The Rule of Contradictory Pairs, Insolubles and Validity *

Stephen Read
Arché Research Centre
17-19 College St.
University of St Andrews
St Andrews KY16 9AL
Scotland, U.K.
e: slr@st-andrews.ac.uk
t: +44-7766-123002
ORCID: 0000-0003-2181-2609

*The present work was funded by Leverhulme Trust Research Project Grant RPG-2016-333.

The Rule of Contradictory Pairs, Insolubles and Validity

Abstract

The Oxford Calculator Roger Swyneshed put forward three provocative claims in his treatise on insolubles, written in the early 1330s, of which the second states that there is a formally valid inference with true premises and false conclusion. His example deployed the Liar paradox as the conclusion of the inference: 'The conclusion of this inference is false, so this conclusion is false'. His account of insolubles supported his claim that the conclusion is false, and so the premise, referring to the conclusion, would seem to be true. But what is his account of validity, that can allow true premises to lead to a false conclusion? We consider Roger's own account, as well as that of Paul of Venice, writing some sixty years later, whose account of the truth and falsehood of insolubles followed Roger's closely. Paul endorsed Roger's three claims. But their accounts of validity were different. The question is whether these accounts are coherent and support Paul's claim in his *Logica Magna* that he endorsed all the normal rules of inference.

Keywords: contradiction, validity, signification, Aristotle, Roger Swyneshed, Paul of Venice.

1 The Rule of Contradictory Pairs

In his treatise on insolubles, that is, the logical paradoxes, written in Oxford in the early 1330s, Roger Swyneshed, one of the Oxford Calculators, put forward three surprising, even paradoxical, claims:[1]

1. There is a false proposition which principally signifies as things are
2. There is a formally valid inference with true premises and false conclusion
3. There is a pair of contradictory propositions both of which are false.[2]

The third claim, (3), if correct, is a counter-instance to the Rule of Contradictory Pairs (RCP), that of any pair of mutually contradictory propositions, one is true and the other false.

[1] For biographical information on Swyneshed, see Weisheipl (1964).

[2] Note that throughout this paper, I will use 'proposition' to refer to what the medievals referred to as '*propositiones*', that is, concrete token sentences, whether spoken, written or mental.

I have discussed Swyneshed's rejection of the (RCP) elsewhere, and whether it is entailed by Aristotle's account of contradictories in his *De Interpretatione*.[3] (RCP) follows immediately from Boethius' definition of 'contradictory' in his Second Commentary on Aristotle's *De Interpretatione*:

> "Contradiction is then the opposition of an affirmation and a denial in which neither can both be false nor both true, but one is always true the other false."[4]

But that is not how Aristotle defined contradictories. He did so syntactically, contrasting affirmations with denials:

> "Since it is possible to state both of what is that it is not, of what is not that it is, of what is that it is and of what is not that it is not and also in the times which are outside the present, it must similarly be possible to deny whatever anyone has affirmed and to affirm whatever anyone has denied. And so it is clear that for every affirmation there is an opposite denial and for every denial there is an opposite affirmation. Let us call this a contradiction when an affirmation and denial are opposed."[5]

Whitaker, in his (1996), where he coined the label *Rule of Contradictory Pairs*, argues that not only is Aristotle's syntactic definition not equivalent to the semantic one, but that Aristotle realised this and presented counter-instances to (RCP) in chs.7-9 of *De Interpretatione*, in ch.7 cases where a pair of contradictories are both true, in ch.8 cases where they are both false, and in ch.9, cases where they are neither—or at least, not definitely either. Aristotle writes at the end of ch.9:

> "Clearly, then, it is not necessary that of every affirmation and opposite denial one should be true and the other false. For what holds for things that are does not hold for things that are not but may possibly be or not be; with these it is as we have said."[6] (19a39-b3)

[3]See Read (2020).

[4]Boethius (2010, p.88).

[5]*De Interpretatione* 17a26-33, as cited by Boethius at (2010, §126: 17-23, pp.83-84). Smith translates 'negatio' as 'negation', but 'denial' is an equally valid translation, and more appropriate in the context.

[6]Aristotle (1963, p.53). Boethius's translation reads: *Quare manifestum est quoniam non est necesse omnes adfirmationes vel negationes oppositarum hanc quidem veram, illam autem falsam esse; neque enim quemadmodum in his quae sunt sic se habent etiam in his quae non sunt, possibilium tamen esse aut non esse sed quemadmodum dictum est* (Aristotle, 1965, pp.17-18).

Quite what Aristotle's claim about future contingents was has puzzled philosophers for millennia. His claims in chs.7-8 that contradictories can both be true or both be false, however, has for the most part been overlooked, or reinterpreted as remarks about contraries or subcontraries, not contradictories, as the text has it. For example, Aquinas notes that Aristotle wrote, when summarising ch.7:

> "We have explained, then, that a single affirmation has a single denial as its contradictory opposite, and which these are; that contrary statements are different, and which these are; and that not all contradictory pairs are true or false, why this is, and when they are true or false."[7] (18a8-12)

Aquinas's comment is:

> "He also says here that it has been shown that not every contradiction is true or false, 'contradiction' being taken here broadly for any kind of opposition of affirmation and negation; for in enunciations that are truly contradictory one is always true and the other false."[8]

In other words, he corrects what Aristotle says to ensure that (RCP) is maintained.

2 Swyneshed on Insolubles

It is not surprising, therefore, that Roger Swyneshed ran into fierce opposition when he claimed that some pairs of contradictories are both false. Ralph Strode protested that

> "The opposite [of the third claim] is clear from the first book of the *Postpredicaments*, the fourth book of the *Metaphysics* and the

[7]Aristotle (1963, p.49). Boethius's translation reads: *Quoniam ergo uni negationi una adfirmatio opposita est contradictorie, et quae sint hae, dictum est, et quoniam aliae sunt contrariae et quae sint hae, et quoniam non omnis vera vel falsa contradictio, et quare, et quando vera vel falsa* (Aristotle, 1965, p.12).

[8]Aquinas and Cajetan (1962, Lecture 12 §6): *Deinde cum dicit: quod igitur una affirmatio etc., epilogat quae dicta sunt, et concludit manifestum esse ex praedictis quod uni affirmationi opponitur una negatio; et quod oppositarum affirmationum et negationum aliae sunt contrariae, aliae contradictoriae; et dictum est quae sint utraeque ... Dictum est etiam quod non omnis contradictio est vera vel falsa; et sumitur hic large contradictio pro qualicumque oppositione affirmationis et negationis: nam in his quae sunt vere contradictoriae semper una est vera, et altera falsa.*

first book of *De Interpretatione*, where [Aristotle] clearly means that it is impossible for two mutually contradictory contradictories to be true together or false together."[9]

Roger's central example was a case of the Liar paradox, 'This is false' (*Hoc est falsum*), referring to itself. This is false, he says, because it falsifies itself. But its contradictory, 'This is not false', referring to 'This is false', is also false, for he has just argued that 'This is false' is false. First, note that Roger makes a strong case that these really are contradictories, for one affirms of 'This is false' that it is false, and the other denies it, just as Aristotle required in his definition of contradictories. Aristotle wrote:

> "Every such pair of propositions we, therefore, shall call contradictories, always assuming the predicates and subjects are really the same and the terms used without ambiguity. These and some other provisos are needed in view of the puzzles propounded by importunate sophists." (17a33-37)

Roger's solution to the Liar paradox turns on two notions. First, a proposition falsifies itself if it entails its own falsehood.[10] This is clearly true of 'This is false', he says, for from 'This is false', 'This is false' immediately follows, provided 'this' refers to the same proposition, in this case, to the premise.[11] Accordingly, Roger tightens up the definition of truth to require of true propositions that not only should they signify as things are, but in addition, they should not falsify themselves. Then a false propostion is one that

[9] Spade (1978, p.76). Translations from Spade (1978) and (1979) are my own.

[10] See Spade (1979, §17, p.186): "Next there follow eight basic principles (*suppositiones*), of which the first is this: every proposition relevant to inferring itself to be false is one falsifying itself. This is clear from the intention of the Philosopher in the fourth book of the *Metaphysics*, where he claims that some utterances destroy themselves. And it is plain there that the only utterances that he takes to destroy themselves are utterances relevant to inferring themselves to be false, which is signified by this phrase 'falsifying itself'. And it sounds better in the Latin language to say that such a proposition falsifies itself than destroys itself in that no utterance is destructive of itself nor of anything other than itself. But it sounds better in the Greek language to say that such a proposition destroys itself than falsifies itself. And so the translator uses the verb 'destroy itself' where we use 'falsify itself' in that it applies more truly to such a proposition."

