Aristotle's Theory of the Assertoric Syllogism

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June 19, 2017

Abstract

Although the theory of the assertoric syllogism was Aristotle's great invention, one which dominated logical theory for the succeeding two millenia, accounts of the syllogism evolved and changed over that time. Indeed, in the twentieth century, doctrines were attributed to Aristotle which lost sight of what Aristotle intended. One of these mistaken doctrines was the very form of the syllogism: that a syllogism consists of three propositions containing three terms arranged in four figures. Yet another was that a syllogism is a conditional proposition deduced from a set of axioms. There is even unclarity about what the basis of syllogistic validity consists in. Returning to Aristotle's text, and reading it in the light of commentary from late antiquity and the middle ages, we find a coherent and precise theory which shows all these claims to be based on a misunderstanding and misreading.

1 What is a Syllogism?

Aristotle's theory of the assertoric, or categorical, syllogism dominated much of logical theory for the succeeding two millenia. But as logical theory developed, its connection with Aristotle because more tenuous, and doctrines and intentions were attributed to him which are not true to what he actually wrote. Looking at the text of the *Analytics*, we find a much more coherent theory than some more recent accounts would suggest.[1]

Robin Smith (2017, §3.2) rightly observes that the prevailing view of the syllogism in modern logic is of three subject-predicate propositions, two premises and a conclusion, whether or not the conclusion follows from the premises. Such a view is found in, e.g., Quine (1962, p. 73). Łukasiewicz (1951, p. 2) claimed that a syllogism is really a single conditional proposition with a conjunctive antecedent, again, either logically true or not. Corcoran (1974, p. 92) argued that for Aristotle a syllogism is "a deductive argument (premises, conclusion, plus a chain of reasoning)." In contrast, John Buridan, writing in the fourteenth century, declared:

> "It seems to me that Aristotle takes a syllogism not to be composed of premises and conclusion, but composed only of premises

[1] At least Stebbing (1930, p. 81)—apparently the source of the account of the syllogism in Lemmon (1965)—admitted that her account departed from Aristotle's.

> from which a conclusion can be inferred; so he postulated one power of a syllogism [to be] that from the same syllogism many things can be concluded." (Buridan, 2015, III i 4, p. 123)

He referred, in particular, to Aristotle's remark at the start of the second book of the *Prior Analytics* (II 1), where he says:

> "Some syllogisms ... give more than one conclusion." (53a4-6)

Aristotle's own description of the syllogism is at the start of the first book (I 1):[2]

> "A syllogism is an argument (λόγος) in which, certain things being posited, something other than what was laid down results by necessity because these things are so." (24b19-20)

But Striker's translation here of 'λόγος' is contentious and prejudicial. Other translations render it as 'discourse' (Tredennick in Aristotle, 1938) or 'form of words' (Jenkinson in Aristotle, 1928) and (Smith in Aristotle, 1989). In his translation of the *Prior Analytics* (Aristotle, 1962), Boethius rendered it in Latin as 'oratio', a genus covering anything from a word to a paragraph, or even a whole speech. Moreover, in his commentary on Aristotle's *Topics*, Boethius made further distinctions, drawn apparently from Cicero's *Topics*:

> "An argument is a reason (*ratio*) producing belief regarding a matter [that is] in doubt. Argument and argumentation are not the same, however, for the sense (*vis sententiae*) and the reason enclosed in discourse (*oratio*) when something [that was] uncertain is demonstrated is called the argument; but the expression (*elocutio*) of the argument is called the argumentation. So the argument is the strength (*virtus*), mental content (*mens*), and sense of argumentation; argumentation, on the other hand, is the unfolding of the argument by means of discourse (*oratio*)." (*De Topicis Differentiis*: Boethius, 1978, p. 30)

He repeats the last clause in Book II, and continued:

> "There are two kinds of argumentation; one is called syllogism, the other induction. Syllogism is discourse in which, when certain things have been laid down and agreed to, something other than the things agreed to must result by means of the things agreed to." (Boethius, 1978, p. 43)

As one can see, Boethius' definition of the syllogism repeats (with the addition of agreed to') Aristotle's description in the *Prior Analytics*, which itself repeated his earlier account in the *Topics* (100a25-27).

[2]Translations from *Prior Analytics* I are those by Gisela Striker in Aristotle (2009) unless otherwise stated. Those from *Prior Analytics* II are by Hugh Tredennick in Aristotle (1938).

Smiley (1973, p. 138) invoked the "Frege point" to argue that Corcoran's interpretation will not work. For in different arguments (as chains of reasoning), one and the same proposition may have a different force, as assumption or assertion or question, but the same syllogism is in play. Smith, in the 'Introduction' to his edition of the *Prior Analytics* (Aristotle, 1989, pp. xv-xvi) argued that what Aristotle says at 24b19-20, repeating the same formula from the *Topics*, is not so much a description of the syllogism as he will come to develop it in the *Prior Analytics* but of deduction or valid argument in general. For Aristotle later tries to show that every deduction can be reduced to a succession of syllogisms. The most we can say is that a syllogism for Aristotle must, as Buridan realised, include a set of premises from which one or more conclusions can be shown to follow validly. This is the interpretation given by Al-Farabi:

> "A syllogism is, at a minimum, composed of two premises sharing one common part."[3]

So what Aristotle is ultimately interested in is which pairs of assertoric, subject-predicate propositions are productive, that is, yield conclusions of the same sort. That's compatible with his including in that quest an examination of the argumentation by which those conclusions are produced, with investigating which triples, quadruples of propositions, and so on, are productive, and conversely, with discovering what premises will substantiate a given conclusion.

This explains how, if one does include the conclusion, the resulting "syllogism" is by definition valid, since the conclusion "results by necessity" from the premises. A demonstrative syllogism is then a productive set of premises each of which is in fact true, while a dialectical syllogism is such a set not necessarily satisfying this restriction.[4] In the simplest case, a syllogism is a pair of premises from which a syllogistic conclusion can be inferred. More generally, a simple or compound syllogism is a set of two or more premises yielding a distinct syllogistic conclusion pairwise, that is, by taking the premises in pairs to yield intermediate conclusions which can be paired with further members of the set.

