THEORIES OF PARADOX FROM THOMAS BRADWARDINE TO PAUL OF VENICE[1]

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Abstract. We can divide medieval discussions of the insolubles—logical paradoxes such as the Liar—into two main periods, before and after Bradwardine, who wrote his treatise on *Insolubles* in Oxford in the early 1320s. Bradwardine's aim was to develop a solution to the insolubles which, unlike the then dominant theories, *restrictio* and *cassatio*, placed no restriction on self-reference or the theory of truth. He claimed to be able to prove that insolubles signify not only that they are false but also that they are true, and so are false. Few subsequent writers on insolubles followed him completely. Nonetheless, Heytesbury's solution agrees with Bradwardine's that there is an additional signification, though he was agnostic what that additional signification was; and a popular solution commonly found in the teaching manuals at Oxford modified Heytesbury's solution to incorporate aspects of Bradwardine's. There were remarkably similar developments at Paris, where Buridan's solution also claimed that propositions have an additional signification or implication of their own truth. Gregory of Rimini claimed that (spoken and written) insolubles correspond to a conjunction of two mental propositions, one of which says that the other is false. Gregory's solution was taken up and adapted by Peter of Ailly, arguing that the phenomena are better explained by realising that insolubles are equivocal, both true and false, corresponding to two different mental propositions. In contrast, Roger Swyneshed's aim was to provide a solution without the postulation of hidden meanings, but taking the expressions at face value. At the end of the century, Paul of Venice subscribes to the modified Heytesbury solution in his *Logica Parva*, but in his *Logica Magna* he defends a version of Swyneshed's solution.

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[2]Much of the content of this survey paper is taken from §§3-5 of the 'Introduction' to Paulus Venetus 2022.

1. Insolubles[2]

Although the Liar paradox and similar puzzles were well known and much discussed in antiquity, the medieval interest in them seems to be quite independent and largely in ignorance of those discussions.[3] On the one hand, their paradoxical nature seems not even to have been properly recognised until the end of the twelfth century, and the only reference from antiquity which is regularly cited is Aristotle's discussion of the oath-breaker in his *Sophistical Refutations*, ch.25. The oath-breaker first says that he will break his oath and then proceeds to fulfil that oath by breaking a subsequent one. To be truly paradoxical, it would needs be one and the same oath which he both fulfils and breaks. This is what we find in the classic case of the Liar paradox, when someone says 'I am lying' (where this is all he says, or at least he means to refer to that utterance) or 'This utterance is false' (referring to that very utterance). For if it is true then it must be false (for that is what was said), so it is not true (since it cannot be both), and consequently by *reductio ad absurdum* it really is not true, and so is false (assuming it is either true or false, and so if not true, then false). But given, as we have just proved, that it is false, it is surely true (since that is what was said). Thus we have proved both that it is true and that it is false (indeed, that it is both true and not true), and that conclusion is paradoxical (literally, beyond belief). Something has surely gone wrong. But what is the mistake?

We can divide medieval discussions of the insolubles—logical paradoxes such as the Liar—into two main periods, before Bradwardine and after Bradwardine. Thomas Bradwardine wrote his treatise on *Insolubles* in Oxford in the early 1320s and it seems to mark a sea change in the solutions which were mainly favoured. Up until the 1320s two types of solution were the focus of attention, *restrictio* and *cassatio* (though only two treatises are known which favoured the latter). Supporters of *restrictio* (restrictivism, aka restrictionism) proposed an outright ban on self-reference—in some authors, a blanket ban, in most a limited ban under which self-reference may not be combined with a negative (or privative) expression. Walter Burley, a prime advocate of the latter form of restrictivism, wrote:

> "According to earlier writers, there are three sources of insolubles. The first source comes from the combination of an intentional verb with the expression 'false' or anything convertible with it or with the denial of 'true'

[3]See Spade and Read 2021, §§1.1-1.2.

> ... Concerning the first source it should be recognised that whenever the same act has reflection on itself with a privative determination, namely, with the determination 'false' or anything convertible with it, then the act is restricted ... For a term is restricted when it does not imply its superior [that is, any term it falls under] ... It should be recognised that a part can never supposit for a whole of which it is part when, putting the whole in place of the part, there results reflection of the same on itself with a privative determination. So if one says 'I say a falsehood', the term 'falsehood' does not supposit for 'I say a falsehood' because, if the whole is put in place of the term 'falsehood', there results reflection on itself with a privative determination."[4]

Bradwardine attacked several versions of restrictivism mercilessly in chs.3-5 of his *Insolubles*. While this was the pre-eminent proposal for solving the insolubles before his attack, it had few proponents thereafter.[5] Ralph Strode later wrote, around 1359-60:

> "Indeed, the aforesaid opinions were ⟨those⟩ of the earlier generation, who correctly understood little or nothing about insolubles. After them arose the prince of modern natural philosophers, namely, master Thomas Bradwardine, who was the first to come across anything of value about insolubles, for which reason it deserves to be quoted more extensively for the use of the young. So this

[4]Walter Burley 1970, §§3.01-3.03 (in Roure, 1970, 271-2), correcting the text against De Rijk 1966, 90: "*Et secundum antiquos, tres sunt radices insolubilium. Prima radix provenit ex conjunctione verbi pertinentis ad motum animi cum hac dictione falsum vel cum suo convertibili vel cum negatione veri. ... Circa primam radicem est sciendum quod quandocumque idem actus reflectitur supra se ipsum cum determinatione privativa, scilicet cum hac determinatione falsum vel cum suo convertibili, tunc actus ponitur diminutive ... Terminus enim ponitur diminutive quando non infert suum superius ... sciendum quod nunquam supponit pars pro toto cuius est pars, quando, posito toto loco partis, accidit reflexio eiusdem supra se ipsum cum determinatione privativa. Ideo sic dicto dico falsum, iste terminus falsum non supponit pro hac: dico falsum, quia, posita hac tota loco huius termini falsum, accidit reflexio supra se cum determinatione privativa.*"

[5]A couple of exceptions can be found in Walter Segrave's 'Insolubles' (Spade, 1975, item LXVIII, 113-6) and Richard Brinkley (listed as "anonymous" in Spade 1975, item X, 33).

> reverend doctor precedes the plainer description of Aristotle's opinion with first some divisions, definitions, assumptions and conclusions, from which everything which follows shines more clearly."[6]

Bradwardine also dismissed cassationism (*cassatio*) as nonsense, just as its supporters dismissed insolubles as nonsense. The verb 'to cass' (archaic, and derived from the Latin '*cassare*') means to render null and void. Cassationism is regularly rejected in thirteenth-century treatises (indeed, as mentioned, it is defended in only two that survive). Bradwardine concluded after only a brief discussion: "the view of the nullifiers is sufficiently nullified."[7] Cassationism received its most extensive and well-argued defence in John Dumbleton's *Summa Logicae*, written in the late 1330s or 1340s.[8]

2. Thomas Bradwardine

Other theories were proposed and defended before Bradwardine's revolution. He considers others in the fifth chapter of his treatise. But it was Bradwardine's own proposal, dubbed the "multiple-meanings" solution in Read (2015, §1), along with his detailed and extensive criticism of restrictivism, which initiated a whole new era in discussions of the insolubles. His aim was to develop a solution to the insolubles which placed no restriction on self-reference or the theory of truth. His first and third postulates were:

(1) "Every proposition is true or false" (and, implicitly, not both)
(3) "The part can supposit for the whole of which it is part and its opposite and for what is convertible with them."[9]

Along with his definition of a true proposition as "an utterance signifying only as it is," a false one as "signifying other than it is," and some basic

[6]Ralph Strode, *Tractatus de Insolubilibus* (ms Erfurt Amploniana, Quarto 255, fol.3vb): "*Predicte namque opiniones fuerunt antiquitus antiquorum, qui parum vel nihil de insolubilibus recte sapuerunt. Post quos surrexit princeps modernorum physicorum videlicet magister thomas braduardyn qui aliquid quod valuerit de insolubilibus primitus adinvenit, quapropter eius opinionem pro utilitate minorum reputo diffusius recitandam. Premittit ergo iste reverendus doctor ad opinionem Aristotelis planius declarandam primo aliquas divisiones, diffinitiones, suppositiones et conclusiones, ex quibus omnia que secuntur clarius elucescunt.*" For the date, see Maierù 1982b, 89.