[11] Spade (1979, §5, p.182): "A proposition falsifying itself directly is a proposition signifying principally as things are or other than things are, relevant to inferring itself to be false. And it is of two kinds. Some are relevant sufficiently, some are relevant insufficiently. Relevant sufficiently are propositions signifying principally as things are or other than things are from which, signifying in this way, it directly follows or is apt to follow that they are false. An example: let the proposition 'This is false' signify principally that this is false, referring to itself. Then it directly follows 'This is false, therefore, this is false'. And in this way it is relevant sufficiently to inferring itself to be false."

either does not signify as things are or, even if it does signify as things are, falsifies itself. All self-falsifying propositions, like the self-referential ‘This is false’, are false, though not all self-verifying propositions, that is, those which entail their own truth, are true—think of ‘Every proposition is true’, for example, which entails its own truth, but is clearly falsified by every false proposition.

‘This is false’ also serves to establish Roger’s first iconoclastic claim, that some false proposition signifies as things are. For ‘This is false’ signifies that it is false, and it is, he says, false (since it falsifies itself), so it is a false proposition that signifies as things are. It also provides a neat example to establish his second claim, that there is a formally valid argument with true premises and false conclusion:

> “This inference is formally valid (*bona et formalis*): ‘The conclusion of this inference is false, so the conclusion is false’.”[12]

Call it (†). The conclusion is an insoluble, falsifying itself, and the premise truly and correctly says that it is false. So the premise is true and the conclusion false. Nonetheless, (†) appears to be a transparent case of validity, in that the conclusion merely repeats what the premise said. Recall that propositions are, for the medievals, concrete token utterances, whether spoken, written or mental.[13] Different tokens of the same type can have different truth-values, as in the inference (†) above. Call its premise N and its conclusion M. Then M falsifies itself. But N does not falsify itself: there is no (obvious) way to infer that N is false from N itself, even though N and M signify the same thing—they both predicate falsity of M. According to Roger’s definitions, N is true and M is false.

Roger’s text reveals little about how Roger understands ‘signify’, though he usually accompanies it by the adverb ‘principally’. As Spade (1983, p.106) observes, nothing seems to be lost by ignoring the qualification, but it does serve to connect Roger’s terminology with comments by his contemporaries and critics, such as William Heytesbury. As we will see in §5, Roger’s solution to the insolubles was taken up by Paul of Venice (though with some changes). As Paul makes clear, the motivation for Roger’s solution is to avoid the postulation, by Thomas Bradwardine and Heytesbury, among others, of an additional signification beyond what Heytesbury (see Pozzi, 1987, p.236) characterises as “what the words usually mean” or “as the words commonly

[12] Spade (1979, §26, p.189).

[13] Klima (2004, p.96), for example, notes that the medievals commonly conceived of propositions “as being singular, contingent, temporary occurrences, whether in writing, speech, or in the mind.”

suggest” (*sicut verba communiter pretendunt*).[14] Roger’s proposal is to take the terms in insolubles at face value and amend the account of truth and falsehood, rather than propose hidden significations in a seemingly ad hoc manner, as Heytesbury does.

Indeed, there is a common approach in Bradwardine’s, Heytesbury’s and Roger’s solutions to the insolubles. The contradiction which appears to ensue from admitting self-reference is internalised in such a way as to render the insoluble false and so disclaim anything that might follow. This is literally so in Bradwardine’s and Heytesbury’s case: they both impute a contradictory signification to the insolubles that renders them false.[15] But a similar phenomenon lies at the heart of Roger’s account: the ensuing contradiction shows that the insoluble can’t be true, so it falsifies itself, and accordingly is false.

One might pause, however, when Roger says that (†) is formally valid. What is the account of validity which he appeals to here? In this case, the conclusion says the same thing about the same thing as the premise does. But not all valid inference steps are instances of identity. What is the general account which allows a valid inference to fail to preserve truth? It can’t be truth-preservation, often taken as a criterion of validity in both medieval and modern times,[16] since Roger claims that (†) fails to preserve truth. Does Roger have an account of validity which explains how such inferences can be valid but fail to preserve truth?

3 Validity

Among medievals, Roger was not alone in this rejection of the truth-preservation criterion. We find a similar rejection in, e.g., John Buridan, Albert of Saxony and Pseudo-Scotus—though the much-repeated counterexample seems to have originated with Buridan in his early *Questions on the Prior Analytics*.[17] Consider the inference

(*) Every proposition is affirmative, so no proposition is negative.

This seems to be valid, given the meanings of ‘affirmative’ and ‘negative’, as is the contrapositive inference

[14] Spade’s translation of ‘*sicut termini communiter pretendunt*’ as ‘as the terms commonly pretend’ or ‘... commonly pretend to signify’—see, e.g., Heytesbury (1979, p.81)—is unhelpful. No pretence is implied.

[15] See, e.g., Spade and Read (2017, §§3.1, 3.3).

[16] See, e.g., Dutilh Novaes (2016, §1.1).

[17] See Buridan (nd, Q.39). See also Buridan (2001, p.952), Albert in Fitzgerald (2002, p.356) and Pseudo-Scotus in Yrjönsuuri (2001, p.227).

> Some proposition is negative, so not every proposition is affirmative.

But whereas this second inference preserves truth, the first does not, Buridan observes. For the premise can be true (if, for example, God has annihilated all negative propositions), but the conclusion cannot, for it falsifies itself in the non-Swyneshedian but natural sense, that it is a counter-instance to itself. It is itself a negative proposition, but it must exist in order to be true or false, yet if it does exist, it is false. Nonetheless, Buridan says (2001, p.952), the rule of Contraposition "is common to every valid consequence."[18] So validity of inference cannot be a matter of preserving truth.[19]

Example (*) is not, for Buridan, formally valid, which he defines as validity in all (non-logical, that is, categorematic) terms. Nor are any of the above inferences syllogistic inferences, for a syllogism requires (at least) two premises. However, consider the following syllogism in the second figure:

> Every true proposition is a premise
>
> Not every conclusion is a premise
>
> So not every conclusion is true

and suppose that God has annihilated all propositions other than these three. Then the premises are true (the first, since the premises are the only true propositions) and the conclusion is false because it falsifies itself by Roger's criterion—it entails its own falsehood, since it is the only conclusion. So the premises of this syllogism can be true and the conclusion false, yet it is a formally valid syllogism in Baroco.

However, as Klima (2016, p.318) notes, the Parisian account of validity of consequence represented by Buridan and others was radically different from the English tradition. Moreover, in both cases, formal consequence was a special case of validity in general.[20] So the Parisian definition of formal consequence as truth-preservation under substitution of all non-logical terms was a definition of 'formality', not of validity. Where truth was preserved but not formally, the inference was said to be materially valid.[21] Validity itself was truth-preservation. So too in the English tradition: in Ockham, for example, we find many divisions of consequence, but what is common to

[18] Buridan also endorses it in (2014, I 8, p.76).

[19] Buridan's example is discussed at length in Prior (1969).

[20] Note that I sometimes speak of 'inference', sometimes of 'consequence' in translating the Latin term '*consequentia*'. As Buridan (2001, §7.4.5, p.575) observes, consequences (for the medievals) include not only arguments (where premises and conclusion are asserted) but also conditionals (where antecedent and consequent are not assserted).

[21] See, e.g., Buridan (2014, I 4, p.68).

them is that in a valid consequence the premises cannot be true without the conclusion.[22] Similarly, Burley wrote in around 1302 before he left Oxford for Paris:

> "It must be known that for the correctness of a conditional no more is required than that if the antecedent is true, the consequent is true ... Some conditionals hold as-of-now and others simply ... and there is consequence as-of-now when the premise cannot be true as-of-now without the conclusion."[23]

Burley proceeds to distinguish what he called natural from accidental consequences, in the way later used to distinguish formal from material consequence:[24]

> "Simple consequences are divided like this: some are natural and others accidental. Natural consequence is when the conclusion is understood in the premises (*de intellectu antecedentis*), nor can the premises be true unless the conclusion is true, e.g., 'If it is a man it is an animal'. Accidental consequences are of two sorts: some hold on account of the terms or on account of the matter, e.g., 'It is true that God exists, so it is necessary that God exists', which holds on account of the terms or the matter, for in God truth and necessity are the same. Other accidental consequences are 'From the impossible anything follows' and 'The necessary follows from anything' ... Accidental consequence is when the conclusion is not understood in the premises."

Weber (2003, §I.1.1, esp. pp.133-4) shows how the phrases *intelligitur in antecedente* and *de (formale) intellectu antecedentis* pervade English treatments of consequences in the fourteenth century, in particular in the works of Richard Billingham, Robert Eland,[25] Ralph Strode, Richard Lavenham and others.

