‘Everything true will be false’: Paul of Venice’s two solutions to the insolubles*

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Abstract

In his *Quadratura*, Paul of Venice considers a sophism involving time and tense which appears to show that there is a valid inference which is also invalid. His argument runs as follows: consider this inference concerning some proposition A: A will signify only that everything true will be false, so A will be false. Call this inference B. Then B is valid because the opposite of its conclusion is incompatible with its premise. In accordance with the standard doctrine of ampliation, Paul takes A to be equivalent to ‘Everything that is or will be true will be false’. But he proceeds to argue that it is possible that B’s premise (‘A will signify only that everything true will be false’) could be true and its conclusion false, so B is not only valid but also invalid. Thus A and B are the basis of an insoluble.

In his *Logica Parva*, a self-confessedly elementary text aimed at students and not necessarily representing his own view, and in the *Quadratura*, Paul follows the solution found in the *Logica Oxoniensis*, which posits an implicit assertion of its own truth in insolubles like B. However, in the treatise on insolubles in his *Logica Magna*, Paul develops and endorses Swyneshed’s solution, which stood out against this “multiple-meanings” approach in offering a solution that took insolubles at face value, meaning no more than is explicit in what they say. On this account, insolubles imply their own falsity, and that is why, in so falsifying themselves, they are false. We consider how both types of solution apply to B and how they complement each other. On both, B is valid. But on one (following Swyneshed), B has true premises and false conclusion, and contradictories can be false together; on the other (following the *Logica Oxoniensis*), the counterexample is rejected.

Keywords: paradox, multiple-meanings, self-falsification, Heytesbury, Swyneshed.

1 A Temporal Paradox

Paul of Venice’s *Quadratura* is not a treatise on squaring the circle or on determining area in any way. Rather, it is an introductory logic text framed around four doubtful questions (*dubia*), each of which provokes fifty sophisms in a wide variety of philosophical areas, whose resolution allows Paul to present his students with a whole gamut of theories and arguments. The quadrature is a pun, since each of the fifty sophisms provoked by the

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four doubtful questions is resolved by four conclusions (or theses) and at least as many corollaries.[1] The first question asks whether the same inference can be both valid and invalid. The fifteenth chapter, invoking an insoluble as sophismatic puzzle (*Capitulum de insolubilibus*), argues as follows:

> "Regarding the ⟨first⟩ doubtful question, one argues like this:
>
> > This inference ⟨call it B⟩ is valid: A will signify only that everything true will be false, so A will be false; and this inference ⟨B⟩ is invalid. So the question is true.
>
> The argument is valid and I prove the premises, and first, the second premise: for there is a possible scenario in which the premise of inference ⟨B⟩ is true and its conclusion false, so inference ⟨B⟩ is not valid. I prove the premise: let us assume that as long as A will exist, A will signify only that everything true will be false, and ⟨that⟩ it will be the case that everything true will be false as long as A will exist. On this assumption, the premise ⟨of B⟩ is true according to the scenario; and I prove that the conclusion ⟨of B⟩ is false, because as long as A will exist, A will be true, so it will not be false. I prove the premise, because as long as A will exist, it will be the case that everything true will be false, and as long as A will exist, A will signify only that everything true will be false, therefore as long as A will exist, A will be true.
> But now I prove the first premise ⟨of the argument⟩, namely, that inference ⟨B⟩ is valid, because the opposite of the conclusion is incompatible with the premise, for these are incompatible:
> A will signify only that everything true will be false
> and
> A will not be false.
> Proof: I form this syllogism:
>
> > Everything true will be false, A will not be false, therefore A will not be true.
>
> This syllogism holds in Baroco, and as long as A will exist, it will be as the premises signify. Hence, as long as A will exist, it will be as the conclusion signifies. So A will exist and will be neither true nor false, which will be impossible. That is shown in this way: A will not be false, and A will exist, so A will be true as long as it will exist. Hence I argue like this:
>
> > Everything true will be false, A will be true, therefore A will be false.
>
> This inference holds, and as long as A will exist, it will be as the premises signify, therefore as long as A will exist, it will be as the conclusion signifies,

[1] "I will formulate four doubtful questions ... first, whether the same inference can be both valid and invalid; secondly, whether the same proposition can be both true and false; thirdly, whether disparate things are true of the same thing; fourthly, whether two incompatibles can be both true or both false" (*Quatuor formabo dubia ... primo utrum eadem consequentia sit bona et mala; secundo utrum eadem propositio sit vera et falsa; tertio utrum de eodem sint verificabilia disparata; quarto utrum duo repugnantia possint esse simul vera vel simul falsa*). Quotations from Paul's *Quadratura* are drawn from Appendix A to Paulus Venetus (20xx).

which is incompatible with the second conjunct of the conjunction composed of the premise and the opposite of the conclusion of the original ⟨inference B⟩."[2]

This needs careful analysis. At its heart is a self-referential proposition A, where we assume that A will signify only that everything true will be false. This is the premise of inference (*consequentia*) B, whose conclusion is that A will be false. Paul argues first that B is invalid and then that B is valid. It follows, he says, that our overall question is true: the same inference B can be both valid and invalid.