But Corcoran was right to emphasize the deductive character of syllogistic reasoning. Recognising that, first, a (simple) syllogism consists simply of a pair of premises, secondly, that the premises constitute a syllogism just when a suitable conclusion can be deduced from them (as assumptions) avoids the unnecessary dispute we find in, e.g., Łukasiewicz (1963, ¶4) and (1951, §8) and Kneale and Kneale (1962, pp. 80-1) as to whether a syllogism is a conditional proposition or an inference.[5] It is neither. But of course, if the premises do constitute a syllogism, then there is an associated valid inference and it can be expressed in a conditional with a conjunctive antecedent.[6]

[3] Rescher (1963, §(vi), p. 59). See also Duerlinger (1968) and Rescher (1965).
[4] See *Topics* 100a25-31.
[5] See also Duerlinger (1968, pp. 488-90) and Thom (1981, §2).
[6] See also Smiley (1973, p. 139).

So a syllogism, or at least the associated inference, is, by its very definition, valid, contrary to the modern view cited above from (Quine, 1962, p. 74).

Nonetheless, this still leaves important questions open. One of them concerns existential import. I've argued elsewhere (Read, 2015) that there is a coherent account of syllogistic propositions which satisfies all the relationships in the traditional square of opposition and at the same time allows the inclusion of empty and universal terms; moreover, that this was Aristotle's intention. On this interpretation, affirmative propositions are false if their subject is empty; the corresponding negative propositions are accordingly true on that same condition. Existence goes with quality, not with quantity. This interpretation becomes more plausible when particular negative propositions are expressed, following Aristotle's own form of words, as 'Not all S are P', or better, 'P does not belong to all S' (equivalently, 'P does not belong to some S'), rather than 'Some S is not P'.

2 Syllogistic Validity

Once we are clear about the truth-conditions of syllogistic propositions, we can start to consider the basis of validity in Aristotle's theory. The core theory of the assertoric syllogism is contained in *Prior Analytics* I 4-6. The syllogisms there all consist in two subject-predicate premises containing three terms, two extremes (or "outer" terms) and one middle term shared between the premises. The premise containing the predicate of the conclusion is called the major premise (and that term, the major term), that containing the subject of the conclusion the minor premise (and that term, the minor term). *Prior Analytics* I 4 describes the first figure, in which the middle term is subject of one premise and predicate of the other. Let us write 'PxS' to represent 'P belongs to x S', where 'x' is a: 'every', e: 'no', i: 'some' or o: 'not every', that is:[7]

PaS**:** 'P belongs to every S'

PeS**:** 'P belongs to no S'

PiS**:** 'P belongs to some S'

PoS**:** 'P does not belong to every S'

Then the form of the first figure is:[8] AxB
ByC

Syllogisms of the first figure are perfect because the middle term, B, links the premises immediately and evidently (Aristotle, 1938):

[7] I will leave aside so-called "indefinite" or "indeterminate" propositions since I read Aristotle as treating them not as a separate class or type of propositions but as indeterminately universal or particular, and so implicitly included in the fourfold classification.

[8] Note that 'A', 'B', 'C' here are schematic letters, not variables, as, e.g., Bochenski (1951, 1962) repeatedly claims. But Aristotle is concerned with form, contrary to Corcoran (1994, pp. 12-13). In fact, Aristotle's word for 'figure' is 'schema'.

> "Whenever, then, three terms are related to one another in such a way that the last is in the middle as a whole and the middle either is or is not in the first as in a whole, it is necessary for there to be a perfect syllogism with respect to the extremes ... It is also clear that all the syllogisms in this figure are perfect, for they all reach their conclusion through the initial assumptions." (25b32, 26b29-30)

We say that pairs are productive when a syllogistic conclusion follows.

Aristotle identifies four syllogisms in the first figure: the pairs aa, ea, ai and ei. If we include the strongest conclusion each yields, we obtain the four traditional forms; known by their traditional names, they are:[9]

$$\text{Barbara} \quad \frac{\begin{array}{c} AaB \\ BaC \end{array}}{AaC} \qquad \text{Celarent} \quad \frac{\begin{array}{c} AaB \\ BeC \end{array}}{AeC}$$

$$\text{Darii} \quad \frac{\begin{array}{c} AaB \\ BiC \end{array}}{AiC} \qquad \text{Ferio} \quad \frac{\begin{array}{c} AeB \\ BiC \end{array}}{AoC}$$

These four moods are "evident" in virtue of what is traditionally called the *dictum de omni et nullo*:

> "For one thing to be in another as in a whole is the same as for the other to be predicated of all of the first. We speak of 'being predicated of all' when nothing can be found of the subject of which the other will not be said, and the same account holds for 'of none'." (24b28-31)

For example, he shows how the pairs *ai* and *eo* are productive and derives the strongest conclusion:

> "For let A belong to every B and B to some C. Now if 'being predicated of all' is what was said at the beginning, it is necessary for A to belong to some C. And if A belongs to no B and B belongs to some C, it is necessary for A not to belong to some C. For it was also defined what we mean by 'of none'." (26a24-26)

Thus the perfect syllogisms are, we might say, analytically valid, valid in virtue of the meaning of the logical terms in them, namely, 'all', 'no', 'some'

[9] The names of the moods in the medieval mnemonic are (see, e.g., Peter of Spain, 2014, p. 191):

> Barbara Celarent Darii Ferio Baralipton
> Celantes Dabitis Fapesmo Frisesomorum;
> Cesare Camestres Festino Baroco; Darapti
> Felapton Disamis Datisi Bocardo Ferison.

and ‘not’. They are self-evident, and do not need any further or more elaborate demonstration that they are productive. Once recognised, they will become themselves rules of inference whereby the validity of further syllogisms (in the second and third figures) is demonstrated.

2.1 Invalidity in the First Figure

There are 16 possible combinations of syllogistic premises in each figure (restricting ourselves just to particular and universal premises). Aristotle has established that four of these combinations yield a valid conclusion in the first figure, namely, *aa*, *ea*, *ai* and *ei*. In fact, he observes, each of the four types of proposition, *a*, *e*, *i* and *o*, can be established by a first-figure syllogism. He proceeds to show that each of the other 12 combinations does not yield a valid conclusion, and so is not a (valid, first-figure) syllogism.

His method is the method of counterexamples:[10] he specifies substituends for A, B and C in each pair of premises such that, first, the premises are true as well as AaC, then substituends making the premises true as well as AeC. Since the premises are thus consistent with AaC, that means that AoC cannot follow from the premises, and since they are consistent with AeC, neither can AiC follow. Consequently, neither can AeC follow (or its subaltern AoC would follow), nor can AaC follow (or its subaltern AiC would too).

Thus, Aristotle takes syllogistic validity to be formal. In fact, he does more than this. Many authors have been puzzled to determine what is the actual basis of syllogistic validity. It might appear that all validity is based on the perfect syllogisms to which all others are reduced (as we will see below). But the basis of the validity of the perfect syllogisms is not their perfection: that explains their self-evidence, as described at 26b29, but not their validity. Rather their validity consists in the lack of any counterexample. Thus Aristotle adopts what Etchemendy (1990) calls an interpretational, as opposed to a representational, account of validity, as found in Bolzano and Tarski.

