that skilled workers can operate either the basic technology or the innovative technology, whereas unskilled workers cannot operate the innovative technology. When an innovating (high-technology)
firm is matched with a skilled worker, the match is productive because skilled workers can operate the innovative technology. The match is also efficient, because both the worker and the firm can remunerate their investment by sharing a higher level of output. If the innovating firm is matched with an unskilled worker the match is not productive, because the worker is not able to operate the high-technology. In this case, the match breaks immediately and the worker and the firm involved in this match remain inactive. A firm retaining the old technological profile (low-technology) can employ either a skilled or an unskilled worker. When an unskilled worker is assigned to a low-technology firm, the match is productive and efficient. When a skilled worker is assigned to a low-technology firm, the match is productive but not efficient, because the additional skills acquired by the worker cannot be employed to increase the productivity of the low-technology. We refer to this case as a mismatch, since the worker has invested resources in skill-acquisition, but receives the same wage of an unskilled worker.\footnote{This assumption could be relaxed, assuming that a low-technology firm matched with a skilled worker displays higher productivity than a low-technology firm matched with an unskilled worker. This extension is considered by Gautier (2002). Our fundamental results would not be significantly affected by this extension.} Productively matched (and mismatched) worker-firm pairs bargain over the proportion of final output to be distributed between wages and profits and engage in production.

The model articulates in two sequential stages. In the first stage, agents take their investment decisions and contribute to the formation of an endogenous distribution of skills in the labour force and an endogenous distribution of technologies across firms. In the second stage, agents enter the labour market, and the matching mechanism allocates workers to firms affecting the degree of technological diffusion, economic expansion, and unemployment. All the agents have common knowledge about the parameters affecting skill-acquisition and technology adoption activities, about the matching process, and about the distribution rule.

We outline the main features of our model by moving backwards. In the following section, we provide a more accurate description of the algorithms that regulate matching, production, and distribution in this economy: the second stage of the model. Subsequently, we present the analysis of the process of skill-acquisition and technology adoption taking place at the first stage.

## 3 Matching, production and distribution

When workers and firms access the labour market, their investment decisions, $\{h^*_i\}_{i \in I}$ and $\{t^*_j\}_{j \in J}$, have already been taken. Hence, the distribution of skills and technologies, $(\phi^*, \mu^*)$, is also determined. The labour market is not coordinated. Workers ignore whether they are applying for a high-technology or for a low-technology job, as well as firms cannot discriminate between applications received from skilled or un-
skilled workers. This information is revealed only after a match. Since the matching process is anonymous, wages and profits can only be determined after a match is formed.

3.1 Random matching

Each agent entering the labour market has to engage in search activities to find a compatible production partner. In this simple model, we assume that each worker can apply to one firm only, and that each firm can consider the first application received only. We represent this process through a random matching algorithm that allocates a worker to each firm.\(^6\) Denote with \(x_{i,j} = (h_i, t_j)\) a generic worker-firm pair. Each pair \(x_{i,j}\) is characterised by a skill-technology profile that depends on the investment decisions previously taken by the worker, \(h_i\), and the firm, \(t_j\), composing the pair. In a random matching configuration each worker-firm pair assumes a specific skill-technology profile according to a stochastic rule. This rule associates a worker-firm pair to a skill-technology profile with given probabilities. These probabilities depend endogenously on the proportion of skilled and unskilled workers, and on the proportion of high-technology and low-technology firms, determined at the first stage of the model.

**Definition 3.1: Endogenous random matching configuration**

Consider the set of all the possible worker-firm pairs: \(x_{i,j} \in (I \times J)\) and their associated skill-technology profiles. An endogenous random matching configuration \(M \{x_{i,j}, (\phi^*, \mu^*) \}, p(x_{i,j})\}_{(i,j) \in (I \times J)}\) is an allocation of one-to-one worker-firm pairs induced by the distribution of skills and technologies \((\phi^*, \mu^*)\) and a probability distribution over the skill-technology profile of each worker-firm pair, such that:  
\[
\begin{align*}
p(h_{us}, t_L) &= (1 - \mu^*) (1 - \phi^*), \quad p(h_{us}, t_H) = (1 - \mu^*) \phi^*, \quad p(h_s, t_L) = \mu^* \phi^*, \quad p(h_s, t_H) = \mu^* (1 - \phi^*),
\end{align*}
\]
for every \(x_{i,j} \in (I \times J)\).

3.2 Production

For each worker-firm pair \(x_{i,j}\) the outcome of the production process is determined through a fixed coefficients production function \(y_{i,j} = f(x_{i,j})\) that combines the skill-technology profile of worker \(i\) and firm \(j\) to generate the final output. We assume that a firm retaining the low-technology profile produces one unit of output, independently

---

\(^6\)Suppose that the names of all workers, skilled and unskilled, are placed in an urn and all firms’ identification tags are placed in another urn. The random matching mechanism that we are describing simultaneously draws workers’ names and firms’ tags from the two urns and allocates a worker’s name to a firm’s tag.
from the skill-profile of its worker: $y^L (h^{us}, t^L) = y^L (h^s, t^L) = 1$. A firm investing in the high-technology profile produces $y^H (h^s, t^H) = \psi > 1$ units of output only if matched with a skilled worker. If matched with an unskilled worker, the high-technology firm cannot produce any output: $y^H (h^{us}, t^H) = 0$.

### 3.3 Distribution

In a matching model, prices do not play any allocative or distributive role. Moreover, workers and firms ignore what is the skill or technological profile of their production partner until a match is formed. For this reason, workers and firms engage in the bargaining and distribution problem only after a (successful) match has taken place. As standard in the literature on random matching, we assume that workers and firms bargain over the proportion of final output to be allocated between wages, $w$, and profits, $\pi$, according to a Nash-bargaining scheme:

$$\begin{align*}
\max_{w_i, \pi_j} & \quad (w_i)^\beta (\pi_j)^{1-\beta} \\
\text{s.t.} & \quad y_{i,j} = w_i + \pi_j
\end{align*}$$

We denote with $\beta \in (0, 1)$ the bargaining power of workers and we assume that this parameter is exogenously given and identical for both skilled and unskilled workers. The solution to this bargaining problem is a distribution of the final output produced by each worker-firm pair in proportions $\beta$ and $(1 - \beta)$ to wages and profits, respectively. Thus, the output produced by a high-technology firm, $y^H = \psi$, is distributed as: $w^H = \beta \psi$ and $\pi^H = (1 - \beta) \psi$. The output produced by a low-technology firm, $y^L = 1$, is distributed as: $w^L = \beta$ and $\pi^L = (1 - \beta)$.

### 4 Investment decisions of workers and firms

Agents take their irreversible investment decisions at the first stage of the model, prior to accessing the labour market. Nevertheless, both workers and firms display a forward-looking behaviour as they (i) form expectations about the matching outcome emerging in the labour market, and (ii) have knowledge about the outcome of the bargaining game, also taking place in the second stage of the model. Thus, workers

---

7This assumption could be relaxed, assuming that a low-technology firm matched with a skilled worker displays higher productivity than a low-technology firm matched with an unskilled worker. This feature is considered in Gautier (2002) However, our fundamental results would not be significantly affected by this extension.

8An obvious extension to the model would be to allow $\beta$ to vary between skilled and unskilled workers. We note that there is an argument that the more specifically skilled workers might be expected to exert greater individual bargaining power. On the other hand, this might be offset by greater collective bargaining of the less skilled workers. The non-cooperative model of bargaining, associated with Binmore, Rubinstein and Wolinsky (1986), implies that $\beta$ is to be interpreted as a discount rate: there is no necessary reason why this should vary systematically across sectors.
and firms evaluate costs and benefits of investing in skill-acquisition and technology adoption. With regards to the benefits emerging from investment in either skills or technology, we have assumed that the bargaining power, $\beta$, is identical and constant for both skilled and unskilled workers. If we consider the productivity differential between the high-technology and the low-technology: $y^H - y^L = \psi - 1$ we can observe that the wage-differential and the profit differential paid by high-technology and low-technology firms are both constant: $w^s - w^{us} = \beta (\psi - 1)$, $\pi^H - \pi^L = (1 - \beta) (\psi - 1)$. Nevertheless, each worker $i$ and each firm $j$ display a specific cost of investment in skill-acquisition, $c_i$, and technology diffusion, $d_j$. Moreover, the equilibrium proportions of skilled workers and high-technology firms, $(\phi^s, \mu^s)$, affect each agent’s probability of finding a compatible production partner, and the congestion costs of investment in skill-acquisition and technology adoption. Therefore, four elements characterise the investment decision faced by each agent: wage (or profit) differential, specific investment costs, congestion costs, and matching probabilities.

### 4.1 Skill-acquisition

We consider now the decision problem faced by a worker who has to determine his/her desired amount of skills prior to entering the labour market. In the beginning all workers are naturally endowed with an identical amount of human capital $h^{us}$ and they are all unskilled. Workers can invest part of their income to increase their skill endowment and qualify as skilled.\(^9\)

Workers aim to acquire the skill-profile that maximises their expected net income. The net income of each worker is computed as the difference between the wage earned and the investment costs associated with the worker’s skill-profile. If worker $i$ decides to invest in skill-acquisition, s/he incurs a specific cost of investment, $c_i$, and a congestion cost $\gamma \phi_i^2$, which is increasing in the proportion of other workers investing in skills. In order to evaluate the expected wage, each worker forms beliefs about the distributions of skills and technologies in the economy, $(\phi^e, \mu^e)$. In this way, the worker estimates the probability of being matched with either a high-technology firm, $\mu^e$, or a low-technology firm, $(1 - \mu^e)$. Estimating the value $\phi^e$, the worker is also able to form an expectation about the congestion cost of investing in skill-acquisition.

For a given configuration of beliefs over the distribution of skills and technologies $\{(\phi^e_i, \mu^e_i)\}_{i \in I}$, the payoff associated with the two investment strategies available to each worker is identified by the following equations:

$$
h_i = \begin{cases} 
    h^s: E_i(U^s | (\phi^e_i, \mu^e_i)) = \mu^e_i w^H + (1 - \mu^e_i) w^L - c_i - \gamma (\phi^e_i)^2 \\
    h^{us}: E_i(U^{us} | (\phi^e_i, \mu^e_i)) = (1 - \mu^e_i) w^L
\end{cases}
$$

If the worker chooses to invest in skill-acquisition s/he expects to be matched

\(^9\)We assume that, at the first stage, workers can borrow the resources necessary to invest in skill-acquisition at a rate zero. At the end of the second stage, workers receive their wage and re-pay their investment costs. A similar assumptions holds for firms.
with a high-technology firm with probability $\mu_i^e$ or with a low-technology firm with probability $(1 - \mu_i^e)$, earning respectively $w^H = \beta \psi$ or $w^L = \beta$. In this case, the worker expects to face the investment costs $c_i + \gamma (\phi_i^e)^2$. If worker $i$ chooses to remain unskilled, s/he expects to be matched with a low-technology firm with probability $(1 - \mu_i^e)$ and earn the wage $w^L = \beta$, otherwise the worker will be unemployed.

Each worker maximises his/her expected income, given a configuration of beliefs over the distribution of skills and technologies determined in the economy:

$$\text{Max}_{h_i^* \in (h^e, h^us)} \{ E_i(U_i^s | (\phi_i^e, \mu_i^e)), E_i(U_i^{us} | (\phi_i^e, \mu_i^e)) \}$$

We assume that workers form expectations about the distribution of skills and technologies in the same way, so that: $E_i(U_i^s | (\phi_i^e, \mu_i^e)) = E(U_i^s | (\phi_i^e, \mu_i^e))$ and $E_i(U_i^{us} | (\phi_i^e, \mu_i^e)) = E(U_i^{us} | (\phi_i^e, \mu_i^e))$ for all $i \in I$.

The decision rule adopted by each worker can be formalised as:

$$h_i^* = \begin{cases} h^e & \text{if } E(U_i^s | (\phi_i^e, \mu_i^e)) \geq E(U_i^{us} | (\phi_i^e, \mu_i^e)) \\ h^{us} & \text{if } E(U_i^s | (\phi_i^e, \mu_i^e)) < E(U_i^{us} | (\phi_i^e, \mu_i^e)) \end{cases}$$

Therefore, substituting the equilibrium value of wages paid by high-technology firms and low-technology firms, a worker will invest in skill-acquisition activities if:

$$c_i \leq \mu_i^e \beta \psi - \gamma (\phi_i^e)^2 \equiv c^e (\phi_i^e, \mu_i^e)$$

Condition (3) implies that each worker compares his/her specific cost of investment in skill-acquisition with a threshold cost $c^e (\phi_i^e, \mu_i^e)$, which depends on his/her beliefs over the distribution of skills and technologies in the economy. If the worker expects more firms to invest in the high-technology s/he also expects that it is easier to be matched with a high-technology vacancy: the estimated threshold cost $c^e$ is lower. Conversely, if the worker expects more workers to invest in skill-acquisition, the estimated cost of congestion is higher and the estimated threshold cost $c^e$ is also higher.

We assumed that the costs of investment in skill-acquisition are uniformly distributed as: $c_i \sim U[0, C]$. From condition (3) we know that all the workers who display an investment cost lower than the threshold value $c^e$ will invest in skill-acquisition. Therefore, the proportion of skilled workers (1) can be expressed as:

$$\phi = \frac{1}{C} \int_0^{c^e} di = W(\phi_i^e, \mu_i^e)$$

Relation (4) is obtained by integrating over the distribution of investment costs of all the workers who display an investment cost lower than –or equal to– the threshold value $c^e (\phi_i^e, \mu_i^e)$. We can interpret relation (4) by observing that workers form an expectation about congestion externalities generated by potential competitors in the labour market. At the same time, workers contribute to generate congestion externalities with their own investment decision. This determines a correspondence between
the *expected* proportion of skilled workers – on the right-hand side – and the *actual* proportion of skilled workers generated in the economy. For any given expected proportion of innovating firms $\mu^e$, the equilibrium proportion of skilled workers is identified by the fix-point value $\phi = \phi^e$, where the expectations of all workers are mutually consistent. Hence, imposing a consistency condition on the beliefs of workers:

$$\phi = W(\phi, \mu^e)$$  \hspace{1cm} (5)

Solving (5), we can express the actual proportion of skilled workers, $\phi^R$, as a function of workers’ expected proportion of innovating firms, $\mu^e$:

$$\phi^R = \phi^R(\mu^e) = \min \left\{ \frac{-C + \sqrt{C^2 + 4\gamma \beta \psi \mu^e}}{2\gamma}, 1 \right\}$$  \hspace{1cm} (6)

Equation (6) summarises the response of workers to the diffusion of the new technology. It is easy to observe that when no firm invests in the new technology, no worker acquires skills, so that: $\phi^R(\mu^e = 0) = 0$. On the other hand, the response function $\phi^R(\mu^e)$ has to be bounded to 1 above. This happens because, under particular parameterisations of the economy, all workers invest in skill-acquisition, even when the expected proportion of innovating firms is lower than one (and the diffusion of the high-technology is not complete).\(^{10}\) The following proposition characterises its properties:

**Proposition 4.1**

The skill-upgrading response function $\phi^R(\mu^e)$ is:

(i) increasing and concave in the expected proportion of high-technology firms, $\mu^e$;

(ii) decreasing in the cost of investment in skill-acquisition, $C$, and of the congestion cost parameter, $\gamma$;

(iii) increasing in the productivity of the high-technology, $\psi$, and in the bargaining power of workers, $\beta$.

