# Symmetry and paradox

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The ‘no’-‘no’ paradox (so-called by Sorensen) consists of a pair of propositions each of which says of the other that it is false. It is not immediately paradoxical, since it has a solution in which one proposition is true, the other false. However, that is itself paradoxical, since there is no clear ground for determining which is which. The two propositions should have the same truth-value. The paper shows how a proposal by the medieval thinker Thomas Bradwardine solves not only the Liar paradox, but also symmetric paradoxes like the ‘no’-‘no’, the descending ‘no’-‘no’, and the Truth-teller paradoxes.

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# 1 The ‘No’-‘no’ Paradox

In the eighth sophism of ch. 8 of his *Sophismata*, John Buridan (*2001*, 971) considers what Roy Sorensen (*2001*, 165) has recently dubbed the ‘No’-‘no’ paradox:

> Socrates says: (1) ‘What Plato is saying is false’
>
> Plato says: (2) ‘What Socrates is saying is false’,

where neither, it appears, says anything else. It seems clear that (1) and (2) cannot both be true. For if (1) is true, then (2) is false and so not true, and similarly, if (2) is true, (1) is false and so not true. However, neither can they both be false, for if (1) is false, (2) is true and so not false, and if (2) is false, (1) is true and so not false. Buridan is happy to endorse Bivalence, but anyone minded to deny it, might consider amending (1) and (2) by replacing ‘false’ by ‘not true’. Then not only can (1) and (2) not both be true, they cannot both fail to be true either.

The conclusion would seem to be that one is true, the other false. But which? There appear to be no good grounds for assigning truth to one or the other. One might be tempted by a position similar to the supervaluationist claim that a disjunction can be true without either disjunct being true, or an existential claim (‘there is $n$ such that $\phi n$ and $\neg\phi n + 1$’) can be true without any instance being true: perhaps one of (1) and (2) is true, the other false, without either being determinately true or false. Or perhaps one might adopt an epistemicist position of ignorance, as Sorensen does: we can prove (and so know) that one or other of (1) and (2) is true, the other false, but we cannot know which is which, even though one is one, the other the other. But the situation here appears more puzzling than the simple Truth-teller, as Buridan recognises. Even if one were willing to let (1) or (2) be true without it being determinate which, or that (1) or (2) be true without good reason, an asymmetric assignment of truth and falsity to (1) and (2) is in itself paradoxical. For Socrates' and Plato's utterances are symmetrical. An asymmetric assignment of truth-value, with or without good reason, is in itself paradoxical, for they have said exactly the same thing about each other. (1) and (2) should receive the same truth-value, on pain of paradox. But it seems that they cannot, again on pain of paradox. Is paradox unavoidable?

When faced with a similar (indeed, more general) situation, Tarski (*1969*, 66) proposed to reject the use of universal languages, in which, for example, (1) and (2) can be formulated, as showing "a symptom of disease". His approach was uncommonly popular. We will see, however, that a solution is possible without such a drastic curtailment of the expressive power of language.

## 2 The Truth-entailment Principle

Buridan's own solution to the paradox is to show that (1) and (2) are both false. His reason is based on the claim that every proposition implies its own truth provided it exists (for if it doesn't exist, it can't be true or false). For Buridan believed that propositions were, like everything else, singular existents, which exist only when uttered. This peculiarity can be (with care) set aside, and, assuming all (or all relevant) propositions exist, Buridan's proposal is that all propositions imply their own truth. He then claims, plausibly, that a proposition cannot be true unless every proposition it implies is also true. So whatever the account of truth (and he has already discussed this at length in ch. 2 of the *Sophismata*), for a proposition to be true not only must it satisfy the appropriate truth-condition, but so too must any proposition it implies.

Take propositions (1) and (2). By the symmetry condition, they must either both be true, or both be false. We have already seen that they cannot both be true. But the truth-entailment principle (as Hughes calls it—*1982*, 22), that each proposition entails its own truth, blocks the derivation in § 1 of contradiction from the assertion that they are each false. For suppose (1) is false. It does not immediately follow that (2) is true, for the truth of (2) requires not only that (1) be false, but that (2) be true, and the falsity of (2) can result not only from the falsity of (1) but also from the falsity of (2). Indeed, Buridan (*2001*, 972) argues that (1) entails both that (1) is true (by the truth-entailment principle) and that (1) is false, for if (1) were true, so too would be (2) (by symmetry) and so (1) would be false. Since (1) cannot be both true and false, (1) cannot be true, and so (1) is false, and so too is (2) by the same reasoning.

This may seem like sophistical reasoning of the highest degree, and so it is. The truth-entailment principle renders Buridan's account of truth nugatory. Some propositions must be false, on this account, such as the Liar paradox, since their truth requires not only that they be false (as they say) but that they be true too (by the principle), and no proposition can be both. But no proposition can satisfy the condition for truth without circularity: if things are as it (otherwise) says they are (Buridan's worries from ch. 2 about this formula aside), then if it is true, what it implies holds too, and so it is true; while if it is false, something that it implies (that it is true) does not hold, and so it is false. Buridan's truth-principle gives only a necessary, but no sufficient, condition for truth.