The contrast between the Parisian and English notions of formal consequence is not as extreme as it may seem. Indeed, to contrast formal validity as "truth-preservation under all substitutions of non-logical terms"

[22]Ockham (1974, III-3 ch.1): "Some consequences are 'as-of-now', others are simple. An 'as-of-now' consequence is when the premise can be true at some time without the conclusion but not at this time ... A simple consequence is when the premise cannot be true at any time without the conclusion."

[23]Green-Pedersen (1980, p.128).

[24]See, e.g., Martin (2004, pp.134ff.), Dutilh Novaes (2016, §2.1).

[25]Following Spade (1976) she refers to him as 'Robert Fland'. On the identity of Robert Eland, see Read and Thakkar (2016).

with "a containment principle ... requiring that the understanding of the antecedent should contain the understanding of the consquent", as Klima (2016, p.318) and others do, is to some extent misleading. The containment principle explains why truth is preserved (when it is), whereas (despite the arguments given by Tarski, 2002) truth-preservation under substitution does not.[26] Rather, in both cases, validity is ensured by the connection of meaning between premises and conclusion, in the Parisian case the meanings of the logical, or syncategorematic, terms alone, in the English case, all the constituent terms. Both accounts give a reason why the consequence is valid, though the English account of formal consequence includes more consequences, with *ad impossibile sequitur quodlibet* and *necessarium sequitur ad quodlibet* being for the most part the only inferences classed as materially valid in the English tradition, where the understanding of the conclusion is notoriously not contained in the understanding of the premises.[27]

4 Firmness

But what of Roger? Weisheipl (1964, p.245) records a reference in a fourteenth-century commentary on William Sutton's treatise on consequence to a treatise on consequences by Roger, but this treatise seems never to have been identified, even if it still exists. So we have no direct, explicit evidence of Roger's preferred account of validity. In its absence, Paul Spade (1983, p.105) claims that Roger's alternative to truth-preservation as the account of validity is preservation of "signifying as is the case" or signifying as things are (*significans sicut est*)—Spade invents the neologism 'firm' for the property of signifying as things are.[28] He infers this (1983, p.113 n.35) from Roger's response to the objection that, since Roger's opinion "claims that some false proposition principally signifies as things are, then for the same reason it should claim that some true proposition signifies other than things are."[29] Roger countered:

> "The inference is not valid. But the premise [that every proposition falsifying itself is false] is true. And even though, if from some propositions each of which signifies principally as things are, some proposition follows, it signifies as things are, however, if from some propositions one of which signifies other than things are

[26]On the problems with Tarski's argument, see, e.g., Etchemendy (2008).
[27]We will return to the matter of formality in §6.
[28]See Heytesbury (1979, p.81). Klima (2016, pp.321) refers to it as "correspondence".
[29]Spade (1979, §33, p.191)

and all others as things are, some proposition follows, it doesn't follow that it signifies as things are."[30]

Here Roger certainly seems to equate validity with preservation of signifying as things are (*significans sicut est*)—firmness-preservation, we might say—though it does literally only give firmness-preservation as a necessary condition of validity. Note that (†) preserves firmness, even though it doesn't preserve truth: the premise and the conclusion both signify that the conclusion is false, which it is.

A similar move was made by John Buridan. Buridan was responding to the sophism (*) mentioned above: 'Every proposition is affirmative, so no proposition is negative'. In the fifth conclusion in ch.8 of his *Sophismata* (repeating similar remarks in his *Treatise on Consequences*), Buridan wrote:

> "The fifth conclusion is that for the validity of a consequence it does not suffice for it to be impossible for the antecedent to be true without the consequent, if they are formed together ... Therefore, something more is required, namely, that things cannot be as the [premise] signifies without being as the [conclusion] signifies."[31]

Preservation of firmness as the generic criterion for validity is also found in some authors in the English tradition, e.g., in Strode:

> "A consequence is said to be valid (*bona*) when things cannot be as is exactly signified by the premises unless they are as is exactly signified by the conclusion ... But a consequence is said to be valid in two ways: for some consequences are said to be valid in form (*bona de forma*) and some valid only in matter (*bona de materia*). A consequence is said to be valid in form when, if the way things are exactly signified by the premises is understood, the way things are signified by the conclusion is also understood ... and so it is said that in such consequences the conclusion is formally understood in the premises."[32]

[30] Spade (1979, §35, p.191), reprinted in Spade (1988).

[31] Buridan (2001, p.955). Cf. Buridan (2014, I 3, p.67). In fact, Buridan dislikes what he sees as the rather loose talk of "how a proposition signifies" and prefers to replace it by talk of the causes of truth (Buridan, 2014, I 1-2). Buridan is also able to ground the notion of causes of truth in the basic subject-predicate (aka "categorical") propositions using Ockham's nominalist formula concerning the various supposita of the subject and the predicate. See Buridan's *Sophismata* (Buridan, 2001, ch.2 conclusion 14, pp.858-9).

[32] Seaton (1973, pp.1-2): *Consequentia bona dicitur cuius non potest esse sicut adequate significatur per antecedens, nisi sit ita sicut adequate significatur per consequens suum ... Sed dupliciter dicitur consequentia bona: quedam enim dicitur consequentia bona de*

Miroslav Hanke (2014, p. 150) rejects the attribution of preservation of firmness as a criterion of validity to Roger.[33] For consider

(††) The conclusion of this inference is not firm, so the conclusion is not firm.

This inference is not considered by Roger, though he does consider its conclusion later in his treatise (§§103-109) and realises it is paradoxical: for if it were firm (that is, if things were as it signifies), it would not be firm, so by *reductio ad absurdum*, it is not firm. However, since the conclusion signifies that it is not firm, it does signify as things are and so is firm. Contradiction.

Roger does not class the paradox, strictly speaking, as an insoluble, since it does not falsify itself (see §100).[34] In fact, it undermines itself more radically, for Roger believes it neither signifies as things are nor other than they are.[35] So it isn't firm.

But there is an equivocation here over the phrase 'not firm', between 'signifying other than things are' and 'not signifying as things are'. When we infer that the conclusion is not firm, we are concluding that it does not signify as things are. But although Roger concedes that if it signifies other than things are then it does not signify as things are, he rejects the converse implication:

> "[The proposition 'This proposition signifies other than things are'] should be denied. And this inference should be denied: "it does not signify other than things are, and it signifies complexly, therefore, it signifies as things are". But it is necessary to add that it is not relevant to inferring itself not to signify as things are. And if that is added, it should be denied. For it follows directly from 'it signifies other than things are' that it does not signify as things are."[36]

On the other hand, 'This proposition does not signify as things are' should be granted, he says (§108), since for the same reason, it does not signify as things are. Even so, both 'This proposition signifies other than things are' and 'This proposition does not signify as things are' do signify in some way.

---

forma et quedam bona de materia tantum. Consequentia bona de forma dicitur cuius si sicut adequate significatur per antecedens intelligatur, sicut etiam adequate significatur per consequens intelligitur ... Et ideo dicitur quod in tali consequentia consequens est de formali intellectu antecedentis. My translation.

[33]See also Hanke (2013, §3.2, pp. 61-65) and Hanke (2017, p. 6).

[34]See also Read (2016, §3, pp. 339-44).

[35]See Spade (1979, §2, p.180); see also Read (2020, §3).

[36]Spade (1979, §104, p.218).

He claims that the conclusion of (††) neither signifies as things are nor other than things are, and so, he infers, it is neither true nor false.

Hanke finds the inference (††) itself in the work of John Mair, written some two centuries after Roger.[37] Mair considered two arguments against defining valid consequence as preservation of things being as is signified (that is, firmness):

> "I argue secondly: this consequence is valid, 'This conclusion signifies other than things are, therefore, this conclusion signifies other than things are', referring to the conclusion each time. For it proceeds from one synonym to another, but the conclusion signifies other than things are and the premise as things are. Proof: because either the conclusion signifies as things are or other than things are. If the second, we are done; if the first, and it signifies that the conclusion signifies other than things are, and so it is (you agree), so the conclusion signifies other than things are. From this it is clear that the premise signifies as things are, so [the definition is bad]."[38]

But that means that (††) is problematic for Roger, for the premise is firm (and indeed true, since it does not undermine itself). Thus the inference has a firm premise and infirm conclusion. If Roger really took preservation of firmness to be necessary for validity, he would have to say the inference was invalid.

Hanke (2013, pp. 64-65) backs up this judgment by considering Roger's reaction to an objection he considers (§36), namely, that his second claim is incompatible with Aristotle's rules in the first book of the *Prior Analytics* that if the conclusion is false, so is the premise, and if the premise is true so is the conclusion.[39] Roger replies that Aristotle's rules must be understood to concern firmness, not truth:

---

[37]Ashworth (1974, p.125 n.19) also finds it in Enzinas, Celaya and Caubraith.

