“Self-Reference and Validity revisited”

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1. An argument is valid if its conclusion follows from its premises; it is invalid if it is possible for its premises to be true while its conclusion is false. How can we be certain of these claims?

(1) That the conclusion follows from the premises is a sufficient condition of validity because an argument is a piece of discourse which purports to deduce a conclusion from certain premises. Its success (its validity) is measured by its succeeding in that derivation. Of course, this condition is pretty vacuous until we give some account of the methods of deduction: that is the central task of logic. But (1) is nonetheless right.

(2) That the premises cannot be true while the conclusion is false is a necessary condition of validity because it is essential to the notion of validity of an argument that it guarantee to take one from truth to truth. It is for this reason that sustaining *modus ponens* is required of any connective expressing entailment which corresponds to valid argument.

But might there not appear a gap between (1) and (2)? Perhaps to give different necessary and sufficient conditions for validity will permit an argument which can neither be shown to be valid, for its conclusion cannot be deduced from its premises, nor shown to be invalid, for its conclusion could not be false while its premises were true. A natural way to prevent this situation arising is to take just one condition to be both necessary and sufficient for validity.

One such account of validity takes (2) to express both a necessary and a sufficient condition. I shall call it ‘the Classical Account of Validity’. It states that an argument is valid if and only if it is impossible for its premises to be true while its conclusion is false.

What is distinctive of the classical account is that it takes the impossibility of true premises and a false conclusion to be sufficient for validity. But can this be accepted? The paradoxes of strict implication are often put forward as a counterexample to this claim. But perhaps they just show that any argument whose premises cannot be true together or whose conclusion must be true is, often despite appearances, valid. To support the objection we need to produce an argument which is clearly invalid and yet has, for example, a necessarily true conclusion. For if its conclusion must be true, then it would indeed be impossible for its conclusion to be false jointly with the truth of its premises. So by the classical account a clearly invalid argument would be valid.

2. Such an argument was put forward by Pseudo-Scotus.[1] The epithet ‘Pseudo-Scotus’ derives from the fact that the treatise by which we know the author, though not by Scotus, was included in the collected works of John Duns Scotus in the seventeenth century.[2] This treatise is a commentary on the *Prior Analytics* of Aristotle. The collected works also contain a commentary on Aristotle’s *Posterior Analytics*. Neither commentary is by Scotus himself. So both are attributed to ‘Pseudo-Scotus’.

The following argument comes from the commentary on the *Prior Analytics.* It dates after 1331 (Scotus died in 1308), since it discusses the notion of the complexly signifiable (*complexe significabile*: Pseudo-Scotus 1891-5, question 8, pp. 98b-101b), a notion introduced in that year by Adam Wodeham (see Gál 1977, pp. 70-71) in his *Sentences* commentary. Bendiek (1952, p. 206) used this fact to argue for a date after 1344, since at the time he was writing, it was thought that the notion of the complexly signifiable was due to Gregory of Rimini. Gál (1977), in editing a question from Wodeham, showed that he had anticipated Gregory by some years. Nonetheless, it may well be that our author took the notion from

[1]See Pseudo-Scotus 1891-5, question 10 (pp. 103a-108a), p. 104b. The relevant text is reproduced in Spade 1975, pp. 44-5 and the whole question in Pozzi 1978, pp. 150-60. An English translation is given in the Appendix to the present volume (pp. ??-??). The objection was discussed in Mates 1965a, and also retold, with little or no comment, in Moody 1953, p. 69, Kneale and Kneale 1962, pp. 287-8, Mates 1965b, p. 213, McDermott 1972, pp. 288-90, Ashworth 1974, p. 184, and Boh 1982, pp. 308-9.

[2]Indeed, all the printed texts of the treatise, from 1500 onwards, attribute it to Scotus.

Gregory, and that the correct date is indeed in the decade or so after 1344. Boh (1982) dates it around 1350, but gives no reason.

In an Oxford manuscript, the commentary on the *Posterior Analytics* is attributed to John of St Germain of Cornwall. Emden, in his list of Oxford scholars, identifies John of St Germain as studying at Oxford from 1298-1302 and teaching at Paris from 1310-15 (Emden 1959, col. 1626). Some modern commentators have chosen to cite the author of the questions on the *Prior Analytics* as John of Cornwall.[3] However, there is no reason to suppose the two treatises have the same author. Indeed, the late date of the questions on the *Prior Analytics* shows that the work cannot be by the John of St Germain listed by Emden, who also reveals no connection with Cornwall. Perhaps the Cornish St Germain is a scholar of the next generation; or our Pseudo-Scotus is not him at all. We do not know; we must continue, therefore, to call its author ‘Pseudo-Scotus’.