[7]Thomas Bradwardine 2010, ¶5.6: "*Opinio ergo cassantium satis est cassata.*"

[8]See Spade 1975, item XXXVI, 63-65 and Bartocci 2023 (this volume).

[9]Thomas Bradwardine 2010, ¶6.3: "*Suppositiones autem sunt sex. Quarum prima est ista: quelibet propositio est vera vel falsa . . . Tertia est ista: pars potest supponere pro suo toto et eius opposito et convertibilibus earundem.*"

logical principles, he was able to establish his second conclusion, that "any proposition that signifies itself not to be true or to be false, also signifies itself to be true and is false."[10] The basis of this claim was his novel proposal, encapsulated in his second postulate:

(2) "Every proposition signifies or means given how things are now or unrestrictedly everything which follows from it given how things are now or unrestrictedly."[11]

The ingenious use of this postulate in proving his second conclusion is well worth studying, as is his application of it in analysing a succession of insolubles.

However, although most if not all subsequent writers on insolubles owe a debt to Bradwardine, few followed him completely, and in particular, it seems to have been his powerful second postulate that was not popular.[12] Two alternative proposals presented in Oxford in the 1330s, both responding to Bradwardine's idea but in different ways, dominated subsequent discussion of the insolubles. They were due to William Heytesbury and Roger Swyneshed.

Discussion of the insolubles was shot through by the language of obligations. But in Heytesbury's case, it was not just the language (of granting, denying etc) but the whole structure of obligational disputations which frames the discussion. Most insolubles need a background scenario—minimal as in the Liar, which assumes that the Liar utterance is self-referential (and not just apologising for a lie just told, for example), or more extensive in such cases as the common medieval scenario where a stock character called 'Socrates' says one and only one thing, namely, 'Socrates says a falsehood'; or elaborate examples where, say, a landowner, troubled by vagabonds, has set up a gallows by a bridge over a river dividing his lands, decreeing that everyone who wishes to cross the bridge must declare their business and where they are going. They are to be let across if they speak truly but hung on the gallows if

[10]Thomas Bradwardine 2010, ¶6.4: "*Secunda ⟨conclusio⟩ est ista: si aliqua propositio significet se non esse veram vel se esse falsam, ipsa significat se esse veram et est falsa.*"

[11]Thomas Bradwardine 2010, ¶6.3: "*Secunda ⟨suppositio⟩ est ista: quelibet propositio significat sive denotat ut nunc vel simpliciter omne quod sequitur ad istam ut nunc vel simpliciter.*"

[12]For example, Dumbleton points out that it has the unacceptable consequence that every proposition signifies that the first cause exists, which, being necessarily true, follows from every proposition. See ms Merton College 306, fol.2ra.

they lie: only to be confronted by Socrates (yet again) saying that his sole business is to be hung on the gallows.[13]

There are two parties to an obligational disputation, an Opponent and a Respondent. The Opponent presents the Respondent with a background scenario (*casus*),[14] briefly sketched, and an *obligatum*, a proposition which is usually false in the scenario. Burley distinguishes six types of obligational disputation, the main one of which is called *positio*, and the proposition at its heart is the *positum*. The Respondent was expected to accept the *positum* provided it was not impossible, otherwise to reject it. The obligation on the Respondent was then to respond in accordance with strict rules and without contradicting himself, to a series of propositions put forward by the Opponent. His response could be to grant, to deny or to doubt each of them in turn.

3. Heytesbury

Heytesbury takes each insoluble to be the *positum* in an obligation. He first distinguishes an insoluble scenario from an insoluble proposition:

> "[A] scenario of an insoluble is one in which mention is made of some proposition such that if in the same scenario it signifies only as its words commonly suggest, from its being true it follows that it is false and vice versa ... [A]n insoluble proposition is one of which mention is made in some insoluble scenario such that if in the same scenario it signifies only as its words commonly suggest, from its being true it follows that it is false and vice versa."[15]

Heytesbury's solution turns on the specification of the precise signification of the insoluble. The scenario may specify that the insoluble signifies only as its terms commonly suggest (*praecise sicut termini communiter pretendunt*) or it may leave it open, signifying as the terms commonly

[13]The example about crossing the bridge was a real stock in trade: see, e.g., Thomas Bradwardine 2010, ¶8.8.1, Paul's 'Insolubles' edited in Paulus Venetus 2022, §5.4 and Cervantes, *Don Quixote*, Part II ch.45.

[14]See Pironet 2008, 257.

[15]"*[C]asus de insolubili est ille in quo fit mentio de aliqua propositione quae, si cum eodem casu significet praecise sicut verba illius communiter praetendunt, ad eam esse veram sequitur eam esse falsam et e converso ... [P]ropositio insolubilis est de qua fit mentio in casu insolubili quae, si cum eodem casu significet praecise sicut verba illius communiter praetendunt, ad eam esse veram sequitur eam esse falsam et e converso*." William Heytesbury 1987, 236; 2008, 284. For an alternative translation, see William Heytesbury 1979, 46.

suggest but not necessarily only in that way (*non sic praecise*).[16] If it were left completely open, John Hunter (aka Johannes Venator) and others realise that the respondent could then do no more than doubt the *positum*, since he would not know how it signified.[17] If the opponent adds '*precise*' ('only'), then the scenario should not be accepted, for, as we noted, one of the first rules of obligations is that no intrinsically impossible obligation should be accepted, and with this qualification ('only') the usual contradiction, that the *positum* should be both granted and denied, is immediately forthcoming. Heytesbury's third Rule applies when the Opponent does not add the '*precise*' qualification. In that case, he says, the obligation should be accepted, the insoluble should be granted as following, but that it is true should be denied:

> "The third ⟨rule⟩ is that given an insoluble scenario, and supposing that the insoluble signifies as its terms suggest, but not however only in that way, the scenario should be accepted and the insoluble should be granted as following in whatever place it is proposed and it should be denied that it is true as inconsistent."[18]

Recall the earlier proof that 'I am lying' is both true and false. What is common to Bradwardine's and Heytesbury's solutions (and most others) is that they accept the first leg of the argument, using *reductio ad absurdum* to infer that the insoluble is false, but they find some way to block the second leg, arguing from its falsity, already granted, to its truth. Bradwardine, for example, blocks this move by reminding his reader that the truth of a proposition requires that the proposition signify only as things are, and since it signifies both that it is false (by the meaning of 'lie' or 'false') and that it is true (by his second conclusion), it cannot signify only as things are (since it cannot be both true and false), so it is false. Heytesbury is more cautious. He writes:

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[16]See William Heytesbury 1987, 238; 2008, 284-5. Spade's translation of '*sicut termini communiter praetendunt*' as 'as the terms commonly pretend' or '... commonly pretend to signify'—see, e.g., William Heytesbury 1979, 81, Pironet 2008, 256—is unhelpful. No pretence is implied.

[17]See, e.g., Hunter in Pironet 2008, 303.

[18]William Heytesbury 1987, 238-40: "*Tertio: si fiat casus de insolubili et cum hoc supponatur quod illud insolubile significet sicut termini ipsius praetendunt non tamen sic praecise, admisso casu, concedendum est illud insolubile quocumque loco propositum fuerit tamquam sequens et negandum est illud esse verum tamquam repugnans.*" See also William Heytesbury 2008, 285, and for an alternative translation, William Heytesbury 1979, 49.

> "But if someone asks what the proposition uttered by Socrates signifies in this scenario other than that Socrates says a falsehood, I say that the Respondent will not have to respond to that question, because from the scenario it follows that the proposition will signify other than that Socrates says a falsehood, but the scenario does not specify what that is and so the Respondent does not have to respond any further to what was asked."[19]

The respondent is only required to say 'I grant it', 'I deny it', 'I doubt it' and so is not obliged to enter into discussion of what the *positum* may or may not signify—that is for the Opponent to do. Later authors came to distinguish the exact or primary signification of a proposition (what the words commonly suggest, or "according to the common institution of the idiom," or "the common institution of grammar")[20] from its secondary or additional signification, the two combining to make up its principal or total signification. But caution is needed, for these terms are often used somewhat differently by different authors (and are often translated differently by different translators).