Let us first consider Paul's argument that B is valid. He claims that the opposite of its conclusion is incompatible with its premise. The reason is that Paul takes proposition A to be equivalent to 'Everything that is or will be true is false'. This was standard practice according to the medieval doctrine of ampliation, whereby a future-tense copula ampliates its subject from the present to the future. That is, any proposition of the form

Every S will be P

is equivalent to

Everything that is or will be S will be P,

and similarly for 'Some S will be P', 'No S will be P' and 'Not every S will be P'. For example, 'Everything white will be black' is true if everything which is now white will be black and everything which will at any future time be white, will (at some future time, before, at or after the other) be black. If anything that is now or at some future time will be white will not also at some future time be black, the proposition 'Everything white will be black' was taken to be false.[3]

[2] Paulus Venetus (20xx, Appendix A §1.15.1): *Quintodecimo principaliter ad questionem arguitur sic: ista consequentia est bona: a significabit precise quod quodlibet verum erit falsum, ergo a erit falsum: et hec eadem non valet, igitur questio vera.*

(1.15.1.1) *Tenet consequentia et antecedens probatur. Et primo pro secunda parte, nam casu possibili posito antecedens est verum et consequens falsum, igitur consequentia non valet. Antecedens probatur: et pono quod quamdiu a erit, a significabit precise quod quodlibet verum erit falsum, et ita erit quod quodlibet verum erit falsum quamdiu a erit. Isto posito antecedens est verum per casum, et quod consequens sit falsum probatur, nam quamdiu a erit a erit verum, igitur non erit falsum. Antecedens probatur: nam quamdiu a erit, ita erit quod quodlibet verum erit falsum, et quamdiu a erit a significabit precise quod quodlibet verum erit falsum, igitur quamdiu a erit a erit verum.*

(1.15.1.2) *Sed iam probatur prima pars antecedentis, videlicet quod illa consequentia est bona, quoniam oppositum consequentis repugnat antecedenti, hec enim repugnant a significabit precise quod quodlibet verum erit falsum et a non erit falsum. Probatur, et facio istam consequentiam: quodlibet verum erit falsum, a non erit falsum, igitur a non erit verum. Ista consequentia tenet in quarto secunde figure, et quamdiu a erit, erit ita sicut significatur per antecedens, igitur quamdiu erit a, erit ita sicut significatur per consequens, et ita a erit et non erit verum nec falsum, quid erit impossibile. Confirmatur sic: a non erit falsum, et a erit, igitur a erit verum quamdiu erit; arguo ergo sic: omne verum erit falsum, a erit verum, igitur a erit falsum. Ista consequentia est bona, et quamdiu a erit, erit ita sicut significatur per antecedens, igitur quamdiu a erit, erit ita sicut significatur per consequens, quod repugnat secunde parti principalis copulative facte ex antecedente et opposito consequentis.* On the meaning of '*significat precise*' (which is rendered here as 'signify only') see De Rijk (1982, p.177).

[3] See, e.g., Paulus Venetus (1984, ch.2 §8): "Every term standing in initial position with respect to a verb of future time or to its participle stands for that which is or which will be, e.g., in 'A man will be generated', 'man' stands for only that which is or which will be. Thus it signifies this proposition: 'Whoever is a man or whoever will be a man will be generated'." (p.161) See more generally, e.g., Kann (2016, §9.3.3).

The proof that B is valid is straightforward. Take the contradictory opposite of its conclusion, *viz* 'A will not be false'. So assuming A exists, A will be true.[4] But then according to the premise, everything that will be true will be false, so A will be false, contradicting the assumption that A will not be false. So as long as A will exist, and as part of B it does exist, the premise of B is incompatible with the contradictory of its conclusion, so B is valid.

This is not exactly how Paul presents the argument, but formulating the reasoning in this way brings out how remarkably similar it is to that in Yablo's paradox. Yablo's paradox explicitly invokes a sequence of propositions, each referring to all subsequent propositions in sequence:[5]

S_1: for all $k > 1$, S_k is untrue

S_2: for all $k > 2$, S_k is untrue

...

Suppose that for some n, S_n is true. Then for all $k > n, S_k$ is untrue, in particular, S_{n+1} is untrue and for all $k > n+1, S_k$ is untrue. But that is what S_{n+1} says, so S_{n+1} is (also) true. Accordingly, there can be no n such that S_n is true, that is, for all n, S_n is untrue. But Yablo points out that if every S_n is untrue, then "the sentences subsequent to any given S_n are all untrue, whence S_n is true after all." (p.252) Contradiction.

Yablo's intermediate conclusion that every S_n is untrue matches Paul's intermediate conclusion above that given that everything true will be false, A will not be false and so B is valid. So far, so good. But then Paul, like Yablo, realises that this conclusion will lead to paradox. For there is a counterexample to the validity of B: suppose that as long as A exists, it will continue to signify that everything true will be false, and that in fact it will continue to be the case that everything true will be false. Then A will continue to be true, but B's premise will be true and its conclusion false. So B is invalid. Paradox.

2 Theories of Insolubles from Bradwardine to Paul of Venice

Paul's *Quadratura* is preserved in three manuscripts and an incunabulum of 1493 (Paulus Venetus, 1493). The Vatican manuscripts (Vat.Lat.2133 and 2134) have a colophon to the fourth doubt, which reads:

> "Here end the sophistical Determinations with their tables composed by me, brother Paul of Venice of the Order of the Brother Hermits of St Augustine while I was teacher in the Convent at Padua and Bachelor of the same most Holy Order."[6]

[4]The medievals took propositions to be concrete, token utterances, which were neither true nor false if they were not uttered, and (for the most part) were either true or false if they did.