The source of this problem is the truth-entailment principle, and for that, Buridan gives no argument. He simply asserts it. It renders the paradoxes harmless, but at the cost of undermining Buridan's account of truth, an unacceptable paradox itself. Surely some propositions are true. But if every proposition entails its own truth, and everything a proposition entails must obtain for it to be true, propositions (unless they entail a contradiction) are merely true, if true, false if false. That's a non-starter as a theory of truth.

Actually, Buridan has a way out, which he exploits shamelessly. Although by the truth-entailment principle, every proposition entails its own truth (so Buridan believes), Buridan applies only to the paradoxical cases the stricter condition for the truth of a proposition that requires not only that it itself satisfy the appropriate condition but that whatever it entails do so too. (See *2001*, 857.) That lets the unparadoxical truths be true, but only at the cost of outrageous discrimination. The paradoxical cases are excluded simply by raising the standard for truth for them, and that simply because they are paradoxical.

## 3 Closure under Entailment

A more subtle analysis was offered by Thomas Bradwardine, in a treatise written in Oxford a generation earlier than Buridan. Some twenty-five years later, he died from the Black Death in August 1349, a month after returning from being made Archbishop of Canterbury by the Pope in Avignon, and a few years before Buridan, writing in Paris, completed the final version of his *Sophismata*. Bradwardine did not claim that all propositions entail their own truth: rather, he proved from more basic principles that certain propositions, in particular, those saying of themselves that they are not true, also say of themselves that they are true. Consequently, by the argument of § 2, all such propositions are false. (See *Read 2002*.) A central, and highly tendentious, principle in the argument is the claim that every proposition says (implicitly) everything which what it says entails (or implies), that is, that saying that is closed under entailment.

In a recent paper, Miroslava Andjelković and Timothy Williamson (*2000*) discuss the connection between saying that and truth (and falsity). They say a great deal about truth and falsity, but relatively little about saying that. One possibility which they reject out of hand is that a proposition might say several inequivalent things. They endorse the principle of Uniqueness:

(U) $(\forall s)(\forall P)([\forall Q)[\mathbf{Say}(s,P) \bullet \mathbf{Say}(s,Q)] \to [P \leftrightarrow Q]]$

(they relativize saying that and truth to a context, $c$, but I will omit this qualification for brevity, as inessential to my topic here).

Andjelković and Williamson also speak of sentences, not propositions, but this is merely terminological, for ‘proposition’ here means simply ‘declarative sentence’. Note that (U) and similar formulae to follow should be thought of as expressions in type theory, where $s$ has type $\iota$ (iota, of individuals) and $P$ and $Q$ have type $o$ (omicron, of propositions). See, e.g., *Church 1940*. The use of $\bullet$ for conjunction and $\rightarrow$ for implication (and later $\Rightarrow$ for entailment), and $\leftrightarrow$ and $\Leftrightarrow$ for the respective equivalences, rather than Andjelković and Williamson's notation, is intended to leave open the interpretation of these notions, whether classical, intuitionist, relevant or whatever.[1]

It surprises me that Slater (*2004*, 60) and others continue to challenge the user of propositional quantification with the dilemma: ‘objectual or substitutional?’ This is a false dichotomy. For all quantifier instantiation involves substitution. But only instantiation of individual variables involves objects. To complain that, e.g., ‘$(\exists P)$(Prior says that $P \bullet \neg P$)’ cannot be read, or not without invoking an inappropriate completion such as ‘is true’, is by the by. The question is whether any propositions are usefully represented formally in this way. The claims of the proponents of Protothetic, or of second- and higher-order logic and type theory, is that many are. For example, the above wff represents ‘Something Prior says is not the case’ (cf.

[1] All we need assume is that $\rightarrow$ is the residual of $\bullet$, that is, that $P \bullet Q \Rightarrow R$ iff $P \Rightarrow (Q \rightarrow R)$.

*Prior 1971*, 103).

Despite rejecting (U), Andjelković and Williamson note (*2000*, 230) that a conjunction might be thought to say what is said by each of its conjuncts. More generally (as Andjelković and Williamson note, and as Bradwardine claimed), a proposition might be thought to say everything that anything it says entails:

(E) $$(\forall s)(\forall P)(\forall Q)[[\mathbf{Say}(s,P) \bullet (P \Rightarrow Q)] \rightarrow \mathbf{Say}(s,Q)].$$

Thus, for example, a universal generalization says what each of its instances says, a singular proposition says what its existential generalization says (at least on the assumption that its singular term denotes), and so on.