4. The Modified Heytesbury Solution

A later author, identified by Spade as 'Robert Fland', but arguably properly called 'Robert Eland',[21] presented Bradwardine's and Heytesbury's solutions and invited the reader to choose between them:

> "So ⟨these⟩ two responses [sc. Bradwardine's and Heytesbury's] are better than the others for solving insolubles. Therefore the respondent should choose one of them for his solution to the insolubles."[22]

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[19]William Heytesbury 1987, 240: "*Si autem quaeratur in casu illo quid significabit illa propositio dicta a Sorte aliter quam quod Sortes dicit falsum, huic dicitur quod respondens non habebit illud seu illam quaestionem determinare, quia ex casu isto sequitur quod ista propositio aliter significet quam quod Sortes dicit falsum, sed casus ille non certificat quid illud sit et ideo non habet respondens quaesitum illud ulterius determinare.*" See also Pironet 2008, 286, and for an alternative translation, William Heytesbury 1979, 49-50.

[20]See Pironet 2008, 324-5: "*dum solum significat juxta communem institutionem idiomatis in quo est disputatio"; "significet secundum communem institutionem grammaticae.*"

[21]See Read and Thakkar 2016.

[22]Spade 1978, 65: "*Ideo duae responsiones sunt meliores aliis ad insolubilia solvenda. Eligat ergo respondens unam istarum pro sua solutione ad insolubilia.*"

What seems to have happened, however, is that the popular solution was to combine them. We find such a solution in a number of treatises, several anonymous, including that of pseudo-Heytesbury, so called by Spade (1975, 35-36) because his treatise is so closely modelled on that of Heytesbury, and in treatises ascribed to John of Holland and to John Hunter (Johannes Venator).[23] It is the solution commonly found in the teaching manuals at Oxford now known as the *Logica Oxoniensis*,[24] and is the basis of Paul of Venice's solution in his *Logica Parva* (see §8 below). When it comes to the third Rule, instead of refusing to specify what the additional signification is, pseudo-Heytesbury writes (where 'A falsehood exists' is the only proposition): "It must be said that ... 'A falsehood exists' signifies conjunctively, namely, that a falsehood exists and that that very proposition is true."[25]

Hunter spells it out more clearly, adopting Bradwardine's solution to the "revenge" problem.[26] Suppose Socrates says 'Socrates says a falsehood' and no other proposition, call it A. The solution claims that A is false. Then we might be tempted to argue as follows:

> "A is false, Socrates says A, so Socrates says a falsehood. The inference is valid, the premises are true, so the conclusion is true too. But Socrates says the conclusion. So Socrates says the truth."

Not so, Hunter replies:

> "It should be denied that Socrates says the conclusion. Rather, he says another proposition similar in sound but not in signification, because what was said by Socrates signifies that Socrates says a falsehood (and thus that it itself is false, since Socrates said nothing else) and that it itself is true. But any other similar proposition which is not said by Socrates signifies only that Socrates says a falsehood."[27]

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[23]Those of pseudo-Heytesbury, John Hunter and another anonymous treatise are edited in Pironet 2008; that of John of Holland in Bos 1985, 123-46; and another anonymous treatise in Spade 1971.

[24]See De Rijk 1977.

[25]Pironet 2008, 292: "*Et eodem modo dicendum est ... ⟨quod⟩ 'falsum est' significat similiter copulative, videlicet quod falsum est et quod eadem propositio est vera.*"

[26]See, e.g., Thomas Bradwardine 2010, 'Introduction', 20.

[27]Pironet 2008, 305: "*Aliter arguitur sic, et probo illam propositionem esse veram quam Socrates dicit, et pono quod A sit illa. Tunc arguitur sic: A est falsum, Socrates dicit A, ergo Socrates dicit falsum. Illa consequentia est bona, et antecedens est verum, ergo et consequens; et Socrates dicit consequens, ergo Socrates dicit verum. Solutio:*

We might call this the modified Heytesbury solution. That John of Holland subscribes to the modified view is not immediately obvious, but clear enough when his text is examined carefully. In response to a counter-argument, he writes:

> "I deny the first inference, namely 'A is a falsehood, and Socrates says A, therefore Socrates says a falsehood', because it is a fallacy of the restricted and unrestricted. For the conclusion signifies many things conjunctively, namely that Socrates says a falsehood and something else (according to some people, *viz* that 'Socrates says a falsehood' is true)."[28]

That parenthetical clause contains the modified Heytesbury solution. At this point, John has mentioned it but hasn't yet committed himself to it. But in his response to the fourth counter-argument, he writes:

> "I reply that 'Socrates does not say a falsehood' is not the contradictory of the insoluble, because the insoluble is not a singular proposition. Hence it is not necessary that this proposition contradict it. But if one asks, what then is the contradictory of this insoluble, I say that it is 'Not-(Socrates says a falsehood)' signifying disjunctively that either Socrates does not say a falsehood or that 'Socrates says a falsehood' is not true. Then this disjunction is true if one of its parts is, namely: 'Socrates says a falsehood' is not true. And the reason is that from the fact that the insoluble signifies conjunctively that Socrates says a falsehood and that 'Socrates says a falsehood' is true (at least as many people say), it is necessary that its contradictory is a proposition in which the negation is preposed to the whole."[29]

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negandum est quod Socrates dicit consequens, sed dicit unam aliam sibi consimilem in voce et non in significatione, quia illa quae dicta est a Socrate significat quod Socrates dicit falsum, et sic, quod ipsamet est falsa, cum Socrates nihil aliud dicat, et quod ipsa eadem est vera. Sed quaecumque alia consimilis quae non est dicta a Socrate significat praecise quod Socrates dicit falsum."

[28]Bos 1985, 130:10-14: "*Hic dicitur negando primam consequentiam, istam sc.: A est falsum, et Sortes dicit A, igitur Sortes dicit falsum. Quia est fallacia secundum quid et simpliciter. Consequens enim significat plura copulative, sc. quod Sortes dicit falsum et aliquid aliud (secundum aliquos: videlicet quod hec sit vera: Sortes dicit falsum).*"

[29]Bos 1985, 130:28 - 131:7: "*Dicitur quod ista: Sortes non dicit falsum non est contradictoria illius insolubilis. Quia insolubile non est propositio singularis. Ergo non oportet quod talis sibi contradicat. Et si queratur, que tunc est contradictoria*

That clearly commits John of Holland to the modified view, and stands (to my mind) significantly in contrast to Heytesbury's view, which refuses to specify what the additional meaning is (and is why other thinkers reject Heytesbury's view, e.g. Paul of Venice in his *Logica Magna*[30]). But none of those who hold the modified view seems to offer any argument, as Bradwardine had done, to show that this additional signification is that the proposition itself is true.

Ralph Strode clearly distinguishes the modified Heytesbury solution from Heytesbury's own solution, about which he writes:

> "Regarding this third opinion, namely, that of Heytesbury, in so far as it agrees with Thomas Bradwardine's opinion, I consider it to be true, namely, in that it claims that it is impossible for an insoluble proposition to signify only as the words commonly suggest. For example, supposing that the proposition 'A falsehood exists' is the only proposition, it is impossible that it only signifies that a falsehood exists. But in so far as it is claimed that, in the given scenario, it is not decided or stated by the Respondent what else that proposition signifies, or in what other way that proposition signifies, I do not consider it to be true."[31]

Strode proceeds in the Third Part of his treatise to apply his preferred solution to a range of insolubles. His response to the widely discussed scenario in which Socrates says only 'Socrates says a falsehood', labelled 'A', he writes:

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illius insolubilis, dicitur quod talis: non-(Sortes dicit falsum) significans disiunctive quod Sortes non dicit falsum vel quod hec non est vera: Sortes dicit falsum. Et tunc ista disiunctiva verificatur pro altera eius parte, sc. pro ista: hec non est vera: Sortes dicit falsum. Et ratio est: ex quo insolubile significat copulative quod Sortes dicit falsum et quod hec est vera: Sortes dicit falsum (saltem ut plures dicunt), oportet quod sua contradictoria sit una in qua negatio preponitur toti."