3. Let A be the argument:

> God exists
>
> Hence this argument is invalid

Pseudo-Scotus took the premise to be necessarily true. (Atheists may substitute ‘1 = 1’.) We can then reason as follows.

If the argument A is valid, A has a true premise and a false conclusion. But every argument with a true premise and a false conclusion is undoubtedly invalid. (This follows from the necessity of the condition offered earlier. What is under attack is its sufficiency.) So A is invalid. That is, if A is valid, then it is invalid.

[3] See Read 1993, p. 236 n. 10. Indeed, McDermott (1972, p. 274) and Bäck (1996, pp. 205-7, 274-8) take the questions on the *Sophisticis Elenchis* (also printed in volume 2 of Wadding’s *Opera Omnia* of Scotus, pp. 1-80), on the *Prior Analytics* (*ibid.*, pp. 81-197) and on the *Posterior Analytics* (*ibid.*, pp. 199-347) all to have the same author, *viz* John of Cornwall or Cornubia. Note that both extant mss. of the questions on the *Sophisticis Elenchis*, unlike those of the other two commentaries, explicitly ascribe them to Scotus.

So A is invalid, by *reductio ad absurdum*.

Our only assumption (if we may call it that) in demonstrating the invalidity of A was that God exists (or that 1 = 1). And that is necessarily true. By a plausible thesis concerning modal terms, what is deduced from a necessarily true proposition is itself necessarily true.[4]

So it is necessarily true that A is invalid.

But that shows that A has a necessarily true conclusion. If the classical account of validity were correct, and the necessary truth of the conclusion of an argument were sufficient for the argument's validity, it would follow that A was valid.

Hence, if the classical account is correct, A is both valid and invalid. The classical account leads to contradiction, and so must be wrong. A is an argument which is clearly invalid, yet which the classical account maintains is valid.

So the classical account is incorrect.

4. One may, however, have reservations about the self-reference present in argument A. On taking the classical account of validity, we find that A leads to contradiction. But certain sentences exhibiting self-reference lead to contradiction anyway. Various ways of dealing with those paradoxes were suggested both in the Middle Ages and in more recent times.

Two kinds of solution popular in the twelfth and thirteenth centuries were *cassatio* and *restrictio*. The idea of the former was that such self-referential utterances as the Liar paradox, 'What I am saying is false', simply say nothing at all; the latter went further, in claiming that strictly speaking, self-reference is impossible, and that if such an utterance as the Liar means anything at all, it is that, e.g., one's previous utterance was false.[5] There is considerable similarity between both these ideas and Tarski's familiar claim that semantic closure (a

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[4] The principle alluded to is K: L(p   q)   (Lp   Lq). See, e.g., Hughes and Cresswell 1996, p. 25.

language’s containing its own truth-predicate) leads to incoherence, and his recommendation of a hierarchy of object language and metalanguage.[6]

Cassatio and *restrictio* became less popular in the fourteenth century, and had few proponents in the later fourteenth, fifteenth and sixteenth centuries. An extremely influential alternative theory seems first to have been proposed by Thomas Bradwardine in his treatise on insolubles (*insolubilia*—problems which are not strictly insoluble but only “solved with difficulty”, as William Ockham put it). Bradwardine’s treatise was written in the 1320s, many years before he became Archbishop of Canterbury. Forty years later, Ralph Strode called Bradwardine that “prince of modern natural philosophers who first came upon something of value concerning insolubles”. This idea was that every insoluble not only has a primary standard meaning, but also means that it itself is true. Bradwardine proved this by an extended argument hinging on the postulate that every proposition includes in its meaning whatever logically follows from it (see Roure 1970, p. 297).