Andjelković and Williamson argue that this conception of saying that must be rejected. For a 'plausible principle of compositional semantics', that any proposition saying that two propositions are equivalent says that whatever they say are equivalent (*2000*, 233), in conjunction with a 'standard proposal' concerning the definition of truth, entails (U). The standard definition of truth they refer to is

(T) $$(\forall s)(\mathbf{True}(s) \Leftrightarrow [(\exists P)\mathbf{Say}(s,P) \bullet (\forall P)[\mathbf{Say}(s,P) \rightarrow P]]).$$[2]

That is, a proposition is true if and only if it says something, and whatever it says is the case. The conjoined existential clause is added to prevent vacuous truth for lakes and mountains, for example, which say nothing. If

[2]Andjelković and Williamson call it (TDEF2*): *2000*, 230.

truth and falsity are to be exclusive and exhaustive, we need to match (T) with a definition of falsity:

(F) $$(\forall s)(\mathbf{False}(s) \Leftrightarrow [(\exists P)\mathbf{Say}(s, P) \bullet \neg P]).$$[3]

Andjelković and Williamson reject (E), (T) and (F) as incompatible with (U), and with their notion of a proposition 'saying *just* that $P$'. For their notion, for example, I might doubt what you say, but not doubt all that what you say entails, so there is a narrower notion of saying that which is not governed by (E) (and possibly is governed by (U)). Nonetheless, they admit, there is a 'non-technical notion of saying that on which to say something can also be to say ... its ... consequences' (*2000*, 230). This notion clearly does not obey the compositional principle that Andjelković and Williamson propose. If $s$ says two things, $P$ and $Q$ say, $s$ can be equivalent to itself without $P$ and $Q$ themselves being equivalent. Instead, this multivalent notion is governed by (E), and Bradwardine takes it to enter the account of truth and falsity defined by (T) and (F). We might call this notion of saying that 'Carnapian', in recognition of Carnap's account of the content (or sense) of a proposition in *1937* (§ 14) as the class of its non-analytic consequences.[4]

[3] Andjelković and Williamson call it (FDEF1): *2000*, 226.

[4] Carnap excludes the analytic consequences in order to render analytic propositions empty of content (*1937*, § 49), and because he took the analytic propositions to be equivalent and entailed by all others. A stronger account of '⇒' will allow us to distinguish differences in what analytic propositions say.

Before we turn to consider (1) and (2), we need to look closely at Bradwardine's argument for his main thesis, that every proposition which says of itself that it is not true, or false, also says of itself that it is true. Bradwardine's argument can be put succinctly as follows:[5] suppose $\mathbf{Say}(s, \mathbf{False}(s))$, that is, suppose some proposition, $s$, says of itself that it is false, and suppose that it is false. By (F), it follows that something $s$ says fails to obtain: $(\exists P)(\mathbf{Say}(s, P) \bullet \neg P)$, if not that $s$ is false then something else $s$ says, call it $Q$. Then if it's not $Q$ that fails to hold, it must be $\mathbf{False}(s)$ that fails to hold, i.e., $\mathbf{False}(s) \Rightarrow (Q \rightarrow \neg\mathbf{False}(s))$, indeed, by Residuation and Bivalence, $(\mathbf{False}(s) \bullet Q) \Rightarrow \mathbf{True}(s)$. But $\mathbf{Say}(s, \mathbf{False}(s))$ and $\mathbf{Say}(s, Q)$, so by (E), $\mathbf{Say}(s, \mathbf{True}(s))$. Thus any proposition which says of itself that it is not true (or false), also says of itself that it is true. Finally, by (F) it must be false, since at least one thing it says must fail to hold, for it cannot be both true and false.

Now consider (1) and (2). It appears that Socrates and Plato each say just one thing—and so they do, explicitly. But given the closure principle (E), that appearance may be misleading—indeed, we will find that it is. Suppose that what Plato is saying is false. We cannot infer that what Socrates is saying is true, for although one thing that Plato is saying is that

[5] *Bradwardine 1970*, § 6.054, 299; translated in *Read 2002*, 192-3. Bradwardine's argument uses the Disjunctive Syllogism explicitly, but as the following reasoning shows, it uses an arguably relevant version. See, e.g., *Read 2004*. Indeed, relevantly, we need an intensional version of Bivalence, $\neg\mathbf{False}(s) \rightarrow \mathbf{True}(s)$, which is stronger than $\mathbf{False}(s) \vee \mathbf{True}(s)$.

what Socrates is saying is false, that may not be all that Plato is saying, and by (F), what Plato is saying is false if anything that Plato is saying fails to obtain. But what we can infer is that what Socrates is saying is false, for symmetry considerations tell us that Plato's and Socrates' utterances are true or false together.