[38]See, e.g., Mair (1527, f.142rb): *Secundo argumentor, hec consequentia est bona, hoc consequens significat aliter esse quam est, ergo hoc consequens significat aliter esse quam est, demonstrando consequens utrobique. Proceditur enim a synonimo ad synonimum, et tamen consequens significat aliter esse quam est, et antecedens sicut est. Probatio quia vel consequens significat taliter sicut est vel aliter esse quam est. Si secundum intentum habetur, si prius et ipsum significat consequens significare aliter esse quam est, et ita est per te, ergo consequens significat aliter esse quam est. Et ex illo patet quod antecedens significat taliter esse qualiter est in re, igitur.* Mair proceeds to repeat the argument with 'does not signify as things are' (*non significat sicut est*) in place of 'signifies other than things are' (*significat aliter esse quam est*).

[39]Spade refers to Book II, 57a36-37 and 53b7-8 respectively.

> "Here it must be said that what Aristotle means by the first rule is this: if the conclusion does not signify as things are and neither the premise nor the conclusion is relevant to inferring itself not to signify as things are, then the premise does not signify as things are. The second rule may be understood like this: if the premise signifies as things are and neither the premise nor the conclusion is relevant to inferring itself not to signify as things are, then the conclusion signifies as things are."[40]

Hanke's point is that if Roger really believed that firmness-preservation sufficed for validity, then the qualification in the repeated clause 'and neither the premise nor the conclusion is relevant to inferring itself not to signify as things are' would be unnecessary. Its presence implies that an inference preserving firmness which did not satisfy the condition, such as inference (††) above, might not be valid. So is (††) valid or not, and what was (or should be) Roger's view? Is his account of validity given in §35, where he writes "if from some propositions each of which signifies principally as it is, some proposition follows, it signifies as it is" (that is, validity preserves firmness), as Spade inferred (see n.30 above), or in §37, where he adds the qualification?

One problem with Hanke's proposal that we should follow what is given in §37 is that it seems to leave it open whether inferences like (*) are valid or not—Roger's theory will be incomplete. Indeed, reading §37 as saying that validity is preservation of firmness provided neither premise nor conclusion undermine themselves suggests that firmness-preservation may not be necessary or sufficient when they do, and so Roger might reject (††) as invalid. Mair (1527, f.141vb, 142rb) states several times that (††) is valid because it proceeds from one synonym to another (*nempe illa consequentia est valida, quia a synonimo ad synonimum proceditur*) even though it does not preserve firmness. However, we noted at the end of §2 that while proceeding from one synonym to another, that is, propositions with the same signification, might plausibly be sufficient for validity, it can hardly be necessary.[41]

But Strode and Eland, in arguing passionately against Roger's position, claim that it has nine impossible consequences, the fifth of which is that "Some inference is formally valid whose premise is true and conclusion neither true nor false" (Strode) and "Some inference is formally valid where the

[40] Spade (1979, §37, p.193).

[41] Hanke (2014) discusses in detail how George of Brussels, Thomas Bricot and John Mair differ over their accounts of synonymy and whether synonymy should suffice for validity, given that propositions with the same signification—and so arguably synonymous—can differ in truth-value and indeed in whether or not they are firm. See also Ashworth (1977) and Hanke (2013).

premise is true[42] and the conclusion neither signifies as things are nor other than they are" (Eland).[43] They don't say what this inference was. They may, of course, be speaking in propria persona when they say the inference is formally valid (*bona et formalis*), and not attributing this view to Roger, but when they cited Roger's second claim (they dubbed it the third: "In a formally valid inference the false follows from the true") they were voicing Roger's own view. So it is reasonable to assume the same for the fifth impossibility they attribute to him. In that case, Roger cannot be wedded to the unqualified account in §35, whereby (††) would be invalid. Strode and Eland's fifth impossibility also suggests that inference (††), or something like it, may have already been known in the fourteenth century, if not when Roger wrote his treatise, at least soon afterwards.

There is good reason, then, to conclude that Roger's view of (††), which he does not himself consider explicitly, is that it is valid, and presumably formally so. This means that he will be committed to the claim that some formally valid argument has a firm premise and infirm conclusion, as Strode and Eland aver. However, it does leave us wondering what Roger's generic account of validity was, given that it is neither truth-preservation nor firmness-preservation. The example is at least consistent with the idea that synonymy between premise and conclusion does suffice for validity. We noted that we had such identity of what premise and conclusion signified in inference (†).

But quite how the English inclusion criterion ("if the way things are exactly signified by the premises is understood, the way things are signified by the conclusion is also understood", as articulated by Strode above) should be applied in the case of insolubles is still unclear. Recall the distinction between formally and materially valid consequences: they are different ways of being valid, but there is nonetheless a common feature to their validity, be it preservation of truth or of firmness or whatever. The danger with Roger's or Mair's account is that validity for insolubles can come apart from that for non-insolubles, so that, e.g., firmness-preservation is necessary and sufficient for validity for non-insolubles while synonymy is sufficient, but presumably not necessary, for insolubles. Then what does validity for insolubles have to do with validity for non-insolubles? Why call it 'validity' at all? What we need is a uniform account of validity, which is then satisfied differently by formal consequences and material consequences, and so too for non-insolubles and insolubles. It turns out that such an account is found in Paul of Venice's *Logica Magna*.

[42] Spade follows the (single) manuscript in reading *falsum* here. But *verum* not only matches Strode's text, but makes better sense.

[43] Spade (1978, Strode §1, p. 76; Eland, there referred to as 'Fland', §35, p. 68). Cf. Read (2016, pp. 345-7).

5 Paul of Venice on Insolubles and Consequence

As mentioned above, Paul of Venice follows Roger in claiming that insolubles falsify themselves, though he amends the surrounding theory and updates it, especially to make it accord with his own theory of signification. Unlike Roger, Paul believes that the total significate of a proposition includes far more than what the terms commonly mean (*sicut termini communiter pretendunt*),[44] but Paul restricts the criterion for truth to the proposition's exact significate (*significatum adequatum*). Paul's exact significate matches Roger's signifying principally:

> "The first assumption is this: A true proposition is one whose exact significate is true and for which it is not incompatible that the proposition is true. This is clear from what has been said in the treatise 'On the truth and falsity of propositions'.[45]
>
> The second assumption: A false proposition is one which either falsifies itself or whose falsity does not arise from its terms, but from its false exact significate."[46]

Thus the truth or falsehood of propositions is dependent on the truth or falsehood of their exact significates.[47] In including that extra clause, 'and it is not incompatible that the proposition, signifying exactly in that way, is true' in his definition of truth in the treatise 'On the truth and falsity of

[44]See the treatise *De Significato Propositionis* in Paulus Venetus (1978, Thesis 3, pp. 192-3). Paul does not use the phrase '*sicut termini communiter pretendunt*' in that treatise, but he does use it, in propria persona, repeatedly in his treatise on insolubles in the *Logica Magna* (see, e.g. Paulus Venetus, 1499, f.195vb) and in his *Logica Parva* (Paulus Venetus, 2002, pp. 128 ff.).

[45]See Paulus Venetus (1978, p.62), where he writes: "If the exact significate of a proposition is true and it is not incompatible that the proposition, signifying exactly in that way, is true, the proposition is true ... Note that I say 'it is not incompatible that the proposition ... is true'. For, as I said earlier, the proposition 'This is false' (referring to itself) is false, even though its exact significate is true." (my translation)

[46]Paulus Venetus (1499, f.194vb), corrected where necessary against Biblioteca Apostolica Vaticana lat. 2132, f.239rb: *Prima suppositio est ista: quod propositio vera est illa cuius adequatum significatum est verum et non repugnat ipsam esse veram. Patet ex dictis in de veritate et falsitate propositionum. Secunda suppositio: propositio falsa dicitur esse illa que falsificat se, aut cuius falsitas non consurgit ex terminis sed ex adequato significato falso.* Text and translation from Paul's treatise on 'Insolubles' are from an edition currently in preparation by Barbara Bartocci and myself (Paulus Venetus, 20xx).