Albert of Saxony and others went further. Every proposition, they said, signifies itself to be true. Once again, this was proved from a set of postulates analysing what exactly it is to be true (Albert of Saxony 1988, pp. 339-40). John Buridan, Albert’s teacher at Paris in the 1340s and ’50s, whose development of these ideas has been most frequently commented on and analysed (e.g., Prior 1962, Scott 1966, Hughes 1982), and who at one time adopted Albert’s expressed view, qualified this claim. We can’t say that every proposition means that it itself is true, for then we would either have a use/mention confusion (not every proposition refers to itself) or at least a reference to the proposition’s own truth, and so no adequate account of falsehood, for there would be nothing to refer to when it was false (Hughes 1982, §7.7.1 and commentary). Buridan preferred to say that every proposition “virtually implies” its own truth.

Whatever the exact detail of Bradwardine’s and his followers’ theses about the meaning or implication of each insoluble, they agree on this: that each insoluble is false. For it means, or implies, something which is not the case, namely, that it itself is true. It can’t be, for it also

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[5]See, e.g., Martin 1993 and Panaccio 1993.

[6]See Tarski 1956, §1.

(primarily) means that it's not true. Whatever entails a contradiction is false, and the insolubles entail the contradiction that they are both true and false. So they are all false.

5. It is time to return to argument A. If A should be found to lead to contradiction, independently of acceptance of the classical account of validity, then whatever solution one takes to the paradoxes of self-reference will undercut the demonstration in §3 that the Classical Account of Validity is mistaken. Whatever solution removes the contradiction resulting from A independently of the classical account will certainly remove that resulting from it in conjunction with that account.

So does A result in contradiction independently of the classical account?

Let B be the argument:

> This argument is valid
>
> Hence this argument is invalid.

If the argument B is valid, then it has a true premise and a false conclusion. Therefore B is invalid. That is, if B is valid, then it is invalid.

So B is invalid, by *reductio*.

But what have we shown? Look again at that sentence: 'if B is valid, then it is invalid'. We have deduced the invalidity of B from the premise that B is valid. That is precisely what B says we can do.

So B is valid.

And that is a contradiction. B is both valid and invalid. Any solution to the paradoxes of self-reference must deal with argument B as well. When we deduced a contradiction from A in §3, we had made an assumption, namely that the classical account of validity was correct. Hence we were able to evade the contradiction (we thought) by denying the correctness of the classical account. With B, however, we have deduced a contradiction—by unquestionable facts about validity: 1) that any argument whose conclusion follows from its premises is valid; and 2) that any argument whose premises might be true and conclusion false is invalid. If the establishment above of the validity of B is to be faulted that can only be done by faulting some step in the derivation of B's invalidity from the hypothesis of its validity. If that deduction is sound, as I claim it is, then it is immediate, from 1) that B is valid.

6. We can now diagnose the fault in Pseudo-Scotus' example. The classical account of validity is not needed to establish that argument A is valid, and so to derive a contradiction from it.[7] In establishing that A was valid, Pseudo-Scotus reasoned as follows.

We take as our premise that God exists. Then suppose A is valid. In that case, A has a true premise and a false conclusion. Therefore it is invalid. That is, given that God exists, then if A is valid, it is invalid.

So, if God exists, A is invalid.

(From this, by *modus ponens* for 'if', Pseudo-Scotus deduced that A was indeed invalid, since God exists.)

What we have done is precisely to deduce A's conclusion (that A is invalid) from its premise (that God exists).

So A is valid.

Regardless of our acceptance of the classical account of validity, A is both valid and invalid. Nor does A's premise need to be necessarily true. If A's premise is true, then if A is valid, it is invalid. Hence, by *reductio ad absurdum*, if A's premise is true, A is invalid. Since we have deduced A's conclusion from its premise, A is valid. And since its premise is true, A is invalid.

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[7]One has also of course to ensure that the deduction of the contradiction does not use methods of argument only justifiable classically. The deduction which follows is acceptable in a relevance logic such as FE of Anderson and Belnap 1975.

7. The paradoxical nature of argument A was in fact recognised in the medieval period by Albert of Saxony (1988, pp. 360-1), also writing in about 1350. Clearly A is invalid, Albert says, by an argument similar to that we gave in §3. But then, suppose A is invalid. It follows that its consequent is true. So the antecedent cannot be true without the consequent's also being true. Hence (see below) A is valid, i.e., if we suppose A is invalid, it follows that it's valid. So it's valid.