[30]See Paulus Venetus 2022, §1.12.5.

[31]Strode, *Tractatus de Insolubilibus*, ms Erfurt Amploniana Q 255, fol.10va: "*Circa vero tertiam opinionem, videlicet ipsius Hentisberi, quantum ad hoc quod concordat cum opinione magistri Thome Bradwardijn, ipsam reputo esse veram, videlicet in hoc quod ponit quod inpossibile est propositionem insolubilem precise significare sicud verba illius communiter pretendunt. Verbi gratia, posito quod ista propositio 'falsum est' sit omnis propositio, tunc inpossibile est istam precise significare falsum esse. Sed quantum ad hoc quod ponitur quod, isto casu posito, non est determinandum uel dicendum a respondente quid aliud ista propositio significet uel qualiter aliter quam ista propositio significet, ipsam non reputo esse veram.*"

> "Regarding the solution to this insoluble it should be realised that close attention be given whether in the presentation of the scenario it is supposed that the insoluble proposition signifies only as the words *prima facie* suggest, or it is supposed that they signify in that way but not with the addition of the adverb 'only'. If it was given in the first way, the scenario should in no way be accepted, because the scenario is impossible, as was clearly stated above. If it was given in the second way, then the scenario should be accepted, and generally so in every insoluble scenario. Furthermore, one should deny that A is true and grant that A is false and also that the proposition uttered by Socrates is false."[32]

He spells out the reason for those verdicts about the truth and falsehood of the insoluble in response to the next insoluble he considers, namely, where all and only those who speak the truth will receive a penny, and Socrates pipes up, 'Socrates will not receive a penny':

> "And so, just as ⟨in the case of⟩ the proposition 'Socrates says a falsehood', supposing that he says only that, the proposition is insoluble, the proposition 'Socrates will not receive a penny' is an insoluble proposition in the scenario described, and consequently in line with what was established earlier, it signifies itself to be false and itself to be true."[33]

5. Swyneshed

Roger Swyneshed's solution was at root very different. His aim was to provide a solution without the postulation of hidden meanings, but taking the expressions at face value, so that the principal signification

[32] *loc.cit.*: "*Circa solutionem istius insolubilis est aduertendum quod diligenter est notandum in positione casus, vtrum ponatur quod talis propositio insolubilis precise significet sicud verba illius prima facie pretendunt, vel ponatur quod sic significet, non tamen addendo istud aduerbium precise. Si primo modo fiat positio, nullo modo est casus admittendus, quia casus est inpossibilis, sicud diffuse patet per superius dicta. Si vero secundo modo fiat positio, tunc admittendus est casus, et istud generale est in omni casu insolubili. Et vlterius dicitur negando a esse verum, et conceditur quod a est falsum et etiam quod propositio dicta a Sorte est falsa.*"

[33] *ibid.*, fol.10vb: "*Et ideo, sicud ista propositio 'Sor dicit falsum', posito quod solam istam dicat, est propositio insolubilis, ita ista propositio 'Sor non habebit denarium' est propositio insolubilis, casu predicto posito, et per consequens secundum predeterminata significat se esse falsam et se esse veram.*"

is just what it is commonly taken to be, what the words commonly suggest (though this phrase, which seems to originate with Heytesbury, is not used by Roger). Rather, he focussed on the fact that all the insolubles entail their own falsehood (though, of course, not only insolubles do that). Where Bradwardine and Heytesbury demanded for truth that everything that a proposition signified, including any hidden secondary or additional signification, should obtain, Swyneshed proposed that truth should require that a proposition not entail its own falsehood:

> "There are four definitions ... The second is this: a true proposition is a proposition not falsifying itself, signifying principally as things are either naturally or by an imposition by which it was last imposed to signify.
> Third definition: a false proposition is an utterance falsifying itself or an utterance not falsifying itself signifying principally other than things are either naturally or by an imposition by which it was last imposed to signify."

Then Swyneshed defines an insoluble:

> "[A]n insoluble as put forward is a proposition signifying principally as things are or other than things are ⟨which is⟩ relevant to inferring itself to be false or unknown or not believed, and so on."[34]

Swyneshed derives three famous iconoclastic conclusions from his theory:

(1) There is a false proposition that signifies principally as things are
(2) There is a formally valid inference where the false follows from the true
(3) There are two mutual contradictories that are both false.[35]

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[34]Roger Swyneshed 1979, 185-6; 1987, 182: "*Post illa sequuntur quattuor definitiones seu descriptiones ... Secunda est haec: Propositio vera est propositio non falsificans se ipsam, principaliter sicut est significans naturaliter aut ex impositione vel impositionibus, qua vel quibus ultimo fuit imposita ad significandum. Tertia definitio: propositio falsa est oratio falsificans se vel oratio non falsificans se principaliter aliter quam est significans naturaliter aut ex impositione vel impositionibus, qua vel quibus ultimo fuit imposita ad significandum. Quarta est haec: Insolubile ad propositum est propositio significans principaliter sicut est vel aliter quam est pertinens ad inferendum se ipsam fore falsam vel nescitam vel non creditam, et sic de singulis.*" Spade omits '*non*' before '*creditam*'.

[35]Roger Swyneshed 1979, 188-9; 1987, 186-8: "*Aliqua propositio falsa significat principaliter sicut est ... In aliqua consequentia bona formali ex vero sequitur falsum ... [D]uo contradictoria sibi mutuo contradicentia sunt simul falsa.*"

The Liar serves to illustrate and support all three claims. For 'This very proposition is false' does signify as things are, for it signifies that it is false (and, for Swyneshed, that is all it signifies) and by Swyneshed's account of truth and falsehood, it is false, for it falsifies itself, that is, it entails its own falsehood. It also serves to establish the second claim: for consider the inference:

> The conclusion of this inference is false
> So the conclusion of this inference is false.

The conclusion is a version of the Liar, asserting of itself that it is false. Swyneshed claims that the inference is valid, for the premise signifies exactly the same as the conclusion, namely, it predicates the same property (being false) of the same thing (the conclusion). Spade (1983, 105) claims that Swyneshed, like several fourteenth-century authors, takes preservation of signifying as things are, rather than truth-preservation, as the criterion of validity.[36] Nonetheless, the premise is true, since it correctly (by his lights) says that the conclusion is false, and the conclusion is false, because it falsifies itself.

The third claim is in some ways the most puzzling and surprising. How can contradictories both be false? Did Aristotle not introduce the notion of contradictories as pairs of propositions that cannot both be true and cannot both be false? Not so, according to Whitaker 1996, who reminds us that what Aristotle actually wrote was:

> "As men can affirm and deny the presence of that which is present and the presence of that which is absent and this they can do with reference to times that lie outside the present: whatever a man may affirm, it is possible as well to deny, and whatever a man may deny, it is possible as well to affirm. Thus, it follows, each affirmative statement will have its own opposite negative, just as

[36]Spade cites in evidence Swyneshed's remark at Roger Swyneshed 1979, 191, §35 that "if from some propositions each of which signifies principally as things are, some proposition follows, it signifies as things are" ("*Si ex aliquibus propositionibus quarum quaelibet significat principaliter sicut est sequitur aliqua propositio, ipsa significat sicut est*"). However, Hanke 2014, 150 challenges Spade's claim, referring to §37 where Swyneshed qualifies this claim, saying it holds of non-insolubles: "If the premise signifies as things are and neither the premise nor the conclusion is relevant to inferring itself not to signify as things are, then the conclusion signifies as things are" ("*Si antecedens significat sicut est et nec antecedens nec consequens est pertinens ad inferendum se ipsum non significare sicut est, igitur consequens significat sicut est*"). See further Read 2020, 286-91.

> each negative statement will have its affirmative opposite. Every such pair of propositions we, therefore, shall call contradictories, always assuming the predicates and subjects are really the same and the terms used without ambiguity."[37]

Whitaker claims that Aristotle proceeds in the subsequent chapters of De Interpretatione to argue against what Whitaker (p.79) calls the Rule of Contradictory Pairs (RCP): that, of any pair of contradictories, one is true and the other false, giving examples in ch.7 of pairs both of which are true, in ch.8 pairs each of which is false, and in ch.9 (regarding the future sea-battle) that (RCP) fails for future contingents.