Consider Property (i) of Proposition 4.1. For any expected proportion of high-technology firms, each worker estimates the expected income associated with the choice of investing in skill-acquisition and with the choice of remaining unskilled.

\(^{10}\)From a technical point of view, the value $\frac{-C + \sqrt{C^2 + 4\gamma \beta \psi \mu^e}}{2\gamma}$ can be greater than one for particular configurations $\{C, \gamma, \beta, \psi\}$ and $\mu^e < 1$. For these cases, we restrict the skill-upgrading response function, $\phi^R(\mu^e)$, to assume value 1. The proportion of skilled workers will assume value 1, even when the diffusion of the high-technology is not complete.
The higher the expected proportion of high-technology firms, the higher the estimated probability of being matched with a high-technology vacancy. Thus, if more firms are expected to invest in the high-technology, more workers invest in skill-acquisition activities. The skill and technology complementarity introduced in the model generates strategic complementarity between the investment decisions of workers and firms. Nevertheless, each worker is also aware that a higher number of innovating firms induces a higher proportion of skilled workers, increasing the congestion costs in skill-acquisition. Therefore, as a second order effect, congestion externalities become stronger and reduce the incentive to invest in skill-acquisition. The higher the expected proportion of innovating firms, the stronger the second order effect generated by congestion costs in skill-acquisition, the lower the aggregate response of workers investing in skill-acquisition. Therefore, investment decisions in skill-acquisition are strategic complements across the two sides of the labour market, but strategic substitutes within the workers’ side of the labour market. As a consequence, the skill-upgrading response function is increasing in the expected number of high-technology firms, but less than proportionally. The intuition for properties (ii) and (iii) is straightforward: an increase in the specific cost of investment, or in the congestion cost parameter, discourages workers from acquiring skills. On the other hand, an increase in workers’ bargaining power, or in the output to be shared between wages and profits, constitute a further incentive to skill-acquisition.

4.2 Technology adoption

The description of firms’ investment behaviour is analogous to the analysis conducted on workers. To evaluate their expected net profit, firms form beliefs about the distribution of skills and technologies in the economy, \((\phi^e, \mu^e)\). Each firm estimates the probability of being matched with either a skilled worker, \(\phi^e\), or with an unskilled worker, \((1 - \phi^e)\).

For a given configuration of beliefs over the distribution of skills and technologies \((\phi^e, \mu^e)\), each firm faces a choice between investing in technology adoption or retaining the low-technology profile. The net profit associated with either investment decision is identified by the following equations:

\[
\begin{align*}
t_j &= \begin{cases} 
  t^H : & E_j \left( \prod_{j}^{H} \left( \phi^e_j, \mu^e_j \right) \right) = \phi^e_j \pi^H - d_j - \rho (\mu^e_j)^2 \\
  t^L : & E_j \left( \prod_{j}^{L} \left( \phi^e_j, \mu^e_j \right) \right) = \pi^L 
\end{cases}
\]

Firms retaining the low technology profile are aware that they are going to be matched with either a skilled or unskilled worker. In both cases they would be able to produce the output \(y^L = 1\) and gain, as profit, the output share \(\pi^L = (1 - \beta)\). A firm \(j\) that invests in technology adoption expects to be matched with a skilled worker with probability \(\phi^e_j\). In this case the firm would be able to produce the output \(y^H = \psi\) and earn the profit share \(\pi^H = (1 - \beta) \psi\). If a high-technology firm \(j\) is matched with an unskilled worker, with estimated probability \((1 - \phi^e_j)\), the skill-
technology profile of the match is not compatible for production and both the firm and the worker remain inactive. Each firm $j$ adopting the high-technology incurs a total cost of innovation $d_j + \rho \mu^2$.

Firms maximise their expected income by choosing a technological profile associated with the higher payoff. Thus, given a configuration of beliefs about the distribution of skills and technologies that is going to be determined in the economy:

$$\max_{t_j \in \{t^H, t^L\}} \{ E_j \left( \Pi^H_j \mid (\phi_j^e, \mu_j^e) \right), E_j \left( \Pi^L_j \mid (\phi_j^e, \mu_j^e) \right) \}$$

As in the case of workers, we assume that all firms form expectations about the distribution of skills and technologies in the same way, so that: $E_j \left( \Pi^H_j \mid (\phi_j^e, \mu_j^e) \right) = E \left( \Pi^H_j \mid (\phi^e, \mu^e) \right)$ and $E_j \left( \Pi^L_j \mid (\phi_j^e, \mu_j^e) \right) = E \left( \Pi^L_j \mid (\phi^e, \mu^e) \right)$ for all $j \in J$.

The decision rule adopted by each firm is formalised as:

$$t^*_j = \begin{cases} t^H & \text{if } E \left( \Pi^H_j \mid (\phi^e, \mu^e) \right) \geq E \left( \Pi^L_j \mid (\phi^e, \mu^e) \right) \\ t^L & \text{if } E \left( \Pi^H_j \mid (\phi^e, \mu^e) \right) < E \left( \Pi^L_j \mid (\phi^e, \mu^e) \right) \end{cases}$$

Therefore, substituting the equilibrium value of profits gained by high-technology firms and low-technology firms, a firm will innovate if:

$$d_j < \phi^e (1 - \beta) (\psi - 1) - \rho (\mu^e)^2 \equiv d^* (\phi^e, \mu^e) \quad (7)$$

Condition (7) states that each firm compares its specific cost of investment in technology adoption with a threshold cost, $d^* (\phi^e, \mu^e)$. The threshold cost depends on the firm’s beliefs over the distribution of skills and technologies in the economy. If more skilled workers are expected to enter the labour market, each firm expects that it is easier to be matched with a skilled worker. Therefore, the estimated threshold cost $d^*$ is lower. Nevertheless, if a firm expects many other firms to adopt the high-technology, the firm also expects facing higher congestion costs. Therefore, the estimated threshold cost $d^*$ is higher.

The distribution of investment costs in technology adoption is uniform over the interval $[0, D]$. From condition (7), all the firms displaying an investment cost lower than the threshold value $d^*$ will invest in technology adoption. Thus, the proportion of high-technology firms (2) can be expressed as:

$$\mu = \frac{1}{D} \int_0^{d^*} d_j = F (\phi^e, \mu^e) \quad (8)$$

For any given expected proportion of skilled workers $\phi^e$, the equilibrium proportion of high-technology firms is identified by the fix-point value $\mu = \mu^e$, where the expectations of all firms are mutually consistent. Thus, imposing a consistency condition on the beliefs of firms:

$$\mu = F (\phi^e, \mu) \quad (9)$$
Solving (8), we can express the proportion of innovating firms, $\mu^R$, as a function of firms’ expected proportion of skilled workers, $\phi^e$:

$$\mu^R = \mu^R(\phi^e) = \min \left\{ \frac{-D + \sqrt{D^2 + 4\rho(1-\beta)(\psi-1)\phi^e}}{2\rho}, 1 \right\}$$

Equation (10) relates the degree of diffusion of the high-technology to workers’ skill-acquisition activities. The properties of this relationship are summarised by the following proposition:

**Proposition 4.2**

The innovation response function $\mu^R(\phi^e)$ is:

(i) increasing and concave in firms’ expected proportion of skilled workers $\phi^e$;

(ii) decreasing in the cost of investment in technology adoption, $D$, of the congestion cost parameter, $\rho$, and of the bargaining power of workers, $\beta$;

(iii) increasing in the productivity of the high-technology, $\psi$.

The interpretation of the properties of Proposition 4.2 is analogous to the one provided for Proposition 4.1. Property (i) states that for any expected proportion of skilled workers, each firm estimates the expected income associated with the choice of investing in technology adoption and to the choice of retaining the low-technology profile. The higher the expected proportion of skilled workers, the higher the estimated probability of being productively matched to a skilled worker. Thus, if more workers are expected to acquire skills, more firms invest in technology adoption. However, the higher the expected number of other firms investing in the new technology, the higher the congestion costs of technology adoption faced by each firm. Therefore, the innovation response function is increasing in the expected proportion of skilled workers because of the effect of strategic complementarities across the two sides of the labour market. Moreover, the innovation response function is concave because of the effect of strategic substitutability within the firms’ side of the labour market. Property (ii) of Proposition 4.2 states that increases in the cost of investment and in the congestion parameter $\rho$, as well as increase in workers’ bargaining power, discourage firms from investing in technology adoption. On the other hand, an increase in the productivity of the high-technology increases the profit-share for firms and generates a further incentive to innovate.
5 Equilibrium distributions of skills and technologies

In the previous section we have derived the skill-upgrading response function, $\phi^R(\mu^e)$, and the innovation response function, $\mu^R(\phi^e)$, under the assumption that agents belonging to the same side of the labour market form consistent expectations on the distribution of skills and technologies. In equilibrium, not only are agents’ expectations consistent within the same side of the labour market, but they are also consistent across the two sides of the labour market. Thus, the equilibrium configuration of investment decisions in skills and technology is summarised by the following definition:

**Definition 5.1: Equilibrium**

Given an economy $\Omega = \{\beta, \psi, \rho, \gamma, C, D\}$, an equilibrium for the economy is a configuration:

$$\left(\{h^*_i\}_{i \in I}, \{t^*_j\}_{j \in J}, (\phi^*, \mu^*), M \{x_{i,j}, (\phi^*, \mu^*), p(x_{i,j})\}_{(i,j) \in (I \times J)}\right),$$

such that:

(i) $h^*_i = \arg \max_{h^*_i \in \{h^*, h^{\mu*}\}} E[U_i(\phi, \mu)]$ for all $i \in I$

(ii) $t^*_j = \arg \max_{t^*_j \in \{t^*, t^{\mu}\}} E[\Pi_j(\phi, \mu)]$ for all $j \in J$

(iii) $\phi^R(\mu^*) = \phi^*$ and $\mu^R(\phi^*) = \mu^*$.

In equilibrium, agents take investment decisions in skill-acquisition and technology adoption that maximise their expected payoffs. Moreover, workers’ expectations about the equilibrium proportion of high-technology firms, as well as firms’ expectations about the equilibrium proportion of skilled workers, are mutually consistent. Agents’ expectations are mutually consistent when the skill-upgrading response function crosses the innovation response function. At the crossing point $(\phi^*, \mu^*)$ the expectation formed by workers is fulfilled by the actual proportion of innovating firms and the expectation formed by firms is fulfilled by the actual proportion of skilled workers. This situation is represented in Figure 1.

Figure 1 shows that, in the most general case, our model generates two equilibria: an unstable equilibrium, located in $(0, 0)$, and a stable equilibrium, located in point $E = (\phi^*, \mu^*)$. The position of equilibrium $E$ is determined by the slope and the degree of concavity of the two response functions. The slope of each response function measures the strength of the strategic complementarity between workers’ and firms’ investment decisions, whereas the concavity of the response functions reflects the strength of strategic substitutability between investment decisions within either
side of the labour market. The slope and the concavity of each response function are affected by the fundamental parameters of the model: the distribution of investment costs, the congestion cost parameters, the productivity of the high-technology and the bargaining power of workers. Thus, according to parameter values, the equilibrium $E$ can be located anywhere in the $(\phi, \mu)$ space, including cases where the proportion of skilled workers is significantly higher or significantly lower than the proportion of high-technology firms. This represents, in our opinion, an interesting feature of the equilibrium derived by our model. In fact, in related literature, the few contributions that endogenise simultaneously the distribution of skills and the distribution of technologies are developed as a pure-coordination game, where agents belonging to the same side of the market take the same investment decisions. Thus, the typical coordination problem described by this kind of models generates two equilibria in which if all workers invest in skills all firms invest in technology and if no worker invests in skills no firm invests in technology. This type of configuration, implies that equilibrium unemployment is entirely frictional, because agents entering in the

Figure 1: Equilibrium distributions of skills and technologies.
labour market display always compatible skill-technology profile.\textsuperscript{11} In our model, instead, there can be substantial imbalances between the equilibrium distributions of skills and technologies. Therefore unemployment cannot be classified as an exclusively frictional phenomenon. Since \textit{ex-ante} agents’ investment decisions affect the determination of the skill-technology profile of each worker-firm matched pair, the outcome of the matching mechanism operating in the labour market displays a structural component in our model.

The comparison of our results with those derived in the related literature uncovers another point of strength of our approach. The majority of models presented in related literature are based on pure-coordination games where agents belonging to the same side of the labour market take identical investment decisions and all agents belonging to the opposite side of the labour market coordinate on the corresponding appropriate investment response. Thus, in equilibrium, the economy is always composed of a single sector: a low-technology sector, if nobody invests, or a high-technology sector, where everybody invests. The two technologies never operate simultaneously in the economy.\textsuperscript{12} In our model, instead, agents’ investment decisions in skill-acquisition and technology adoption can differ in equilibrium. Therefore, in the most general case, the economy is composed of two sectors, one producing with the low-technology and one producing with the high-technology.

6 Labour market performance, unemployment, and economic expansion

In the previous two sections we have described how workers and firms take their investment decisions in skill-acquisition and technology adoption prior to accessing the labour market. This analysis has characterised the first stage of the model, providing microeconomic foundations to the behaviour of agents. In this section we focus on the second stage of the model, and we examine the macroeconomic implications of our analysis.

6.1 The analysis of unemployment

When workers and firms have made their investment decision in skill-acquisition and technology adoption, the equilibrium proportions of skilled workers and high-technology firms, \((\phi^*, \mu^*)\), emerge endogenously in our economy. At this stage, workers and firms enter the labour market. Skilled workers can be productively matched

\textsuperscript{11}For instance, in the no-investment equilibrium, all workers are unskilled and all firms retain the low-technology and in the investment equilibrium all workers are skilled and all firms have adopted the high-technology.

\textsuperscript{12}In Acemoglu (1997), heterogeneity in the distribution of skills and technology characterises the transition dynamics, but not the equilibrium outcome. In Redding (1996) not even the dynamics of the model are affected by heterogeneity.
with either high-technology or low-technology firms. Conversely, unskilled workers cannot operate the innovative high-technology. Thus, when an unskilled worker is matched with a high-technology firm, the match breaks immediately. According to this algorithm, the rate of (unskilled) unemployment in this economy corresponds to the proportion of unskilled workers matched with high-technology firms:

$$u^* = (1 - \phi^*) \mu^*$$

The rate of unemployment is zero if no firm invests in the new technology, or if all workers are skilled. The same rate of unemployment, $$\bar{u}$$, can be generated by different combinations of equilibrium proportions of skilled workers and high-technology firms $$(\phi^*, \mu^*)$$, belonging to the same iso-unemployment relationship:

**Definition 6.1**

The iso-unemployment relationship $$IU(\bar{u})$$ is the locus defined by all the configurations of equilibrium proportions of skilled workers and high-technology firms $$(\phi^*, \mu^*)$$ that generate the same level of unemployment, $$\bar{u}$$.

Therefore: $$IU(\bar{u}) = \{(\phi^*, \mu^*) : (1 - \phi^*) \mu^* = \bar{u}\}$$.