Let’s look at a couple of counterexamples to first-figure invalidity. First, consider the pair *ae*, that is, AaB, BeC:

- For A, B, C take the triple ‘animal’, ‘human’, ‘horse’: ‘Every human is an animal’, ‘No human is a horse’ and ‘Every horse is an animal’ are all true (26b25)
- Now for A, B, C take the triple ‘animal’, ‘human’, ‘stone’: this time, ‘Every human is an animal’, ‘No human is a stone’ and ‘No stone is an animal’ are all true (26b26)

Thus both AaC and AeC are consistent with AaB and BeC. So, by the argument above, no syllogistic conclusion follows in the first figure from the pair *ae*. Note that Aristotle does not suppose that no horse is white, say, but

[10] Corcoran (1974, p. 105) calls Aristotle’s method that of “contrasting instances”.

takes substituends for A, B and C such that the premises are (actually) true and the conclusion false. For example, take the pair of premises 'Every white thing is coloured, No horse is white'. To show that it does not follow that not every horse is coloured, one might postulate a possible world in which no horse is white, but all are, say, black. Then the premises are made true, but every horse is still coloured. This is representational semantics, taking the basis of validity to be the impossibility of the premises being true and the conclusion false. In constrast, what Aristotle does is reduce the intuitive or representational account of invalidity, of the failure of the premises to necessitate the conclusion, to the interpretational account, the existence of a counter-instance.[11] Moreover, he does this not only for the assertoric syllogisms in *Prior Analytics* I 4-6, but also for the modal syllogisms in I 9-22. Note, however, that when he writes, e.g.,

> "Nothing prevents one from choosing an A such that C may belong to all of it," (30b30)

he is not claiming that the conclusion might be false, as in representational semantics, but that the conclusion is false, that is, that its contradictory, 'C possibly does not belong to every A', is true. For this is the contradictory of 'C necessarily belongs to every A'. So once again, what Aristotle does is to provide a substitution-instance where the premises and the contradictory of the putative conclusion are in fact true.

Now take any pair of particular premises, that is, *ii*, *io*, *oi*, *oo*. A similar pair of substitutions will show that no syllogistic conclusion follows:

- For A, B, C take the triple 'animal', 'white', horse': some white things are animals and some aren't; some horses are white and some aren't; but every horse is an animal
- Now for A, B, C take the triple 'animal', 'white', 'stone': some white things are animals and some aren't; some stones are white and some aren't; but no stone is an animal

So nothing follows in the first figure from two particular premises.

Aristotle proceeds systematically through the remaining seven pairs of premises, producing substituends for A, B, C to show that none of these pairs yields any i or o conclusion, and hence no a or e conclusion can follow.

2.2 Validity in the Second, or Middle Figure

The form of the second figure is: $\begin{array}{l} MxN \\ MyX \end{array}$

that is, the middle term is predicate in both premises. Aristotle identifies four more valid syllogisms in the second figure. Again showing the strongest conclusion that can be drawn, we have:

[11] Corcoran (1974, p. 103) calls this "one-world semantics".

$$\text{Cesare}\quad \frac{\begin{array}{c}MeN\\ MaX\end{array}}{NeX} \qquad \text{Camestres}\quad \frac{\begin{array}{c}MaN\\ MeX\end{array}}{NeX}$$

$$\text{Festino}\quad \frac{\begin{array}{c}MeN\\ MiX\end{array}}{NoX} \qquad \text{Baroco}\quad \frac{\begin{array}{c}MaN\\ MoX\end{array}}{NoX}$$

However, he describes these syllogisms all as imperfect, that is, as not self-evident. Referring to Cesare and Camestres, he says:

> "It is evident, then, that a syllogism comes about when the terms are so related, but not a perfect syllogism, for the necessity is brought to perfection not only from the initial assumptions, but from others as well." (27a16-19)

Aristotle employs three methods to establish (or to perfect—see Corcoran, 1974, p. 109) the imperfect syllogisms. The main method he calls "ostensive", and contrasts with "hypothetical". Although Corcoran (1974, p. 89) was right to call Aristotle's methods of proof "natural deduction" methods, that is, using rules to derive a conclusion from certain premises, he mischaracterizes the essential feature of such systems. It is not just that in such systems "rules predominate" over axioms (though they do). They also predominate in sequent calculus systems, but those are not natural deduction systems. What characterizes natural deduction is that one proceeds from assumptions to conclusion.[12] Accordingly, in ostensive proof, Aristotle assumes the premises of the putative syllogism, then uses simple or accidental conversion to infer the premises of a first-figure syllogism, draws the first-figure conclusion, and then, if necessary, uses further conversions to obtain a second-figure conclusion.[13] Setting the proof out in the manner of Fitch (1952) follows Aristotle's text almost to the letter. For example, here is his proof of Cesare, by reduction to Celarent:[14]

[12] See, e.g., Jaśkowski (1934, p. 5) and Gentzen (1969, p. 75).

[13] The medieval mnemonic uses certain consonants, following the vowels, to record the moves needed to demonstrate the imperfect moods:

- the initial letter (A,B,C,D) records which perfect mood will be used
- 's' following a vowel marks simple conversion
- 'p' following a vowel marks accidental, or partial, conversion (i.e., *per accidens*)
- 'm' tells us to invert the order of the premises
- 'c' following a vowel marks a proof using *reductio per impossibile* on that premise.

See Peter of Spain (2014, IV 13). William of Sherwood (1966, p. 67) gives a slightly different, and perhaps muddled account of the mnemonic.

[14] See also Corcoran (1974, p. 111) and Barnes (1997, p. 70). We could also represent the proof in tree form (see von Plato, 2016):

$$\dfrac{\dfrac{MeN}{NeM}\text{ Simple Conversion} \qquad MaX}{NeX}\text{ Celarent}$$

MeN	Premise
MaX	Premise
NeM	Simple conversion
MaX	Repetition
NeX	Celarent

He writes:

> "For let M be predicated of no N and of all X. Now since the privative premiss converts, N will belong to no M; but it was assumed that M belongs to all X, so that N will belong to no X—this was proved before." (27a6-9)

2.3 *Reductio per Impossibile*

The ostensive method also shows the validity of Camestres and Festino. But, Aristotle observes, it cannot be used to show the validity of Baroco. So he uses a different method, that of *reductio per impossibile*, which he describes as a special case of hypothetical proof. This is a further feature of so-called "natural deduction": that assumptions may be discharged in the course of a deduction. Such discharged assumptions are (temporary) hypotheses, made solely "for the sake of proof", or "for the purpose of reasoning" (Corcoran, 1974, p. 70).