[47]I discussed this dependency at some length in Read (2020, §3) and defended it against a charge of regress.

propositions', Paul had anticipated the problems thrown up by the insolubles. In the treatise on insolubles, Paul proceeds to draw several iconoclastic Conclusions from his account of truth and falsehood, including the three famous claims of Roger's that we noted earlier. In particular, he states in his fifth Conclusion that

> "There are some formally valid inferences which signify exactly by the composition of their principal parts, where the premise is true and the conclusion false."[48]

Again, we may wonder what account of validity Paul endorses which permits such a situation, and how it is still possible to reason and argue coherently. His account of validity is, in fact, different from Roger's. Paul writes in the chapter in the *Logica Magna* on consequence and inference (*De Rationali*):

> "A valid inference which signifies in accordance with the composition of its parts may be defined as one where the conclusion's contradictory would be incompatible with its premise, given that these signify as they do; and by 'as they do' I refer to what they customarily signify (*ipsorum significata consueta*)."[49]

He reiterates this in the treatise on insolubles:

> "Briefly, I assume all the rules of formal inferences in which it is stated that an argument from this to that is formally valid. This assumption is clear since, otherwise, it would be possible both that some inference is not valid and that the conclusion's opposite is formally incompatible with the premise, the opposite of which, however, I assume strongly. For, in agreement with everybody, I mean that if there is an inference that signifies by the composition of its parts and the conclusion's opposite is formally incompatible with the premise, that inference is formally valid."[50]

[48]Paulus Venetus (1499, f.195rb), ms 239vb: *Quinta conclusio: aliqua consequentia est bona et formalis significans adequate ex compositione suarum partium principalium et antecedens est verum et consequens falsum.*

[49]Paulus Venetus (1990, p.80). The significance of the qualification "which signifies in accordance with the composition of its parts" is that Paul believes that, e.g., 'conversely' in 'A man is an ass so an ass is a man and conversely' is an inference (*consequentia*) but does not signify by the composition of its parts, and has neither premises nor conclusion.

[50]Paulus Venetus (1499, f.195ra), ms 239rb: *Et breviter suppono omnes regulas consequentiarum formalium quibus asseritur formaliter valere argumentum ab hoc ad illud. Hec suppositio patet quia aliter staret aliquam consequentiam non valere et oppositum*

Such an account of validity may look implausible for one who endorses Roger's third claim (that pairs of contradictories can both be false), as Paul does in his second Conclusion. Indeed, the fifth Conclusion appears flatly to contradict what he had written in the fourth section of the chapter on consequence earlier in the *Logica Magna*. He wrote there:

> "The Third Rule is this:[51] If the premise of a valid inference which signifies primarily in accordance with the composition of its parts is true, then the conclusion is also true. The Rule is proved as follows: If the premise of a valid inference is true and its conclusion is false, and if the conclusion's contradictory exists, then it is true; it is therefore consistent with the premise, since every independent truth is compatible with any other independent truth; but if that is so, the inference is not valid."[52]

The proof of this third rule appeals overtly to the (RCP): "if the conclusion is false ... then its contradictory is true." But his second Conclusion in the treatise on insolubles denies this, repeating Roger's third claim:

> "Some contradictories, that is, which contradict one another, are simultaneously false."[53]

We noted above that when formulating his account of truth in the treatise 'On Truth and Falsehood', Paul anticipated the issue with insolubles, but at this point in his chapter on consequence, he fails to do so. He does, however, partially foresee it later in the chapter when faced with the Knower paradox. The example considered there is the inference:

> (**) This is unknown to you, therefore this is unknown to you,[54]

where each occurrence of 'this' refers to the conclusion. In the treatise on insolubles, Paul cites this inference in proof of his fourth Conclusion:

---

consequentis formaliter repugnare antecedenti, cuius oppositum premaxime suppono. Volo namque concordanter cum omnibus quod si est aliqua consequentia significans ex compositione suarum partium, et oppositum consequentis repugnat formaliter antecedenti, quod illa consequentia sit bona et formalis.

[51]Note that under the heading of 'rules' (*regulae*) the medievals grouped not only rules of inference (as we would now call them) but also meta-rules, like the Third Rule here.

[52]Paulus Venetus (1990, p.140).

[53]Paulus Venetus (1499, f.195rb), ms 239vb: *Secunda conclusio est ista: aliqua contradictoria inter se contradicentia sunt simul falsa.*

[54]Paulus Venetus (1990, p.200).

> "There are some inferences that are formally valid, and known by you to be so, which signify by the composition of their parts, where the premise is known by you, yet the conclusion is not known by you."[55]

His proof runs:

> "For it is evident that this inference (**) is formally valid, because one cannot see how the opposite of the conclusion can be compatible with the premise. But the premise is known by you, because you know that the conclusion is not known, since it is an insoluble that asserts that it itself is unknown. And indeed the conclusion is not known by you. Proof: because if it is known by you, then it is true and it signifies exactly that it is unknown to you, therefore it is true that it is unknown to you, therefore you do not know the conclusion. And thus whatever one says, it follows that the conclusion is not known by you."[56]

The conclusion ('This is unknown to you') is an example of the Knower Paradox.[57] If you know it, then it must be true, and so you don't know it—and so, by *reductio ad absurdum*, you don't know it. But that's what it says, and you've just proved it, so you do know it. Paradox.

Paul is rather conflicted about this sophism. In ch.2 of the 'Insolubles', just quoted, he calls it an insoluble. But he includes it in ch.8 of the same treatise, which is headed 'On merely apparent insolubles'.[58] Moreover, whereas in ch.2 he says that the sophismatic inference is known by you to be valid, that its premise is known by you, and that the conclusion is not known by you (and he agrees in ch.8 that indeed the conclusion cannot be known by you), in the earlier chapter on 'Consequences' he infers from the

[55]Paulus Venetus (1499, f.195va), ms 240ra: *Quarta conclusio: aliqua consequentia est bona et formalis, scita a te esse talis, significans ex compositione suarum partium, et antecedens est scitum a te et consequens non est scitum a te.*

[56]*Patet namque consequentiam illam esse bonam et formalem, quia non videtur quomodo stabit oppositum consequentis cum antecedente et antecedens est scitum a te, quia scis illud consequens non sciri, cum sit insolubile asserens se nesciri. Et tamen consequens non est scitum a te. Probatur: quia si est scitum a te, igitur est verum et significat adequate ipsum nesciri a te, igitur verum est ipsum nesciri a te, igitur tu non scis illud consequens. Et sic qualitercumque dicatur sequitur illud consequens non esse scitum a te.*

[57]See, e.g., Anderson (1983). We might call Paul's example the *Inferential Knower Paradox.* The Knower Paradox proper is its conclusion.

[58]*De apparentibus insolubilibus*: the reason to call it merely apparent is that it doesn't strictly meet his criterion for being an insoluble, namely, implying its own falsehood, that is, that it is not true. Rather, it implies that it is not known. He also discusses it in Sophism 50 in his *Sophismata Aurea* (Paulus Venetus, 1493, f.53vb).

fact that the conclusion is not known by you that the premise is not known by you either. For his Ninth Rule states:

> "Suppose that a certain inference is valid, is known by you to be valid, is understood by you, and signifies primarily in accordance with the composition of its elements; suppose too that its premise is known by you, and that you know that what is false does not follow from anything that is true; then its conclusion is also known by you."[59]

Paul retains the validity of the Ninth Rule in the face of the inferential Knower Paradox by denying the premise:

> "In the given posited case, ⟨I⟩ grant that all the clauses in the [Ninth] Rule are satisfied, but ⟨I⟩ deny that the premise is known, the reason being that just as it is incompatible for the conclusion to be known, so it is for the premise. But more about this when we come to deal with the insolubles."[60]

In the *Insolubles*, it transpires that the final qualification in the Ninth Rule ("you know that what is false does not follow from anything that is true") fails, as noted above in his Fifth Conclusion.

It remains to be seen, however, whether anything hangs on the inconsistency between the chapter on consequence and the treatise on insolubles. For the Third Rule cited above only follows from his account of validity given (RCP): so once that is rejected in the second Conclusion of ch.2 of the *Insolubles*, perhaps the way is clear for the fifth Conclusion to accord with the earlier account of validity.

Paul considers three objections to the Third Rule of consequences, but none raises the issue of insolubles. Hughes notes that Paul is following Strode's *Consequences* here and responding to some of Strode's problems with the Rule,[61] but Strode was not a follower of Roger on insolubles, indeed, he was a major critic, as we saw in §2. As we noted in §4, Strode defined validity in terms of preservation of signifying as things are, so Strode's proof of the rule (his first, Paul's third) is different from Paul's, and in fact does not invoke (RCP).[62]

Let us, then, return to the earlier example from Roger discussed in §2:

[59] Paulus Venetus (1990, p.195).
[60] Paulus Venetus (1990, p.200).
[61] Paulus Venetus (1990, p.282 n.289).
[62] See Seaton (1973, §1.2.03, p.6).

> (†) The conclusion of this inference is false, so this conclusion is false.

The premise is (for Paul, as for Roger) true and the conclusion false, but Paul (like Roger) endorses the validity of the inference. For the conclusion's contradictory (*viz* 'This conclusion is not false') is also false, and clearly incompatible with the premise, which truly and correctly says that the conclusion is false.