Again, given that what Socrates is saying is false, it does not follow that what Plato is saying is true, for that may not be all that Socrates is saying—indeed, we are about to discover that both Plato and Socrates are also saying that what each is saying is true. But, paraphrasing Bradwardine's own argument, we can infer that something Socrates is saying fails to obtain, either that what Plato is saying is false or something else. So if it's not that other thing which fails to obtain, then what Plato is saying is not false, that is, assuming that what Plato is saying is false, it follows that what Socrates is saying is false, so if it's not something else which Socrates is saying which doesn't obtain, then what Plato is saying is true. So if what Plato is saying is false and whatever else Socrates is saying obtains, then what Plato is saying is true. But what Socrates is saying is that what Plato is saying is false together with whatever else he, Socrates, may be saying. So by (E), Socrates is also saying that what Plato is saying is true. So Socrates is saying that what Plato is saying is both true and false. But that is impossible, whence by (F) what Socrates is saying is false. A similar argument shows that Plato is saying that what Socrates is saying is both true and false, and so Plato's utterance is also false.

The argument has the following form: suppose **False**(2). Then **False**(1) by symmetry, so $(\exists P)(\mathbf{Say}(1, P) \bullet \neg P)$, whence $Q \rightarrow \mathbf{True}(2)$, where $Q$ abbreviates everything that (1) says other than $\mathbf{False}(2)$. That is, $\mathbf{False}(2) \Rightarrow (Q \rightarrow \mathbf{True}(2))$, assuming that $\mathbf{False}(2) \Rightarrow \mathbf{False}(1)$, that symmetry of truth-value is not just contingent, but necessary. So $\mathbf{False}(2) \bullet Q \Rightarrow \mathbf{True}(2)$. But $\mathbf{Say}(1, \mathbf{False}(2) \bullet Q)$, so by (E), $\mathbf{Say}(1, \mathbf{True}(2))$ and so $\mathbf{Say}(1, \mathbf{True}(2) \bullet \mathbf{False}(2))$. So (1) is (implicitly) contradictory, and so by (F) is false.

This reasoning is sufficient to block the reasoning in § 1 that (1) and (2) cannot both be false. For it does not follow from the fact that (2) is false that (1) is true. For (1) to be true we require not only that (2) be false, but also that (2) be true. That conjunction is impossible. Similarly, for (2) to be true, we require not only that (1) be false, but also that (1) be true. So by (F), (1) and (2) are both false, for something each says of the other, namely, that it is true, fails to hold.

What exactly is the appeal to Symmetry here? We have seen that the 'no'-'no' paradox is a second-order paradox. There is a consistent assignment of truth-values—one true, the other false. But that in itself is paradoxical. The solution should be symmetrical, and (1) and (2) should have the same truth-value. For no reason can be given why their values should be different other than the threat of (first-order) paradox. In the absence of any good (i.e., non-*ad hoc*) reason to the contrary, if one is true, so should the other be. The assignment should be symmetrical.

The denial of (U), and recognition that what one says may only be implicit, solves a puzzle which has long dogged paradoxes such as that of Epimenides the Cretan, who is reported to have said 'All Cretans are liars'. Prior (*1958*) and others have plausibly arged that it follows that that cannot be all that Cretans said—at least one Cretan must have said something true and what Epimenides said is false. Prior is amazed:

> 'Yet how can there be a *logical* impossibility in supposing that some Cretan asserts that no Cretan ever says anything else, and that this is the only assertion ever made by a Cretan?' (*Prior 1958*, 261; cf. *Prior 1971*, 101 and 105)

The puzzle is solved by realizing that in uttering 'All Cretans are liars', Epimenides said two things, both (explicitly) that all Cretans are liars and (implicitly) that what he was saying was true. So what he said was simply false, because not all that what he said entailed was the case.

In fact, Bradwardine considers a puzzle very similar to Buridan's 'no'-'no' paradox, and offers a similar diagnosis:

> 'Suppose Socrates and Plato begin to speak at the same time, both uttering 'Something false is uttered by Socrates and Plato', and let what Socrates utters be $A$, what Plato utters $B$, and they utter only $A$ and $B$. Then no reason can be given why $A$ and $B$ are not equivalent, and so $A$ and $B$ are equivalent, and $A$ is either true or false. If true, then $B$ is true too and so nothing false was said, and so $A$ and $B$ are false. If false, then $B$ is false

too, and so the subject of each stands for the other, and so each has one simple negative instance, and so each is true.