[5]Yablo (1993). Hanke (2014, §3.2) finds Yablo's paradox, or something very similar to it, in Lax (1508) and Celaya's *Insolubilia* (see Roure, 1962). Yablo's declared aim was to formulate paradox without the circularity of self-reference. Whether Yablo's formulation really does avoid self-reference is a matter of contention: see, e.g., Priest (1997) and Read (2006).

[6]BAV Vat.Lat.2133, f.141rb, 2134, f.161rb: *Expliciunt determinaciones sophistice cum tabulis earundem acte per me fratrem Paulum de Veneciis ordinis fratrum heremitarum sancti Augustini dum essem lector in conventu Paduano ac bacellarius eiusdem sacratissimi ordinis.*

The *Determinations* were an exercise which Paul completed for his Magister Artium at Padua between October 1399 and July 1400, after which he enrolled as a Bachelor of Theology.[7] In the *Quadratura* he draws on the solution to the insolubles which he had presented in his *Logica Parva*, composed shortly after his return to Italy from three years' study at the Augustinian Convent in Oxford, a solution derived from one he describes in his *Logica Magna* as that "which is now generally maintained by everyone."[8] But it is different from his favoured solution in the *Logica Magna*. We need to look back to Thomas Bradwardine's iconoclastic solution presented in his *Insolubilia* in the early 1320s to trace the origins of these three solutions.

Bradwardine's revolutionary idea was that insolubles, indeed, all propositions, might mean (*denotare*) or signify (*significare*) more than is immediate at first glance, or as Heytesbury would put it ten years later, more than the words commonly suggest (*verba communiter pretendunt*). Bradwardine proposed a principle to govern this multiplicity of overt and hidden meanings, his second postulate: every proposition signifies or means anything which follows from anything it signifies or means.[9] Then he was able to provide a rather clever and subtle proof that any proposition which signifies its own falsity also signifies its own truth. Since what seems to be characteristic of many insolubles is that they signify their own falsity, it follows that they are implicitly contradictory in signifying both (overtly) that they are false and (covertly) that they are true. So if truth requires, as seems most plausible if one thinks of multiple meanings as being essentially conjunctive, that everything a proposition signifies must obtain, it is impossible for everything these insolubles signify to obtain, and so something they signify fails to obtain and they are all false. Moreover, although they are false and they signify that they are false, it does not follow (as the standard argument goes) that they are true, for their being false is only part of what they signify, so though one might say they are partly true (true *secundum quid*), they are also partly false, and so are as a whole (*simpliciter*) false.

Most of Bradwardine's successors, in a flurry of treatments of the insolubles in the fourteenth century, took up the idea that there might be hidden, additional, meanings to propositions, but few were willing to endorse Bradwardine's second postulate and the proof using it to show that insolubles also signify their own truth. Bradwardine's fellow Calculator, William Heytesbury,[10] notoriously suggested that if someone presented an apparent insoluble saying that what it appeared to signify is all it signifies, one should reject it outright, whereas if it was presented without that stipulation, it should be accepted but that it is true should be denied, on the grounds that it must have some hidden meaning whch failed to obtain.[11] Heytesbury was able to act in this seemingly cavalier way because he framed his solution in the language of obligations, whereby the Respondent, to whom he was offering this advice, was only allowed to accept or reject the initial obligation (or *positum*) presented to him by his Opponent, and to grant, deny or doubt

[7] On the role of Determinations *de sophismatibus* in the Arts curriculum, see, e.g., Weisheipl (1971).

[8] Paulus Venetus (20xx, §2.12.1): *Duodecima opinio, que iam communiter ab omnibus sustinetur . . . .*

[9] Bradwardine (2010, §6.3): *Secunda est ista: quelibet propositio significat sive denotat ut nunc vel simpliciter omne quod sequitur ad istam ut nunc vel simpliciter.* On the justification for interpreting this as a closure condition, see Bradwardine (2010, 'Introduction' §5, p.17).

[10] Both Bradwardine and Heytesbury are more famous in the history of science as leading members of the Oxford Calculators than they are as logicians: see, e.g., Sylla (1982).

[11] See Heytesbury's second and third rules in Heytesbury (1979, §§49-50) (Pozzi, 1987, V §§3.06-3.071).

subsequent propositions which the Opponent proposed.[12] In particular, when challenged as to what this hidden meaning might be, on which the whole success of the solution turned, Heytesbury could invoke the framework of obligations theory to say that the Respondent was under no obligation to specify what it might be, but only to respond by granting or denying.[13] He was thus able to trace the narrow but consistent line of granting the insoluble but denying that it was true.