As with ostensive proof, he starts by assuming the premises of the syllogism whose validity needs to be demonstrated. But then he makes a further assumption, taking as hypothesis the contradictory of the putative conclusion. For example, to demonstrate Baroco:

MaN		Premise
MoX		Premise
	NaX	Hyp
	MaN	Reiteration
	NaX	Repetition
	MaX	Barbara
	MoX	Reiteration
NoX		*Reductio per impossibile*

He writes:

> "Again, if M belongs to all N but does not belong to some X, it is necessary for N not to belong to some X. For if it belongs to all X and M is predicated of every N, it is necessary for M to belong to every X. But it was assumed that it did not belong to some. And if M belongs to every N but not to every X, there will be a syllogism to the effect that N does not belong to all X; the proof is the same." (27a36-b2)

But the representation of the proof given in the text in Fitch style seems better to accord with Aristotle's reasoning, as we will see.

Contrary to what Łukasiewicz (1951, §18) says (endorsed by Bochenski, 1962, pp. 77-8), this is a valid deduction of the conclusion NoX, by deducing MaX by Barbara, contradicting the minor premise MoX, as Corcoran (1994, p. 12) notes. Aristotle's method of reduction is indeed a so-called "natural deduction" method of proof by deduction from assumptions.

The passage suggests that although Aristotle distinguishes 'N does not belong to some X' from 'N does not belong to every X', he also thinks they are equivalent and play the same role in proof. We might say that 'some' "scopes out", that is, takes wide scope over 'not'.

Aristotle only appeals to *reductio per impossibile* twice, once in figure II, as above, and once in figure III. But he mentions several times, after an ostensive proof, that the syllogisms in question could also have been proved by *reductio per impossibile*. In his examples, he never embeds *reductio* in itself, or uses it as the main proof, but only as a hypothetical subordinate proof, as noted by Corcoran (1974, p. 116). Again, a proof by *reductio per impossibile* always concludes with that step. So he seems not to conceive of it as a general method of proof, but only restricted to the establishment of syllogistic conclusions. Note that the subproof in a *reductio* proof need only conclude in contraries (though often, as above, they are in fact contradictories). But the assumption for *reductio* must, of course, be the contradictory of the ultimate conclusion to be proved.

2.4 Invalidity in the Second Figure

Invalidity by counterinstance proceeds in the second figure as for the first, by giving triples of substituends for M, N, X. The only point of novelty arises with the pair *eo*, that is MeN, MoX:

> "Terms for not belonging: black, snow, animal. For belonging to all one cannot find terms if M belongs to some of the X, but not to others. For if N belongs to all X and M to no N, then M will belong to no X; but it was assumed that it did belong to some. It is not possible, then, to find terms in this way, and one must prove the point from indeterminacy. For since it is true that M does not belong to some X even if it belongs to none, and there was no syllogism when it belonged to none, it is evident that there will not be one in this case either." (27b15-23)

Thus to show that NiX, and consequently NaX, does not follow from MeN and MoX we note that 'No snow is black', 'Not every animal is black' and 'No animal is snow' are all true. But to show that NoX, and so also NeX, does not follow from the same pair, MeN and MoX, Aristotle appeals to the fact that that he has already shown that NoX does not even follow from the stronger premises MeN and MeX, by citing the triple 'line', 'animal', 'human' (27a21): 'No animal is a line', 'No human is a line' and 'Every human is an animal' are all true.

2.5 Validity in the Third, or Last Figure

The form of the last figure is: $\begin{array}{l} PxS \\ RyS \end{array}$

that is, the middle term is subject in both premises. Aristotle identifies a further six valid syllogisms in the third figure:

$$\text{Darapti} \quad \begin{array}{c} PaS \\ RaS \\ \hline PiR \end{array} \qquad \text{Felapton} \quad \begin{array}{c} PeS \\ RaS \\ \hline PoR \end{array}$$

$$\text{Datisi} \quad \begin{array}{c} PaS \\ RiS \\ \hline PiR \end{array} \qquad \text{Disamis} \quad \begin{array}{c} PiS \\ RaS \\ \hline PiR \end{array}$$

$$\text{Bocardo} \quad \begin{array}{c} PoS \\ RaS \\ \hline PoR \end{array} \qquad \text{Ferison} \quad \begin{array}{c} PeS \\ RiS \\ \hline PoR \end{array}$$

Once again, all these syllogisms are imperfect, requiring establishment by some form of reduction.

Aristotle reduces five of the third figure moods to the first figure by the familiar ostensive method. For example, Disamis is proved as follows:

$$\begin{array}{|l l}
PiS & \text{Premise} \\
RaS & \text{Premise} \\
\hline
SiP & \text{Simple conversion} \\
RaS & \text{Repetition} \\
SiP & \text{Repetition} \\
RiP & \text{Darii} \\
PiR & \text{Simple conversion}
\end{array}$$

Aristotle writes:

> "For since the affirmative premise converts, S will belong to some P, so that since R belongs to all S and S to some P, R will belong to some P and hence P will belong to some R." (28b8-11)

Once again, ostension fails in the case of Bocardo, which requires proof by *reductio per impossibile* (see Corcoran, 1974, p. 111):

$$\begin{array}{|l l l}
PoS & & \text{Premise} \\
RaS & & \text{Premise} \\
\hline
 & \begin{array}{|l} PaR \\ \hline RaS \\ PaS \\ PoS \end{array} & \begin{array}{l} \text{Hyp} \\ \text{Reiteration} \\ \text{Barbara} \\ \text{Reiteration} \end{array} \\
PoR & & \textit{Reductio per impossibile}
\end{array}$$

As Aristotle says:

> "For if R belongs to all S but P does not belong to some S, it is necessary for P not to belong to some R. For if it belongs to all R and R belongs to all S, then P will also belong to all S; but it did not belong to all." (28b17-20)

2.6 Exposition, or *Ecthesis*

Aristotle introduces yet a third method of proof for three of the third-figure moods, that of *ecthesis*, or the expository syllogism. Note that all the third-figure conclusions are particular, simply requiring exhibition of an R which is or isn't P:

> "The demonstration [of Darapti] can also be carried out through the impossible or by setting out [*ecthesis*]. For if both terms belong to all S, and one chooses one of the Ss, say N, then both P and R will belong to it, so that P will belong to some R." (28a24-6)

Ross (1949, p. 311), commenting on this passage and following Einarson (1936, pp. 161-2), notes that Aristotle uses the term '*ecthesis*' in two senses, both for the general procedure of choosing the terms of a syllogism in order to formalize the argument, and in order to pick out "a particular instance of the class denoted by the middle term." Both uses seem to derive from their application in geometry. Einarson (1936, p. 156) shows in detail how the way the terms are set out in the basic mood Barbara matches the manner of reasoning about propositions found, e.g., in the *Sectio Canonis* (Barbera, 1991, pp. 118-21). Such analogy pervades Aristotle's formulation of syllogistic reasoning. But it is the second sense of '*ecthesis*' which underlies Aristotle's third method of proof, sketched in the proof of Darapti just cited.