Reply: $A$ and $B$ are equivalent, and the subject of each does stand for the other, and each has one true instance. But it does not follow from that, that each of them is true, because each of them says more, namely, that it itself is not true, for $A$ says that a falsehood was uttered, and from 'A falsehood is uttered' it follows that the falsehood ... was $A$ or $B$. Now from its being false that $A$ or $B$, it follows that everything which says exactly what $A$ does is false ⟨or⟩ everything which says exactly what $B$ does is false, and from each part of this disjunction it follows that $A$ and $B$ are false, because $A$ and $B$ are equivalent and say exactly the same. Hence from the disjunction it follows that $A$ and $B$ are false and it follows from the fact that $A$ and $B$ are false that $A$ is false. Hence, by postulate (E), $A$ says that $A$ is false, and so [by Bradwardine's main thesis], $A$ says that it itself is true, and the same for $B$.'[6]

Bradwardine uses the closure principle (E) to show that each of Socrates' and Plato's utterances says of itself that it is false, and then invokes his main result, that any proposition which says of itself that it is false, also says of itself that it is true, to conclude that each utterance is false. Thus both $A$ and $B$ are false, being equivalent propositions, that is, by symmetry, there being no reason why they should have different truth-value.

[6] *Bradwardine 1970*, § 7.10, 304-5.

## 4 Iterated Paradox

Sorensen (*2001*, 181) claims that Yablo's paradox (see *Yablo 1993*) and a descending form of the 'no'-'no' paradox cannot be explained by the Buridanian approach. The descending form of the 'no'-'no' paradox is as follows:

($1_d$) ($2_d$) is false

($2_d$) ($3_d$) is false

($3_d$) ($4_d$) is false

and so on

The consistent assignments are asymmetrical: either all the odd-numbered propositions are true and the even-numbered false, or vice versa. But that is paradoxical. Being odd- or even-numbered is an accidental property, and each proposition says equally of its identical successor that it is false. Is there not a symmetrical solution?

Let us approach the solution by first considering Yablo's paradox:

($1_y$) All subsequent propositions are false

($2_y$) All subsequent propositions are false

($3_y$) All subsequent propositions are false

and so on

$(1_y)$ cannot be true. For if it were, then everything it said would hold, so all subsequent propositions would be false, in particular, $(2_y)$ would be false, so what it said would fail to obtain. So some subsequent proposition would be true. But given that $(1_y)$ is true, all subsequent propositions are false, a contradiction. So $(1_y)$ is false. But given that $(1_y)$ is false, it follows that some subsequent proposition must be true, yet by the same argument it cannot be.

Once again, the questionable step is the use of (F), first, to infer that if $(2_y)$ is false, then some subsequent proposition is true, and later that if $(1_y)$ is false, once more some subsequent proposition is true. For although, in the first case, one thing that $(2_y)$ says is that every subsequent proposition is false, that some subsequent proposition be true is not necessary for the falsehood of $(2_y)$. For $(2_y)$ may say something else—indeed, we will find that it does, and it is the failure of that to obtain which is what makes $(2_y)$ false, not the truth of some subsequent proposition.

To solve the puzzle, we need to adapt Bradwardine's reasoning, as described in § 3, and there adapted to the 'no'-'no' paradox. First, suppose that for all $n > m$, $\mathbf{False}(n_y)$. Then by symmetry, $\mathbf{False}(m_y)$. So something $(m_y)$ says fails to hold, either that for all $n > m$, $\mathbf{False}(n_y)$ or something else, call it $Q$. So if it's not $Q$ which fails to hold, then for some $n > m$, $\mathbf{True}(n_y)$. That is, $((\forall n > m)\mathbf{False}(n_y) \bullet Q) \Rightarrow (\exists n > m)\mathbf{True}(n_y)$. But $\mathbf{Say}(m_y, (\forall n > m)\mathbf{False}(n_y) \bullet Q)$. So by (E), $\mathbf{Say}(m_y, (\exists n > m)\mathbf{True}(n_y))$. So $\mathbf{Say}(m_y, (\forall n > m)\mathbf{False}(n_y) \bullet (\exists n > m)\mathbf{True}(n_y))$, whence something $(m_y)$ says must fail to hold, and so $\mathbf{False}(m_y)$.

Thus, each proposition in the Yablo hierarchy is false. Accordingly, the paradox is blocked. All the propositions in Yablo's list are false, not because one of them is true, but because they are all implicitly contradictory.

The descending 'no'-'no' paradox succumbs to the same diagnosis. Each proposition in the chain is false not because the succeeding proposition is true, but because each is inherently contradictory. Suppose ($2_d$) is false. Then by symmetry, ($1_d$) is also false. So something ($1_d$) says fails to hold, either that ($2_d$) is false or something else, call it $Q$. The same reasoning as hitherto shows that $(\mathbf{False}(2_d) \bullet Q) \Rightarrow \mathbf{True}(2_d)$. But $\mathbf{Say}(1_d, \mathbf{False}(2_d) \bullet Q)$. So $\mathbf{Say}(1_d, \mathbf{True}(2_d))$, whence $\mathbf{Say}(1_d, \mathbf{True}(2_d) \bullet \mathbf{False}(2_d))$, so by (F), $\mathbf{False}(1_d)$, and so on for each proposition in the chain. ($1_d$) is false, not because ($2_d$) is true, but because ($1_d$) is implicitly contradictory, and the same for each of the propositions in the chain.