Swyneshed argues for his third claim by again taking 'This proposition is false' and its pair 'This proposition is not false', each referring to the former. The latter, he says, denies of the former what the former affirms of itself. So by Aristotle's account, they are a contradictory pair. But the former is false because it falsifies itself, and the latter is false because it says, falsely, that the former is not false. So we have a pair of contradictories both of which are false.

Swyneshed adds "or unknown or not believed and so on" at the end of his definition of insolubles in order to include what are now called epistemic paradoxes, which the medievals included under the title 'insoluble'. The most famous example is perhaps the Knower paradox, in the forms 'This proposition is not known', or 'You do not know this proposition'. Suppose it were known. Then it would be true, and so not known. Hence by *reductio ad absurdum*, it is unknown. That is, we have just proved that it is unknown, which is what it signifies. So it is true, and moreover, since we have proved it, we know that it is true and so it is known. Swyneshed's response is to question the second leg of the argument. Let A be the proposition 'A is unknown':

> "'A is unknown' should be granted, and it should be granted that I know A to signify principally in this way. And the inference, "therefore, I know A" should be denied. But it is necessary to add that A is not relevant to inferring itself not to be known. And if that is added, it should be denied. For it follows directly, 'A is unknown, therefore, A is unknown'."[38]

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[37] *De Interpretatione* ch.6, 17a27-33, cited from Aristotle 1938, 123-5.

[38] Roger Swyneshed 1979, 209: "*Admisso casu, concedenda est illa a nescitur. Et concedendum est quod ego scio a sic principaliter significare. Et neganda est consequentia 'igitur, scio a'. Sed oportet addere quod a non est pertinens ad inferendum se*

6. Buridan

So far we have looked only at developments at Oxford reacting to Bradwardine's proposed solution. There were remarkably similar developments at Paris, remarkable not least for the fact that their differences suggest that there may not have been any very direct influence. We can perhaps divide them again into two branches, the first stemming from John Buridan's ideas, the second from Gregory of Rimini's.

Buridan's solution to the insolubles is nowadays perhaps the most famous of all medieval solutions, having been discussed extensively over the past fifty years.[39] In fact, Pironet 1993 showed that Buridan's ideas developed over the course of three or four decades in some five or six works. His early suggestion was that every proposition signifies its own truth, as Burley and before him, Bonaventure, had also suggested.[40] So too had Gerald of Odo, writing in Paris in the early 1320s, not long before Buridan's first discussion of the insolubles:

> "Fourthly, that this proposition 'I am saying a falsehood' ... has these four defects ... The fourth, where the difficulty is lurking, is that it signifies the predicate to be united and not united with the subject at the same time. This is clear because any affirmative signifies union of this sort, and this proposition is affirmative, so it signifies the predicate to be united with the subject. But the predicate 'falsehood' signifies in an affirmative proposition that the predicate is not united with the subject to which it is attributed. How this is the case here because the proposition is affirmative and its subject supposits for the whole, and its predicate denotes that whole. So it signifies by the form of the expression that the extremes are united, and by the sense of the predicate that they are not united."[41]

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ipsum fore nescitam. Et si illud addatur, illud est negandum. Nam sequitur immediate 'a nescitur; igitur, a nescitur'."

[39]See, e.g., Zupko 2018, §4.

[40]See Walter Burley 1970, §3.02 and Spade and Read 2021, §3.8.

[41]See Giraldus Odonis 1997, 397: "*Quarto quod hec propositio 'ego dico falsum' ... habet has quatuor malitias ... Quarto vero, ubi latet lepus, notat predicatum uniri et non uniri subiecto simul et semel. Quod patet quia: quelibet affirmativa notat huiusmodi unionem; hec autem propositio est affirmativa; quare notat predicatum uniri cum subiecto; sed istud predicatum 'falsum' notat non uniri predicatum cum subiecto in propositione affirmativa, cui attribuitur. Sic autem est hic quoniam hec propositio est affirmativa et eius subiectum supponit pro tota ea, et eius predicatum*

This is strongly in contrast with the Oxford tradition, both with Bradwardine and with the modified Heytesbury solution, which only claimed that insolubles signify their own truth. Like Bradwardine, but unlike the modified Heytesbury solution, Buridan offered a proof of his claim that every proposition signifies its own truth:

> "For every proposition is affirmative or negative. But each of them signifies itself to be true or at least from each it follows that it is true. This is clear first concerning affirmatives, for every affirmative proposition signifies that its subject and predicate supposit for the same, and this is for it to be true ... Secondly, it is clear concerning negatives, for a negative does not signify that the subject and predicate supposit for the same, and this is for the negative proposition to be true."[42]

Then propositions such as the Liar are self-contradictory, signifying both that they are true and that they are false, and so are simply false:

> "Regarding this proposition, 'I say a falsehood', I grant that it is false ... because it not only signifies that it itself is false, but also, from the general condition of a proposition, it signifies that it itself is true, and is not true."[43]

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denotat eam totam. Quare ipsa notat ex forma enuntiandi extrema uniri, et ex ratione predicati notat ea non uniri."

[42]John Buridan 1994, 92: "*[N]am omnis propositio est affirmativa vel negativa. Modo quelibet illarum significat se esse veram vel saltem ad quamlibet illarum sequitur eam esse veram. Patet hoc primo de propositione affirmativa, nam propositio affirmativa significat subiectum et predicatum supponere pro eodem. Et hoc est ipsam esse veram ... Secundo patet hoc de negativa, nam negativa non significat esse idem pro quo supponunt subiectum et predicatum. Et hoc est propositionem negativam esse veram.*" Although Buridan claimed that a proposition is true if it signifies as it is, he thought this phrase seriously misleading and cashed it out in terms of supposition: an affirmative proposition is true if subject and predicate supposit for the same, and a negative proposition is true if subject and predicate do not supposit for the same. See, e.g., Buridan, *Sophismata*, ch.2: 'On the Causes of the Truth and Falsity of Propositions' John Buridan 2001, 845-62, especially the Fourteenth Conclusion (858-9).

[43]Buridan, *Quaestiones in primum librum Analyticorum Posteriorum*, Q.10, cited in Pironet 1993, 295 n.4: "*De ista propositione, 'ego dico falsum', concedo quod est falsa ... quia ipsa non solum significat se esse falsam. Sed etiam ex commune condicione propositionis, significat se esse veram, et non est vera.*"

However, in later works, Buridan had doubts about this claim. In his *Sophismata*, composed some twenty or more years later, in the 1350s, he rehearsed his earlier view before criticising and revising it:

> "For some people have said, and so it seemed to me elsewhere, that although this proposition ['Every proposition is false'] does not signify or assert anything according to the signification of its terms other than that every proposition is false, nevertheless, every proposition by its form signifies or asserts itself to be true ... But this response does not seem to me to be valid, in the strict sense ... rather, I [am going to] show that it is not true that every proposition signifies or asserts itself to be true."[44]

His objection was that this claim either implies that every proposition is metalinguistic, in always talking about the truth of some proposition (which he believes is not so), or commits one to the postulation of significates, some real correlate of the proposition (which to Buridan was anathema).[45] Buridan's revised view was that every proposition implies its own truth, or at least would do so if it existed:

> "Therefore, we put this otherwise, in a manner closer to the truth, namely, that every proposition virtually implies another proposition in which the predicate 'true' [would be] affirmed of the subject that supposits for [the original proposition]; and I say 'virtually implies' in the sense in which the antecedent implies that which follows from it."[46]

Buridan's account is one of the well-known solutions to the insolubles which Paul of Venice does not include in his survey of fifteen alternative solutions in the treatise on 'Insolubles' in his *Logica Magna*. But he does

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[44]John Buridan 2001, 967-8; 2004, 154-5: "*Aliqui enim dixerunt, et ita visum fuit [mihi alias corr.] quod licet ista propositio secundum significationem suorum terminorum non significet vel asserat nisi quod omnis propositio est falsa, tamen omnis propositio de forma sua significat vel asserit se esse veram ... Ista tamen responsio non videtur mihi valere de proprietate sermonis ... Sed ostendo illud non esse verum, scilicet quod omnis propositio significat vel asserit se esse veram.*"

[45]These correlates are the notorious *complexe significabilia*, whose existence Buridan strongly contested. See, e.g., Klima 2009, §10.2.