Proclus (1970, p. 159) (see Friedlein, 1873, p. 203) famously enumerated the six parts to a Euclidean demonstration: "enunciation, exposition [*ecthesis*], specification, construction, proof, and conclusion." This is illustrated by Bos (1993, pp. 142, 156) for the case of Euclid's proof of Pythagoras' Theorem. It opens with the enunciation, or statement (*protasis*) of the theorem: "In right-angled triangles the square on the side subtending the right angle is equal to the squares on the side containing the right angle," and then proceeds directly to the exposition: "Let ABC be a right-angled triangle having the angle BAC right." That is, the exposition [*ecthesis*], or "setting out", takes an arbitrary case (here, right-angled triangle) and gives it a designation, 'ABC', named from its vertices. Euclid proceeds to show that the sum of the squares on the opposite sides satisfy Pythagoras' result, and so, generally, since ABC was just an argitrary right-angled triangle, that every such triangle has the Pythagorean property.

Aristotle's use of *ecthesis* can be seen in a mathematically simpler example, but one that is logically more subtle, in Euclid's very first Proposition

(Heath, 1908, p. 262): "On a given finite straight line to construct an equilateral triangle," that is, given a finite straight line, there is an equilateral triangle. Euclid starts with the *ecthesis*: "Let AB be the given finite straight line" (or better—cf. Heath (1920, p. 162)—"let AB be a given finite straight line"). Euclid then constructs an equilateral triangle on AB, thus showing that there is such a triangle.

Aristotle's sketch of a proof of Darapti follows this model. Suppose P and R both belong to S. Then, given an S, call it n, it follows that some R is P—that is the *protasis* or enunciation, i.e., what is to be proved. For if n is both P and S, something (namely, n) is both P and R (since every S is R), so some R is P. But what is the basis of that final step?

The medievals identified Aristotle's method of *ecthesis* as the "expository syllogism". But the epithet is misleading. Its basis is not syllogistic. Rather, they said, its basis is a principle found in Aristotle's *Sophistical Refutations* 6, at 168b32: "Things that are the same as one and the same thing are the same as one another."[15] Buridan writes:

> "Every affirmative syllogism holds by virtue of the principle 'whatever things are said to be numerically identical with one and the same thing, are also said to be identical between themselves ... [N]egative syllogisms ... are valid by virtue of that other principle, namely: 'whatever things are so related that one of them is said to be identical and the other is said to be not identical with one and numerically the same thing, they necessarily have to be said not to be identical with each other'."(Buridan, 2001, pp. 313, 315)[16]

The first of these is similar to the first of Euclid's Common Notions (Heath, 1908, p. 222): "Things which are equal to the same thing are also equal to one another." But for these principles to be valid, it is crucial that the subjects are identical in reference, that is, that the premises are singular propositions.

It has been claimed that Aristotle does not include singular propositions in his syllogistic theory, indeed, that it only includes universal and particular propositions.[17] I have argued against this claim elsewhere.[18] In *De Interpretatione* 7, he describes four classes of proposition, universal, particular, indefinite and singular. Indefinite propositions are indeterminately universal or particular (as noted in footnote 7 above), sometimes best interpreted as universal (e.g., 'Men are animals'), sometimes as particular (e.g., 'Men are white'). Singular propositions could be taken *sui generis*, thus forming a

[15]See also *Physics* 185b15-16, and Hamesse (1974, p. 140): *"Quaecumque uni et eidem sunt eadem, inter se sunt eadem."*

[16]"*Dico ergo quod omnes syllogismi affirmativi tenent per hoc principium 'Quaecumque dicuntur eadem uni et eidem in numero, illa sibi invicem dicuntur eadem' ... Nunc de syllogismis negativis dicendum est. Qui tenent per illud principium 'Quaecumque sic se habent quod uni et eidem in numero unum eorum dicitur idem et alterum non idem, necesse est inter se illa dici non idem'.* (Buridan, 2009, pp. 17, 19)

[17]E.g., Ross (1923, p. 30).

[18]See Read (2015, pp. 536-7).

hexagon of opposition with the others;[19] or as universal, as Aristotle seems to do at *Prior Analytics* II 27 (70a27), for example: Pittacus is good, Pittacus is a wise man, so (some) wise men are good. The argument is (he says) in the third figure, and so by Darapti, with the singular premises interpreted as universal, and the indefinite conclusion is particular. The premises must be universal, since they are of the same kind, and as Aristotle says at *Prior Analytics* I 24, nothing follows from two particular propositions.

There are, therefore, two distinctive moves in an ecthetic proof, distinct from, but supporting, syllogistic inference (just as conversion and *reductio per impossibile* are not themselves syllogistic inferences but support, that is, validate, syllogistic argument): *ecthesis* itself, that is, taking an arbitrary instance; and the principle of Expository Syllogism. But as Aristotle points out in *Sophistical Refutations* 6, the latter requires strict identity. Partial identity will not suffice:

> "Suppose that A and B are 'the same' as C *per accidens* [i.e., partially]—for both 'snow' and 'swan' are the same as something 'white'." (168b34)

But snow and swan are certainly not the same. The principle requires that A and B be wholly or strictly identical, and for that, the premises must be singular propositions.

But at the same time, the instances must be syllogistic propositions, so that we can apply syllogistic reasoning to them. In the proof of Darapti, knowing there is an S, given that PaS is true, we take a particular S which is P and call it n. We apply Barbara to this instance, San, introduced by *ecthesis*, which with the other premise, RaS, yields Ran. Thus we know that n is both P and R, and so we infer that something is both P and R. The conclusion here, both in its form, and in its assumptions, is free of reference to n, and so the term n is arbitrary and the inference is valid, as in Existential Instantiation.[20]

To emphasize the need to treat 'n' as arbitrary and so ensure that it cannot belong to the conclusion, let us indent the subproof involving it, not based on a hypothesis, but on *ecthesis*, and labelled with the arbitrary name, in this case, 'n'.[21] Then the proof runs:

[19] See, e.g, Czeżowski (1955).