## 5 The Truth-Teller

Even if Sorensen is right, therefore, in claiming that Buridan's theory cannot deal with iterated paradox, Bradwardine's account, based on (E), does provide a satisfactory and symmetrical solution. But in fact, the account is even more powerful. As we saw, although the 'no'-'no' paradox is not immediately paradoxical, it has a second-order paradoxicality, in that what seems to be the only consistent assignment is asymmetrical, consequently offending against the evident symmetry in (1) and (2). Using Bradwardine's truth-condition, coupled to his principle (E), we find that there is a symmetrical and consistent assignment, making each false. A similar problem affects (3):

> Aristotle says: (3) 'What I am saying is true.'

This is the Truth-teller. Again, there are, it seems, two consistent assignments: (3) can be true, or it can be false. But which is it? If one is committed to Bivalence, it must be one or the other, but since it cannot be both, it seems to defy any attempt to discover which it is. This might attract the epistemicist, but then what defies comprehension is not so much our inability to tell which it is, rather, what could constitute its being asymmetrically one rather than the other.

Hence, once again, we are led to a second-order paradox, a need to accept that (3) is true, or false, but not both, when it could be either. What breaks the impasse? A clue is contained in another medieval author, this time anonymous, writing around the year 1200, that is, more than a hundred years before Bradwardine. He observes that some say that the Truth-teller is not an 'insoluble', that is, a puzzle case that is difficult to solve. For if true, it is true, and if false, it is false, and there is no contradiction in that. He considers the following response:

> 'But to the contrary: suppose there are two men, and when I utter this proposition, 'I say something true', one of them responds: 'That's true', the other: 'That's false'. He who responds 'That's true' responds well, in as much as from his response no

> contradiction follows. Similarly, the one who responds 'That's false' responds well, in as much as from his response a contradiction does not follow [either]. So each responds well or each badly. But they cannot both respond well, because then the same proposition would be both true and false, which is impossible. So neither responds well, and neither response is good. Hence this proposition, 'I say something true', is a proposition to which it is not possible to respond well. So it is an insoluble, which some accept for this reason.' (*De Rijk 1966*, 106-7)

Unimpressed by this argument, the anonymous author's own view was that the Truth-teller says nothing.[7] But what is interesting is the suggestion of symmetry: if it is either true or false then it is both. That cannot be quite right, for the conditions (T) and (F) are not themselves symmetrical: (T) requires for the truth of a proposition that everything it says hold, while (F) requires for its falsity only that something it says fail to obtain. But what more does (3) say besides its own truth? If the two responses are symmetrical, that is, equally good or bad, then if it is right to say that it is true it must also be right to say that it is false, that is, if it's true it's false, and so by (E), since it says it is true, it must also say that it is false. So it says both that it is true and that it is false. But it cannot be both. So by (F) it is false.

[7] This doctrine, known as *cassatio*, was a popular theory in the thirteenth century, one which Bradwardine criticizes powerfully.

Note that the converse reasoning is blocked: we cannot infer that since it is false, then it is also true. For to repeat, (T) and (F) are not themselves symmetrical: for (3) to be true, more is required than that something (3) says obtain—everything it says must hold, and that is impossible, for it says both that it is true and that it is false, and it cannot, as our anonymous author noted, be both.

## 6 Dispensing with Closure

We have seen that Buridan's diagnosis of the 'no'-'no' paradox is vindicated, but by a deeper analysis of truth and saying that than he himself offered. But one might balk at the strength of the Carnapian principle (E), and worry that the solution depends crucially on it. Suppose that one rejects (E). One might still capture the force of (T) and (F) by use of Buridan's principle that for the truth of a proposition, not only must what it says hold, but so too must everything that what it says entails:

$$(\mathrm{T}') \ (\forall s)(\mathbf{True}(s) \Leftrightarrow [(\exists P)\mathbf{Say}(s,P) \bullet (\forall Q)[(\exists P)(\mathbf{Say}(s,P) \bullet (P \Rightarrow Q)) \to Q]]$$

$$(\mathrm{F}') \qquad (\forall s)(\mathbf{False}(s) \Leftrightarrow (\exists Q)(\exists P)[\mathbf{Say}(s,P) \bullet (P \Rightarrow Q) \bullet \neg Q]),$$

(T′) and (F′) capture the idea that no proposition is true unless everything that what it says entails obtains, and that if what it says entails something which fails to hold, it must be false. Then (1) and (2) can both be false, without contradiction, because the falsity of (1) results not from the truth of (2), but from what the truth of (2) entails, namely, the falsity of (1). (1)

and (2) cannot consistently both be true (given (T) and (F), or (T′) and (F′)), but they can both be false, and in that there is no paradox.