The following proposition summarises the fundamental properties of the iso-unemployment relationships:

**Proposition 6.1**

For any given configuration of equilibrium proportions of skilled workers and high-technology firms, $$(\phi^*, \mu^*)$$, and associated unemployment rate, $$\bar{u}$$, the iso-unemployment relationships $$IU(\bar{u}) = \{(\phi^*, \mu^*) : (1 - \phi^*) \mu^* = \bar{u}\}$$ are:

(i) positively sloped in the $$(\phi, \mu)$$ space: $$\left. \frac{d\mu^*}{d\phi^*} \right|_{(\phi^*, \mu^*) \in IU(\bar{u})} > 0$$;

(ii) convex in the $$(\phi, \mu)$$ space: $$\left. \frac{d^2\mu^*}{d\phi^2} \right|_{(\phi^*, \mu^*) \in IU(\bar{u})} > 0$$.

Turning to the interpretation of the results of Proposition 6.1,

When the economy experiences a wave of technological change, the proportion of high-technology firms increases. This promotes Neo-Schumpeterian creative destruction, increasing the number of unproductive matches in the labour market and generating unskilled unemployment. Thus, in order to neutralise the negative effects of creative destruction, the proportion of skilled workers must also increase.
Property (i) of Proposition 6.1 captures this fact and states that, for any given distribution \((\phi^*, \mu^*)\), the slope of the iso-unemployment relationship represents the rate of substitution between the proportion of skilled workers and the proportion of high-technology firms that holds unemployment constant. Moreover, the matching process in the labour market occurs at random. Thus, heterogeneities in the distributions of skills and technologies cause frictions in the process of allocation of workers to firms. Suppose that there are not many skilled workers looking for a job. If the proportion of innovating firms increases, an equivalent increase in the proportion of skilled workers would not be enough to maintain the overall rate of unemployment constant. A number of additional skilled workers would be mismatched with low-technology firms, and a proportion of unskilled workers would be unsuccessfully matched with high-technology firms, generating more unemployment. Therefore, the proportion of skilled workers has to increase more than the proportion of high-technology firms in order to maintain a constant level of unemployment. In this case, a larger increment in the proportion of skilled workers balances out two phenomena: (i) an increase in the proportion of high-technology vacancies, (ii) plus an increase in the number of mismatches generated by the anonymous random matching process. Property (ii) of Proposition 6.1 states that, in our model, the same increment in the proportion of innovating firms generates more unemployment in an economy where the proportion of skilled workers is small, and less unemployment if the proportion of skilled workers is large.

Consider now the configuration of parameters that characterises the economy: \(\Omega = \{\beta, \psi, \rho, C, \gamma, D\}\). Suppose one were to bring a marginal change to one of these parameters and analyse the impact on the unemployment outcome \(u^*\). Evidently, a change in any of the fundamental parameters of the economy alters the equilibrium distributions of skills and technologies, triggering a change in the equilibrium allocation of workers to firms and in the unemployment outcome. Using definition (11), for a small variation of any \(k \in \Omega\), the impact on unemployment can be decomposed in the following way:

\[
\frac{d u^*}{d k} = \frac{d \mu^*}{d k} (1 - \phi^*) - \frac{d \phi^*}{d k} \mu^*
\]  

(12)

The first term of equation (12) measures the creative destruction effect as the change in the proportion of high-technology firms. Holding constant the proportion of unskilled workers, a positive creative destruction effect generates more unemployment because each unskilled worker has higher probability to be unproductively matched with a high-technology firm. The second term of equation (12) defines the skill-upgrading effect, which quantifies the change in the proportion of skilled workers. Holding constant the proportion of high-technology firms, a positive skill-upgrading effect contributes to reduce the rate of unemployment because the number of productive matches increases. The total effect on the unemployment rate depends: (i) on the combination of the creative destruction effect and the skill-upgrading effect, and
(ii) on the initial equilibrium distributions of skills and technologies \((\phi^*, \mu^*)\).

Re-arranging the terms of equation (12), Proposition 6.2 characterises the sign of the change in the unemployment outcome.

**Proposition 6.2**

Consider an economy \(\Omega\), and its associated equilibrium distribution of skills and technologies, \((\phi^*, \mu^*)\). For a marginal change in any of the parameters, \(k \in \Omega\), the sign of the change in rate of unemployment, \(du^*\), can be determined comparing the relative strength of the creative destruction effect and of the skill-upgrading effect with the slope of the the iso-unemployment relationship \(IU(\pi)\) evaluated at the initial equilibrium, \((\phi^*, \mu^*)\). Therefore:

(i) if \(\frac{d\phi^*}{dk} > 0\): \(\frac{du^*}{d\phi^*/dk} > \frac{\mu^*}{(1-\phi^*)} \Rightarrow \frac{du^*}{dk} > 0\);

(ii) if \(\frac{d\phi^*}{dk} < 0\): \(\frac{du^*}{d\phi^*/dk} > \frac{\mu^*}{(1-\phi^*)} \Rightarrow \frac{du^*}{dk} < 0\).

Properties (i) and (ii) of Proposition 6.2 characterise the impact of a change in any parameter of the economy, \(k\), on the unemployment rate. To provide further intuitions about this characterisation, we discuss the example displayed in Figure 2.

Following a change in any parameter, \(k\), the skill-upgrading response function \(\phi_1^R(\mu^*)\) shifts to position \(\phi_2^R(\mu^*)\). At the same time, the innovation response function shifts from \(\mu_1^R(\phi^*)\) to \(\mu_2^R(\phi^*)\). The configuration of equilibrium proportions of skilled workers and high technology firms changes from \(E_1\) to \(E_2\), and the variation of these equilibrium proportions is measured by \(d\phi^*\) and \(d\mu^*\). Thus, the ratio \(d\mu^*/d\phi^*\) provides information about the number of new high-technology vacancies created per additional skilled worker, and corresponds to the left-hand side of the inequalities presented in properties (i) and (ii). The right-hand side of these inequalities, instead, represents the slope of the iso-unemployment relationship \(IU(\pi)\) evaluated at the initial equilibrium \(E_1\). As discussed above, the slope of the iso-unemployment relationship determines the number of high-technology firms necessary to compensate the entry of an additional skilled worker in the labour market to preserve the rate of unemployment constant. In the case represented in Figure 2, both the skill-upgrading effect and the creative destruction effect display a positive sign and, according to equation (12), they compete against each other in affecting unemployment. Nevertheless, employing Property (i) of Proposition 6.2, we can observe that the response of workers and firms to a change in parameter \(k\) generates an additional number of high-technology vacancies per skilled worker that is lower than the slope of the iso-unemployment relationship \(IU(\pi)\) at the initial equilibrium. In other words, the creation of additional high-technology vacancies increases the risk of unemployment for each unskilled worker, but the number of these workers is substantially reduced. Thus, in the new equilibrium \(E_2\), the labour market generates a number of unsuccessful matches that is lower than the one generated by the equilibrium \(E_1\). In Figure
we can observe that the new equilibrium configuration $E_2$ is located below the iso-unemployment $IU(\bar{u})$, such that $u_2^*(\phi_2^*, \mu_2^*) < \bar{u}$. The rate of unemployment is lower.

The left-hand side of properties (i) and (ii) of Proposition 6.2 reflects Stage 1 of the model – the ex-ante stage – when investment decisions are taking place. The right-hand side corresponds to Stage 2 of the model – the ex-post stage – when workers and firms have entered the labour market, and the matching mechanism allocates workers to firms. Thus, given an initial equilibrium allocation $E_1$, we can observe that the criterion presented in Proposition 6.2 is based on the comparison between how agents react to a change in the parametric configuration, and how the labour market performance responds to this change. Using this result, the following propositions summarise the effect of a change in the fundamental parameters of the economy on the unemployment rate. We are particularly interested in investigating the effects of policies aimed at reducing the costs of investment in skill-upgrading and innovation, which are described in Proposition 6.3 and Proposition 6.4, respectively. Proposition 6.5 describes the effect of the alteration of workers’ bargaining power.
Proposition 6.3

Consider an economy $\Omega$, its equilibrium distribution of skills and technologies, $(\phi^*, \mu^*)$, and its associated unemployment rate, $u^*$. If the equilibrium configuration $(\phi^*, \mu^*)$ displays a sufficiently high (low) proportion of skilled workers, $\phi^*$, and high-technology firms, $\mu^*$, a reduction in the investment cost parameter, $C$, or in the congestion cost parameter, $\gamma$, generates a reduction (increase) in the unemployment rate. In particular:

(i) $\frac{\partial R(\phi)}{\partial \phi} \bigg|_{(\phi^*, \mu^*)} < 1$ and $\mu^* > 1 - \phi^* \implies -\frac{du^*}{dC} < 0$;

(ii) $\frac{\partial R(\phi)}{\partial \phi} \bigg|_{(\phi^*, \mu^*)} > 1$ and $\mu^* < 1 - \phi^* \implies -\frac{du^*}{dC} > 0$;

(iii) In all the other cases, a reduction in the costs of investment in technology adoption displays an ambiguous effect on the unemployment rate.

Proposition 6.3 considers the effect of interventions aimed at reducing the costs of investment in skill-acquisition. We focus the discussion of this proposition on Property (i), as the intuition for Property (ii) can be derived following an analogous reasoning. The effects of a reduction of the investment costs parameters, $C$ or $\gamma$, affect only the skill-upgrading response function, leaving unchanged the innovation response function. The skill-upgrading response function shifts right (e.g. from $\phi_1^R(\mu^c)$ to $\phi_2^R(\mu^c)$ in Figure 2) because, for any given expectation of being productively matched with a high-technology firm $\mu^c$, lower costs of investment incentivise more workers investing in skill-acquisition. The innovation response function does not shift, as the costs of investment in skill-acquisition do not directly affect the investment decisions of firms. Nevertheless, firms expect that more skilled workers will enter the labour market, and estimate an increase in the probability of a productive match with a skilled worker. This expectation triggers an increase in the number of firms adopting the high-technology as a second order effect. Both the equilibrium proportions of skilled workers and high-technology firms increase in the new equilibrium. Therefore, both the creative destruction effect and the skill-upgrading effect display a positive sign, and generate competing effects on unemployment. In the most general case, the final effect on unemployment is ambiguous.\(^{13}\) Property (i) of Proposition 6.3 identifies two sufficient conditions to resolve this ambiguity. In fact, (i) when the congestion externalities generated by technology adoption activities are sufficiently strong, and (ii) when the number of innovating firms is higher than the number of unskilled workers at the initial equilibrium $E1$, a reduction of the costs of investment in skill-acquisition determines a reduction of equilibrium unemployment. Note that

\(^{13}\)Taking as reference the diagram represented in Figure 2, this means that the equilibrium configuration $E2$ could be located either above or below the iso-unemployment relationship $IU(\pi)$. 

22
these two conditions are both satisfied when the initial proportions of skilled workers and innovating firms are sufficiently high.

**Proposition 6.4**

Consider an economy $\Omega$, its equilibrium distribution of skills and technologies, $(\phi^*, \mu^*)$, and its associated unemployment rate, $u^*$. If the equilibrium configuration $(\phi^*, \mu^*)$ displays a sufficiently low (high) proportion of skilled workers, $\phi^*$, and high-technology firms, $\mu^*$, a reduction in the investment cost parameter, $D$, or in the congestion cost parameter, $\rho$, generates a reduction (increase) in the unemployment rate under the following conditions:

(i) $\frac{\partial \phi^R(\mu)}{\partial \mu} \bigg|_{(\phi^*, \mu^*)} > 1$ and $\mu^* > 1 - \phi^* \implies \frac{du^*}{dD} < 0$;

(ii) $\frac{\partial \phi^R(\mu)}{\partial \mu} \bigg|_{(\phi^*, \mu^*)} < 1$ and $\mu^* < 1 - \phi^* \implies \frac{du^*}{dD} > 0$;

(iii) In all the other cases, a reduction in the costs of investment in technology adoption determines ambiguous effects on the unemployment rate.

Proposition 6.4 considers the effects of interventions aimed at reducing the costs of investment in technology adoption. In this case, the innovation response function is expected to shift up, whereas the skill-upgrading response function is not affected by this policy. The intuition for the properties of this proposition is based on analogous reasoning to the one conducted for Proposition 6.2. For this reason, we do not discuss these results in detail and we proceed, instead, to analyse the effects of a modification of workers’ bargaining power.

**Proposition 6.5**

Consider an economy $\Omega$, its equilibrium distribution of skills and technologies, $(\phi^*, \mu^*)$, and its associated unemployment rate, $u^*$. A change in workers’ bargaining power, $\beta$, determines an unambiguous effect on the unemployment rate if the creative destruction effect and the skill-upgrading effect display opposite sign. In particular:

(i) $\frac{d\phi^*}{d\beta} > 0$ and $\frac{du^*}{d\beta} < 0 \implies \frac{du^*}{d\beta} < 0$.

(ii) $-\frac{d\phi^*}{d\beta} < 0$ and $-\frac{du^*}{d\beta} > 0 \implies -\frac{du^*}{d\beta} > 0$.

(iii) In all the other cases, a change in workers’ bargaining power determines ambiguous effects on unemployment.
Proposition 6.5 provides a characterisation of the effects of policies aimed at altering the distribution rule that allocates the final output of each firm between wages and profits. As in propositions 6.3 and 6.4, this characterisation builds on the criterion introduced by Proposition 6.2. Nevertheless, the analysis of a change in parameter $\beta$ is slightly more complicated as this parameter affects directly both the skill-upgrading response function and the innovation response function. A variation in workers’ bargaining power, $\beta$, can generate (i) either an increase or a decrease in the equilibrium proportion of skilled workers, $\phi^*$ and, at the same time, (ii) either an increase or a decrease in the equilibrium proportion of high-technology firms, $\mu^*$. This twofold ambiguity is originated by the interaction of a direct effect and of an indirect strategic effect. Consider, for instance, an increase in workers’ bargaining power, $\beta$. The increase in the wage paid to skilled workers generates a direct positive effect on workers’ investment decisions in skill-acquisition. This effect alone would cause an increase in the proportion of skilled workers. On the other side of the labour market, though, firms’ investment decisions are affected by a direct negative effect, as their profit-share is reduced. This negative effect alone would reduce the proportion of high-technology firms. Nevertheless, indirect strategic effects have also to be considered. If workers expect a significant drop in the proportion of high-technology firms, they also estimate a much lower probability of being productively matched with a high-technology firm. This expectation could discourage investment in skill-acquisition. Thus, combining the direct effect and the indirect strategic effect, the proportion of skilled workers could either increase or decrease in the new equilibrium. At the same time, on the opposite side of the labour market, the determination of the new equilibrium proportion of high-technology firms is also affected by the combination of the direct effect on expected earnings and the indirect effect on expected matching probabilities. However, properties (i) and (ii) of Proposition 6.5 confirm that, when the changes in the equilibrium proportions of skilled workers and high-technology firms display an opposite sign, policy-intervention on the bargaining parameter, $\beta$, displays unambiguous effects on unemployment.