[20] See Parsons (2014, pp. 24-29). On Existential Instantiation, see, e.g., Quine (1950, p. 96); but cf. Prawitz (1967) and Pelletier (1999).

[21] This follows Fitch's practice in Fitch (1952, ch. 5, see especially p. 131). We can liken 'n' to an arbitrary name, as Lemmon (1965, pp. 106-7) does. See also the comments by Smith (1982, p. 126).

$$
\begin{array}{l|l|l}
PaS & & \text{Premise} \\
RaS & & \text{Premise} \\
\hline
{}^{n} & San & \left.\begin{array}{l}\\ \\ \end{array}\right\} \text{Ecthesis} \\
 & Pan & \\
 & RaS & \text{Reiteration} \\
 & San & \text{Repetition} \\
 & Ran & \text{Barbara} \\
 & Pan & \text{Repetition} \\
 & Ran & \text{Repetition} \\
PiR & & \text{Expository Syllogism}
\end{array}
$$

In the case of Disamis, with premises PiS and RaS, we again need to take one of the Ss which is P—call it n:

$$
\begin{array}{l|l|l}
PiS & & \text{Premise} \\
RaS & & \text{Premise} \\
\hline
{}^{n} & Pan & \left.\begin{array}{l}\\ \\ \end{array}\right\} \text{Ecthesis} \\
 & San & \\
 & RaS & \text{Reiteration} \\
 & San & \text{Repetition} \\
 & Ran & \text{Barbara} \\
 & Pan & \text{Repetition} \\
 & Ran & \text{Repetition} \\
PiR & & \text{Expository Syllogism}
\end{array}
$$

That is, given PiS, we take one of the Ss which is P, call it n, show that R also belongs to n, hence concluding that some R is P. Aristotle notes this method for proving Disamis at 28b15, but without giving any details.

In fact, *ecthesis* and Expository Syllogism form a pair, giving the meaning of AiB: $AiB \Leftrightarrow Aan, Ban$ (where n is fresh, that is, arbitrary, and treating the RHS conjunctively).[22] From AiB we can infer Aan and Ban by *ecthesis*, and from Aan and Ban we can infer AiB by Expository Syllogism.

Aristotle also suggests using *ecthesis* to prove Bocardo:

> "If R belongs to all S but P does not belong to some S, it is necessary for P not to belong to some R ... (This can also be proved without reduction to the impossible if one chooses one of the Ss to which P does not belong.)" (28b17-21)

The negative version of the rule for *ecthesis* will permit the inference of Aen and Ban from AoB. However, one might worry about the validity of this rule, since by the interpretation in Read (2015), the major premise, AoB,

[22] It was not until the twelfth century that a conjunctive proposition was even recognised as such. Martin (2012, pp. 295-8) notes that Boethius denies that conjunctive constructions produce single propositions (rather than a complex of propositions), and cites Abelard's recognition of the conjunctive proposition (*propositio copulativa*) as "a fundamental turning point in the history of logic".

lacks existential import, and so is true if there is no B. In the case of Bocardo, however, we do know that the term is non-empty, for the second premise, RaS, is true only if there are Ss. Hence we safely can take one of the Ss which is not P.

Conversely, we can extend Expository Syllogism to its negative version, as in the quotation above from Buridan: $AoB \Leftrightarrow Aen, Ban$, provided there is a B (n fresh). That is, not only does *ecthesis* allow us to take some arbitrary n that is B and not A, but conversely, if there is such an n which is B and not A, then A and B are distinct. The proof of Bocardo then runs:

$$
\begin{array}{|r|ll}
\multicolumn{2}{|l}{PoS} & \text{Premise} \\
\multicolumn{2}{|l}{RaS} & \text{Premise} \\ \cline{1-1}
{}^{n} & San & \rbrace\ \text{Ecthesis} \\
 & Pen & \rbrace\ \text{Ecthesis} \\
 & RaS & \text{Reiteration} \\
 & San & \text{Repetition} \\
 & Ran & \text{Barbara} \\
 & Pen & \text{Repetition} \\
 & Ran & \text{Repetition} \\
\multicolumn{2}{|l}{PoR} & \text{Expository Syllogism}
\end{array}
$$

As Aristotle says, we choose an S, namely n, which is not P. Then R must belong to n, since it belongs to every S, and so P cannot belong to every R.

However, we do not have the same guarantee of the non-emptiness of the term set out in the case of Baroco:

$$
\begin{array}{c}
MaN \\
MoX \\ \hline
NoX
\end{array}
$$

What can we do? Smith (1982, p. 117) notes that Aristotle himself does not mention *ecthesis* in connection with assertoric Baroco, despite appealing to it in the case of modal Baroco (*Prior Analytics* I 8, 30a4-14). Although the argument by *ecthesis* can be made to work, we need to deal with the case where X is empty separately, and perhaps this is why Aristotle does not suggest *ecthesis* here. For if X is empty, NoX is true for the same reason that MoX is; while if X is not empty, we can take an X, call it c, which is not M: MaN and Mec yield Nec by Camestres, and Xac and Nec yield the conclusion NoX by Expository Syllogism.

In fact, as Thom (1981, ch. X) observes, it is possible to demonstrate all the valid syllogisms by the expository method of *ecthesis*, including the perfect moods of the first figure. Moreover, in *Prior Analytics* I 2, Aristotle uses *ecthesis* to prove the conversion of E-propositions: That is,

AeB			Premise
	BiA		Hyp
		c Bac	Ecthesis
		Aac	
		Bac	Rep
	AiB		Expository Syllogism
	AeB		Reiteration
BeA			*Reductio per impossibile*

> "Now if A belongs to none of the Bs, then neither will B belong to any of the As. For if it does belong to some, for example, to C, it will not be true that A belongs to none of the Bs, since C is one of the Bs." (25a15-17)

Supposing that some A is B, take an A which is B, call it c, then we know both that c is A and that c is B, so some B is A, contradicting the assumption that no B is A. Aristotle then uses E-conversion to establish the accidental conversion of A-propositions and the simple conversion of I-propositions—which is in fact a corollary of the proof, lying at its heart.