[46]John Buridan 2001, 969; 2004, 155: "*Ideo dicitur aliter, propinquius veritati, scilicet quod quaelibet propositio implicat virtualiter aliam propositionem qua de subiecto pro ea supponente affirmaretur hoc praedicatum 'verum': dico 'implicat virtualiter' sicut antecedens implicat illud quod ad ipsum sequitur.*"

include Albert of Saxony's account, which is very similar to Buridan's early view.[47]

7. Gregory of Rimini and his successors

Paul also omits Gregory of Rimini's solution from his survey of previous opinions, which is again surprising, since Gregory is one of the few authors to whom Paul refers by name in the *Logica Magna*, and indeed, Gregory had been Prior General of the Order of Augustinians in the 1350s, Paul's own order. But Gregory's solution was taken over and adapted by Peter of Ailly, and Peter's view is discussed, commented and criticised at length by Paul.

To understand Gregory's approach, we need to recall that the medievals, following Aristotle's lead, divided language into three levels, written, spoken and mental.[48] Boethius (writing in the sixth century CE) cited Aristotle as saying: "[S]poken [words] are signs (*notae*) of impressions (*passionum*) in the soul, and the written ones are those of the spoken ones."[49] By the fourteenth century, it was common to speak of a complete parallelism of mental, spoken and written propositions. Spoken and written propositions signify by human imposition (*ad placitum*), mental propositions signify naturally. Gregory seems to have inferred that only spoken and written propositions can be insolubles—the paradoxical situation cannot infect mental language. Consequently, (spoken and written) insolubles correspond to non-insoluble mental propositions, in fact, to a conjunction of two such mental propositions, the first of which captures the primary or customary signification of the insoluble, and the second of which says that the first conjunct is false. For example, taking the Liar again, the spoken proposition 'This proposition is false', referring to itself, call it A, corresponds to the conjunctive mental proposition whose first conjunct says that (the spoken proposition) A is false, and whose second conjunct says that the first conjunct is false. Neither conjunct of the mental proposition is self-referential, nor is either of them insoluble or contradictory. In fact, the first conjunct is true (A is false) and the second conjunct is false (since it says falsely that the first conjunct is false), so the whole mental conjunction is false, and so the corresponding spoken proposition A is false too.[50]

[47] See, e.g., Spade and Read 2021, §3.9.

[48] See Aristotle, *De Interpretatione* ch.1, and Read 2014.

[49] Boethius 1877, 36: "*sunt ergo ea quae sunt in voce earum quae sunt in anima passionum notae et ea quae scribuntur eorum quae sunt in voce.*"

[50] No text on insolubles by Gregory survives, and so this is a reconstruction of Gregory's view by Spade and others. See Spade's 'Introduction' to Spade 1980, 6-7.

In Spade 1980, 6, Spade suggests that Gregory's solution was a development of Bradwardine's, and that Marsilius of Inghen's was also. However, Marsilius' solution (probably developed in the 1360s) has strong similarities to the modified Heytesbury solution discussed above. Spade quotes Marsilius (in translation), discussing the common example where there is only one Socrates, who only says 'Socrates says a falsehood':

> "The reply is that the sophism is false. For it amounts to the conjunction 'Socrates says a falsehood and it is false that Socrates says a falsehood'. But that is false on account of its second part. Therefore, although things are always as is signified by its first signification, nonetheless things are not as is signified by its second signification, namely by the reflection of falsity, that is, that it is false that it is false."[51]

So in general, an insoluble is expounded as a conjunction whose first conjunct expresses what the terms commonly suggest and whose second conjunct contradicts this and says that is false.[52] But since insolubles falsify themselves, that second conjunct says that it is false that it is false, that is, that it is true, just as Bradwardine, Heytesbury and their followers proposed.

Gregory's solution, probably dating from the 1340s, was taken up and adapted by Peter of Ailly in his treatise on *Concepts and Insolubles*, written in 1372. Where Gregory had claimed that insolubles were false, corresponding to false mental conjunctions, Peter argued that the phenomena are better explained by realising that insolubles are equivocal, both true and false, corresponding to two different mental propositions, the two conjuncts from Gregory's theory, now to be seen not as conjoined but as each separately corresponding to the insoluble. For example, A, that is, 'This proposition is false', in saying of itself that A is false, is true (in so far as it corresponds to the true mental proposition that says that A is false), but in saying that what it (i.e., the mental proposition that says that A is false) says is false, it is false. Now you see it, now

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[51]Spade 1980, 98 n.56. See ms Vatican Pal. Lat. 995, fol.73v: "*Responditur quod sophisma est falsum, valet enim tantum sicut hac copulativa, sortes dicit falsum et falsum est sortem dicere falsum, modo ista est falsa pro secunda parte. Et ideo licet semper ita sit sicut ipsa significat prima significatione, tamen non est ita sicut ipsa significat secunda significatione, scilicet reflexione falsitatis, scilicet quod falsum sit sortem dicere falsum.*"

[52]See Pal. Lat. 995, fol.71r: "when it is said of some utterance that it is false, the sense is that is not as the utterance suggests" ("*quando de aliquo dicto dicitur esse falsum sensus est quod non est ita sicut istud dictum pretendit*").

you don't; Peter tries to capture the flip-flop behaviour that insolubles exhibit.[53]

8. Paul's Theories of Insolubles

The authenticity of the *Logica Magna* is a matter of some contention. Assuming that Paul was the author, we can find discussions by him of the insolubles in at least four works, his *Logica Parva*,[54] his *Logica Magna* (in the treatise on 'Insolubles' presented in Paulus Venetus 2022), in the *Quadratura* (in at least four of its 200 chapters), and in the final sophism, no.50, in his *Sophismata Aurea*.[55]

We saw in §4 that two of the leading solutions to the insolubles in the fourteenth century were the modified Heytesbury solution (adapting Heytesbury's distinctive solution) and Roger Swyneshed's. Paul follows both of these solutions in different works: the modified Heytesbury solution in the *Logica Parva* and the *Quadratura*, and that of Swyneshed in the *Logica Magna* and the *Sophismata Aurea*.[56]

Recall that Heytesbury, and his followers such as pseudo-Heytesbury, Hunter and Holland, distinguish an insoluble scenario in which it is specified that the insoluble signifies only as the terms suggest, in which case the scenario (or *obligatio*) should not be accepted, from a scenario where that exclusion clause is omitted, in which case the insoluble and its scenario can be accepted and the insoluble denied. Paul follows this division at the start of the chapter on insolubles in his *Logica Parva*, marking the distinction as that between what is simply, or unrestrictedly, an insoluble (*insolubile simpliciter*) and what is an insoluble restrictedly

[53] See Spade's comment in Spade 1980, 12-13.

[54] Translated in Paulus Venetus 1984 and edited in Paulus Venetus 2002.

[55] Relevant passages from Paul's *Quadratura* and *Sophismata Aurea* are included in Appendices A and B of Paulus Venetus 2022.

[56] There are few summaries and presentations of Paul's solution(s). A very brief account of that in the *Logica Magna* is given in Spade 1975, 83-4, and a slightly fuller, but confused and misleading one in Bottin 1976, 148-51, who conflates the solutions in the *Logica Magna* and the *Logica Parva*. The account of Paul's view in Bochenski 1962, 247-51 is also muddled and misleading: after correctly reproducing a selection of Paul's divisions and assumptions, he writes: "Paul's own solution is very like that of the eleventh [*viz* Albert of Saxony's] and twelfth [*viz* Heytesbury's] opinions, and so we do not reproduce his long and difficult text", giving instead a one-page summary. This summary bears no relation whatever to what Paul writes in the *Logica Magna*, not even to the passages Bochenski has cited from it, nor to Heytesbury, but is similar in many ways to Albert's solution (see Albert of Saxony, 1988, 346-7).