To conclude, we observe that proposition 6.3, 6.4, and 6.5, provide information to the policy-maker about how the performance of the labour market is affected by changes in agents’ investment decisions through changes in the composition of the labour force, and in the distribution of technologies across firms. The characterisation offered by these three propositions is restricted to a limited number of cases, where the effect of a change in any of the fundamental parameters of the economy on unemployment is predictable ex-ante. The source of ambiguity arising in all the other cases is twofold. First, one has to consider the direct impact of a change in the economy’s parameters on the position, the slope, and the degree of concavity of the response functions. Second, adding the indirect effects caused by strategic complementarities and substitutabilities, one can gather information about the relative changes in the composition of the labour force and in the distribution of technologies across firms. It is important to emphasize that, in this model, the performance of the labour market
depends on the initial proportion of skilled workers and high-technology firms. This feature implies that, in the most general case, the adoption of the same policy (e.g. a reduction of the cost of investment in skill-acquisition) can display different effects on the rate of unemployment of two different economies, depending on their initial distribution of skills and technologies.

6.2 Economic expansion

In this section we focus on the effects of the introduction and the diffusion of the innovative high-technology on aggregate production. We organise our analysis following the same structure adopted for the case of unemployment. First, we focus on the relationship between labour market performance and economic expansion (the second stage of the model). Then, we work our way back to the first stage of the model to describe how a change in the parametric configuration of the economy impacts on the equilibrium distributions of skills and technologies and on economic expansion.

We begin our analysis assuming that an initial equilibrium configuration \((\phi^*, \mu^*)\) has emerged in the labour market. Our reference framework is static and, for this reason, it does not allow for a description of the dynamics of aggregate output over the path of technological diffusion. To construct a measure of economic expansion, we consider a benchmark economy where workers are all unskilled, and firms all operate the low-technology equipment. Thus, we compare economies experiencing technological diffusion with this benchmark case.

After the introduction of the skill-biased high-technology, and the determination of the equilibrium matching allocation, the economy produces the final good through two different technologies. Out of the overall proportion of high-technology firms, \(\mu^*\), only a fraction, \(\mu^*\phi^*\), of this group of firms is productively matched with a skilled worker. Each of these firms produces \(\psi > 1\) units of output. Low-technology firms are productively matched with either a skilled or an unskilled worker and produce one unit of output in either case. The aggregate production of the whole economy is:

\[
Y^* = \mu^* \phi^* \psi + (1 - \mu^*) \tag{13}
\]

The first term of equation (13) represents the contribution to aggregate production of the high-technology sector, while the second term measures the contribution of the low-technology sector. The proportion of high-technology firms, \(\mu^*\), defines the size of the high-technology sector and of the low-technology sector of the economy. The proportion of skilled workers, \(\phi^*\), affects the number of productive matches in the high-technology sector. The aggregate production of the benchmark economy—which is characterised by low-profile matches only—is \(Y_0 = 1\). Thus, compared to the benchmark case, we can define the degree of economic expansion of an economy experiencing technological diffusion as:

\[
\Delta Y^* = Y^* - Y_0 = \mu^* (\phi^* \psi - 1) \tag{14}
\]
The maximum degree of economic expansion that an economy can achieve corresponds to the case of complete diffusion of the high-technology and complete skill-upgrading in the labour force: \( \Delta Y_{(\phi^*=1, \mu^*=1)} = \psi - 1 \), which is the productivity differential between the high-technology and the low-technology. From equation (14), we can immediately derive the following result:

**Proposition 6.6**

For a sufficiently high proportion of skilled workers and/or for a sufficiently high productivity of the high-technology, technological diffusion generates an economic expansion. Otherwise, technological diffusion generates an economic contraction. More precisely: \( \phi^* > \frac{1}{\psi} \iff \Delta Y^* > 0 \).

The intuition for this proposition is straightforward. If the proportion of skilled workers is sufficiently high the number of successful matches between innovating firms and skilled workers increases significantly. At the same time, if the productivity of the innovative technology is sufficiently high, the contribution of the high-technology sector to aggregate production is also high. Under these circumstances, the contribution to aggregate production generated by the high-technology sector is sufficient to outweigh the loss in production caused by unsuccessful matches between unskilled workers and high-technology firms. However, if the conditions of Proposition 6.6 are not satisfied, we can register positive correlation between technological diffusion and economic contraction. This eventuality represents an extreme case, and it is driven by the assumption that the matching mechanism taking place in the labour market is completely anonymous. Nevertheless, it is important to remark that, in an economy characterised by labour market frictions and skill and technology complementarities, technological diffusion does not automatically generate economic expansion. What really matters is the number of high-profile productive matches, not just the number of firms adopting the new technology.

Given an equilibrium distribution of skills and technologies, \((\phi^*, \mu^*)\), the labour market generates an endogenous matching allocation that sustains a particular level of aggregate production, \(Y^*\), and a corresponding degree of economic expansion (or contraction), \(\Delta Y^*\). The same degree of economic expansion could be generated by alternative distributions of skills and technologies belonging to the same iso-expansion relationship, whose properties are summarised in Proposition 6.7.

**Definition 6.2**

The iso-expansion relationship IE \((\Delta Y)\) is the locus defined by all the configurations of equilibrium proportions of skilled workers and high-technology firms, \((\phi^*, \mu^*)\), that generate the same level of economic expansion (or contraction), \(\Delta Y\). Therefore: \(IE (\Delta Y) = \{ (\phi^*, \mu^*) : \mu^* (\phi^* \psi - 1) = \Delta Y \} \).
Proposition 6.7

For any initial configuration of equilibrium proportions of skilled workers and high-technology firms, \((\phi^*, \mu^*)\), and the associated level of economic expansion, \(\Delta Y\), the iso-expansion relationships \(IE(\Delta Y) = \{(\phi^*, \mu^*) : \mu^* (\phi^* \psi - 1) = \Delta Y\}\) are:

(i) increasing in the \((\phi, \mu)\) space, if the economy experiences a contraction:

\[\phi^* < \frac{1}{\psi} \implies \frac{d\mu^*}{d\phi^*}|_{(\phi^*, \mu^*) \in IE(\Delta Y)} > 0;\]

(ii) decreasing in the \((\phi, \mu)\) space, if the economy experiences an expansion:

\[\phi^* > \frac{1}{\psi} \implies \frac{d\mu^*}{d\phi^*}|_{(\phi^*, \mu^*) \in IE(\Delta Y)} < 0;\]

(iii) convex in the \((\phi, \mu)\) space:

\[\frac{d^2\mu^*}{d\phi^2}|_{(\phi^*, \mu^*) \in IE(\Delta Y)} > 0.\]

According to the properties of Proposition 6.7, the iso-expansion relationships can be represented on the \((\phi, \mu)\) space as a set of increasing and convex curves for \(\phi^* < \frac{1}{\psi}\), and as a set of decreasing and convex curves for \(\phi^* > \frac{1}{\psi}\), as displayed in Figure 3.\(^{14}\)

Properties (i) and (ii) of Proposition 6.7 characterise the slopes of the iso-contraction and iso-expansion relationships. The slopes of these two relationships depend on three fundamental factors: (i) the productivity of the high-technology, \(\psi\), (ii) the features of the matching technology, and (iii) the assumption of skill, and technology complementarity. In the case of economic contraction, an increase in the proportion of high-technology firms can only be counter-balanced by an increase in the proportion of skilled workers. Otherwise, the amount of frictions generated by the matching mechanism would cause a further loss in aggregate production. Therefore, the slope of the iso-contraction relationships is positive. In the case of economic expansion, an increase in the proportion of high-technology firms would still generate a higher number of mismatches and unproductive matches in the labour market. Nevertheless, the productivity of the high-technology is sufficiently high to boost an overall increase in aggregate production by employing more resources in the high-technology sector of the economy. In these circumstances, in order to preserve the same level of aggregate production, the proportion of skilled workers should be reduced. Thus the iso-expansion relationships display a negative slope.

\(^{14}\)The iso-expansion curves in Figure 3 are ranked from the most severe degree of economic contraction, \(\Delta^{-}Y^1\), to the highest degree of economic expansion, \(\Delta^{+}Y^6\). In the case of \(\phi^* = \frac{1}{\psi}\) the economy generates the same level of aggregate production of the benchmark case, so: \(Y^* = Y_0\), and \(\Delta Y^* = 0\).
Property (iii) of Proposition 6.7 characterises the shape of the iso-contraction and iso-expansion relationships, which depends on the characteristics of the matching mechanism, as we have already noted discussing the convexity of the iso-unemployment relationships. In analogy with the analysis of unskilled unemployment, we remark that, also in this case, for a given level of productivity $\psi$, the degree of convexity of the iso-contraction and of the iso-expansion relationships provides information about the performance of the matching mechanism for alternative equilibrium distributions of skills and technologies, $(\phi^*, \mu^*)$. This measure can be used to evaluate how the labour market substitutes innovating firms with skilled workers to preserve the same level of economic expansion (or contraction).

We are now ready to analyse how a change in the parameterisation of the economy affects the degree of economic expansion through a change in agents’ investment decisions in skill-acquisition and technology adoption. We focus on a reference economy, $\Omega$, and we consider a marginal change in any of its fundamental parameters, $k \in \Omega$. This change triggers a variation of the initial equilibrium distributions $(\phi^*, \mu^*)$, which, in turn, induces a change in the matching allocation emerging in the labour market,
and a consequent change in the degree of economic expansion, $\Delta Y^*$. The impact of a change in any parameter $k$, on economic expansion can be decomposed as:

$$\frac{d\Delta Y^*}{dk} = \left[ \frac{d\psi}{dk} \mu^* \phi^* + \left( \frac{d\mu^*}{dk} \phi^* \psi + \frac{d\phi^*}{dk} \mu^* \psi \right) \right] - \frac{d\mu^*}{dk} \text{ effect on the low-technology sector} (15)$$

The first term of equation (15), in brackets, represents the contribution to economic expansion generated by a change in the structure of the high-technology sector. The second term of equation (15) measures the contribution to economic expansion generated by a change in the structure of the low-technology sector. This term is negative because, over the process of technological diffusion, firms switch from the low-technology to the high-technology sector. There is no entry of new firms in this model. The contribution to economic expansion generated in the high-technology sector can be further decomposed into three effects: (i) the direct expansion effect, (ii) the creative destruction effect, and (iii) the skill-upgrading effect. In this simple model, the direct expansion effect only operates through changes in the productivity of the high-technology, $\psi$. A positive creative destruction effect increases aggregate production by allowing more high-technology firms to be matched with the existing proportion of skilled workers. Given an existing stock of high-technology plants, a positive skill-upgrading effect also increases aggregate production by allowing more skilled workers in the labour market to reduce the number of unproductive matches.\(^{15}\)

We now impose a restriction on the parametric configuration of the economy in order to rule out the extreme case of technological change associated to economic contraction. Thus, on the basis of the result of Proposition 6.6, we provide the following definition.

**Definition 6.3**

Consider the set of all the economies $\Omega : \Omega = \{\beta, \psi, \rho, C, \gamma, D\}$ and define: $\Omega^+ = \{\Omega : \phi^* > \frac{1}{\psi}\}$. $\Omega^+$ identifies the set of all the economies associated to an endogenous distribution of skills and technologies that generates economic expansion.

We restrict our attention to changes of the investment costs in skill-acquisition and technology adoption, and to changes in the bargaining power of workers:\(^{16}\) $k \in$

---

\(^{15}\)The creative destruction also displays effects in the low-technology sector because an increase in the number of firms in the high-technology sector is mirrored by a reduction in the number of firms operating in the low-technology sector, and *vice versa*.

\(^{16}\)This restriction does not appear to us as a narrow limitation of our analysis. The productivity of the high-technology was assumed exogenous to the model, and cannot be affected by the intervention of any economic agent. We believe it is more interesting to focus on the structure of the investment costs, and on the parameter regulating distribution, as these are the parameters that can be more easily targeted by the policy-maker.
\{\beta, \rho, C, \gamma, D\}. Under this restriction, the direct expansion effect from equation (15) is always null, and equation (15) can be re-written as:

\[
\frac{d\Delta Y^*}{dk} = \frac{d\mu^*}{dk} (\phi^* \psi - 1) + \frac{d\phi^*}{dk} \mu^* \psi
\]  

(16)

Re-arranging equation (16) we can derive the following:

Proposition 6.8

Given an economy \( \Omega \), with \( \Omega \in \Omega^+ \), and its associated equilibrium distribution of skills and technologies, \((\phi^*, \mu^*)\). For a marginal change in any of the parameters, \( k \in \Omega \), the sign of the change in the degree of economic expansion, \( d\Delta Y^* \), can be determined comparing the relative change in the equilibrium proportions of skilled workers and high-technology firms with the slope of the iso-expansion relationship \( IE(\Delta Y^*) \) evaluated at the initial equilibrium, \((\phi^*, \mu^*)\). Therefore:

(i) if \( \frac{d\phi^*}{dk} > 0 \) : \( \frac{d\mu^*}{dk} > -\frac{\mu^* \psi}{(\phi^* \psi - 1)} \implies \frac{d\Delta Y^*}{dk} > 0 \);

(ii) if \( \frac{d\phi^*}{dk} < 0 \) : \( \frac{d\mu^*}{dk} > -\frac{\mu^* \psi}{(\phi^* \psi - 1)} \implies \frac{d\Delta Y^*}{dk} < 0 \).

Using Figure 4, we can provide an intuition for the properties of Proposition 6.8. The response functions \( R^1(\mu^e) \) and \( R^1(\phi^e) \) generate the equilibrium configuration \( E1 \) on the iso-expansion relationship \( IE(\Delta Y_1) \). The intercept of the iso-expansion relationship, located on the right edge of the \((\phi, \mu)\) space, identifies the actual level of economic expansion,\(^{17} \) relative to its maximum potential \( \psi - 1 \). A change in any parameter of the economy \( k \in \Omega \) causes a revision of agents’ optimal investment decisions in skill-acquisition and technology adoption. Thus, following a change in \( k \), the skill-upgrading response function, \( R^1(\mu^e) \), and the innovation response function, \( R^1(\phi^e) \), shift and identify a new equilibrium \( E2 \).\(^{18} \) When \( E2 \) is located above (or below) the iso-expansion relationship \( IE(\Delta Y_1) \), the economy associated to \( E2 \) generates a higher (or lower) degree of economic expansion, compared to the economy identified by \( E1 \).

The left-hand sides of the inequalities presented in Proposition 6.8 represent the ratio between the variation in the equilibrium proportion of high-technology firms per additional skilled worker. The right-hand sides of these inequalities measure the

\(^{17}\)Note that, for any given level of economic expansion \( \Delta Y > 0 \), the iso-expansion relationship can be represented on the \((\phi, \mu)\) space as: \( \mu^* (\phi^*)|_{IE(\Delta Y)} = \frac{\Delta Y}{\phi^* \psi - 1} \). Thus, for \( \phi^* = 1 \), the vertical intercept of the iso-expansion relation assumes value \( \mu^* = \frac{\Delta Y}{\psi - 1} \).

\(^{18}\)In the particular example considered in Figure 4, the skill-upgrading response function is the only one shifting.
rate of substitution between high-technology firms and skilled workers required to generate the same degree of economic expansion associated to equilibrium $E_1$. In Proposition 6.8 we provide a criterion to determine whether a variation in any of the fundamental parameters of the economy will generate a higher or a lower level of economic expansion. Given an exogenous shock to the economy, this criterion relates the impact on agents’ investment decisions – the left-hand side – to the labour market performance, on the right-hand side.