But care is needed here in interpreting Paul's formula '*contradictorium consequentis repugnat antecedens*', which I have translated as 'the conclusion's contradictory is incompatible with the premise(s)'. Paul explained '*repugnat*' ('is incompatible') in the chapter on conditionals:

> "Two propositions are mutually incompatible when, signifying as they do, they cannot be nor can have been nor could be true together, or at least their significates cannot be nor can have been nor could be true together."[63]

and he emphasizes in the chapter on inference that "two things ⟨are⟩ incompatible with each other when the conjunction formed from them is *per se* impossible."[64] How does this relate to the treatment of validity as firmness-preservation? As we noted in §3, propositions were taken by the medievals to be concrete tokens, and that they subscribe to this conception is clear from one of the objections that is standardly made to the definition of validity as the impossibility of the premises being true and the conclusion false, by stipulating a case where the conclusion does not exist. Paul (1990, pp.141-2) considers such an objection to his Third Rule in the chapter on consequences. His response does not question the assumption that propositions can cease to exist (if written, when erased; if spoken, as the sound dies away; if mental, when no longer thought).

Given this conception, each proposition has many contradictories. For most thinkers, this is not a problem, for they will all be equivalent and have the same truth-value. But for Paul, this will not be true. As Paul points out in the seventh Conclusion in ch.2 of his 'Insolubles', there are pairs of convertible propositions (which for him means those with the same exact signification, predicating the same thing of the same subject) one of which can be true and the other false:

> "Seventh Conclusion: some pairs of propositions are convertible, but one is true or possible and the other is false or impossible. It

[63] Paulus Venetus (1990, p.21).
[64] Paulus Venetus (1990, p.81).

> is clear as regards these propositions: ‘This is false’ and ‘This is false’, both referring to the second.”[65]

Nonetheless, though some mutually convertible propositions will be true and some false, they will all be contradictories of another proposition, in predicating the same thing of the same thing, and so affirming of it what the other denies or vice versa.

This Aristotelian, syntactic conception of contradictory opposition, denying of the same what the other affirms and vice versa, serves for Paul to define validity, despite contradictory pairs sometimes having the same truth-value, or pairs of convertible propositions sometimes having different truth-values. What are incompatible are their exact significates, where the exact significates are what are customarily signified by the premise and conclusion. So the requirement for validity is that the contradictory significate of the conclusion and the significate of the premise cannot both be true—where being true or false for significates is their obtaining or not.[66] Recall that, for Paul as well as Roger, contradictories cannot both be true, though they can both be false. Hence any inferences, such as (†) or (††), where the exact significates of premise and conclusion are the same, must be valid, for the contradictory significate of the conclusion will also be a contradictory significate of the premise. This account of validity cannot guarantee either truth-preservation or firmness-preservation for valid inferences, for the contradictory significate of the conclusion of (†), *viz* that the conclusion is not false, is incompatible with the significate of the premise, *viz* that the conclusion is false, so (†) is valid; and similarly, the contradictory significate of the conclusion of (††), *viz* that the conclusion signifies as things are, is incompatible with the significate of its premise, *viz* that the conclusion does not signify as things are, so (††) is valid. Nonetheless, as Paul remarks:

> “But one can respond to all these arguments in another way, always admitting the scenario, by denying both contradictories, namely,
>
> | | |
> |---|---|
> | It is as Socrates says it is | It is not as Socrates says it is |
> | It is as A signifies | It is not as A signifies |
> | Some proposition signifies other than it is | No proposition signifies other than it is |
>
> For just as it is not impossible for two contradictories to be false

[65]Paulus Venetus (1499, f.195va), ms 240ra: *Septima conclusio: alique due propositiones adinvicem convertuntur et tamen una est vera vel possibilis et reliqua falsa vel impossibilis. Patet de talibus: hoc est falsum et hoc est falsum, secunda demonstrata.*

[66]See Read (2020, §3).

> at the same time in the case of insolubles, so it is not impossible for the same thing to be denied at the same time in the same case, and especially when the insolubles principally have reflection on their own signification."[67]

In Read (2020, §3) I questioned whether this is compatible with Paul's realism, but if it is, and things can be neither as a proposition signifies nor other than it signifies, then (††) is not only valid, but its premise signifies as things are while its conclusion does not, that is, firmness is not preserved. Thus Paul's account of validity captures the notion of validity for which Roger seemed to be searching.

Hanke (2017, p.6) appears to contest this conclusion. He writes:

> "Inferences such as [(††)] seem to challenge the criterion of propositional truth-preservation, if propositional paradoxes are neither-true-nor-false: this type of inferences [*sic*] is not addressed by either Swyneshed or Paul of Venice."

By 'propositional truth-preservation', Hanke means "if the exact significates of the premises are true then so is the exact significate of the conclusion". Though firmness-preservation is how Roger defines validity (self-undermining propositions aside), it is not how Paul defines it. His definition, which applies to non-insolubles and insolubles alike, turns on the incompatibility of the premise with the contradictory of the conclusion. The premise of (††) and the contradictory of its conclusion are indeed incompatible, since their exact significates cannot both be true. So (††) is valid for Paul. But it does not follow that if the exact significate of the premise is true then so is that of the conclusion. For as we have just seen Paul observe, the premise's exact significate is true—the conclusion neither signifies as things are nor other than they are—so the conclusion is neither true nor false. Here Paul offers an alternative response to the paradox generated by the conclusion of (††), closer to Roger's, denying both that its significate is true and that its significate is false. So the inference is valid, the premise is firm and the conclusion is infirm.

[67]Paulus Venetus (1499, f.196rb), ms f.241ra: *Potest tamen ad hec omnia aliter responderi negando semper admisso casu utrumque contradictorium, videlicet: ita est sicut sortes dicit, non est ita sicut sortes dicit; ita est sicut a significat, non est ita sicut a significat; aliqua propositio significat aliter quam est, nulla propositio significat aliter quam est. Sicut enim non est inconveniens duo contradictoria esse simul falsa in materia insolubilium ita non est inconveniens eadem simul negari in eadem materia, et precipue quando insolubilia habent principaliter reflexionem ad significationem propriam.*

6 Validity in Form

Recall from §3 that Roger's and Paul's account of insolubles can be adapted to yield syllogistic counterexamples to truth-preservation as necessary for validity. How then do they avoid inconsistency? Note that the standard medieval view is that the syllogistic is an example of formal validity, that is (for Paul), where the contradictory of the conclusion is formally incompatible with the premises.[68] Paul states this explicitly:

> "The regular syllogism has three figures containing moods such that an argument in any of these moods is formally valid."[69]

He notes that both Baroco and Bocardo are reduced to the first figure *per impossibile*, as Aristotle proposed in *De Interpretatione* chs.5-6:

> "The fourth mood of [the second] figure, namely Baroco, is composed implicitly or explicitly from a universal affirmative and a particular, indefinite, or singular negative constructed normally, concluding directly or indirectly in a particular, indefinite, or singular negative either implicitly or explicitly. For example, from these premises: 'Every man is an animal' and 'Some stone is not an animal', it follows that some stone is not a man, and conversely indirectly. Now this syllogism is reduced to the first [mood] of the first [figure] *per impossibile*, by taking the major with the opposite of the conclusion and inferring the opposite of the minor, namely:
>
> Every man is an animal, every stone is a man, therefore every stone is an animal."[70]

In fact, Aristotle's method of reduction *per impossibile* is weaker than *reductio ad absurdum* (RAA), equivalent rather to *modus tollendo tollens* (MTT) or Negation-Introduction in the form: from 'not-B' and a proof that B follows from A, infer 'not-A' (discharging the assumption of A). (RAA),

[68]See, e.g., Paulus Venetus (1990, pp.89, 91).

[69]Paulus Venetus (1499, f.172ra), ms f.158va: *Syllogismus regularis tres habet figuras modos continentes in quorum quolibet argumentum existit [consistit* ms*] formale.*

[70]Paulus Venetus (1499, f.174vb), ms f.161va: *Quartus modus istius figure, videlicet, Baroco constat implicite vel explicite ex universali affirmativa et particulari indefinita vel singulari negativa modi consueti, particularem indefinitam vel singularem negativam implicite vel explicite directe vel indirecte concludentibus. Verbi gratia, ex hiis premissis: Omnis homo est animal et quidam lapis non est animal, sequitur quod quidam lapis non est homo, et econtra indirecte. Iste autem syllogismus reducitur ad primum prime per impossibile, sumendo maiorem cum opposito conclusionis et inferri oppositum minoris, videlicet: Omnis homo est animal, omnis lapis est homo, igitur omnis lapis est animal.*

read as licensing inference of 'not-A' from a derivation of a contradiction of the form 'B and not-B' from A, entails (MTT) and Negation-Introduction, but not vice versa.[71] Indeed, the use of the phrase 'reduction *per impossibile*' to describe Aristotle's method of indirect proof is something of a misnomer, as Robin Smith points out in Aristotle (1997, pp.120-1): first, nothing is reduced to impossibility—rather, one mood is reduced to another; secondly, the contradictory of the conclusion is "reduced to", or better, is shown to entail, in conjunction with one premise, something false, namely, the contradictory, or contrary, of the other premise. What is impossible is that this pair of contradictories or contraries hold together.[72] Roger and Paul agree to that part of (RCP).