But contradiction is not far away. Suppose $\neg\mathbf{True}(s)$ is all $s$ says. Then by (T′),

$$\mathbf{True}(s) \Leftrightarrow (\forall P)((\neg\mathbf{True}(s) \Rightarrow P) \rightarrow P)$$

so
$$\neg\mathbf{True}(s) \Rightarrow (\exists P)((\neg P \Rightarrow \mathbf{True}(s)) \bullet \neg P)$$

whence
$$\neg\mathbf{True}(s) \Rightarrow \mathbf{True}(s).$$

So if $\neg\mathbf{True}(s)$ is all $s$ says, and $s$ is not true, then $s$ is also true, and contradiction has returned with a vengeance.

Hence, even if one abandons (E) and (T) in favour of (T′), it is crucial to Bradwardine's solution that $s$ say more than just $\neg\mathbf{True}(s)$, even if it does not say as much as (E) would entail. For without some such principle, it will be possible to incur paradox (even inadvertently) if a proposition could say just that, for example, it was not true. In particular, whatever any such sentence says must entail that it is true: rehearse the reasoning of the previous paragraph with $Q \bullet \neg\mathbf{True}(s)$ in place of $\neg\mathbf{True}(s)$ to obtain $Q \bullet \neg\mathbf{True}(s) \Rightarrow \mathbf{True}(s)$.

This finally shows the untenability of Buridan's proposed solution to the paradoxes. Buridan rejected (E) and the suggestion that every proposition says of itself that it is true (recall that he did not restrict the feature just to paradoxical propositions) on the ground that that would make every proposition metalinguistic. Instead, his idea was to use the truth-entailment principle (in § 2 above) together with the claim that every proposition implies or entails its own truth to show that the paradoxes are simply false, and not true. In fact, he did not endorse (T′) in quite the form expressed above. His version relies on a specific feature of medieval theory:

$$(\mathrm{T}'')\ (\forall s)(\mathbf{True}(s) \Leftrightarrow [(\exists P)\mathbf{Say}(s,P)\ \bullet\ (\forall Q)[(\exists P)(\mathbf{Say}(s,P)\ \bullet\ (P \Rightarrow Q)) \rightarrow S(Q)]]$$

where $S(Q)$ abbreviates a complex condition depending on the exact form of $Q$ and incorporating the medieval theory of 'supposition of terms'. But applied to the case of the truth-entailment principle, the condition amounts to the requirement that '**True**' and '$s$' (subject and predicate of the entailed proposition) denote the same thing(s), i.e., that $s$ be true.

We can rehearse the argument: take any Buridanian proposition $s$ which says only $\neg\mathbf{True}(s)$. Then

$$\neg\mathbf{True}(s) \Rightarrow (\exists Q)((\neg\mathbf{True}(s) \Rightarrow Q)\ \bullet\ \neg S(Q)).$$

We only need the condition that $Q \Rightarrow S(Q)$ to infer

$$(\exists Q)\mathbf{True}(s)$$

i.e.,

$$\mathbf{True}(s).$$

So if $s$ is false (as Buridan claims) and so not true, then it is true too, and contradiction has returned.

Is Buridan committed to the principle that $Q \Rightarrow S(Q)$ for all $Q$, used here? Certainly, he denies its converse, specifically for insoluble propositions:

$S(Q)$ is not sufficient to infer $Q$ (or at least, that $Q$ is true). But $Q \Rightarrow S(Q)$ seems an immediate consequence of the theory of supposition. To take the simplest example, where $Q$ is an affirmative singular proposition of the form '$A$ is $B$'. Then $S(Q)$ says that what '$A$' denotes (supposits for) is what '$B$' denotes, and that clearly follows from '$A$ is $B$'.

There may yet be a residual doubt. We have used (E) to show that if a proposition says that it's false then it says that it's true; or we've used (F′) to show that if it says that it's false then it entails that it's true. Either way, it cannot be both. Yet at the heart of the argument seems to lie an entailment from its falsity to its truth. So if we have indeed shown that the proposition is false, surely, we must infer that it is also true, and paradox has returned.

This is not so. To be sure, what a diagnosis of the paradoxes does not need is a proof that they are false. Proofs abound, that they are false and that they are true. What is needed is a diagnosis of a fallacy somewhere in the argument, and the present diagnosis is precisely that the inference from falsity to truth fails. So what suggests otherwise? Take the proof of Bradwardine's main thesis at the end of § 3. He supposes that $s$ is false and infers that $s$ is true, using (E) to infer that $s$ says of itself that it is true, given that it says of itself that it is false. But he does not show that $\mathbf{False}(s) \Rightarrow \mathbf{True}(s)$. Rather, he shows that what $s$ says, which is not just that $s$ is false, entails $\mathbf{True}(s)$. If $s$ said only that $s$ is false then we could infer that $\mathbf{False}(s) \Rightarrow \mathbf{True}(s)$, as we have just seen. But the conclusion is that $s$ does not say only that $s$ is false, but also that $s$ is true, so all we have is the jejune entailment $\mathbf{False}(s) \bullet \mathbf{True}(s) \Rightarrow \mathbf{True}(s)$.[8]

In the case of the 'no'-'no' paradox and the Truth-teller, the story is somewhat different. For the falsity of (2) doesn't entail its truth *simpliciter*, but only given that (1) and (2) are true together. So in neither case is there an inference from a proposition's falsity (*simpliciter*) to its truth. Rejecting that move is the touchstone of Bradwardine's idea.