On the basis of the criterion presented in Proposition 6.8, the following two propositions characterise the impact of a change in the costs of investment in skill-acquisition and technological adoption, as well as of a change in workers’ bargaining power, on the degree of economic expansion.

**Proposition 6.9**

Consider an economy $\Omega$, with $\Omega \in \Omega^+$, its equilibrium distribution of skills and technologies, $(\phi^*, \mu^*)$, and its associated level of economic expansion, $\Delta Y^*$. A reduction of either the costs of investment in skill-acquisition, $C$ or $\gamma$, or the
costs of investment in technology adoption, $D$ or $\rho$, determines an increase in the economy’s degree of expansion. Therefore:

(i) $-\frac{d\Delta Y^*}{dC} > 0$, $-\frac{d\Delta Y^*}{d\gamma} > 0$;

(ii) $-\frac{d\Delta Y^*}{dD} > 0$, $-\frac{d\Delta Y^*}{d\rho} > 0$.

The results derived in Proposition 6.9 are intuitively straightforward. A reduction of the costs of investment in skill-acquisition induces a positive skill-upgrading effect, as well as a reduction of the costs of investment in technology adoption enhances a positive creative destruction effect. Either effect contributes to promote the diffusion of the high-technology and to increase aggregate production, fostering economic expansion. Using the diagram presented in Figure 4, Property (i) of Proposition 6.9 is represented by a shift to the right of the skill-upgrading response function, $\phi^R(\mu^*)$, where we can observe that the new equilibrium $E2$ is located above the iso-expansion relationship $IE(\Delta Y_1)$.

Proposition 6.10 considers the effects of policy-intervention on workers’ bargaining power.

**Proposition 6.10**

Consider an economy $\Omega$, with $\Omega \in \Omega^+$, its equilibrium distribution of skills and technologies, ($\phi^*, \mu^*$), and its associated level of economic expansion, $\Delta Y^*$. A change in workers’ bargaining power, $\beta$, determines an unambiguous effect on the economy’s degree of economic expansion if the creative destruction effect and the skill-upgrading effect display the same sign. In particular:

(i) $\frac{d\phi^*}{d\beta} > 0$ and $\frac{d\mu^*}{d\beta} > 0 \implies \frac{d\Delta Y^*}{d\beta} > 0$;

(ii) $\frac{d\phi^*}{d\beta} < 0$ and $\frac{d\mu^*}{d\beta} < 0 \implies \frac{d\Delta Y^*}{d\beta} < 0$.

(iii) in all the other cases, a change in workers’ bargaining power displays an ambiguous effect on the economy’s degree of economic expansion.

As we have observed commenting the results of Proposition 6.5, a change in workers’ bargaining power, $\beta$, generates a direct effect on both workers’ and firms’ investment decisions, as well as an indirect effect on the opposite side of the labour market. Therefore, according to the position and the slope of the skill-upgrading response function and the innovation response function, the skill-upgrading effect and the creative destruction effect can either display a positive sign or a negative sign. Proposition 6.10 states that when these two effects display simultaneously the same sign, a variation of parameter $\beta$ determines unambiguous effects on economic expansion.\(^{19}\)

\(^{19}\)In other words, the creative destruction effect and the skill-upgrading effect will both contribute to foster (or to discourage) economic expansion.
In this section, we present a classification of the macroeconomic outcomes that can emerge in the economy considered in our model.

Figure 5 represents the equilibrium configuration $E_1$, identified by the response functions $\phi^R(\mu^e)$ and $\mu^R(\phi^e)$. The equilibrium proportions of skilled workers and high-technology firms, $(\phi^*, \mu^*)$, located at $E_1$, generate an endogenous matching allocation $M \{x_{i,j}, (\phi^*, \mu^*), p(\cdot)\}_{(i,j)\in(I\times J)}$. According to this matching allocation, some unskilled workers are assigned to high-technology firms and cannot be employed. The rate of unemployment of this economy can be read on the intercept of the iso-unemployment relationship $IU(\bar{\pi}_1)$, on the $\mu$-axis. The diffusion of the high-technology generates a positive level of economic expansion that locates the economy on the iso-expansion relationship $IE(\Delta Y_1)$. The intercept of the iso-expansion relation represents the actual rate of economic expansion relative to its maximum potential level $\psi - 1$. The iso-unemployment and the iso-expansion relations, crossing at the equilibrium configuration $E_1$, divide the $(\phi, \mu)$ space in four regions. Equilibria located in these regions display either higher or lower levels of unemployment and economic expansion, when compared to the levels induced by equilibrium $E_1$.

In particular, we can observe that the Pissarides Region (P) comprehends all the equilibrium configurations located simultaneously above the iso-expansion relationship, $IE(\Delta Y_1)$, and below the iso-unemployment relationship $IU(\bar{\pi}_1)$. The configurations of equilibrium located in the P-zones fulfil the prediction of Pissarides’ (1990) model because moving from equilibrium $E_1$ to the P-zone implies that a higher degree of economic expansion is associated to a lower unemployment rate. The Aghion and Howitt Region (A&H), instead, identifies equilibrium configurations generating higher unemployment along with higher levels of economic expansion—the effects of creative destruction—when compared to the initial equilibrium $E_1$. Thus, even in its simplicity, we observe that our framework can reproduce the contrasting predictions of Pissarides (1990), and Aghion and Howitt (1994). Our way to capture Pissarides’ capitalisation effect is based on different microeconomic foundations. More precisely, in our model, economic expansion is negatively correlated with unemployment when the skill-upgrading effect dominates the creative destruction effect. On the other hand, our model fulfils the predictions of Aghion’s and Howitt’s Neo-Schumpeterian approach, when the number of workers who engage in skill-acquisition activities is not sufficient to sustain the diffusion of the new technology at the same rate of unemployment. In this case, the skill-upgrading effect is dominated by the creative destruction effect.

Consider a reference economy $\Omega$ and its associated equilibrium configuration $E_1$. A change in any of the fundamental parameters of $\Omega$ causes a revision of agents’ investment decisions, which could be represented in Figure 5 as a shift of the skill-upgrading response function, $\phi^R(\mu^e)$, and/or of the innovation response function, $\mu^R(\phi^e)$. This shift generates a change in the equilibrium proportions of skilled work-
ers and high-technology firms. In the case of economic expansion, the new equilibrium configuration, $E_2$, could be located either in the P-Region or in the A&H-Region. Thus, economic expansion can be associated with either lower or higher levels of unemployment. The relative changes in the equilibrium proportions of skilled workers and high-technology firms are determined by the two response functions, which represent the investment decision stage of the model. The classification of equilibria is determined by the iso-unemployment and the iso-expansion relationship, which provide the link between labour market performance and the macroeconomic outcomes.

The classification of possible effects induced by policy-intervention aimed at reducing the costs of investment in skill-acquisition and technology adoption are summarised in Table 7.1 and Table 7.2, respectively. Both these policies generate a higher degree of economic expansion, because both the creative destruction effect and the skill-upgrading effect display a positive sign. Nevertheless, the effects on unemployment are ambiguous.
Table 7.1: Macroeconomic effects of $C$ ↓ and $\gamma$ ↓

<table>
<thead>
<tr>
<th>$\frac{\mu^<em>}{1-\phi^</em>}$</th>
<th>$\frac{\partial \mu^R(\phi)}{\partial \phi}(\phi^<em>,\mu^</em>) &lt; 1$</th>
<th>$\frac{\partial \mu^R(\phi)}{\partial \phi}(\phi^<em>,\mu^</em>) &gt; 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$&lt; 1$</td>
<td>$\Delta Y \uparrow \ u ?$</td>
<td>$\Delta Y \uparrow \ u \uparrow$</td>
</tr>
<tr>
<td>$&gt; 1$</td>
<td>$\Delta Y \uparrow \ u ?$</td>
<td>$\Delta Y \uparrow \ u$</td>
</tr>
</tbody>
</table>

Table 7.1 highlights that (i) if the initial equilibrium proportion of innovating firms is relatively small and, (ii) firms’ innovation activities are very sensitive to an increase in the proportion of skilled workers, subsidies to skill-acquisition generate economic expansion with more unemployment. Conversely, a large initial proportion of innovating firms and a weak response of firms to workers’ skill-acquisition activities generate economic expansion with less unemployment.

Table 7.2: Macroeconomic effects of $D$ ↓ and $\rho$ ↓

<table>
<thead>
<tr>
<th>$\frac{\mu^<em>}{1-\phi^</em>}$</th>
<th>$\frac{\partial \phi^R(\mu)}{\partial \rho}(\phi^<em>,\mu^</em>) &lt; 1$</th>
<th>$\frac{\partial \phi^R(\mu)}{\partial \rho}(\phi^<em>,\mu^</em>) &gt; 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$&lt; 1$</td>
<td>$\Delta Y \uparrow \ u \uparrow$</td>
<td>$\Delta Y \uparrow \ u \uparrow$</td>
</tr>
<tr>
<td>$&gt; 1$</td>
<td>$\Delta Y \uparrow \ u ?$</td>
<td>$\Delta Y \uparrow \ u \downarrow$</td>
</tr>
</tbody>
</table>

In a similar way, Table 7.2 shows that policy measures aimed at sustaining innovation activities have to be evaluated considering workers’ strategic reaction to an increase in the proportion of high-technology firms. These policies can foster economic expansion, and reduce unemployment, if (i) the initial proportion of skilled workers is sufficiently high, and (ii) workers’ investment decisions in skill-acquisition are highly sensitive to an increase in the number of high-technology firms. In the opposite situation—that is, a low initial proportion of skilled workers and weak workers’ strategic response to firms’ innovation activities—the diffusion of the high-technology generates an economic expansion with higher unemployment.

We can conclude that policy measures aimed at reducing the costs of investment in skill-acquisition and technology adoption foster economic expansion but display ambiguous effects on unemployment. Policies aimed at subsidising skill demand or skill supply may generate different outcomes, and the choice of the most opportune intervention should be based on the initial parametric configuration of the economy, which affects: (i) the equilibrium allocation of workers to firms, and (ii) the relative strength of strategic complementarities and strategic substitutabilities between agents’ investment decisions, and (iii) the performance of the labour market.

Table 7.3: Macroeconomic effects of $\beta$ ↑

<table>
<thead>
<tr>
<th>$\frac{d\phi^*}{d\beta}$</th>
<th>$\frac{d\mu}{d\beta} &gt; 0$</th>
<th>$\frac{d\mu}{d\beta} &lt; 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$&gt; 0$</td>
<td>$\Delta Y \uparrow \ u ?$</td>
<td>$\Delta Y \downarrow \ u \downarrow$</td>
</tr>
<tr>
<td>$&lt; 0$</td>
<td>$\Delta Y \uparrow \ u ?$</td>
<td>$\Delta Y \downarrow \ u \uparrow$</td>
</tr>
</tbody>
</table>
Table 7.3 summarises the possible consequences of intervention on workers’ bargaining power. An increase in workers’ bargaining power determines unambiguous effects on economic expansion only when the skill-upgrading effect and the creative destruction effect display the same sign. More skilled workers and high-technology firms generate a higher degree of economic expansion, as the number of productive high-technology matches increases. Nevertheless, an increase in workers’ bargaining power reduces unemployment only when a positive skill-upgrading effect is combined with a negative creative destruction effect. In this case, the number of high-technology firms decreases and the number of skilled workers increases. Since skilled workers can be employed by either high or low-technology firms, and the number of high-technology vacancies is lower, unskilled unemployment falls. The economy registers a higher number of productive matches with high-technology firms, but the overall number of innovating firms is lower. Therefore aggregate production can either increase or decrease. Table 7.3 highlights that when the effects on economic expansion are unambiguous, effects on unemployment are ambiguous. Vice versa, when effects on unemployment are well defined, effects on economic expansion are ambiguous. This result is not surprising because intervention on workers’ bargaining power determines direct effects and strategic complementarity effects both within and across the two sides of the labour market, generating a wide set of possible outcomes.

8 Conclusions

In this paper we have considered an economy that experiences a wave of skill-biased technological change and search and matching frictions in the labour market. Heterogeneous workers and heterogeneous firms engage in skill-acquisition and technology adoption activities prior to searching for a compatible production partner. In this framework, agents form expectations about the matching allocation emerging in the labour market, and make optimal investment in skills or technology consistent with these expectations. We have shown that, under the assumption of skill and technology complementarity, the investment decisions of workers and firms are strategically related. From the aggregation of agents’ investment decisions we have derived an endogenous distribution of skills and technologies. Subsequently, we have examined how these two distributions affect the labour market outcome. Technological diffusion, economic expansion, and unskilled unemployment emerge as the macroeconomic outcomes of agents’ investment activities – taken in the first stage of the model – and labour market performance – emerging in the second stage of the model.

Our framework is able to generate two equilibria. In the first equilibrium no agent invests in either innovation or skill-acquisition. This equilibrium is typically unstable and it represents our benchmark case: an economy that does not promote the intro-

---

20 We have only reported the case of an increase in workers’ bargaining power, as the case of a decrease in $\beta$ simply follows changing signs. The case of $\frac{du^*}{d\sigma^*} > 0$ and $\frac{dG^*}{d\sigma^*} < 0$ never occurs.
duction of the new technology. As soon as technology adoption and skill-acquisition activities take place in the economy, the presence of strategic complementarities leads agents to form mutual expectations about the proportions of skilled workers and innovating firms that will enter the labour market. These expectations are fulfilled in the stable equilibrium \((\phi^*, \mu^*)\). In contrast with many models in related literature, an interesting feature of the stable equilibrium with skill-acquisition and technological diffusion is the presence of an imbalance between the proportion of skilled workers and high-technology firms.\(^{21}\) The case of a larger proportion of skilled workers, \(\phi^* > \mu^*\), is a likely outcome because workers have higher incentive to invest in skill-acquisition to insure themselves against the risk of unskilled unemployment, whereas innovating firms run the risk of being unproductively matched with an unskilled worker. In equilibrium, a higher proportion of skilled workers implies the presence of over-education: a stylised fact of many modern economies.\(^{22}\) Another stylised fact captured by non-extreme values\(^{23}\) of the equilibrium configuration \((\phi^*, \mu^*)\) is the coexistence of two technologies and two types of workers in the economy. Thus, full diffusion of the innovative technology, as well as complete skill-upgrading of the labour force, may never occur.\(^{24}\) From a technical point of view, our model departs from the standard approach adopted by similar frameworks, such as Redding (1996) and Acemoglu (1997). These coordination-game models identify two symmetric Nash-equilibria. In one equilibrium, all agents take the same (symmetric) investment decision, whereas, in the other equilibrium, no agent takes any investment decision. Our model encompasses this symmetric approach, widening the set of possible equilibrium outcomes and considering symmetric equilibria only as a particular case.

Unsurprisingly, we find that the labour market plays a fundamental role in characterising the link between economic expansion and unemployment. More interestingly, we have also emphasized that, when agents act strategically, they internalise the role played by search and matching frictions in their investment decisions in skill-acquisition and technology adoption. Therefore, policy-intervention aimed at pro-

\(^{21}\)Our model still admits equilibria displaying equal proportions of skilled workers and high-technology firms, but these equilibria only represent a special case, holding for particular parameterisations of the economy.

\(^{22}\)For an overview on over-education in the UK the interested reader can refer to: Machin and Vignoles (2005), Chapter 9, and Chevalier and Lindley (2006). For a more general perspective see also Chevalier (2003).