Albert has in fact used the Classical Account of Validity here—which, we have just seen, he didn't need to do. He wrote: "if argument A is not valid, it is possible for [its antecedent] to be true while [its consequent] is false" (*ibid.*, p. 361). That is, condition (2) in §1 is taken to be sufficient for validity (because necessary for invalidity) in the penultimate step (marked 'see below'), where he concludes that A is valid on hypothesis that it is invalid. Nonetheless, Albert does not proceed to reject (or revise) the Classical Account, as did Pseudo-Scotus. Instead, he applies his analysis of insolubles, rejecting the move from the supposition that A is invalid to the conclusion that the consequent is true. For supposing the consequent is true, it follows that A is valid, and so it means that A is valid (as well as meaning, primarily, that A is invalid). But we have supposed A wasn't valid, so "things are not however [the consequent] signifies them to be". So A is invalid, but the consequent is not true.

One may not agree with Albert's solution.[8] It appears to achieve consistency at the expense of preventing us saying that what is the case is true. What is important is the recognition of the connection between A and the other insolubles such as the Liar. Indeed, Albert considers some propositional variants of the paradox. The propositional form of A is the conditional α:

If God exists then this (conditional) sentence is false.[9]

If α is true, then it is a true conditional with true antecedent, so its consequent is true, and so if α is true, it's false. So α is false. Albert proceeds to show that the supposition that α is false leads to contradiction, relying on the principle that a conditional is true only if it is impossible

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[8]I have expressed by own reservations about this manner of solution in Read 1984, p. 425.

[9]Albert's example is 'If God exists, some conditional is false' on hypothesis that this is the only conditional. The effect is the same. Similarly with χ and δ below.

for the antecedent to be true and the consequent false—the analogue for conditionals of the Classical Account of Validity. But we can simplify his argument as we did in §6. For in showing that α was false, we relied on the fact that 'God exists' was true. That is, what we showed was that if God exists then α is false. But that is what α says. So α is true (as well as false).

A further variation, which both Albert (1988, pp. 357-9) and Buridan (Hughes, pp. 60-1) consider, gives the paradox a conjunctive or disjunctive form. Let χ be the conjunction:

God exists and this (conjunctive) sentence is false

and δ the disjunction:

God does not exist or this (disjunctive) sentence is false.[10]

Suppose δ is true. Then either God does not exist or δ is false. But God does exist. So δ is false, i.e., if δ is true it's false. So δ is false. Hence either God does not exist or δ is false, which is what δ says. So δ is true too. A similar argument shows that χ is also paradoxical.

Albert and Buridan both use their theories of insolubles to diagnose the error, claiming that α, χ and δ are all false. Paradox is rife here, and Pseudo-Scotus and others (e.g., Priest and Routley 1982) should hesitate to use argument A to question any account of validity. If the Classical Account is wrong, a different proof of that fact must be found.

8. An argument which could (superficially) be called the contrapositive of A was considered by a number of fifteenth and sixteenth century authors.[11] Let C be the argument:

This argument is valid

Hence God does not exist.

(C is not strictly the contrapositive of A since 'this argument' now refers to C, not to A.)

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[10]Albert and Buridan use 'A man is a jackass' in place of 'God does not exist'.

[11]Ashworth 1974, p. 125 and Roure 1962, pp. 275-6. Again, 'A man is a jackass' is commonly used in place of 'God does not exist'.

If C is valid, then since God exists, C has a true premise and a false conclusion. Therefore C is invalid. That is, if C is valid, then C is invalid.

So C is invalid, by *reductio.*

On the other hand, if C is valid, then its conclusion can be deduced from its premise, and its premise is true. So its conclusion is true. That is, if C is valid, then God does not exist.

So C is valid, since its conclusion has been deduced from its premise.

Pseudo-Scotus' own solution to his objection (argument A) to the classical account of validity was to add to that account an exceptive clause. He said that an argument was valid if and only if it is impossible for the premises to be true and the conclusion false together, except when the conclusion explicitly denies the connecting particle (here 'hence'), that is, when it denies that the argument is valid. Argument C serves to show this clause insufficient. Pseudo-Scotus' revision covers argument A, but not its 'contrapositive', C.

On the other hand, of course, neither A nor C in fact need be allowed for by an account of validity. They will already have been excluded from consideration by the account of self-referential paradox.

9. Argument C corresponds to Curry's paradox. For C is the inferential version of a conditional, γ:[12]

If this (conditional) sentence is true, then God does not exist.