Unsurprisingly, many subsequent writers were frustrated by Heytesbury's caution, though they were happy to adopt his framework of obligations theory. They adapted his solution so that the hidden meaning was in fact specified as asserting the truth of the insoluble. That claim is false, since it is inconsistent with its overt meaning, and accordingly the insoluble is granted but its truth is denied and its falsehood granted. This solution became very popular, at least in Oxford, and was incorporated in most of the Oxford logic textbooks of the late fourteenth century, the *Logica Oxoniensis* as it has been dubbed by De Rijk (1977). Among later proponents were John of Holland and John Hunter (aka Venator).[14] We might call it the "modified Heytesbury solution". A possible link between Heytesbury's and these later treatises is that of Ralph Strode (see Spade, 1975, item LIII, pp.87-91), whose solution is explicitly based on combining Bradwardine's and Heytesbury's, and was composed in Oxford, probably in the late 1350s, and so before Holland's and Hunter's treatises.[15] Strode writes (f.10va):

> "Regarding this third opinion, namely, that of Heytesbury, in so far as it agrees with Thomas Bradwardine's opinion, I consider it to be true, namely, in that it claims that it is impossible for an insoluble proposition to signify only as the words commonly suggest. For example, supposing that the proposition 'There is a falsehood' is the only proposition, it is impossible that it only signifies that there is a falsehood. But in so far as it is claimed that, in the given scenario, it is not decided or stated by the Respondent what else that proposition signifies, or in what other way that proposition signifies, I do not consider it to be true."[16]

[12]On the theory of obligations and the ubiquity of its terminology in logical treatises of the fourteenth century, see, e.g., Dutilh Novaes and Uckelman (2016).

[13]"But if anyone asks what in this scenario the proposition uttered by Socrates [*viz* 'Socrates says a falsehood'] will signify other than that Socrates says a falsehood, I reply that the Respondent does not have to answer that question, because it follows from that scenario that his proposition signifies other than that Socrates says a falsehood, but the scenario does not specify what it is and so the Respondent does not have to give any further answer to the question" (*Si autem quaeratur in casu illo quid significabit illa propositio dicta a Sorte aliter quam quod Sortes dicit falsum, huic dicitur quod respondens non habebit illud seu illam quaestionem determinare, quia ex casu isto sequitur quod ista propositio aliter significet quam quod Sortes dicit falsum, sed casus ille non certificat quid illud sit et ideo non habet respondens quaesitum illud ulterius determinare* (Pozzi, 1987, V §3.072)). For an alternative translation, see Heytesbury (1979, §51).

[14]John of Holland's treatise on insolubles is edited in Bos (1985); John Hunter's in Pironet (2008, §15.8, pp.301-23).

[15]See Maierù (1982b, p.89). Whether it is earlier than the anonymous treatises cannot be determined until their dates are known.

[16]Ralph Strode, *Tractatus de Insolubilibus*, ms Erfurt Amploniana Q 255, f.10va: *Circa vero tertiam opinionem, videlicet ipsius Hentisberi, quantum ad hoc quod concordat cum opinione magistri Thome Bradwardijn, ipsam reputo esse veram, videlicet in hoc quod ponit quod inpossibile est propositionem insolubilem precise significare sicud verba illius communiter pretendunt. Verbi gratia, posito quod ista propositio 'falsum est' sit omnis propositio, tunc inpossibile est istam precise significare falsum esse. Sed quantum*

Strode proceeds in the Third Part of his treatise to apply his preferred solution to a range of insolubles. His response to the widely discussed scenario in which Socrates says only 'Socrates says a falsehood' (*Sortes dicit falsum*), labelled 'A', he writes:

> "Regarding the solution to this insoluble it should be realised that close attention be given whether in the presentation of the scenario it is supposed that the insoluble proposition signifies only as the words prima facie suggest, or it is supposed that they signify in that way but not with the addition of the adverb 'only'. If it was given in the first way, the scenario should in no way be accepted, because the scenario is impossible, as was clearly stated above. If it was given in the second way, then the scenario should be accepted, and generally so in every insoluble scenario. Furthermore, one should deny that A is true and grant that A is false and also that the proposition uttered by Socrates is false."[17]

He spells out the reason for those verdicts about the truth and falsehood of the insoluble in response to the next insoluble he considers, namely, where all and only those who speak the truth will receive a penny, and Socrates pipes up, 'Socrates will not receive a penny':

> "And so, just as ⟨in the case of⟩ the proposition 'Socrates says a falsehood', supposing that he says only that, the proposition is insoluble, the proposition 'Socrates will not receive a penny' is an insoluble proposition in the scenario described, and consequently in line with what was established earlier, it signifies itself to be false and itself to be true."[18]

It is this modified Heytesbury solution which Paul presents in the chapter on insolubles in his *Logica Parva*. It is explicitly directed at students, and does not necessarily represent his own view. He writes at the end of the chapter:

> "Notice that not everything I have said here, or in other treatises, have I said according to my own view, but partly according to the view of others, in order to enable young beginners to progress more easily."[19]

Similar solutions attributing an additional signification, or something similar, to insolubles were offered by John Buridan, Albert of Saxony, Gregory of Rimini, Peter of Ailly,

ad hoc quod ponitur quod, isto casu posito, non est determinandum uel dicendum a respondente quid aliud ista propositio significet uel qualiter aliter quam ista propositio significet, ipsam non reputo esse veram.