However, as Ashworth (2016, p.190) notes, Paul distinguishes formal validity (*consequentia bona et formalis*) from validity in form (*consequentia bona de forma*).[73] Validity in form is a species of formal validity, but not all formal validity is validity in form. Indeed, for Paul, most valid inferences are formally valid:[74] the only materially valid inferences are those whose premises or the contradictory of whose conclusion are logically or physically impossible and do not formally entail an explicit contradiction.[75] Validity in form is much tighter:

> "An inference which is valid in form [*de forma*] may be defined as one where any inference of the same form is valid."[76]

Note, however, that this does not mean that the inference holds in all terms, as one finds formal validity or validity in form defined in Buridan or in modern logic. For Paul continues:

[71] Technically, (RAA) involves a structural contraction, which is absent from (MTT) and Negation-Introduction. See, e.g., Routley (1982, ch.3 §9). Note also that in the present discussion it's implicit that each proposition is equivalent to its double-negation (that is, contradictories are mutually contradictory).

[72] See Aristotle's discussion in *Prior Analytics* II 11.

[73] See Paulus Venetus (1990, p.104). Hughes translates '*bona de forma*' as 'valid because of its form', but we will see shortly that this is at best misleading: such an inference is valid because it satisfies the criterion for validity (the incompatibility of the premises with the contradictory of the conclusion), and it so happens that all inferences with the same form do too.

[74] See Read (2010, pp.174-5). Paul's formal validity thus follows the English tradition, while his validity in form is closer to, but does not match Buridan's Parisian formal validity mentioned in §2 above.

[75] Note that whereas for others, e.g., Buridan, formal validity is a species of material validity, for Paul formal validity is contrasted with material validity. See Hughes' comment at Paulus Venetus (1990, p.262 n.196).

[76] Paulus Venetus (1990, p.104).

> "An example [of an inference valid in form is] 'A human being is running, therefore an animal is running'.'

What it is for two inferences to share their form is a complex issue for Paul, and he takes the next six pages to explain it. But one thing is clear, it is not their sharing a form that constitutes their validity, nor even their formality. Validity turns on whether the premise (or premises) are naturally suited (*aptum natum*) to be incompatible with the contradictory of the conclusion; whether that validity is formal or material turns on whether that incompatibility is formal or material. Formal incompatibility comes in three degrees (Paulus Venetus, 1990, pp.91-92), the highest of which turns on whether they can be "understood or imagined without contradiction" (*intelligibile vel imaginabile ... absque contradictione*) together. He then observes:

> "Every valid syllogism is an inference which is valid in form."[77]

But he adds that there are more forms than moods, so a syllogism is not valid in virtue of instantiating a particular mood, or even a particular form, but in virtue of whether the contradictory of the conclusion is formally incompatible with the premises in the highest degree, that is, "when one of them cannot without contradiction be understood or imagined along with the other (given that they signify in the customary way)."[78] Sameness of mood is nonetheless a necessary condition of validity, so if a syllogism is valid so too are all syllogisms in the same mood and figure.[79] Although syllogisms are not valid in virtue of instantiating a mood, moods are valid or not in virtue of all their instances being valid or not. Hence the earlier proof of the validity of a mood is effective; and crucial to the proofs of Baroco and Bocardo by reduction *per impossibile* to Barbara is the fact that, for Roger and Paul, contradictories cannot both be true, though they can both be false. Hence the novel syllogism in Baroco in §3, while valid, can nonetheless have a false conclusion, despite having true premises, in defiance of the Third Rule in §4 of the chapter *De Rationali*. Aristotle's reduction of the paradoxical example of Baroco can be set out like this:

$$\dfrac{\dfrac{\overset{\text{Premise}}{\text{Every } N \text{ is } M} \quad \overset{\text{Hypothesis}}{\text{Every } X \text{ is } N^1}}{\text{Every } X \text{ is } M}\,\text{Barbara} \qquad \overset{\text{Premise}}{\text{Not every } X \text{ is } M}}{\text{Not every } X \text{ is } N}\,\text{Negation-Introduction}(1)$$

[77] Paulus Venetus (1990, p.111).

[78] Paulus Venetus (1990, p.92).

[79] See Paulus Venetus (1990, p.112): "two syllogisms of which one is in a certain mood or figure but the other is not ... differ in form."

where N is 'true proposition', M is 'premise' and X is 'conclusion'. We take the major premise ('Every N is M') with the opposite of the conclusion ('Every X is N'), as Paul said in the quotation above from his treatise on 'Syllogisms',[80] and infer the opposite of the minor premise ('Every X is M') by Barbara. But that contradicts the minor premise ('Not every X is M'), so we can infer the opposite of the hypothesis ('Not every X is N'), the conclusion of Baroco. Of course, in this strange case, both the final conclusion, 'Not every conclusion is true', and its contradictory opposite, the hypothesis, 'Every conclusion is true', are false, the hypothesis because things are not as it signifies (the only conclusion is false), and the conclusion because it falsifies itself.[81]

Paul's statement, cited above, that he "assume⟨s⟩ all the rules of formal inference in which it is stated that an argument from this to that is formally valid" turns out to be rather rash. For it commits him to asserting falsehoods, though only the falsehoods manifested in insolubles. We might grant that there is a certain honesty in bravely asserting the Moorean paradox, 'What I am saying is false. What I have just said was false', asserting a falsehood followed immediately by a truth, namely, that the falsehood was false.[82] This should perhaps be no surprise, given Roger's second claim. For the second claim could be (correctly) qualified as saying that there is a valid inference whose premise is true and whose conclusion is an insoluble implying its own falsehood, and so false. This is no more than an immediate consequence of strengthening the criterion for truth so as to deem all insolubles simply false.

7 Conclusion

Roger Swyneshed rejects the definition of validity of an inference as requiring the preservation of truth. This is because, following his diagnosis of the paradoxes thrown up by the insolubles, a valid inference can issue in a false conclusion from true premises simply because the conclusion falsifies itself, despite things being as it signifies. The classic example is the Liar paradox, 'This is false', which, in saying of itself that it is false, falsifies itself, so that it is false even though things are as it signifies. His three iconoclastic claims come together: the first recognises that the Liar is false despite things being as it signifies; the second that the Liar, though false, follows validly from the

[80] Paul is following Aristotle in *Prior Analytics* 5, 27a37-b1.

[81] Of course, we need now to pretend that the hypothesis and intermediate conclusion don't exist, or find an alternative scenario in which they do and the final conclusion is still an insoluble.

[82] On Moore's paradox, see, e.g., Moore (1993).

true claim that it is false; and the third that its contradictory, ‘This is not false’, saying of ‘This is false’ that it is not false, is also false, so that a pair of contradictories can both be false. This last is contrary to received wisdom endorsing the Rule of Contradictory Pairs, that in any such pair, one is true, the other false, but is arguably in accordance with Aristotle’s remarks in *De Interpretatione*, although their counterexamples to the rule differ.

What account of validity Roger adopted in place of truth-preservation is a matter of speculation. At one point he suggests preservation of firmness is necessary, only to qualify it a few sentences later. Why he does so is unclear, but he may be prescient in recognising that not only does inference (†) show that truth-preservation is not necessary, but inference (††) shows that preservation of firmness is not necessary either. (††) is not mentioned by Roger or Strode or Eland, but one of the latter two’s objections to Roger’s theory is that it accepts that an inference can be valid but fail to preserve firmness, suggesting that they had an example in mind. Indeed, Hanke (2017, p.4) credits to Paul, or perhaps to his reading of Paul, the realisation that whatever value one proposes should be preserved to ensure validity, there will be a paradox like (†) and (††) which will show that its preservation cannot be necessary, assuming that identity (or synonymy) is sufficient.

Paul of Venice follows Roger in endorsing all three claims, although Paul’s theory of signification and account of validity are different. But he shares with Roger the aim of solving the paradoxes without postulating hidden meanings in them, as their contemporaries Bradwardine and Heytesbury had done. Paul agrees that things are as the Liar signifies—as he puts it, its exact significate is true. But that is not enough to make it true, since it falsifies itself. So it is false, while at the same time following validly from the true claim that it is false. For the premise asserting that it is false does not falsify itself, while its exact significate, the same as the exact significate of the conclusion, is true. So the premise is true. Why, then, is the inference valid? For Paul, the reason is that the premise is incompatible with the contradictory of the conclusion, namely, ‘This is not false’. On either definition, Swyneshed’s inference concluding in the Liar turns out to be valid. So too do all instances of (by definition, valid) second- and third-figure moods of Aristotle’s assertoric syllogism, even those whose validation proceeds, for Aristotle, by reduction *per impossibile* to the first figure, and even though their premises may be true and conclusion false—and not only those which are self-falsifying. For Aristotle, this situation cannot arise, since syllogistic propositions obey the (RCP). The reason Aristotle’s method of reduction continues to work for Roger and Paul is that for them contradictory pairs cannot both be true, though they can both be false, contrary to the (RCP).