2.7 The Final Reduction

Finally, Aristotle adds a further twist to the ostensive method, reducing Darii and Ferio to Celarent, via the second figure:

> "But one can also reduce all syllogisms to the universal ones in the first figure ... One can also prove them through the second figure by reduction to the impossible. For example, if A belongs to every B and B to some C, then A belongs to some C. For if it belongs to none, but to every B, B will belong to no C; this we know from the second figure." (29b1, 8-12)

Thus Darii is reduced to Camestres (and so finally to Celarent), and similarly, Ferio is reduced to Cesare (and thence to Celarent too):

AaB		Premise
BiC		Premise
	AeC	Hyp
	AaB	Reiteration
	AeC	Repetition
	BeC	Camestres
	BiC	Reiteration
AiC		*Reductio*

AeB		Premise
BiC		Premise
	AaC	Hyp
	AeB	Reiteration
	AaC	Repetition
	BeC	Cesare
	BiC	Reiteration
AoC		*Reductio*

As Aristotle writes:

> "The demonstration will be similar in the case of the privative syllogism [i.e., Ferio]. For if A belongs to no B and B to some

> C, then A will not belong to some C. For if it belongs to every C but to no B, then B will belong to no C—this was the middle figure [i.e. Cesare]." (29b12-14)

Hence all valid syllogisms can be reduced to the universal perfect syllogisms, Barbara and Celarent, themselves valid by the *dictum de omni et nullo.*

3 The Adequacy of Aristotle's Theory

Thus in *Prior Analytics* I 4-6, Aristotle has shown the validity of 14 (simple) syllogisms. One might, nonetheless, wonder whether this list of complete. In fact, it is not, as Aristotle himself recognises in I 7. Of course, we've already seen that each syllogism can have more than one conclusion. For example, Barbara, inferring AaC from AaB and BaC, also warrants Barbari and Baralipton:

$$\begin{array}{c} AaB \\ BaC \\ \hline AiC \end{array} \qquad \begin{array}{c} AaB \\ BaC \\ \hline CiA \end{array}$$

Barbari follows from Barbara by subalternation, and Baralipton by accidental conversion. Neither of these introduces a new syllogism, that is, a new pair of premises yielding a valid conclusion, since the premises of both are *aa* in the first figure. Indeed, provided the conclusions of the 14 syllogisms already identified are the strongest possible direct conclusions (as in fact they are), drawing further consequences by conversion and subalternation adds no new syllogisms. However, Baralipton alerts us to the possibility of an indirect conclusion. Aristotle has shown by exhibition of counter-instances that no other pairs of premises yield a valid direct conclusion. But we should now ask what further indirect conclusions can be inferred, and indeed, whether further premise pairs, not yielding a valid direct conclusion, perhaps yield an indirect conclusion.

It's easy to show that in the second and third figures, indirect conclusions introduce no new syllogisms. For suppose $\begin{array}{c} MxN \\ MyX \\ \hline XzN \end{array}$ draws an indirect conclusion in Figure II. First, interchange the premises: $\begin{array}{c} MyX \\ MxN \\ \hline XzN \end{array}$

Now reletter, interchanging 'X' and 'N': $\begin{array}{c} MyN \\ MxX \\ \hline NzX \end{array}$

This draws a direct conclusion in the second figure. So either yxz is a valid second-figure mood, and so already counted, or yxz is invalid in the second figure, by a counterinstance already noted, and so the same will be true for xyz. The same argument shows that there are no new syllogisms by drawing indirect conclusions in the third figure.

However, by this reasoning, two pairs of syllogistic premises in the second and third figures normally listed as distinct are arguably the same syllogism. Recall that Aristotle characterizes the second figure as predicating the middle term in both premises (26b34-37). But Cesare and Camestres differ only in the order of the premises and of the terms in the conclusion, so they each show equivalently that premises in which the same term is said to belong universally to one subject and to be excluded universally from the other are productive. They denote the same syllogism.[23] Similarly, he characterizes the third figure as that where the middle term is subject of both premises (28a10-12). But Disamis and Datisi differ only in the order of the premises and of the terms in the conclusion. So they each show that premises in which one predicate is said to belong universally to a subject and another to belong partially to it are productive, and so denote the same syllogism.

Nonetheless, there are two new syllogisms to be obtained by drawing indirect conclusions in the first figure, *viz*:

$$\text{Fapesmo}\quad \begin{array}{c} AaB \\ BeC \\ \hline CoA \end{array} \qquad\qquad \text{Frisesomorum}\quad \begin{array}{c} AiB \\ BeC \\ \hline CoA \end{array}$$

Note that *ae* and *ie* are new first-figure syllogisms, not included among the perfect moods. Aristotle recognises the validity of these syllogisms in I 7, but doesn't seem to appreciate that they are genuinely new:

AaB	Premise
BeC	Premise
CeB	Simple conversion
BiA	Conversion *per accidens*
CoA	Ferio

AiB	Premise
BeC	Premise
CeB	Simple conversion
BiA	Simple conversion
CoA	Ferio

> "It is also clear for all the figures that in those cases where no syllogism comes about, if both terms are positive or privative, nothing necessary comes about at all; but if one term is positive, the other privative and the privative is taken as universal, then a syllogism always comes about of the minor extreme in relation to the major. For example, if A belongs to all or some B and B to no C. For if the premisses are converted, it is necessary for C not to belong to some A. Similarly for the other figures; for a syllogism always comes about through conversion." (29a19-26)

3.1 The Fourth Figure

Many writers since Aristotle, arguably starting with Galen (see Rescher, 1965) claimed that there were in fact four figures in the syllogism. The reason was two-fold: they took a syllogism to consist of two premises *together*

[23] Cf. Buridan (2009, 2001, §5.4.2).

with a conclusion; and they took the order of the premises to matter. Hence what Aristotle took to be an indirect conclusion in the first figure was traditionally counted as a direct conclusion in the fourth figure. For example:

$$\text{Baralipton: } \frac{\begin{array}{c}PaM\\ MaS\end{array}}{PiS} \quad \text{becomes Bramantip: } \frac{\begin{array}{c}MaP\\ SaM\end{array}}{SiP}$$

$$\text{and Frisesomorum: } \frac{\begin{array}{c}PiM\\ MeS\end{array}}{PoS} \quad \text{becomes Fresison: } \frac{\begin{array}{c}MeP\\ SiM\end{array}}{SoP}$$

This doesn't yield any further valid syllogisms, but conceptualizes the syllogism differently. As Aristotle remarked (I 23):

> "Now if it necessary to assume something that is common in relation to both [extremes], and this is possible in three ways (for either one predicates A of C and C of B, or C of both, or both of C), and those form the three figures we have mentioned, it is evident that every syllogism will necessarily come about in one of those figures." (41a13-18)

Buridan (2015, III i 2) agreed:

> "But it should be noted that the fourth figure differs from the first only in the transposition of the premises, and that transposition does not permit inferring another conclusion or prevent that inference, but affects whether the conclusion inferred is diret only when in the first figure and indirect in the fourth and vice versa ... From this it is clear that once the first figure has been explained it will be superfuluous to explain the fourth; so Aristotle does not mention it."