[17] *loc.cit.*: *Circa solutionem istius insolubilis est aduertendum quod diligenter est notandum in positione casus, vtrum ponatur quod talis propositio insolubilis precise significet sicud verba illius prima facie pretendunt, vel ponatur quod sic significet, non tamen addendo istud aduerbium precise. Si primo modo fiat positio, nullo modo est casus admittendus, quia casus est inpossibilis, sicud diffuse patet per superius dicta. Si vero secundo modo fiat positio, tunc admittendus est casus, et istud generale est in omni casu insolubili. Et vlterius dicitur negando a esse verum, et conceditur quod a est falsum et etiam quod propositio dicta a Sorte est falsa.*

[18] *ibid.*, f.10vb: *Et ideo, sicud ista propositio 'Sor dicit falsum', posito quod solam istam dicat, est propositio insolubilis, ita ista propositio 'Sor non habebit denarium' est propositio insolubilis, casu predicto posito, et per consequens secundum predeterminata significat se esse falsam et se esse veram.*

[19] Paulus Venetus (2002, p.150, amended against ms BAV, Vat.Lat. 5363, f.39rb): *Nota quod non quecumque fuit locutus hic, seu in ceteris tractatibus, ego dixi secundum intentionem propriam, sed partim secundum intentionem aliorum, ut iuvenes incipientes proficere facilius introducantur.* For an alternative translation, see Paulus Venetus (1984, pp.255-6).

Marsilius of Inghen and others. But some were unpersuaded.[20] Notable among them was Roger Swyneshed, another Calculator, writing in Oxford in the 1330s. His aim was to find a viable solution to the insolubles by taking them at face value, and his big idea was that insolubles falsify themselves—in an intuitive sense which he set out to make formal and precise. That is, for Swyneshed, the interesting characteristic of insolubles is that they imply their own falsehood. Indeed, that's usually the first leg of a proof of contradiction from them: first, we show that they are false, then feel forced to infer that they must also be true (since that's what they say). Swyneshed avoids this second leg of the paradox argument by broadening the definition of 'false': a proposition is false (he says) if either things are not as it signifies (in the normal *communiter pretendunt* sense of 'signifies') or they falsify themselves (in the sense that they imply their own falsehood). For example, 'This proposition is false' falsifies itself because from 'This proposition is false' we can immediately infer that it is false; 'Every proposition is false' faisifies itself in the sense that it implies that it itself, being a proposition, is false; 'What Socrates says is false' falsifies itself if it is the only proposition uttered by Socrates, since we can then infer that it is itself false. In general, a proposition is true if and only if things are as it signifies and it does not falsify itself.[21] So, given that 'This proposition is false' is false, since it falsifies itself, we cannot infer that it is true (on the grounds that things are as it signifies) since it does not meet the extra condition of not falsifying itself.

3 Paul's Two Solutions to the Temporal Paradox

As we noted in §2, Paul offers different solutions to the insolubles in different works. In the *Logica Parva* and the *Quadratura*, the solution he favours is the modified Heytesbury solution; in the *Logica Magna* and the *Sophismata Aurea*, it is Swyneshed's.

That Paul applies the modified Heytesbury solution to the insolubles in the *Quadratura* is clear from the second and third Conclusions which he sets out in preparing his response to the temporal insoluble we considered in §1. The second Conclusion states:

> "There is some proposition signifying principally purely predicatively which at some time will signify principally in a compound way. Nonetheless, there will be no change in it, nor will any new imposition be added to it."[22]

In proof, Paul claims that 'Every proposition is false' satisfies this claim, assuming that at some time it will be the only proposition, for its principal signification is (now) purely predicatively that every proposition is false. But, he says,

> ". . . when it will be the only proposition it will signify principally that every proposition is false and that it is true, just like other insolubles, whose

[20]Two others who resisted Bradwardine's innovation, and in fact defended earlier views against Bradwardine's criticisms, were Walter Segrave, defending restrictionism (*restrictio*), and John Dumbleton, defending cassationism (*cassatio*): see Spade (1975, items LXVIII and XXXVI, respectively).

[21]See Spade (1979, §§14-15, pp.185-6).

[22]Paulus Venetus (20xx, Appendix A §1.15.2.2): *Secunda conclusio est ista: aliqua est propositio significans solum cathegorice principaliter que aliquando significabit ypothetice principaliter, et tamen nulla in ea fiet mutatio, nec nova adveniet illi impositio.*

significations reflect wholly on themselves."[23]

The point is reiterated and elaborated in discussing his third Conclusion, namely:

> "It is possible for every proposition to be false and for 'Every proposition is false' to signify exactly that every proposition is false."[24]

Again, the relevant scenario is one where 'Every proposition is false' (call it A) is the only proposition. Then, he says,

> "I claim that in this scenario A signifies that every proposition is false and ⟨that⟩ A is true. This conjunctive significate is called the principal significate of A, although it is not the exact ⟨significate⟩, which is only the first part."[25]

That is, when A is the only proposition, it signifies conjunctively and principally that every proposition is false and that A is true, but its exact significate is that every proposition is false, as the third Conclusion claims. This response is clearly very different from Swyneshed's solution and belongs to the tradition started by Bradwardine where an insoluble has a further covert signification. But unlike Heytesbury himself, Paul commits himself squarely to the claim that the additional signification is that the insoluble itself is true, in the way we have seen that the modified Heytesbury solution does.

Paul's response to the temporal insoluble is to accept that B is valid, and to deny that there is any scenario in which its premise is true and conclusion false, as was claimed in the sophism. In particular, the scenario described in the sophism itself is impossible and so fails to show that B is invalid. Recall that the argument was that we could "assume that as long as A will exist, A will signify only that everything true will be false, and ⟨that⟩ it will be the case that everything true will be false as long as A will exist." It follows, it was claimed, that "on this assumption, the premise ⟨of B⟩ is true according to the scenario." Not so. For if A is indeed an insoluble, it will not signify only that everything true will be false, that is, what it standardly signifies, but it will also signify that it itself is true.