(*insolubile secundum quid*).[57] Then he presents two conclusions: the first is that no simple or unrestricted insoluble should be accepted:

> "No scenario from which what is unrestrictedly an insoluble arises should be accepted. E.g., if anyone proposes that 'Every proposition is false', signifying only in that way, is the only proposition, the scenario should not be accepted because a contradiction follows."[58]

On the other hand, any scenario from which there arises what is an insoluble restrictedly, that is, without the exclusion clause, should be admitted:

> "Every scenario from which what is restrictedly an insoluble arises should be accepted; and one grants the proposed insoluble by saying it is false. E.g., suppose that 'This is false' is a proposition referring to itself which signifies as the terms suggest—call it A. Then the scenario is accepted, and A is granted, and it is said that it is false. If one argues like this: 'A is false, therefore it signifies other than is the case', I grant it. 'But A only signifies that it is false, therefore that it is false is not so'. I deny the minor premise, and if it is asked what else it signifies, I say that it signifies that A is true, and that is the reason why A is false. So it should be said that every proposition which is an insoluble restrictedly signifies conjunctively, namely, as its terms suggest and that it is true."[59]

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[57]Note that Perreiah translates this as 'according to a condition'.

[58]Paulus Venetus 2002, 132: "*Prima conclusio est ista. Numquam admittendus est casus quo trahit originem insolubile simpliciter. Ut si poneretur quod illa 'Omnis propositio est falsa' esset omnis propositio sic praecise significans, non est admittendus casus quia sequitur contradictio.*" An alternative translation can be found in Paulus Venetus 1984, 240.

[59]Paulus Venetus 2002, 132-3: "*Secunda conclusio. Omnis casus quo originatur insolubile secundum quid est admittendus. Et conceditur insolubile propositum dicendo ipsum esse falsum ut ponendo quod ista propositio 'Hoc est falsum' demonstrato se ipso significet sicut termini praetendunt quae sit A. Tunc admittitur casus, et conceditur A, et dicitur quod est falsum. Et si arguitur sic 'A est falsum, igitur significat aliter quam est', concedo. Sed A non significat nisi quod hoc est falsum; igitur non est ita quod hoc est falsum. Nego minorem. Et si dicitur quod ergo aliud significat, dicitur quod significat A esse verum, et ratione cuius A est falsum. Unde dictum est quod quodlibet insolubile secundum quid significat copulative, videlicet sicut termini praetendunt, et quod ipsum est verum.*" An alternative translation can be found in Paulus Venetus 1984, 241.

This is the modified Heytesbury solution, following Heytesbury in accepting an insoluble scenario only if it is left open that the insoluble proposition has a secondary or additional signification, and (unlike Heytesbury) specifying that additional signification as asserting its own truth. Paul spells it out towards the end of ch.6:

> "It should be noted that an insoluble has two significates, one exact (*adaequatum*) and one princiPal. The exact significate is a subject-predicate significate similar to the insoluble utterance. E.g., the exact significate of 'Socrates says a falsehood' is Socrates saying a falsehood or that Socrates says a falsehood. But the principal significate is a compound significate, e.g., that Socrates says a falsehood and that the proposition is true."[60]

We find the same approach to insolubles in Paul's *Quadratura*. This work is not about squaring the circle or quadrature, but is a highly formal and artificial series of two hundred sophisms arranged in four parts of fifty chapters each. The reason for the strange title '*Quadratura*' is that each chapter "is fortified with four Conclusions and as many corollaries or more" (*munitur quatuor conclusionibus et totidem aut pluribus correlariis*). Each of the four main parts focuses on a particular question:

> "First, whether the same inference can be both valid and invalid; secondly, whether the same proposition can be both true and false; thirdly, whether disparate things are verifiable of the same thing; fourthly, whether two incompatibles can be both true or both false."[61]

The fifteenth chapter of part 1 is explicitly concerned with insolubles, specifically with the following argument:

> "This inference ⟨call it B⟩ is valid: A will signify only that everything ⟨that is or will be⟩ true will be false, so

[60]Paulus Venetus 2002, 149: "*Notandum quod insolubile habet duo significata: unum adaequatum et unum principale. Adaequatum significatum est significatum categoricum simile dicto insolubilis. Unde adaequatum significatum illius 'Sortes dicit falsum' est istud Sortem dicere falsum vel quod Sortes dicit falsum. Principale autem significatum est significatum hypotheticum ut Sortem dicere falsum et illam propositionem esse veram.*" An alternative translation can be found in Paulus Venetus 1984, 255. Note that the principal significate is not just the second conjunct, as Perreiah says on 256, but the whole conjunction (as Perreiah recognised on 106).

[61]Paulus Venetus 2022, Appendix A (see also 1493, fol.2ra, corrected against the mss): "*Quatuor formabo dubia . . . primo utrum eadem consequentia sit bona et mala; secundo utrum eadem propositio sit vera et falsa; tertio utrum de eodem sint verificabilia disparata; quarto utrum duo repugnantia possint esse simul vera vel simul falsa.*"

> A will be false; and this inference $\langle B \rangle$ is invalid. So the question is true."[62]

Paul shows that the exclusion clause in the scenario ('A will signify only that ...') must be amended. This follows from his second Conclusion:

> "There is some proposition [namely, 'Every proposition is false'] signifying principally purely predicatively which at some time will signify principally in a compound way. Nonetheless, there will be no change in it, nor will any new imposition be added to it."

For "when it will be the only proposition it will signify principally that every proposition is false and that it is true, just like other insolubles, whose significations reflect wholly on themselves."[63] He goes on, in discussing the third Conclusion, to remark that "this conjunctive significate is called the principal significate of A, although it is not the exact ⟨significate⟩ but only the first part ⟨is⟩."[64] Without the exclusive phrase (that is, that A signifies only that everything true will be false), argument B is invalid, but if it is retained, Paul does not accept the scenario, "because it implies a contradiction" (*ibid.*, §ad 1.15.1.2).

In response to a later sophism, the second in part 2, Paul spells out his use of 'exact signficate' and 'principal significate' in greater detail, and links them to the notions of truth and falsity:

> "Finally, then, it should be said that it is because it immediately signifies a truth that any proposition is true, and it is because it immediately signifies a falsehood that any proposition is false, where outside the case of insolubles 'immediately' means the same as 'exactly'. But in the case of insolubles it means the same as 'principally'. Hence 'A man is an animal' is true because it immediately signifies a truth, that is, it exactly ⟨signifies⟩ the

[62]Paulus Venetus 2022, Appendix A §1.15.1 (1493, fol.8rb): "*Quintodecimo principaliter ad questionem arguitur sic: ista consequentia est bona: a significabit precise quod quodlibet verum erit falsum, ergo a erit falsum: et hec eadem non valet, igitur questio vera.*"

[63]Paulus Venetus 2022, Appendix A §1.15.2.2 (1493, fol.8va): "*Secunda conclusio est ista: aliqua est propositio significans solum cathegorice principaliter que aliquando significabit ypothetice principaliter, et tamen nulla in ea fiet mutatio, nec nova adveniet illi impositio ... quando ipsa erit omnis propositio significabit principaliter quod omnis propositio est falsa, et quod ipsa est vera, quemadmodum et alia insolubilia, quorum significationes reflectuntur ad se totaliter.*"

[64]Paulus Venetus 2022, Appendix A §1.15.2.3.

> truth that a man is an animal; but 'This is false', referring to itself, is false because it immediately signifies a falsehood, that is, it principally ⟨signifies⟩ a falsehood, namely, that this is false and that this is not false."[65]

However, whereas in the *Logica Parva* and the *Quadratura* Paul subscribes to the modified Heytesbury solution to the insolubles, in the *Logica Magna* he defends a version of Swyneshed's solution. Surprisingly, the modified Heytesbury solution does not appear among the fifteen solutions that Paul considers in his first chapter (in varying detail) and rejects; but Heytesbury's own solution is considered, being the first solution to which Paul devotes more than a few lines.[66] One of Paul's objections to the solution turns, in fact, on Heytesbury's reluctance to specify what the additional signification is which renders an insoluble false. This, and many of the other objections which Paul levels against Heytesbury's view, are drawn from Peter of Mantua's *Insolubles* (or possibly from a third text on which they both draw). Suppose 'A falsehood is said' signifies principally (that is, wholly and exactly) that God exists. Heytesbury had claimed that a proposition could have a further signification in addition to what it standardly signifies, so presumably it could signify something completely different, such as, that God exists. If so, it would be necessarily true. But Heytesbury's view is that 'A falsehood is said' is false as uttered in the proposed scenario. Yet the conclusion of an inference which is clearly valid and whose premise is in doubt should not be denied—for if one denies the conclusion of an inference one recognises to be valid, one is committed to denying the premise.[67] So Heytesbury's solution proposing such hidden and unspecified significations is unacceptable.