\(^{23}\)We classify the values of the configuration \((\phi^*, \mu^*)\) as ‘non-extreme’ when: \(0 < \phi^* < 1\) and \(0 < \mu^* < 1\).

\(^{24}\)Evidently, our static model of search and production does not consider competition between firms on the final goods market. Competition on the final goods market represents a fundamental rationale for the Neo-Schumpeterian assumption that new technologies are direct substitutes for old technologies. Nevertheless, other contributions in literature, such as Young (1993) and Lloyd-Ellis (2002), claim that human capital accumulation and innovation activities should be simultaneously taken into account within the same framework. Young (1993) shows that, in a model with both innovation and human capital accumulation, old and new technologies can be complements in the short-run and substitutes only in the long-run.
moting economic expansion and lowering unskilled unemployment should also address this strategic interaction between agents. The presence of strategic complementarities across the two sides of the labour market and strategic substitutabilities within either side of the labour market constitutes the fundamental link between agents’ investment decisions in skills and technologies, the degree of economic expansion, and the rate of unemployment. Subsidies to skill acquisition and technology adoption activities affect the performance of the labour market.

The key results of our model are determined by the interaction of the creative destruction effect and of the skill-upgrading effect. Both effects contribute to foster technological diffusion and economic expansion, but they compete in the determination of the unemployment outcome. The introduction of the skill-upgrading effect is one of the fundamental novelties of our analysis. Building on the contribution of Acemoglu (1997), we have observed that the existing literature on matching models for growth, technological change, and unemployment has mainly focused on the mechanism of creative destruction, devoting attention to the microeconomic foundations of firms’ investment decisions, and neglecting the role played by endogenous heterogeneity in the composition of the labour force. Moreover, we argued that even Acemoglu’s (1997) results are biased in the same direction because, in his model, investment decisions in technology adoption and training activities are both taken by firms, within the same side of the labour market. Therefore, the creative destruction effect and the skill-upgrading effect are not independent of each other. In our model, we are able to separate out the skill-upgrading effect from the creative destruction effect by assuming that either side of the labour market faces a separate investment decision: skill-acquisition for workers and technology adoption for firms. In this way, investment decisions in skills and technologies are not strategic complements within the same side of the labour market, but across the two sides of the labour market. In addition, by assuming that skill-acquisition and technology adoption activities are subject to congestion costs, we have introduced strategic substitutabilities within each side of the labour market. This new set-up displays some relevant implications for policy intervention. In particular, we find that the opportunity to subsidise either skill-upgrading or innovation activities has to be evaluated in relation to (i) to the interplay of strategic complementarities and substitutabilities between agents’ investment decisions, and to (ii) the performance of the matching mechanism operating in the labour market. Both these factors depend on the parametric configuration of the economy, including the bargaining power of workers, the degree of ex-ante heterogeneity between workers and firms, and the relative cost of skill-acquisition and technology adoption activities. We believe that this result should be interpreted by accepting that there is no unique prescription to prevent the rising of unskilled unemployment over a wave of skill-biased technological change. The institutional set-up of the labour market –captured in our model by the workers’ bargaining power parameter– consti-

\[25\] In Acemoglu (1997) the attention of the policy-maker is focused on innovation activities, since the decisions about innovation and training are both under firms’ control.
stitutes only one of the dimensions to be examined. Different economies may benefit from differentiated policy-intervention schemes aimed to subsidise either innovation or skill-acquisition. The formulation of these schemes should take simultaneously into account the structure of the economy’s education system, and the efficiency of the process of technology adoption. Thus, rather than suggesting a focus on only one side of the labour market, further effort should be invested in rising the awareness of the policy-makers on the importance of the coordination between skill-demand and skill-supply.

References


Mathematical appendix

Proof of Proposition 4.1

Consider equation (6). Taking derivatives:

(i) \[ \frac{\partial \phi^R(\mu^e)}{\partial \mu^e} = \frac{\beta \psi}{\sqrt{C^2 + 4\gamma \beta \psi \mu^e}} > 0. \]
\[ \frac{\partial^2 \phi^R(\mu^e)}{\partial^2 \mu^e} = -\frac{2 \beta^2 \psi^2 \gamma}{\sqrt{(C^2 + 4\gamma \beta \psi \mu^e)^3}} < 0. \]

(ii) \[ \frac{\partial \phi^R(\mu^e)}{\partial \gamma} = -\frac{1}{2} \left( \frac{C + \sqrt{C^2 + 4 \gamma \beta \psi \mu^e}}{\gamma \sqrt{C^2 + 4 \gamma \beta \psi \mu^e}} \right) < 0. \]
\[ \frac{\partial^2 \phi^R(\mu^e)}{\partial^2 \gamma} = -\frac{1}{2} \frac{2 \gamma \beta \psi \mu^e + C^2 - C \sqrt{C^2 + 4 \gamma \beta \psi \mu^e}}{\gamma^2 \sqrt{C^2 + 4 \gamma \beta \psi \mu^e}} < 0. \]

because: \[ 2 \gamma \beta \psi \mu^e + C^2 > C \sqrt{C^2 + 4 \gamma \beta \psi \mu^e}. \]
\[ 4 \gamma^2 \beta^2 \psi^2 (\mu^e)^2 + C^4 + 4 \gamma \beta \psi \mu^e C^2 > C^2 \left( C^2 + 4 \gamma \beta \psi \mu^e \right). \]
\[ 4 \gamma^2 \beta^2 \psi^2 (\mu^e)^2 > 0. \]

(iii) \[ \frac{\partial \phi^R(\mu^e)}{\partial \psi} = \frac{\beta \mu^e}{\sqrt{C^2 + 4 \gamma \beta \psi \mu^e}} > 0. \]
\[ \frac{\partial^2 \phi^R(\mu^e)}{\partial \psi^2} = \frac{\psi \mu^e}{\sqrt{C^2 + 4 \gamma \beta \psi \mu^e}} > 0. \]

Proof of Proposition 4.2

Consider equation (10). Taking derivatives:

(i) \[ \frac{\partial \mu^R(\phi^e)}{\partial \phi^e} = \frac{(1 - \beta)(\psi - 1)}{\sqrt{D^2 + 4\rho(1 - \beta)(\psi - 1)\phi^e}} > 0. \]
\[ \frac{\partial^2 \mu^R(\phi^e)}{\partial^2 \phi^e} = -\frac{8 \rho (1 - \beta)(\psi - 1)^2}{\sqrt{[D^2 + 4\rho(1 - \beta)(\psi - 1)\phi^e]^3}} < 0. \]

(ii) \[ \frac{\partial \mu^R(\phi^e)}{\partial D} = -\frac{D + \sqrt{D^2 + 4\rho(1 - \beta)(\psi - 1)\phi^e}}{2\rho \sqrt{D^2 + 4\rho(1 - \beta)(\psi - 1)\phi^e}} < 0. \]
\[ \frac{\partial \mu^R(\phi^e)}{\partial \rho} = -\frac{2 \rho (1 - \beta)(\psi - 1)\phi^e + D^2 - D \sqrt{D^2 + 4\rho(1 - \beta)(\psi - 1)\phi^e}}{\rho^2 \sqrt{D^2 + 4\rho(1 - \beta)(\psi - 1)\phi^e}} < 0. \]

because: \[ 2 \rho (1 - \beta)(\psi - 1)\phi^e + D^2 > D \sqrt{D^2 + 4\rho(1 - \beta)(\psi - 1)\phi^e}. \]
\[4\rho^2 (1 - \beta)^2 (\psi - 1)^2 (\phi^e)^2 + D^4 + 4\rho (1 - \beta) (\psi - 1) \phi^e D^2 >
\]
\[> D^2 [D^2 + 4\rho (1 - \beta) (\psi - 1) \phi^e].\]
\[4\rho^2 (1 - \beta)^2 (\psi - 1)^2 (\phi^e)^2 > 0.\]

(iii) \[
\frac{\partial \mu^R(\phi^e)}{\partial \beta} = -\frac{(\psi - 1)\rho\phi^e}{\rho \sqrt{D^2 + 4\rho (1 - \beta)(\psi - 1)\phi^e}} < 0.
\]
\[
\frac{\partial \mu^R(\phi^e)}{\partial \psi} = \frac{(1 - \beta)\rho\phi^e}{\rho \sqrt{D^2 + 4\rho (1 - \beta)(\psi - 1)\phi^e}} > 0.
\]

**Proof of Proposition 6.1**

Consider a generic iso-unemployment relation:

\[IU (\overline{u}) = \{(\phi^*, \mu^*) : (1 - \phi^*) \mu^* = \overline{u}\}.
\]

We can re-write the equation of \(IU (\overline{u})\) in the \((\phi, \mu)\) space as:

\[
\mu^* |_{(\phi^*, \mu^*) \in IU(\overline{u})} = \frac{\overline{u}}{1 - \phi^*}.
\]

Observe that: \(\lim_{\phi^* \rightarrow 0^+} \mu^* |_{(\phi^*, \mu^*) \in IU(\overline{u})} = \overline{u}, \lim_{\phi^* \rightarrow 1^-} \mu^* |_{(\phi^*, \mu^*) \in IU(\overline{u})} = +\infty\), and:

(i) \[
\left. \left. \frac{d\mu^*}{d\phi^*} \right|_{(\phi^*, \mu^*) \in IU(\overline{u})} = \frac{\overline{u}}{(1 - \phi^*)^2} > 0;
\]

(ii) \[
\left. \left. \frac{d^2\mu^*}{d\phi^*} \right|_{(\phi^*, \mu^*) \in IU(\overline{u})} = \frac{2\overline{u}}{(1 - \phi^*)^3} > 0.
\]

**Proof of Proposition 6.3**

Properties (i) and (ii)

According to Property (ii) of Proposition 4.1, a reduction of the investment cost parameter, \(C\), or of the congestion cost parameter, \(\gamma\), generate the same effect on the two response functions. Therefore, we analyse only the case of a reduction in \(C\), as the case of a reduction of \(\gamma\) can be derived in a similar way.

From Property (ii) of Proposition 4.1, we know that: \(\frac{\partial \phi^R(\mu^e)}{\partial C} < 0\). Therefore, the effect of a reduction in \(C\) corresponds to: \(-\frac{\partial \phi^R(\mu^e)}{\partial C} > 0\).
Since \( \frac{\partial \mu^R(\phi^e)}{\partial C} = 0 \), the skill-upgrading response function shifts right, while the innovation response function is not sensitive to a change in \( C \) (or \( \gamma \)). Thus, the change in the equilibrium distributions of skills and technologies takes place shifting the skill-upgrading response function and moving along the innovation response function.

Property (i) of Proposition 4.2 states that: \( \frac{\partial \mu^R(\phi^e)}{\partial \phi} > 0 \) and \( \frac{\partial^2 \mu^R(\phi^e)}{\partial^2 \phi} < 0 \).

Therefore, combining \( -\frac{d\phi^*}{dC} > 0 \) with \( \frac{d\mu^R(\phi^e)}{d\phi} > 0 \), implies that:

\[ -\frac{d\phi^*}{dC} > 0 \quad \text{and} \quad -\frac{d\mu^*}{dC} > 0. \]

Moreover, \( \frac{\partial^2 \mu^R(\phi^e)}{\partial^2 \phi} < 0 \) implies that, when \( \phi^* \) is sufficiently high, the effect of congestions in technology adoption becomes stronger and stronger.

(We also observe that, since \( \frac{\partial \mu^R(\phi^e)}{\partial \phi} > 0 \), higher values of \( \phi^* \) are associated with relatively high values of \( \mu^* \)).

This, by return, implies: \( \frac{\partial \mu^R(\phi)}{\partial \phi} \bigg|_{(\phi^*, \mu^*)} < 1 \iff -\frac{d\phi^*}{dC} > -\frac{d\mu^*}{dC} \iff -\frac{d\mu^*/dC}{-d\phi^*/dC} < 1. \)

Using (12), we can decompose the effects of a reduction in \( C \) on unemployment:

\[ -\frac{d\mu^*}{dC} = -\frac{d\mu^e}{dC} (1 - \phi^*) + \frac{d\phi^*}{dC} \mu^*, \]

and apply Property (ii) of Proposition 6.2:

\[ -\frac{d\mu^*}{dC} < 0 \iff -\frac{d\mu^e}{dC} (1 - \phi^*) < -\frac{d\phi^*}{dC} \mu^* \iff \frac{-\frac{d\mu^*/dC}{-d\phi^*/dC}}{d\phi^*/dC} < \frac{\mu^*}{1-\phi^*}. \]

Combining these results, we can now construct the following table:

| \( \mu^* \) > 1 - \( \phi^* \) | \( \frac{\partial \mu^R(\phi)}{\partial \phi} \bigg|_{(\phi^*, \mu^*)} < 1 \) | \( \frac{\partial \mu^R(\phi)}{\partial \phi} \bigg|_{(\phi^*, \mu^*)} > 1 \) |
|---|---|---|
| \( \mu^* > 1 - \phi^* \) | \( -\frac{d\mu^e}{dC} > \frac{d\phi^*}{dC} > \frac{\mu^*}{1-\phi^*} \) | \( -\frac{d\mu^e}{dC} < \frac{d\phi^*}{dC} > \frac{\mu^*}{1-\phi^*} \) |

| \( \mu^* < 1 - \phi^* \) | \( \frac{\partial \mu^R(\phi)}{\partial \phi} \bigg|_{(\phi^*, \mu^*)} > 1 \) | \( \frac{\partial \mu^R(\phi)}{\partial \phi} \bigg|_{(\phi^*, \mu^*)} < 1 \) |
|---|---|---|
| \( \mu^* < 1 - \phi^* \) | \( \frac{d\mu^e}{dC} > \frac{d\phi^*}{dC} > \frac{\mu^*}{1-\phi^*} \) | \( \frac{d\mu^e}{dC} > \frac{d\phi^*}{dC} > \frac{\mu^*}{1-\phi^*} \) |

From which we can observe that:

(i) \( \frac{\partial \mu^R(\phi)}{\partial \phi} \bigg|_{(\phi^*, \mu^*)} < 1 \) and \( \mu^* > 1 - \phi^* \Rightarrow -\frac{d\mu^e}{dC} > \frac{d\phi^*}{dC} > \frac{\mu^*}{1-\phi^*} \Rightarrow -\frac{d\mu^e}{dC} < 0; \)
(ii) \[ \frac{\partial \mu^R(\phi)}{\partial \phi} \bigg|_{(\phi^*, \mu^*)} > 1 \text{ and } \mu^* < 1 - \phi^* \implies \frac{-\partial \mu^*}{\partial \phi^*} > 1 \implies \frac{-\partial \mu^*}{\partial \phi^*} > 1 \implies -\frac{\partial \mu^*}{\partial C} > 0. \]

Property (iii)

From the table derived in the proof of properties (i) and (ii), we can observe that:

\[ \mu^* > 1 - \phi^* \text{ and } \frac{\partial \mu^R(\phi)}{\partial \phi} \bigg|_{(\phi^*, \mu^*)} > 1 \implies \frac{-\partial \mu^*}{\partial \phi^*} > 1 \implies \frac{-\partial \mu^*}{\partial \phi^*} > 1 \implies -\frac{\partial \mu^*}{\partial C} > 0. \]

\[ \mu^* < 1 - \phi^* \text{ and } \frac{\partial \mu^R(\phi)}{\partial \phi} \bigg|_{(\phi^*, \mu^*)} < 1 \implies \frac{-\partial \mu^*}{\partial \phi^*} < 1 \implies \frac{-\partial \mu^*}{\partial C} < 1 \implies -\frac{\partial \mu^*}{\partial C} > 0. \]

Therefore, in these cases, Property (ii) of Proposition 6.2 does not provide an informative criterion to characterise the effects of a reduction of \( C \) or \( \gamma \) on unemployment.