So is A an insoluble? In his *Logica Parva*, Paul defines an insoluble as a proposition signifying consequentially (*assertive significans*) its own falsehood, later distinguishing insolubles unconditionally (*insolubile simpliciter*) from insolubles (only) conditionally (*insolubile secundum quid*), where an insoluble unconditionally is one to which a scenario is attached which implies a contradiction if admitted.[26] But for the most part, he proceeds in this text by example. If we look back to pseudo-Heytesbury's treatise, we find Heytesbury's original definitions followed more closely:

[23] *loc.cit*: *... quando ipsa erit omnis propositio significabit principaliter quod omnis propositio est falsa, et quod ipsa est vera, quemadmodum et alia insolubilia, quorum significationes reflectuntur ad se totaliter.*

[24] 1.15.2.3: *Tertia conclusio est ista: possibile est omnem propositionem esse falsam, et hanc: omnis propositio est falsa, significare adequate omnem propositionem esse falsam.*

[25] *loc.cit*: *... quia dicitur in casu isto quod a significat omnem propositionem esse falsam, et a esse verum. Et hoc significatum copulativum dicitur principale significatum a, licet non adequatum sed solum prima pars.*

[26] Paulus Venetus (2002, p.128): *Insolubile est propositio se esse falsam assertive significans ... Insolubilium aliquod est insolubile simpliciter, aliquod vero secundum quid. Insolubile simpliciter est illud cui annectitur casus quo admisso sequitur contradictio.* For an alternative translation, see Paulus Venetus (1984, p.237). On the phrase '*significans assertive*' as meaning 'signifying consequentially', or 'implicitly', see Nuchelmans (1980, pp.45-46) and De Rijk (1982, pp.175-7).

"An insoluble proposition is one of which mention is made in some ⟨insoluble⟩ scenario which, if in that scenario it signifies only in that way, it would follow was true and false,"

where an insoluble scenario is one

"in which a proposition is mentioned which, if it signifies in that scenario only as the words suggest, it follows is true and false."[27]

Given that B is valid, then if A does signify only that everything true will be false, A will be false. So if the proposed scenario was possible, A would be both true and false, and so the scenario would be an insoluble scenario and A would be an insoluble. But in that case, A would not only signify that everything true will be false, but also that it itself is true. So the proposed scenario is impossible. Paul concludes this chapter of his *Quadratura* with the words:

"From this it is clear how to respond to the original argument, ⟨namely⟩ by granting this inference: 'A will signify only that everything true will be false, therefore A will be false', and as for the counter-instance, I do not accept the scenario, because it implies a contradiction, as has been clearly seen. Hence etc."[28]

leaving his readers to put the pieces together. The upshot is that B is valid, and the counterexample is rejected, for if A is true and everything true will be false, A will be an insoluble, and so will not signify only that everything true will be false, and so the premise of B will be false, just like its conclusion.

4 Paul's Preferred Solution

In his *Logica Magna*, however, Paul rejects Heytesbury's solution, and passes over the modified version in silence. That is odd, since his main criticism there of Heytesbury's solution is its reluctance to specify what the implicit signification of insolubles is.[29] In any case, having rejected Heytesbury's, together with fourteen other putative solutions, Paul adopts and adapts Swyneshed's solution and applies it at length to a range of insolubles. The temporal paradox is, however, not among them, so it is an interesting exercise to see how Swyneshed's, and Paul's, solution deals with it.

[27] Pironet (2008, p.290): *Est sciendum quod casus insolubilis est ille in quo fit mentio de aliqua propositione quae, si cum eodem casu significat praecise sicut verba illius praetendunt, sequitur eamdem esse veram et falsam … Sed propositio insolubilis est illa de qua fit mentio in aliqua casu quae, si cum eodem casu sic significet praecise, sequeretur ipsam esse veram et falsam.* Cf. Heytesbury's own text at (Pozzi, 1987, V §3.01-02)), translated at (Heytesbury, 1979, §44-45). Note that where some mss of Heytesbury's treatise read '*sequitur eam esse veram et eam esse falsam*' (it follows both that it is true and that it is false), others read '*ad eam esse veram sequitur eam esse falsam et econtra*' (from its being true it follows that it is false and vice versa): see, e.g., Strobino (2012, p.488 n.24). Given natural assumptions, these formulae are equivalent. On the sheer complexity of extant mss of Heytesbury's text, see Spade (1989).

[28] *Per hoc patet responsio ad argumentum principale concedendo illam consequentiam: a significabit precise quod quodlibet verum erit falsum, igitur a erit falsum, et ad improbationem non admitto casum, quia implicat contradictionem ut clare est ostensum. Quare etc.*

[29] See Paulus Venetus (20xx, §§1.12.3.1.2-1.12.3.2.3).