[12]It is often claimed that the medievals, following Aristotle's lead, did not clearly distinguish arguments from conditionals (e.g., Mates 1965b, p. 133, Boh 1982, p. 306). This is certainly not true of Jean of Celaya, who used C and γ to observe that a true conditional could correspond to an invalid argument and a false conditional to a valid one: see Ashworth 1974, p. 125 and Roure 1962, p. 262.

(Of course, the reference changes, from arguments to conditionals—sentences, and truth of conditionals replaces validity of arguments.) Sentences of the form ‘if this sentence is true, then *p*’ can be used to show that any sentence is true.[13]

For suppose γ is true. Then if its antecedent is true, so is its consequent, and its antecedent is true. So its consequent is true, that is, God does not exist. That is, if γ is true, then God does not exist. But that is what γ says. So γ is true.

Hence, if its antecedent is true, so is its consequent, and its antecedent is true. So its consequent is true, that is, God does not exist.

We can use C in the same way to establish its conclusion. (γ and C are, we might say, the ultimate ontological argument.) Indeed, we can use A to show, to Pseudo- Scotus’ dismay, that its premise is false. For A is valid. So its conclusion is false. So its premise must be false too!

And in the manner in which we showed that C is both valid and invalid we can show that γ is both true and false. We have already seen the conditional form, α of A. There will also be a conditional form, β of B. Indeed we can treat B (and A) as we did C: if B is valid, then its conclusion can be deduced from its premise and its premise is true. Therefore its conclusion, that is, ‘B is invalid’, is true, and so B is invalid. (I’ll remark on this final step in §11.) That is, if B is valid, then it is invalid.

That conditional contains all we need to conclude both that B is valid, since we have deduced B’s conclusion from its premise, and that B is invalid, by *reductio*.

10. The conditional form β of the argument B lies behind Rosser’s proof of Gödel's theorem.[14] For β is:

> If this sentence is true, then this sentence is false.

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[13]See Geach 1954-5 and Löb 1955. In the words of my colleague Roger Squires, sentence γ is full-proof.

Suppose β is true. Then it is false. That is, if β is true, then it is false. So β is true, since that is what β says, and β is false, by *reductio*.

Gödel used the Liar paradox, 'This sentence is false', to construct an undecidable sentence G of arithmetic. G says informally: G is not provable. Neither G nor its negation is provable in arithmetic. However, the demonstration that ~G is not provable requires the assumption that arithmetic is ω-consistent.[15]

To reduce this assumption to the assumption only of simple consistency, Rosser considered instead of G a sentence H which says informally: if H is provable, then there is a simpler proof of ~H. (What I am here calling 'simpler' was defined precisely in terms of one proof's having a smaller Gödel number than the other.) It was the condition that the proof of ~H be simpler than that of H which allowed the assumption of ω–consistency to be dropped.

Without this restriction on size of proof, we obtain a sentence J which says informally: if J is provable, then ~J is provable. J corresponds to β as G corresponds to the Liar sentence. It is straightforward to show that both the assumption that J is provable and the assumption that ~J is provable lead to contradiction, given the ω-consistency of arithmetic. Hence, following Gödel, we can conclude that J is an undecidable formula. That is, neither J nor its negation is provable.

If the demonstration that if J is provable then ~J is provable were formalizable in arithmetic, it would constitute a proof of J, by the deduction theorem, and a proof of ~J, by *reductio*, just as we showed B to be both valid and invalid. But although we have that if $\vdash$J then $\vdash$ ~J, we do not have J $\vdash$ ~J; if arithmetic is consistent. On that assumption, the derivation of ~J from J cannot be performed in arithmetic.

11. Does Rosser's use of H rather than J have any significance for us? I think not. From the assumption that J is provable, it follows that '~J is provable' is provable. The problem is to

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[14]See Rosser 1936, p. 89 (Theorem II), and Kleene 1952/71, pp. 204-13 (Theorem 29).

[15]If $\vdash$ ~A(0), $\vdash$ ~A(1), …, then not $\vdash$ *x*A(*x*).

extract a proof of ~J from this. Löb (1955, p. 116) showed that (what informally expresses) 'if S is provable then S' is provable only if S is provable. To obtain the proof of ~J we need to assume ω−consistency, to ensure that the Gödel number which '~J is provable' asserts to exist is in fact one of 0,1,... .