References

Anderson, C. (1983). The paradox of the knower. *Journal of Philosophy*, 80:338–355.

Aquinas, S. T. and Cajetan, T. (1962). *Aristotle on Interpretation*. Marquette University Press, Milwaukee, Wisc. English translation by Jean T. Oesterle.

Aristotle (1963). *Categories and On Interpretation*. Clarendon Aristotle Series. Clarendon Press, Oxford. Translated with Notes by J.L. Ackrill.

Aristotle (1965). *De Interpretatione vel Perihermeias: Translatio Boethii*. Aristoteles Latinus II 1-2. Brill, Leiden. Ed. Laurentius Minio-Paluello.

Aristotle (1997). *Topics: Books I and VIII*. Clarendon Aristotle Series. Clarendon Press, Oxford. Translated with a Commentary by Robin Smith.

Ashworth, E., editor (1974). *Language and Logic in the Post-Medieval Period*. Reidel, Dordrecht.

Ashworth, E. (1977). Thomas Bricot (d. 1516) and the liar paradox. *Journal of the History of Philosophy*, 15:267–80.

Ashworth, E. (2016). The post-medieval period. In Dutilh Novaes and Read (2016), pages 166–91.

Boethius, A. (2010). *On Aristotle* On Interpretation *1-3*. Duckworth, London. Boethius' Second Commentary, tr. Andrew Smith.

Buridan, J. (2001). *Summulae de Dialectica*. Yale U.P., New Haven. English translation by Gyula Klima.

Buridan, J. (2014). *Tractatus de Consequentiis*. Fordham UP, New York. English translation by Stephen Read.

Buridan, J. (n.d.). Questions on the Prior Analytics. H. Hubien ed. URL=`http://individual.utoronto.ca/pking/resources/buridan/QQ\_in\_Prior\_An.txt`.

Dutilh Novaes, C. (2016). Medieval theories of consequence. In Zalta, E. N., editor, *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, fall 2016 edition.

Dutilh Novaes, C. and Read, S., editors (2016). *The Cambridge Companion to Medieval Logic*. Cambridge University Press, Cambridge.

Etchemendy, J. (2008). Reflections on consequence. In Patterson, D., editor, *New Essays on Tarski and Philosophy*, pages 263–99. Oxford UP, Oxford.

Fitzgerald, M. J. (2002). *Albert of Saxony's Twenty-Five Disputed Questions of Logic*. Brill, Leiden.

Green-Pedersen, N. J. (1980). Walter Burley's *De consequentiis*: an edition. *Franciscan Studies*, 40:102–166.

Hanke, M. (2013). John Mair on semantic paradoxes: Alethic modalities and validity in paradoxical contexts. *Studia Neoaristotelica*, 10:50–87.

Hanke, M. (2014). The Bricot-Mair dispute: Scholastic prolegomena to non-compositional semantics. *History and Philosophy of Logic*, 35:148–66.

Hanke, M. (2017). Paul of Venice and realist developments of Roger Swyneshed's treatment of semantic paradoxes. *History and Philosophy of Logic*, 38:299–315.

Heytesbury, W. (1979). *On "Insoluble" Sentences: Chapter One of his Rules for Solving Sophisms*. Pontifical Institute of Mediaeval Studies, Toronto. Translated with an Introduction and Study by Paul Vincent Spade.

Klima, G. (2004). Consequences of a closed, token-based semantics: the case of John Buridan. *History and Philosophy of Logic*, 25:95–110.

Klima, G. (2016). Consequences. In Dutilh Novaes and Read (2016), pages 316–41.

Mair, J. (1527). *Introductorium Perutile in Aristotelicam Dialecticen*. Jean Petit, Paris.

Martin, C. (2004). Formal consequence in Scotus and Ockham: towards an account of Scotus' logic. In Boulnois, O., Karger, E., Solère, J.-L., and Sondag, G., editors, *Duns Scot à Paris*, pages 117–150. Brepols, Turnhout.

Moore, G. (1993). Moore's paradox. In Baldwin, T., editor, *G.E. Moore: Selected Writings*, pages 207–12. Routledge, London and New York.

Ockham, G. de. (1974). *Summa Logicae*. Franciscan Institute Publications, St Bonaventure. Edited by P. Boehner et al.

Paulus Venetus (1493). *Sophismata Pauli Veneti*. Bonetus Locatellus for Octavianus Scotus, Venice.

Paulus Venetus (1499). *Logica Magna.* Bonetus Locatellus for Octavianus Scotus, Venice.

Paulus Venetus (1978). *Pauli Veneti* Logica Magna *Secunda Pars, Tractatus de Veritate et Falsitate Propositionis, et Tractatus de Significato Propositionis.* Oxford UP, Oxford. Published for the British Academy. Edited by Francesco del Punta and Marilyn McCord Adams.

Paulus Venetus (1990). *Pauli Veneti* Logica Magna *Pars II Fascicule 4, Capitula de Conditionali et Rationali.* Oxford UP, Oxford. Published for the British Academy. Edited by G.E. Hughes.

Paulus Venetus (2002). *Logica Parva.* Brill, Leiden. Edited by Alan Perreiah.

Paulus Venetus (20xx). *Logica Magna, Treatise on Insolubles.* Peeters, Leuven. Dallas Medieval Texts and Translations. Edited by Barbara Bartocci and Stephen Read.

Pozzi, L. (1987). *Il Mentitore e il Medioevo.* Edizioni Zara, Parma.

Prior, A. (1969). The possibly true and the possible. *Mind*, 78:481–492.

Read, S. (2010). Inferences. In Pasnau, R., editor, *The Cambridge History of Medieval Philosophy*, pages 173–85. Cambridge UP, Cambridge.

Read, S. (2016). Paradoxes of signification. *Vivarium*, 54:335–55.

Read, S. (2020). Swyneshed, Aristotle and the rule of contradictory pairs. *Logica Universalis*, 14:27–50.

Read, S. and Thakkar, M. (2016). Robert Fland—or Elandus Dialecticus? *Mediaeval Studies*, 78:167–80.

Routley, R. (1982). *Relevant Logics and their Rivals*, volume 1. Ridgeview, Atascadero. With Val Plumwood, Robert K. Meyer and Ross T. Brady.

Seaton, W. K. (1973). *An Edition and Translation of the* Tractatus de Consequentiis *by Ralph Strode, Fourteenth-Century Logician and Friend of Geoffrey Chaucer.* PhD thesis, University of California, Berkeley.

Spade, P. (1976). Robert Fland's *Consequentiae*: an edition. *Mediaeval Studies*, 38:54–84.

Spade, P. (1978). Robert Fland's *Insolubilia*: an edition, with comments on the dating of Fland's works. *Mediaeval Studies*, 40:56–80.

Spade, P. (1979). Roger Swyneshed's '*Insolubilia*': edition and comments. *Archives d'histoire doctrinale et littéraire du moyen âge*, 46:177–220. Reprinted in Spade (1988).

Spade, P. (1983). Roger Swyneshed's theory of 'Insolubilia': a study of some of his preliminary semantic notions. In Eschbach, A. and Trabant, J., editors, *History of Semiotics*, pages 105–13. John Benjamins, Amsterdam/Philadelphia. Reprinted in Spade (1988).

Spade, P. (1988). *Lies, Language and Logic in the Late Middle Ages*. Variorum, London.

Spade, P. V. and Read, S. (2017). Insolubles. In Zalta, E. N., editor, *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, fall 2017 edition.

Tarski, A. (2002). On the concept of following logically. *History and Philosophy of Logic*, 23:155–96. Translated from the Polish and German by Magda Stroińska and David Hitchcock.

Weber, S. (2003). *Richard Billingham "De Consequentiis" mit Toledo-Kommentar*. B.R.Grüner, Amsterdam.

Weisheipl, J. (1964). Roger Swyneshed OSB, logician, natural philosopher, and theologian. In *Studies presented to Daniel Callus*, pages 231–52. Oxford U.P., Oxford.

Whitaker, C. (1996). *Aristotle's* De Interpretatione*: Contradiction and Dialectic*. Oxford Aristotle Studies. Clarendon Press, Oxford.

Yrjönsuuri, M., editor (2001). *Medieval Formal Logic*. Kluwer, Dordrecht.