To start with, the definition of insoluble is different, and indeed, comes in two forms, a narrower and a broader one. Paul gives the narrower one in the second chapter of the treatise on insolubles in his *Logica Magna*:

> "An insoluble proposition is a proposition having reflection on itself wholly or partially implying its own falsity or that it is not itself true."[30]

In brief, insolubles are self-falsifying propositions. Paul comments that his definition excludes many propositions counted as insolubles by others, such as 'Socrates will not cross the bridge' and 'Plato will not have a penny', for he says, they do not have reflection on themselves. But he is not consistent here, for in the fifth chapter he includes them under what he calls "insolubles that don't appear at first glance to be insolubles" (*insolubilia que prima facie insolubilia non apparent*). It is in the eighth chapter that he comes to further cases that he believes only appear to be insolubles, such as 'This proposition is not known to you' and 'This is in doubt for you', which others would include as epistemic insolubles.[31]

Swyneshed himself gave a broader definition which included these epistemic paradoxes:

> "An insoluble as put forward is a proposition signifying principally as things are or other than things are ⟨which is⟩ relevant to inferring itself to be false or unknown or ⟨not⟩ believed, and so on."[32]

Paul himself is tempted to broaden his definition to include the epistemic insolubles, when, for example, he presents the fourth Conclusion in the *Logica Magna*:

> "There is a formally valid inference, known by you to be so, signifying ⟨exactly⟩ by the composition of its parts, where the premise is known by you, yet the conclusion is not known by you."[33]

The example he gives is what may be called the Inferential Knower Paradox:[34]

This is unknown to you, therefore this is unknown to you

where each occurrence of 'this' refers to the conclusion. For, he says, "the premise is known by you, because you know that the conclusion is not known, since it is an insoluble that implies that it itself is unknown."[35] Thus the idea in the broadening of the definition is to say that just as propositions which imply their own falsehood are self-falsifying and

[30] Paulus Venetus (20xx, §2.1.8): *Propositio insolubilis est propositio habens supra se reflexionem sue falsitatis aut se non esse veram, totaliter vel partialiter illativa.*

[31] See, e.g., Bradwardine (2010, ch.9) and Dumbleton in Bottin (1973).

[32] Spade (1979, §16): *Insolubile ad propositum est propositio significans principaliter sicut est vel aliter quam est pertinens ad inferendum se ipsam fore falsam vel nescitam vel ⟨non⟩ creditam, et sic de singulis.* ('*non*' is added for sense in that last clause following the edition in Pozzi, 1987, p.282.)

[33] Paulus Venetus (20xx, §2.3.4): *Quarta conclusio: aliqua consequentia est bona et formalis, scita a te esse talis, significans ⟨adequate⟩ ex compositione suarum partium, et antecedens est scitum a te et consequens non est scitum a te.*

[34] On the Knower Paradox, see, e.g., Sorensen (2018, §5.1). In Spade (1979, §§80-81), Swyneshed argues that 'This proposition is unknown' is unknown.

[35] Paulus Venetus (20xx, §2.3.4): *... antecedens est scitum a te, quia scis illud consequens non sciri, cum sit insolubile asserens se nesciri.*

so are false, so too propositions which imply they are not known are not known and those which imply they are not believed are not believed, and so on.

The fourth Conclusion, in its denial of logical omniscience, may seem attractive. It does indeed seem true that we can know the axioms of some theory, and its rules of inference, but not know all the consequences of those axioms. But that is more a matter of psychology than what lies behind the fourth Conclusion, which is a matter of logic. And that Conclusion is nothing like as dramatic as Paul's fifth Conclusion in the *Logica Magna*, which seems to undermine the whole idea of proof:

> "There are some formally valid inferences which signify exactly by the composition of their principal parts, where the premise is true and the conclusion false."[36]

This was Swyneshed's second iconoclastic Conclusion, and the example is the same:[37]

(*) This is false, therefore this is false,

where each occurrence of 'this' refers to the conclusion. For the conclusion falsifies itself, and the premise truly records this fact. When, in the chapter on 'Consequence' in the *Logica Magna*, Paul mentioned the Inferential Knower paradox, he anticipated the fourth Conclusion about insolubles, by including an important caveat in his Ninth Rule:

> "Suppose that a certain inference is valid, is known by you to be valid, is understood by you, and signifies primarily in accordance with the composition of its elements; suppose too that its premise is known by you, and that you know that what is false does not follow from anything that is true; then its conclusion is also known by you."[38]

That caveat ("you know that what is false does not follow from anything that is true") may have seemed anodyne at the time, but it means he can alert us to expect "more about this when we come to deal with the insolubles."[39] But he was not so careful when he stated his Third Rule in the chapter on 'Consequence':

> "If the premise of a valid inference which signifies primarily in accordance with the composition of its parts is true, then the conclusion is also true."[40]

(*) is a counterexample. As we have seen, the premise is true, the conclusion false. Moreover, the inference is valid, since the opposite of the conclusion is incompatible with the premise. Indeed, for Paul, it is formally valid, as he stated in the fifth Conclusion, for, he says,

[36]Paulus Venetus (20xx, §2.3.5): *Quinta conclusio: aliqua consequentia est bona et formalis significans adequate ex compositione suarum partium principalium et antecedens est verum et consequens falsum.*

[37]Spade (1979, §26, p.189): "This inference is formally valid: 'The conclusion of this inference is false, so the conclusion is false'." (*Haec consequentia est bona et formalis: Consequens illius consequentiae est falsum; igitur, consequens est falsum*)

[38]Paulus Venetus (1990, p.195): *Si aliqua est consequentia bona, scita a te esse bona et intellecta a te, significans primo iuxta compositionem suarum partium, et antecedens est scitum a te, sciendo quod ex nullo vero sequitur falsum, et consequens eiusdem est scitum a te.*

[39]Paul had added this comment at the end of his discussion of the Ninth Rule in the chapter on 'Consequence', Paulus Venetus (1990, p.200): *Sed de hoc magis in materia insolubilium.*

[40]Paulus Venetus (1990, p.140): *Tertia regula est ista: Si alicuius consequentiae bonae, significantis primo iuxta compositionem suarum partium, antecedens est verum, et consequens est verum.*

> “who would claim that these are compatible, ‘This is false’ and ‘This is not false’, referring to the same thing? Surely, no-one who wishes to avoid a greater absurdity.”[41]

If proof consists in validly inferring a conclusion successively from premises already proved, proof now fails, for if Paul is right, it allows us validly to infer prove falsehoods.