In the case of H however, we are given a bound on the Gödel number which indexes the proof of ~H. This yields a proof of ~H with the assumption only of simple consistency. But the bound on size of proof has no analogue in the natural language context of B and β. What might have significance is the analogue of Löb's result, namely that 'if S is true then S' should be true only when S is true. Recall that in §9 we inferred immediately from the fact that B's conclusion, that is, 'B is invalid', was true, that B was invalid. (We used 'if S is true then S' when dealing with B in §5 also, but there it took the form of concluding that 'B is invalid' was false from the hypothesis that B was valid, and so was not invalid.) We did this at precisely the point where in the corresponding arithmetical proof we need to use ω-consistency. We need to make the move in order to show that B is valid (respectively, that if J is provable then ~J is provable). But we would be allowed to make it only if B *was* valid (J *was* provable). So we would never get started.

Suppose we try to treat truth in the natural language examples as we treat provability in arithmetic. Immediately we need to deny the law of excluded middle. Arithmetic is consistent only if it is not negation-complete. Gödel showed that some sentences of (a consistent) arithmetic are neither provable nor refutable. But such a lead from arithmetic would not end with claiming certain self-referential sentences to be neither true nor false. We would have also to reject that half of the truth-equivalence corresponding to Löb's result, permitting the inference of S from 'S is true'. With the Liar sentence we can avoid the establishment by Dilemma that L is both true and false by denying that it is either. With the Curry paradox and arguments A, B and C, excluded middle is not used. It appears that one can *prove* that, for example, γ is true, or B is valid. To refuse to move from, say, "'B is invalid' is true" to "B is invalid" would block the demonstrations both that B is valid and that it is invalid. Yet to do this we have now to deny not excluded middle but 'if S is true then S'. Otherwise any claim that B was neither valid nor invalid would not only be totally unsupported but contrary to fact.

The proposal is the converse of the solution proposed by Bradwardine and others. They rejected the move from S to ‘S is true’ (at least for insolubles, and for all propositions for Albert and Buridan). Their proposal promises to solve the paradoxes, but at the price of compromising one’s theory of truth altogether.[16] The present proposal suggests instead that we block the move from the denial of S to the denial that S is true, at least when S is not true. Again, the paradoxes may be blocked, but at the price of being unable to deny what is plainly not true.

So to deny the truth equivalence is, for me, too high a price to pay. It is constitutive of the notion of truth that if S is true then things are as S states them to be. (Of course, it is constitutive of the notion of proof that if S is provable then there is a proof of S. Löb’s result shows not that the arithmetical predicate which informally expresses ‘provable’ does not have this property, but that only for provable sentences can one show in arithmetic that it has the property.).

Formal arithmetic is (we hope) consistent. That is why we can conclude that it is negation-incomplete. But natural language is at first blush inconsistent. Its deductive power seems unlimited; we cannot easily constrain it in the way we can choose to constrain formal theories. β is both true and false and B is both valid and invalid.

12. Argument B and the others remind us that self-reference can be indirect. B’s premise contains an expression referring to a piece of discourse of which that premise is a part. Further, whether a sentence leads to paradox may depend on how the world is, on whether certain other sentences are true (as Epimenides showed). A is contradictory only if God might exist. Moreover, the semantic paradoxes cannot be evaded simply by denying excluded middle (for truth and for validity). A, B and C can be proven to be valid, just as α, β and γ can be proven to be true. Lastly, semantic closure does not mean simply ‘contains its own truth-predicate’, though that is a useful shorthand for it. There are other semantic concepts besides truth and falsity, and paradox can arise through them too. Validity is one.

[16] See Read 1984 and Spade 1982, p. 249.

That is what Pseudo-Scotus, and others, should have seen. A leads to paradox independently of any account of validity. The proper account of validity has *no more* to deny the validity of A (and B and C) than the proper account of truth has to deny that the Liar sentence is true. (Though perhaps the proper accounts *do* do this.) Unfortunately, the classical account of validity emerges unscathed from Pseudo-Scotus' attack.[17]

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[17]This paper is a revision and expansion of my 'Self-Reference and Validity', *Synthese*, **42**, 1979, pp. 265-74. The major changes consist in the addition of new §§2, 4 and 7. The reader may be interested to see subsequent discussion of the issues in Priest and Routley 1982, Sorensen 1988, pp. 299-310, Drange 1990 and Jacquette 19??.

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