## 7 Conclusion

Central to Bradwardine's solution is the truth-principle, that a proposition is true if and only if everything that what it says entails obtains. One can articulate that thought either by closing saying that under entailment and identifying truth with the holding of everything the proposition says, combining (T) with (E); or more modestly by building the condition into the truth-principle itself, as in (T′). Either way, what prevents paradox is a restricted form of the truth-entailment principle, that the paradoxical propositions not only say they are false (and so not true), but moreover, what they say entails that they are true. But unlike Buridan, Bradwardine does not extend the truth-entailment principle to all propositions. Doing so renders Buridan's theory of truth impotent, for the principle makes truth unachievable for any proposition subject to it—except, ironically, for the paradoxical propositions whose falsity turns out to entail their truth.

[8] If $\bullet$ is the intensional conjunction of relevance logic, however, for which Simplification fails, the entailment is far from jejune, but still holds, by the argument of § 3.

Bradwardine's analysis, preserving ordinary truth, solves not only the Liar, the Strengthened Liar, Curry's paradox, the 'yes'-'no' paradox (Buridan's ninth sophism) and Yablo's paradox (as I showed in *Read 2002*); but also applies to the 'no'-'no' paradox, and the descending 'no'-'no' paradox, once we realise that a symmetrical assignment of truth-values is required. In each case, there is no more reason why one of the class of propositions in question should be true, another false. They must all have the same truth-value. On this basis, Bradwardine's analysis extends to show that the problematic cases, the 'no'-'no', the Truth-teller and so on, are all implicitly contradictory. Hence they are all false, and none of them is true.

The principles (T) and (F), however, require caution in their application, as we saw when applying them to, for example, (1). For if (1) is false, it does not immediately follow that (2) is true, for that is to assume that all (1) says is that (2) is false. From the falsity of (1) we can infer only that something (1) says fails to obtain, and in fact it transpires that (1) also says that (2) is true. So certainly something (1) says fails to obtain, since it says things that cannot hold together.

## References


[1] Andjelković, M. and Williamson, T. 2000. 'Truth, falsity and borderline cases', *Philosophical Topics*, **28**, 211-44.

[2] Bradwardine, T. 1970. *Insolubilia*, in Roure, M.-L. 1970. 'La problématique des propositions insolubles au XIII$^{e}$ siècle et au début

du XIV$^e$, suivie de l'édition des traités de W. Shyreswood, W. Burleigh et Th. Bradwardine', *Archives d'Histoire Doctrinale et Littéraire du Moyen Age*, **37**, 205-326.

[3] Buridan, J. 2001. *Summulae de Dialectica*, tr. G. Klima. New Haven, CT: Yale University Press.

[4] Carnap, R. 1937. *The Logical Syntax of Language*, tr. A. Smeaton. London: Routledge.

[5] Church, A. 1940. 'A formulation of the simple theory of types', *Journal of Symbolic Logic*, **5**, 56-68.

[6] De Rijk, L.M. 1966. 'Some notes on the medieval tract *De insolubilibus*, with an edition of a tract from the end of the twelfth century', *Vivarium*, **4**, 83-115.

[7] Hughes, G.E. 1982. *John Buridan on Self-Reference. Chapter Eight of Buridan's 'Sophismata', translated with an Introduction, and a philosophical commentary.* Cambridge: Cambridge U.P.

[8] Prior, A.N. 1958. 'Epimenides the Cretan', *The Journal of Symbolic Logic*, **23**, 261-66.

[9] Prior, A.N. 1971. *Objects of Thought.* Oxford: Clarendon Press.

[10] Read, S. 2002. 'The Liar paradox from John Buridan back to Thomas Bradwardine', *Vivarium*, **40**, 189-218.

[11] Read, S. 2004. ‘In defence of the dog: response to Restall’, in Rahman, S. et al. ed. *Logic, Epistemology and the Unity of Science*. Dordrecht: Kluwer, 175-80.

[12] Slater, B.H. 2004. ‘Ramseying liars’, *Logic and Logical Philosophy*, **13**, 57-70.

[13] Sorensen, R. 2001. *Vagueness and Contradiction*. Oxford: Clarendon Press.

[14] Tarski, A. 1969. ‘Truth and proof’, *Scientific American*, **220** no. 6, 63-77.

[15] Yablo S. 1993. ‘Paradox without self-reference’, *Analysis*, **53**, 251-2.