**Proof of Proposition 6.4**

Properties (i) and (ii)

According to Property (ii) of Proposition 4.2, a reduction of the investment cost parameter, \( D \), or in the congestion cost parameter, \( \rho \), generate the same effects on the two response functions. Therefore, also in this case, we analyse only the case of a reduction in \( D \), as the case of a reduction in \( \rho \) can be derived in a similar way.

From Property (ii) of Proposition 4.2, we know that: \( \frac{\partial \mu^R(\phi^*)}{\partial D} < 0 \). Therefore, the effect of a reduction in \( D \) corresponds to: \( -\frac{\partial \mu^R(\phi^*)}{\partial D} > 0. \)

Since \( \frac{\partial \phi^R(\mu^*)}{\partial D} = 0 \), the innovation response function shifts up, while the skill-upgrading response function is not sensitive to a change in \( D \) (or \( \rho \)). Thus, the change in the equilibrium distributions of skills and technologies takes place shifting the innovation response function and moving along the skill-upgrading response function.

Property (i) of Proposition 4.1 states that: \( \frac{\partial \phi^R(\mu^*)}{\partial \mu} > 0 \) and \( \frac{\partial^2 \phi^R(\mu^*)}{\partial \mu^2} < 0. \)
Therefore, combining \(-\frac{\partial R(\phi^*)}{\partial D} > 0\) with \(\frac{\partial R(\mu^*)}{\partial \mu} > 0\), implies that:

\[-\frac{d\phi^*}{dD} > 0\] and \[-\frac{d\mu^*}{dD} > 0\].

Moreover, \(\frac{\partial^2 R(\mu^*)}{\partial \mu^2} < 0\) implies that, when \(\mu^*\) is sufficiently large, the effect of congestions in skill-acquisition becomes stronger and stronger.

This, by return, implies: \(\frac{\partial \phi^*}{\partial D} < 1 \iff -\frac{d\phi^*}{dD} < -\frac{d\mu^*}{dD} \iff -\frac{d\mu^*/dD}{-d\phi^*/dD} > 1\).

Using (12), we can decompose the effects on unemployment of a reduction in \(D\) as:

\(-\frac{d\mu^*}{dD} = -\frac{d\mu^*}{dD} (1 - \phi^*) + \frac{d\phi^*}{d\mu^2} \mu^*\), and apply Property (ii) of Proposition 6.2:

\(-\frac{d\mu^*}{dD} < 0 \iff -\frac{d\mu^*}{dD} (1 - \phi^*) < -\frac{d\mu^*}{dD} \mu^* \iff \frac{d\mu^*/dD}{-d\phi^*/dD} < \frac{\mu^*}{1-\phi^*}\).

Combining these results, we can now construct the following table:

<table>
<thead>
<tr>
<th>(\mu^* &gt; 1 - \phi^*)</th>
<th>(\frac{\partial \phi^*}{\partial D})</th>
<th>(\frac{\partial \phi^*}{\partial \mu})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\mu^* &gt; 1 - \phi^*)</td>
<td>(-\frac{d\mu^<em>}{dD} &lt; -\frac{d\mu^</em>}{dD})</td>
<td>(\frac{\partial \phi^<em>}{\partial \mu} \mid_{(\phi^</em> \mu^*)} &lt; 0)</td>
</tr>
<tr>
<td>(\mu^* &lt; 1 - \phi^*)</td>
<td>(-\frac{d\mu^<em>}{dD} &gt; -\frac{d\mu^</em>}{dD})</td>
<td>(\frac{\partial \phi^<em>}{\partial \mu} \mid_{(\phi^</em> \mu^*)} &gt; 0)</td>
</tr>
</tbody>
</table>

From which we can observe that:

(i) \(\frac{\partial \phi^*}{\partial \mu} \mid_{(\phi^* \mu^*)} > 1\) and \(\mu^* > 1 - \phi^* \implies -\frac{d\mu^*}{dD} < \frac{\mu^*}{1-\phi^*} \implies -\frac{d\mu^*}{dD} < 0\).

(ii) \(\frac{\partial \phi^*}{\partial \mu} \mid_{(\phi^* \mu^*)} < 1\) and \(\mu^* < 1 - \phi^* \implies -\frac{d\mu^*}{dD} > \frac{\mu^*}{1-\phi^*} \implies -\frac{d\mu^*}{dD} > 0\).

Property (iii)

From the tables derived in the proof of properties (i) and (ii), we can observe that:

\(\frac{\partial \phi^*}{\partial \mu} \mid_{(\phi^* \mu^*)} < 1\) and \(\mu^* > 1 - \phi^* \implies -\frac{d\mu^*}{dD} \leq \frac{\mu^*}{1-\phi^*} \implies -\frac{d\mu^*}{dD} \leq 1\)

\(\frac{\partial \phi^*}{\partial \mu} \mid_{(\phi^* \mu^*)} > 1\) and \(\mu^* < 1 - \phi^* \implies -\frac{d\mu^*}{dD} \geq \frac{\mu^*}{1-\phi^*} \implies -\frac{d\mu^*}{dD} \geq 1\).
Therefore, in these cases, Property (ii) of Proposition 6.2 does not provide an informative criterion to characterise the effects of a reduction of $D$ or $\rho$ on unemployment.

**Proof of Proposition 6.5**

We begin the proof recalling that, from Property (ii) of Proposition 6.2, we know that:

(a) $\frac{d\phi^*}{d\beta} > 0 : \frac{d\mu^*/d\phi^*}{d\phi^*/d\beta} > \frac{\mu^*}{(1-\phi^*)} \Leftrightarrow \frac{d\mu^*}{d\beta} > 0$;

(b) $\frac{d\phi^*}{d\beta} < 0 : \frac{d\mu^*/d\phi^*}{d\phi^*/d\beta} < \frac{\mu^*}{(1-\phi^*)} \Leftrightarrow \frac{d\mu^*}{d\beta} > 0$.

**Property (i)**

The following table identifies the sign of $\frac{d\mu^*/d\beta}{d\phi^*/d\beta}$ following an increase in workers’ bargaining power, $\beta$, under different scenarios:

<table>
<thead>
<tr>
<th>$\frac{d\phi^*}{d\beta}$</th>
<th>$\frac{d\mu^*}{d\beta}$ &gt; 0</th>
<th>$\frac{d\mu^*}{d\beta}$ &lt; 0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{d\phi^*}{d\beta}$ &gt; 0</td>
<td>$+$</td>
<td>$-$</td>
</tr>
<tr>
<td>$\frac{d\phi^*}{d\beta}$ &lt; 0</td>
<td>$-$</td>
<td>$+$</td>
</tr>
</tbody>
</table>

We can observe that the case of $\frac{d\phi^*}{d\beta} < 0$ and $\frac{d\mu^*}{d\beta} > 0$ is not interesting, as it can never occur.

If $\frac{d\phi^*}{d\beta} > 0$ and $\frac{d\mu^*}{d\beta} < 0$, using (a), we can conclude that: $\frac{d\mu^*/d\beta}{d\phi^*/d\beta} < \frac{\mu^*}{(1-\phi^*)} \Leftrightarrow \frac{d\mu^*}{d\beta} < 0$.

**Property (ii)**

This can be proven changing signs and following the reasoning described for Property (i).

The following table identifies the sign of $\frac{-d\mu^*/d\beta}{-d\phi^*/d\beta}$ following a decrease in workers’ bargaining power, $\beta$, under different scenarios:

<table>
<thead>
<tr>
<th>$-\frac{d\phi^*}{d\beta}$</th>
<th>$-\frac{d\mu^*}{d\beta}$ &gt; 0</th>
<th>$-\frac{d\mu^*}{d\beta}$ &lt; 0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-\frac{d\phi^*}{d\beta}$ &gt; 0</td>
<td>$+$</td>
<td>$( - )$</td>
</tr>
<tr>
<td>$-\frac{d\phi^*}{d\beta}$ &lt; 0</td>
<td>$-$</td>
<td>$( + )$</td>
</tr>
</tbody>
</table>
We can observe that the case of \(-\frac{d\phi^*}{d\beta} > 0\) and \(-\frac{d\mu^*}{d\beta} < 0\) is not interesting, as it can never occur.

If \(-\frac{d\phi^*}{d\beta} < 0\) and \(-\frac{d\mu^*}{d\beta} > 0\), using (b), we can conclude that:

\[
\frac{-d\phi^*/d\beta}{-d\mu^*/d\beta} < \frac{\mu^*}{(1-\phi^*)} \Leftrightarrow -\frac{d\mu^*}{d\beta} > 0.
\]

Property (iii)

As we have already observed, policy-intervention on the bargaining parameter, \(\beta\), can either increase or decrease the equilibrium proportions of skilled workers and high-technology firms. Therefore, different cases have to be considered.

The proof of Property (i) and the proof of Property (ii) have both characterised one case and excluded one case, out of four possible combinations. In all the remaining cases, we can observe that the sign of the relative change in the equilibrium proportion of skilled workers and high-technology firms is positive and the slope of the iso-unemployment relation \(IU(u^*)\) is also positive. Therefore, in these cases, Property (ii) of Proposition 6.2 does not provide an informative criterion to characterise the effect of an increase (or a decrease) in workers’ bargaining power.

Proof of Proposition 6.7

Consider a generic iso-expansion relation: \(IE(\Delta\bar{Y}) = \{(\phi^*, \mu^*) : \mu^* (\phi^* \psi - 1) = \Delta\bar{Y}\}\).

Re-writing the function in explicit form in the \((\phi^*, \mu^*)\) space:

\[
\mu^* \bigg|_{(\phi^*, \mu^*) \in IE(\Delta\bar{Y})} = \frac{\Delta\bar{Y}}{\phi^* \psi - 1}.
\]

Taking derivatives:

\[
\frac{d\mu^*}{d\phi^*} \bigg|_{(\phi^*, \mu^*) \in IE(\Delta\bar{Y})} = -\frac{\psi \Delta\bar{Y}}{(\phi^* \psi - 1)^2},
\]

\[
\frac{d^2\mu^*}{d^2\phi^*} \bigg|_{(\phi^*, \mu^*) \in IU(\Delta\bar{Y})} = \frac{2\psi^2 \Delta\bar{Y}}{(\phi^* \psi - 1)^3}.
\]

From Proposition 6.6 we know that: \(\phi^* > \frac{1}{\psi} \Leftrightarrow \Delta\bar{Y} > 0\). Therefore, to identify the
sign of the derivatives calculated above we need to analyse two separate cases:

If the economy experiences economic expansion: \( \phi^* > \frac{1}{\psi} \) and \( \Delta \overline{Y} > 0 \). Thus:

\[
\lim_{\phi^* \to (1/\psi)^+} \mu^*|_{(\phi^*, \mu^*) \in IU(\pi)} = +\infty, \quad \lim_{\phi^* \to 1^-} \mu^*|_{(\phi^*, \mu^*) \in IU(\pi)} = \frac{\Delta \overline{Y}}{\psi - 1};
\]

\[
\frac{d\mu^*}{d\phi^*}|_{(\phi^*, \mu^*) \in IE(\Delta \overline{Y})} < 0, \quad \frac{d^2\mu^*}{d\phi^2}|_{(\phi^*, \mu^*) \in IU(\Delta \overline{Y})} > 0.
\]

If the economy experiences economic contraction: \( \phi^* < \frac{1}{\psi} \) and \( \Delta \overline{Y} < 0 \). Thus:

\[
\lim_{\phi^* \to 0^+} \mu^*|_{(\phi^*, \mu^*) \in IU(\pi)} = -\Delta \overline{Y} (> 0), \quad \lim_{\phi^* \to (1/\psi)^-} \mu^*|_{(\phi^*, \mu^*) \in IU(\pi)} = +\infty;
\]

\[
\frac{d\mu^*}{d\phi^*}|_{(\phi^*, \mu^*) \in IE(\Delta \overline{Y})} > 0, \quad \frac{d^2\mu^*}{d\phi^2}|_{(\phi^*, \mu^*) \in IU(\Delta \overline{Y})} > 0.
\]

**Proof of Proposition 6.8**

Re-arranging (16), it is immediate to observe that:

if \( \frac{d\phi^*}{dk} > 0 \): \( \frac{d\mu^*/dk}{d\phi^*/dk} > -\frac{\mu^*}{(\phi^* - 1)} \Leftrightarrow \frac{d\Delta \overline{Y}/k}{dk} > 0; \)

if \( \frac{d\phi^*}{dk} < 0 \): \( \frac{d\mu^*/dk}{d\phi^*/dk} < -\frac{\mu^*}{(\phi^* - 1)} \Leftrightarrow \frac{d\Delta \overline{Y}^*/dk}{dk} > 0. \)

From the proof of Proposition 6.7, we know that the slope of the iso-expansion relation, \( IE \left( \Delta \overline{Y}^* \right) \), is:

\[
\frac{d\mu^*}{d\phi^*}|_{(\phi^*, \mu^*) \in IE(\Delta \overline{Y}^*)} = -\frac{\psi \Delta \overline{Y}^*}{(\phi^* - 1)^2}.
\]

We can now observe that, when \( \phi^* > \frac{1}{\psi} \), substituting for \( \Delta \overline{Y}^* = \mu^* (\phi^* \psi - 1) \):

\[
\frac{d\mu^*}{d\phi^*}|_{(\phi^*, \mu^*) \in IE(\Delta \overline{Y}^*)} = -\frac{\psi \Delta \overline{Y}^*}{(\phi^* - 1)^2} = -\frac{\psi \mu^*(\phi^* \psi - 1)}{(\phi^* - 1)^2} = -\frac{\mu^* \psi}{(\phi^* - 1)^2}.
\]

Therefore, the quantity \( \left[ -\frac{\mu^* \psi}{(\phi^* - 1)^2} \right] \), represents the slope of the iso-expansion relation, \( IE \left( \Delta \overline{Y}^* \right) \).

**Proof of Proposition 6.9**

According to Property (ii) of both Proposition 4.1 and Proposition 4.2, the effects
of a reduction in $C$ and $D$, generate the same effects of a reduction in $\gamma$ and $\rho$, on the skill-upgrading response function and on the innovation response function, respectively. Thus, we focus on the effects of a reduction in $C$ and $D$, as the case for $\gamma$ and $\rho$ can be derived in a similar way.

Moreover, we observe that, when $\Omega \in \Omega^+$, the economy experiences economic expansion, so that: $\Delta Y^* > 0$ and the slope of the iso-expansion relation evaluated at $(\phi^*, \mu^*)$ is negative.

Property (i)

From the proof of Proposition 6.3, we know that: $-\frac{d\phi^*}{dC} > 0$ and $-\frac{d\mu^*}{dC} > 0$.