In the case of our temporal paradox, proposition A, as we have seen, does not imply its own falsehood, but its own future falsehood. For suppose everything true will be false, and suppose that A will not be false. Then everything that is or will be true will be false. So A is not true and will not be true, and so is and will be false. Contradiction. So A will be false. Thus, assuming that everything true will be false, A implies its own future falsehood. That is, if things are as A signifies, it will be false. Taking ‘insoluble’ in the narrow sense, A is not strictly an insoluble, but following Paul’s practice in the chapter of *Logica Magna* on merely apparent insolubles (that is, which appear to be insolubles but are not), Paul’s response would be that, just as “each proposition asserting that it itself is unknown is not known,”[42] and “a proposition asserting that it itself should be denied should be denied,”[43] so too A, asserting that it will itself be false, will be false.

Alternatively, Paul could follow Swyneshed, as he does in chapter 5 (‘On Propositions which are not Obviously Insolubles’), and include propositions like A which assert of themselves that they will be false as insolubles. For he there includes the example where all those who speak truly will receive a penny and Socrates says ‘I will not receive a penny’ as insolubles. He writes:

> “I grant that Socrates will not receive a penny and consequently that he says a falsehood. And then in reply to the argument [therefore it is not true that he will not receive a penny, and consequently, he will receive a penny], I deny the inference, since one should add in the premises that what Socrates says does not falsify itself, which I deny since [what Socrates says] falsifies itself.”[44]

Paul’s claim is that Socrates’ statement that he will not receive a penny is false not because he will receive a penny, but because it falsifies itself. And the proof that it falsifies itself is in the paradox. Assuming that Socrates will not receive a penny (that is, what Socrates said), it follows that he will, and so what he said was false.

Similarly, ‘Everything true will be false’ is false not necessarily because something true will not be false (though indeed, that would falsify it) but even when everything true will be false it will be false because it falsifies itself. Thus, whether we class the temporal paradox as an insoluble or not, it will be false if things are as it signifies; while if things are not as it signifies, then it is false. So either it is false, or it will be false. Thus Paul offers two diagnoses of the sophism: in the *Quadratura*, following the modified Heytesbury solution, his answer is that B is valid and the purported counterexample fails since if B’s conclusion were false, A would be an insoluble and so it would not signify only that

[41] Paulus Venetus (20xx, §2.3.5): *Quis enim diceret ista stare simul: hoc est falsum, et: hoc non est falsum, eodem demonstrato? Certe nullus maius volens vitare inconveniens.*

[42] Paulus Venetus (20xx, §8.1.1): *quelibet asserens se nesciri non scitur.*

[43] Paulus Venetus (20xx, §aliter ad 8.2): *propositio asserens se esse negandam est neganda.*

[44] Paulus Venetus (20xx, §ad 5.3.1): *Concedo quod sortes non habebit denarium et consequenter quod ipse dicit falsum. Et tunc ad argumentum negatur consequentia, quia oportet addere quod dictum sortis non falsificat se, quod negatur quia a falsificat se.*

everything true will be false, and so B's premise would also be false; while, though Paul does not discuss the temporal sophism in the *Logica Magna* or the *Sophismata Aurea*, his own solution, following Swyneshed's, lets us choose whether to include the sophism as an insoluble or not. But whichever we do, B is valid, since the contradictory of its conclusion is incompatible with its premise, even though its premise may be true and conclusion false.

Conclusion

Paul of Venice, writing at the end of the fourteenth century, presented two different solutions to the insolubles in his *Logica Parva* and *Logica Magna*. Between them, they serve to illustrate the major lines of approach to insolubles in the preceding century. Those two lines of approach divide between those, following Bradwardine, who aim to solve the insolubles by postulating a hidden, additional signification in insolubles; and those who attempt to solve them simply by taking them at face value. The former include William Heytesbury, Albert of Saxony, John Buridan, Gregory of Rimini, Pierre d'Ailly, John of Holland, John Hunter and many others. At Oxford, Heytesbury simplified Bradwardine's approach by postulating an additional signification, but denying that we need to speculate what it is. This led to the "modified Heytesbury solution", combining Heytesbury's incorporation of the theory of obligations with Bradwardine's claim that the hidden signification is of the insoluble's own truth. Among those rejecting Bradwardine's approach was Roger Swyneshed, who claimed that insolubles falsify themselves and so are false. Swyneshed's solution has three dramatic and themselves paradoxical consequences, including the claim that pairs of contradictories can both be false. Paul posed an intriguing sophism in his *Quadratura*, diagnosing it following the modified Heytesbury solution, following the standard Oxford curriculum for students, according to which propositions can have hidden signification of which speakers may be unaware. The sophism can also be solved by his preferred approach in the *Logica Magna*, following Swyneshed, on pain of accepting that a valid inference can have true premises and false conclusion.

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