Paul devotes the whole of the first chapter of his treatise on insolubles in the *Logica Magna* to the rejection of these other proposed solutions. The treatment can be seen as falling into four groups, the first three

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[65]Paulus Venetus 2022, Appendix A §2.2.3 (1493, fols.26va-b): "*Finaliter est ergo dicendum quod ex eo aliqua propositio est vera quia significat primo verum et ex eo aliqua propositio est falsa quia significat primo falsum, ita quod ly primo extra materiam insolubilium idem sonat quod adequate. Sed in materia insolubilium ⟨idem⟩ sonat quod principaliter. Hec ergo: homo est animal, est vera quia significat primo verum idest adequate verum quod est hominem esse animal, et hec: hoc est falsum, seipso demonstrato, est falsa, quia significat primo falsum idest principaliter falsum videlicet hoc esse falsum et hoc non esse falsum.*"

[66]See Paulus Venetus 2022, §1.12.

[67]This form of inference, occurring first (as far as I know) in Kilvington's *Sophismata*, was christened "Kilvington's disputational meta-argument" in the commentary in Kretzmann and Kretzmann 1990, 316.

groups corresponding to three sources on which Paul draws: first, he runs rapidly through seven of the eight alternative solutions considered by Bradwardine in his treatise,[68] for the most part summarising almost *verbatim* Bradwardine's own criticism. He then turns to Heytesbury's criticism of alternative solutions, starting with the second of the four solutions considered by Heytesbury, that of John Dumbleton. (The first one Heytesbury rejects is Swyneshed's, which Paul will himself accept.) The next (third on Heytesbury's list) is Kilvington's, and then Paul comes to Bradwardine's own solution. Thus the first ten solutions considered are all from Oxford, or at least, those discussed at Oxford in the two decades from the early 1320s to the early 1340s. With the eleventh solution, Paul turns to his third source, namely, Peter of Mantua, the eleventh being Albert of Saxony's solution, presented at Paris in the early 1350s, and possibly the same as John Buridan's own early solution,[69] which is the first view discussed by Mantua; and next to Heytesbury's (Mantua's second), as noted above.[70] Before proceeding to the third view discussed by Mantua, Paul considers Peter of Ailly's solution at some length, seemingly drawing directly on Ailly's own treatise, to the discussion of which Paul appends (without distinguishing it by number) a criticism of Mantua's solution. Finally, Paul turns to a rejection of restrictivism, the first solution rejected by Bradwardine and the third by Mantua. But Paul deals with the specific form given to it by Walter Segrave in Oxford in the 1320s or early 1330s, who in fact defends restrictivism in the face of Bradwardine's objections,[71] attributing the solution to the fallacy of accident, a suggestion not dealt with by either Bradwardine or Mantua.

After this extended discussion of alternatives (occupying a quarter of his treatise), Paul sets out to develop his own solution, based firmly on Roger Swyneshed's proposals from the 1330s. In his second chapter, he systematically lays out his distinctions (*divisiones*) and assumptions (*suppositiones*), then draws seven Conclusions and seven Corollaries. The basic idea is Swyneshed's, namely, to provide a solution which does not depend on postulating tacit or hidden or consequential significates for insoluble propositions beyond what is clearly shown—what they standardly suggest or indicate by the straightforward combination of their

[68]Thomas Bradwardine 2010, chs.3-5.

[69]However, Marsilius of Inghen, whose solution is not discussed by Peter or Paul, appears to have distinguished Albert's solution from Buridan's early view: see Spade 1975, 79.

[70]See Strobino 2012, 484.

[71]See Spade 1975, 113-6 and Read 2023 (this volume).

parts (in Heytesbury's phrase, '*sicut termini communiter pretendunt*'). Instead, as Swyneshed had proposed, Paul tightens the criterion for truth, to exclude those that falsify themselves, weakening the criterion for falsehood to admit those examples that do falsify themselves even if otherwise impeccable. Roger's second and third notorious Conclusions reappear as Paul's fifth and second respectively. Paul will later describe the second Conclusion as a fundamental principle:

> "I deny the inference: 'The contradictory of A is false, therefore A is true,' since in the case of insolubles I uphold as a fundamental principle that two mutually contradictory propositions may both be false."[72]

Consequently, Paul defines an insoluble as a self-falsifying proposition

> "An insoluble proposition is a proposition having reflection on itself wholly or partially implying its own falsity or that it is not itself true, such as 'This is false,' 'This is not true,' referring to themselves, and 'This is true,' referring to its contradictory. And so on for many other examples which can be identified on the basis of what has been said. Therefore ⟨insolubles are self-falsifying propositions⟩."[73]

In the rest of the treatise, Paul considers, in ch.3, the objections which Heytesbury had directed at Swyneshed's solution, many of which had already been addressed in discussing the Conclusions and Corollaries in ch.2; shows at some length, in ch.4, how his solution deals with the much-discussed example where Socrates only says 'Socrates says a falsehood'; extends the account, in ch.5, to deal with other examples, such as that where Socrates says that his sole business is to be hung on the gallows, which are not obviously insolubles until the background scenario is added, in ch.6 to examples like 'A falsehood exists' which become insoluble in a suitable scenario, and in ch.7 to examples involving exclusive

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[72]Paulus Venetus 2022, §ad 4.2.1.3: "*... nego consequentiam: contradictorium a est falsum, igitur a est verum, quoniam in materia insolubilium sustinetur pro fundamento duo contradictoria inter se contradicentia esse duo falsa.*" Two other fundamental principles are mentioned in ch.8, on merely apparent insolubles.

[73]Paulus Venetus 2022, §2.1.8: "*Propositio insolubilis est propositio habens supra se reflexionem sue falsitatis aut se non esse veram totaliter vel partialiter illativa, ut: hoc est falsum, hoc non est verum, seipsis demonstratis; hoc est verum, demonstrando suum contradictorium, et sic de aliis multis que ex predictis possunt assignari. Quare et cetera.*"

and exceptive propositions, such as 'Only a false proposition is exclusive' (assuming it is the only exclusive proposition) and 'No proposition except A is false' (where this is A and is the only exceptive proposition).

Turning to the discussion of insolubles in the *Sophismata Aurea*, we find it is very derivative from that in the *Logica Magna*. The work consists of a collection of fifty sophisms, many very familiar from other collections of sophisms, many turning on an equivocation between compounded and divided senses (what would nowadays be called a scope ambiguity), e.g., 'Every proposition or its contradictory is true' and 'Everything false if it is impossible is not true'. The final sophism, no.50, is 'Socrates says a falsehood', assuming that there is only one Socrates and that is all he says. The sophismatic arguments leading to contradiction are mostly drawn, essentially *verbatim*, from §4.2 of the treatise on insolubles in the *Logica Magna* (with a final argument drawn from §3.1). Paul then presents four Conclusions and associated corollaries in order to defuse the sophismatic arguments. The first of these Conclusions is in effect a statement of the Swyneshed programme, to solve the insolubles without resort to any hidden signification, claiming that "every subject-predicate insoluble signifies exactly according to the composition of its terms,"[74] and its first corollary consequently rejects all those solutions which turn on there being such a secondary signification, describing the standard opinion as specifying that second significate as "that it is true,"[75] that is, the modified Heytesbury solution. The subsequent Conclusions repeat six of the seven Conclusions of the second chapter of the *Insolubles* treatise in the *Logica Magna*. The work concludes with a discussion of the paradox of signification, 'This proposition signifies other than things are', drawn from *Logica Magna* §3.2.

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[74]Paulus Venetus 2022, Appendix B §2.1, (1483, sig.p5va): "*... omne insolubile categoricum significat adequate iuxta compositionem suorum terminorum.*"

[75]*Ibid.*, Appendix B §2.1.1.1: "*Hec, hoc est falsum, significet adequate hoc esse falsum et hoc esse verum, ut communiter dicitur ratione reflexionis.*"

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