Thus, applying Property (i) of Proposition 6.8: $-\frac{d\mu^*/dC}{d\phi^*/dC} > 0$ implies $-\frac{d\Delta Y^*}{dC} > 0$.

Property (ii)

From the proof of Proposition 6.4, we know that: $-\frac{d\phi^*}{dD} > 0$ and $-\frac{d\mu^*}{dD} > 0$.

Thus, applying Property (i) of Proposition 6.8: $-\frac{d\mu^*/dD}{d\phi^*/dD} > 0$ implies $-\frac{d\Delta Y^*}{dD} > 0$.

**Proof of Proposition 6.10**

This proof follows the line of the Proof of Proposition 6.5, but we report all the passages for accuracy. We begin the proof recalling that, from Proposition 6.8, we know that:

(a) $\frac{d\phi^*}{dk} > 0 : \frac{d\mu^*/dk}{d\phi^*/dk} > -\frac{\mu^*\psi}{(\phi^*\psi-1)} \iff \frac{d\Delta Y^*}{dk} > 0$;

(b) $\frac{d\phi^*}{dk} < 0 : \frac{d\mu^*/dk}{d\phi^*/dk} > -\frac{\mu^*\psi}{(\phi^*\psi-1)} \iff \frac{d\Delta Y^*}{dk} < 0$.

Property (i) and (ii)

The following table identifies the sign of $\frac{d\mu^*/d\beta}{d\phi^*/d\beta}$ following an increase in workers’ bargaining power, $\beta$, under different scenarios:
We can observe that the case of $\frac{d\sigma^*}{d\beta} < 0$ and $\frac{d\mu^*}{d\beta} > 0$ is not interesting, as it can never occur. Thus, applying the criterion defined by (a) and (b):

if $\frac{d\sigma^*}{d\beta} > 0$ and $\frac{d\mu^*}{d\beta} > 0$: $\frac{d\mu^*/d\beta}{d\sigma^*/d\beta} > -\frac{\mu^*\psi}{(\phi^*\psi-1)} \implies \frac{d\Delta Y^*}{d\beta} > 0$;

if $\frac{d\sigma^*}{d\beta} > 0$ and $\frac{d\mu^*}{d\beta} < 0$: $\frac{d\mu^*/d\beta}{d\sigma^*/d\beta} \leq -\frac{\mu^*\psi}{(\phi^*\psi-1)} \implies \frac{d\Delta Y^*}{d\beta} \leq 0$;

if $\frac{d\sigma^*}{d\beta} < 0$ and $\frac{d\mu^*}{d\beta} < 0$: $\frac{d\mu^*/d\beta}{d\sigma^*/d\beta} > -\frac{\mu^*\psi}{(\phi^*\psi-1)} \implies \frac{d\Delta Y^*}{d\beta} < 0$.

The following table identifies the sign of $\frac{d\mu^*/d\beta}{d\sigma^*/d\beta}$ following a decrease in workers’ bargaining power, $\beta$, under different scenarios:

<table>
<thead>
<tr>
<th>$-\frac{d\sigma^*}{d\beta}$</th>
<th>$-\frac{d\mu^*}{d\beta}$</th>
<th>$-\frac{d\mu^*}{d\beta}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-\frac{d\sigma^*}{d\beta}$</td>
<td>$&gt; 0$</td>
<td>$(+)$</td>
</tr>
<tr>
<td>$-\frac{d\sigma^*}{d\beta}$</td>
<td>$&lt; 0$</td>
<td>$(+)$</td>
</tr>
</tbody>
</table>

We can observe that the case of $-\frac{d\sigma^*}{d\beta} > 0$ and $-\frac{d\mu^*}{d\beta} < 0$ is not interesting, as it can never occur. Thus, applying the criterion defined by (a) and (b):

if $-\frac{d\sigma^*}{d\beta} > 0$ and $-\frac{d\mu^*}{d\beta} > 0$: $\frac{d\mu^*/d\beta}{-d\sigma^*/d\beta} > -\frac{\mu^*\psi}{(\phi^*\psi-1)} \implies \frac{d\Delta Y^*}{d\beta} > 0$;

if $-\frac{d\sigma^*}{d\beta} < 0$ and $-\frac{d\mu^*}{d\beta} < 0$: $\frac{d\mu^*/d\beta}{-d\sigma^*/d\beta} \geq -\frac{\mu^*\psi}{(\phi^*\psi-1)} \implies \frac{d\Delta Y^*}{d\beta} \leq 0$;

if $-\frac{d\sigma^*}{d\beta} < 0$ and $-\frac{d\mu^*}{d\beta} < 0$: $\frac{d\mu^*/d\beta}{-d\sigma^*/d\beta} > -\frac{\mu^*\psi}{(\phi^*\psi-1)} \implies \frac{d\Delta Y^*}{d\beta} < 0$.

Property (iii)

As observed in Proposition 6.5, policy-intervention on the bargaining parameter, $\beta$, can either increase or decrease the equilibrium proportions of skilled workers and high-technology firms. Therefore, different cases have to be considered.
The proof of Property (i) and the proof of Property (ii) have both characterised one case and excluded one case, out of four possible combinations. In the remaining two cases, we can observe that the sign of the relative change in the equilibrium proportion of skilled workers and high-technology firms is negative and the slope of the iso-unemployment relation $IE(\Delta \bar{Y})$ is also negative. Therefore, in these cases, Proposition 6.8 does not provide an informative criterion to characterise the effect of an increase (or a decrease) in workers’ bargaining power.
ABOUT THE CDMA

The Centre for Dynamic Macroeconomic Analysis was established by a direct grant from the University of St Andrews in 2003. The Centre funds PhD students and facilitates a programme of research centred on macroeconomic theory and policy. The Centre has research interests in areas such as: characterising the key stylised facts of the business cycle; constructing theoretical models that can match these business cycles; using theoretical models to understand the normative and positive aspects of the macroeconomic policymakers' stabilisation problem, in both open and closed economies; understanding the conduct of monetary/macroeconomic policy in the UK and other countries; analyzing the impact of globalization and policy reform on the macroeconomy; and analyzing the impact of financial factors on the long-run growth of the UK economy, from both an historical and a theoretical perspective. The Centre also has interests in developing numerical techniques for analyzing dynamic stochastic general equilibrium models. Its affiliated members are Faculty members at St Andrews and elsewhere with interests in the broad area of dynamic macroeconomics. Its international Advisory Board comprises a group of leading macroeconomists and, ex officio, the University's Principal.

Affiliated Members of the School
Dr Fabio Aricò.
Dr Arnab Bhattacharjee.
Dr Tatiana Damjanovic.
Dr Vladislav Damjanovic.
Prof George Evans.
Dr Gonzalo Forgue-Puccio.
Dr. Michal Horvath
Dr Laurence Lasselle.
Dr Peter Macmillan.
Prof Rod McCrorie.
Prof Kaushik Mitra.
Dr. Elisa Newby
Prof Charles Nolan (Director).
Dr Geetha Selvaretnam.
Dr Ozge Senay.
Dr Gary Shea.
Prof Alan Sutherland.
Dr Kannika Thampanishvong.
Dr Christoph Thoenissen.
Dr Alex Trew.

Senior Research Fellow
Prof Andrew Hughes Hallett, Professor of Economics, Vanderbilt University.

Research Affiliates
Prof Keith Blackburn, Manchester University.
Prof David Cobham, Heriot-Watt University.
Dr Luisa Corrado, Università degli Studi di Roma.
Prof Huw Dixon, Cardiff University.
Dr Anthony Garratt, Birkbeck College London.
Dr Sugata Ghosh, Brunel University.
Dr Aditya Goenka, Essex University.
Dr Michal Horvath, University of Oxford.
Prof Campbell Leith, Glasgow University.
Prof Paul Levine, University of Surrey.
Dr Richard Mash, New College, Oxford.
Prof Patrick Minford, Cardiff Business School.
Dr Elisa Newby, University of Cambridge.

Dr Gulcin Ozkan, York University.
Prof Joe Pearlman, London Metropolitan University.
Prof Neil Rankin, Warwick University.
Prof Lucio Sarno, Warwick University.
Prof Eric Schaling, South African Reserve Bank and Tilburg University.
Prof Peter N. Smith, York University.
Dr Frank Smets, European Central Bank.
Prof Robert Sollis, Newcastle University.
Dr Peter Tinsley, Birkbeck College, London.
Dr Mark Weder, University of Adelaide.

Research Associates
Mr Nikola Bokan.
Mr Farid Boumediene.
Miss Jinyu Chen.
Mr Johannes Geissler.
Mr Ansgar Rannenberg.
Mr Qi Sun.

Advisory Board
Prof Sumru Altug, Koç University.
Prof V V Chari, Minnesota University.
Prof John Driffield, Birkbeck College London.
Dr Sean Holly, Director of the Department of Applied Economics, Cambridge University.
Prof Seppo Honkapohja, Bank of Finland and Cambridge University.
Dr Brian Lang, Principal of St Andrews University.
Prof Anton Muscatelli, Heriot-Watt University.
Prof Charles Nolan, St Andrews University.
Prof Peter Sinclair, Birmingham University and Bank of England.
Prof Stephen J Turnovsky, Washington University.
Dr Martin Weale, CBE, Director of the National Institute of Economic and Social Research.
Prof Michael Wickens, York University.
Prof Simon Wren-Lewis, Oxford University.
<table>
<thead>
<tr>
<th>Number</th>
<th>Title</th>
<th>Author(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CDMA07/16</td>
<td>Arbitrage and Simple Financial Market Efficiency during the South Sea Bubble: A Comparative Study of the Royal African and South Sea Companies Subscription Share Issues</td>
<td>Gary S. Shea (St Andrews).</td>
</tr>
<tr>
<td>CDMA07/17</td>
<td>Anticipated Fiscal Policy and Adaptive Learning</td>
<td>George Evans (Oregon and St Andrews), Seppo Honkapohja (Cambridge) and Kaushik Mitra (St Andrews)</td>
</tr>
<tr>
<td>CDMA07/18</td>
<td>The Millennium Development Goals and Sovereign Debt Write-downs</td>
<td>Sayantan Ghosal (Warwick), Kannika Thampanishvong (St Andrews)</td>
</tr>
<tr>
<td>CDMA07/19</td>
<td>Robust Learning Stability with Operational Monetary Policy Rules</td>
<td>George Evans (Oregon and St Andrews), Seppo Honkapohja (Cambridge)</td>
</tr>
<tr>
<td>CDMA07/20</td>
<td>Can macroeconomic variables explain long term stock market movements? A comparison of the US and Japan</td>
<td>Andreas Humpe (St Andrews) and Peter Macmillan (St Andrews)</td>
</tr>
<tr>
<td>CDMA07/21</td>
<td>Unconditionally Optimal Monetary Policy</td>
<td>Tatiana Damjanovic (St Andrews), Vladislav Damjanovic (St Andrews) and Charles Nolan (St Andrews)</td>
</tr>
<tr>
<td>CDMA07/22</td>
<td>Estimating DSGE Models under Partial Information</td>
<td>Paul Levine (Surrey), Joseph Pearlman (London Metropolitan) and George Perendia (London Metropolitan)</td>
</tr>
<tr>
<td>CDMA08/01</td>
<td>Simple Monetary-Fiscal Targeting Rules</td>
<td>Michal Horvath (St Andrews)</td>
</tr>
<tr>
<td>CDMA08/02</td>
<td>Expectations, Learning and Monetary Policy: An Overview of Recent Research</td>
<td>George Evans (Oregon and St Andrews), Seppo Honkapohja (Bank of Finland and Cambridge)</td>
</tr>
<tr>
<td>CDMA08/03</td>
<td>Exchange rate dynamics, asset market structure and the role of the trade elasticity</td>
<td>Christoph Thoenissen (St Andrews)</td>
</tr>
<tr>
<td>CDMA08/04</td>
<td>Linear-Quadratic Approximation to Unconditionally Optimal Policy: The Distorted Steady-State</td>
<td>Tatiana Damjanovic (St Andrews), Vladislav Damjanovic (St Andrews) and Charles Nolan (St Andrews)</td>
</tr>
<tr>
<td>CDMA08/05</td>
<td>Does Government Spending Optimally Crowd in Private Consumption?</td>
<td>Michal Horvath (St Andrews)</td>
</tr>
<tr>
<td>CDMA08/06</td>
<td>Long-Term Growth and Short-Term Volatility: The Labour Market Nexus</td>
<td>Barbara Annicchiarico (Rome), Luisa Corrado (Cambridge and Rome) and Alessandra Pelloni (Rome)</td>
</tr>
<tr>
<td>CDMA08/07</td>
<td>Seigniorage-maximizing inflation</td>
<td>Tatiana Damjanovic (St Andrews) and Charles Nolan (St Andrews)</td>
</tr>
<tr>
<td>CDMA08/08</td>
<td>Productivity, Preferences and UIP deviations in an Open Economy Business Cycle Model</td>
<td>Arnab Bhattacharjee (St Andrews), Jagjit S. Chadha (Canterbury) and Qi Sun (St Andrews)</td>
</tr>
<tr>
<td>CDMA08/09</td>
<td>Infrastructure Finance and Industrial Takeoff in the United Kingdom</td>
<td>Alex Trew (St Andrews)</td>
</tr>
<tr>
<td>CDMA08/10</td>
<td>Financial Shocks and the US Business Cycle</td>
<td>Charles Nolan (St Andrews) and Christoph Thoenissen (St Andrews)</td>
</tr>
<tr>
<td>CDMA09/01</td>
<td>Technological Change and the Roaring Twenties: A Neoclassical Perspective</td>
<td>Sharon Harrison (Columbia) Mark Weder (Adeleide)</td>
</tr>
<tr>
<td>CDMA09/02</td>
<td>A Model of Near-Rational Exuberance</td>
<td>George Evans (Oregon and St Andrews), Seppo Honkapohja (Bank of Finland and Cambridge) and James Bullard (St Louis Fed)</td>
</tr>
<tr>
<td>CDMA09/03</td>
<td>Shocks, Monetary Policy and Institutions: Explaining Unemployment Persistence in “Europe” and the United States</td>
<td>Ansgar Rannenberg (St Andrews)</td>
</tr>
<tr>
<td>CDMA09/04</td>
<td>Contracting Institutions and Growth</td>
<td>Alex Trew (St Andrews)</td>
</tr>
<tr>
<td>CDMA09/05</td>
<td>International Business Cycles and the Relative Price of Investment Goods</td>
<td>Parantap Basu (Durham) and Christoph Thoenissen (St Andrews)</td>
</tr>
<tr>
<td>CDMA09/06</td>
<td>Institutions and the Scale Effect</td>
<td>Alex Trew (St Andrews)</td>
</tr>
<tr>
<td>CDMA09/07</td>
<td>Second Order Accurate Approximation to the Rotemberg Model Around a Distorted Steady State</td>
<td>Tatiana Damjanovic (St Andrews) and Charles Nolan (St Andrews)</td>
</tr>
</tbody>
</table>

For information or copies of working papers in this series, or to subscribe to email notification, contact:
Jinyu Chen
Castlecliff, School of Economics and Finance
University of St Andrews
Fife, UK, KY16 9AL
Email: jc736@at-andrews.ac.uk; Phone: +44 (0)1334 462445; Fax: +44 (0)1334 462444.