3.2 The Total Number of Assertoric Syllogisms

Thus the total number of two-premise assertoric syllogisms according to Aristotle is 16. Some, e.g., Al-Farabi focussed exclusively on those listed in *Prior Analytics* I 4-6:

> "The [types of] categorical syllogisms are fourteen in number ... This completes the entire collection of categorical syllogisms." (Rescher, 1963, pp. 60, 73)

But this omits *ae* and *ie* in the first figure, which, we have seen, Aristotle specifically mentions in *Prior Analytics* I 7.[24]

Theophrastus is said to have added the five indirect moods to the first figure that we find in the medieval mnemonic: Baralipton, Celantes, Dabitis,

[24] Barnes (1975, p. 65) also omits the two novel imperfect first-figure moods.

Fapesmo, Frisesomorum, making a total of 19.[25] But this includes Baralipton, Celantes and Dabitis, which just draw new conclusions from *aa*, *ea* and *ai* in Figure I, already included. This slip was observed by Buridan:

> "In the first figure in addition to the four moods concluding directly ... Aristotle describes only two other moods that ... conclude indirectly, namely, Fapesmo and Frisesomorum ... Nor did he list Baralipton, Celantes and Dabitis in addition to Barbara, Celarent and Darii, since according to the definition they do not differ from them." (Buridan, 2015, III i 4, pp. 123-4)

Galen and Porphyry are reported to have split Darapti into two moods, Darapti and Daraptis, hence giving a total of 20.[26] But again, Daraptis is not new, being derivable from Darapti by simple conversion, showing that the premises *aa* in figure III "have more than one conclusion".

Traditionally, the 19 Theophrastian moods listed in the medieval mnemonic were augmented by the weakened or "subalternate" moods Barbari and Celaront (in Figure I) and Cesaro and Camestrop (in Figure II)—the full 24 usually listed as 6 valid moods in each of four figures.[27] But again, these just draw further conclusions from syllogistic pairs already recognised.

Buridan includes O-propositions of non-normal form (where the predicate precedes the negation, e.g., 'Some S some P is not') in order to permit conversion of O-propositions (see Read, 2016, p. 460), and so lists 8 syllogisms in the first two figures and 9 in the third figure, 25 in all.[28] He also distinguishes syllogisms with the same premises in different order. But this changes the rules, adding non-normal conclusions.

In Appendix 2 to Barnes et al. (1991),[29] the total reaches 35, adding Camestre, Faresmo, Cesares, Cesaros and Firesmo (in Figure II) and Daraptis, Fapemo, Datisis, Disami and Frisemo (in Figure III). But these are just reletterings of known syllogisms, as shown above.

What Aristotle didn't do in I 7, and needs to be done, is to show that no further indirect conclusions can be drawn from pairs of premises which yield no direct conclusion. There are none in the second and third figures, since indirect conclusions reduce to direct conclusions by interchanging the premises, as we have seen. To show that there are no further indirect conclusions in the first figure from the remaining 11 combinations of premises is straightforward.

Recall the earlier claim that Aristotle's account of validity was interpretational. We can invoke the so-called "squeezing argument" of Kreisel (1967), employed recently by Andrade-Lotero and Dutilh Novaes (2012), to

[25] See, e.g., Kneale and Kneale (1962, p. 100), Barnes et al. (1991, pp. 185-6) and Londey and Johanson (1987, §XIV).

[26] See Alexander's comments in Barnes et al. (1991, 6.2, p. 168) and Boethius' in Thomsen Thörnqvist (2008, pp. 52, 65).

[27] See, e.g., Kneale and Kneale (1962, pp. 74-5) and Bochenski (1951, 9 C) and (1962, pp. 71-2).

[28] Buridan (2015, III i 4, conclusion 8).

[29] See also Barnes et al. (1991, p. 136 footnote 157).

make the point. First, note that Aristotle's method of reduction is sound, that is, whenever there is a reduction to a perfect syllogism in the first figure there is no countermodel: the premises intuitively necessitate the conclusion. Moreover, it is complete: we have just noted that if there is no reduction then there is a counter-instance. Finally, any counter-instance is intuitively a countermodel. Hence there is no counter-instance if and only if there is no countermodel and the premises necessitate the conclusion. Aristotle reduces the intuitive notion of necessitation to the absence of a counter-instance.

Nonetheless, as we have seen, Aristotle overstated the number of distinct syllogisms by distinguishing Cesare from Camesteres, and Disamis from Datisi. Accordingly, there are just 14 distinct Aristotelian assertoric syllogisms, 6 in the first figure (4 direct and 2 indirect), 3 in the second, or middle figure, and 5 in the third and last.

4 Conclusion

My aim in this paper has been to correct misreadings of Aristotle, or at least, misleading accounts of what Aristotle did, e.g., (Smith, 2017; Lagerlund, 2016), and describe acurately the remarkable accoung of deduction which Aristotle constructed in the first few chapters of his *Prior Analytics*. Syllogistic propositions can be particular or universal. Aristotle treats singular propositions as universal, and so-called indeterminate propositions are taken by him as indeterminately universal or particular. Existential commitment goes with quality, not quantity, thus satisfying all the demands of the Square of Opposition: O-propositions can be expressed either as 'P does not belong to every S' or as 'P does not belong to some S', and are true if there is no S (when the corresponding A-proposition is accordingly false). Aristotelian syllogisms are, at their simplest, pairs of syllogistic premises yielding a valid conclusion. There are just three Aristotelian syllogistic figures, depending whether the middle term is subject of one premise and predicate of the other, predicate of both, or subject of both. Such pairs of premises constitute syllogisms just when they yield a syllogistic conclusion, that is, when there is no counterinstance among syllogistic propositions. There are, in total, just 14 pairs of such premises which yield a syllogistic conclusion, that is, there are 14 syllogistic pairs. Their validity can be established either by invoking the meaning of the logical expressions given by the *dictum de omni et nullo*, or by one of Aristotle's methods: by ostensive or hypothetical proof or by *ecthesis*. But they are not made valid by that reduction and proof, which serves rather to demonstrate their validity. The basis of their validity is the lack of any counterexample, that is, the absence of any terms that can be substituted in the schema that will make the premises true and any putative conclusion false.

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