Miller, Bradwardine and the Truth*

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Abstract

In a recent article, David Miller has criticised Thomas Bradwardine's theory of truth and signification and my defence of Bradwardine's application of it to the semantic paradoxes. Much of Miller's criticism is sympathetic and helpful in gaining a better understanding of the relationship between Bradwardine's proposed solution to the paradoxes and Alfred Tarski's. But some of Miller's criticisms betray a misunderstanding of crucial aspects of Bradwardine's account of truth and signification.

John Buridan's diagnosis of the fallacy in the traditional derivation of paradox from the Liar sentence is fairly well known: that the Liar sentence virtually implies its own truth, as does every sentence, and so is implicitly contradictory and hence (simply) false. This is not, however, the theory of Buridan's which David Miller discusses in his careful and revealing study (Miller, 2011), but an earlier and less well known theory of Buridan's, that every sentence, including the Liar, signifies its own truth, but the result is the same. Buridan's earlier diagnosis is somewhat similar to a view developed some twenty years before that, in the 1320s, by Thomas Bradwardine. The more I have thought about and reflected on Bradwardine's account, the more convincing and effective I have found it. Thus I am delighted that it is becoming better known and that serious scholars, such as Miller, are considering it and subjecting it to critical examination. It is, to my mind, at least as good as, and arguably better than, any other proposed solution in explaining the mistake that leads to semantic paradox.

Bradwardine's diagnosis is encapsulated in his second thesis, proved in the sixth chapter of his treatise (Bradwardine, 2010, ¶6.4): that "every

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sentence[1] which signifies itself not to be true or itself to be false, also signifies itself to be true and is false." In that chapter, Bradwardine sets out a number of definitions and postulates from which he derives this thesis. At its heart is the definition of truth (¶6.2): "a true sentence is one signifying only as things are", and "a false sentence is one signifying other than things are."

Bradwardine says little if anything in general about signification apart from his second postulate, that signification is closed under consequence. Miller claims that I attribute three postulates regarding signification to Bradwardine, but I don't think that is quite right. The first of these three is described by Miller as a "postulate of *explicitness*" (Miller, 2011, p. 5), and expressed formally by the schema

$$(E) \qquad\qquad X : p$$

where what replaces 'X' is a structural-descriptive name of a sentence which replaces 'p'. Despite what Spade (1981, p. 125) says, nothing like this is to be found explicitly in Bradwardine, but his practice might justify one in attributing it to him. His general statements, e.g., in his second postulate and in his second thesis, speak only of "any (or every) proposition signifying such-and-such", but he proceeds in later chapters to discuss particular examples, and there it is clear, for example, that he thinks 'Socrates utters a falsehood' signifies (among other things) that Socrates utters a falsehood, and that 'Some proposition is not described by its predicate' signifies that some proposition is not described by its predicate.

Bradwardine's practice also makes clear how he intends his second postulate to be applied. Elsewhere (Read, 2011, n. 13) in fact, I have attributed to him the more general claim:

$$(K') \qquad \forall p, q, r((p \wedge q \to r) \to (s : p \wedge s : q \to s : r))$$

which entails the two principles given by Miller,

$$(K) \qquad\qquad (p \to q) \to (s : p \to s : q)$$

and

$$(C) \qquad\qquad (s : p \wedge s : q) \to s : p \wedge q$$

Bradwardine's practice not only justifies the attribution of these principles; it helps to clarify (and possibly even correct) Bradwardine's expression of his second postulate: "every sentence signifies or means everything which follows from it" (Bradwardine, 2010, ¶6.3). But Miller is right to observe

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[1] I have here translated Bradwardine's Latin term '*propositio*' as 'sentence', though elsewhere (e.g., Bradwardine (2010)) I have translated it as 'proposition'. But for Bradwardine, as for other medieval authors, *propositiones* could be spoken or written, as well as mental (or in the mind), and so 'declarative sentence' is as good a translation.

that Bradwardine's account of signification leaves many questions open, e.g., whether "the rule of ∧-introduction transmits ... truth" (p. 12), indeed, whether one could ever demonstrate categorically that any sentence was true, since the second postulate implies that each sentence signifies many things all of which must obtain, and so be verified, for it to be true and be shown to be true. That ∧-introduction transmits truth is stated in Bradwardine's sixth postulate ("if a conjunction is true, each part is true and conversely"), but this is specifically postulated and does not appear to follow from the accounts of truth and signification.

In fact, in general it would seem that consequence does not, for Bradwardine, entail truth-preservation. This was pointed out explicitly by Albert of Saxony, a younger contemporary of Buridan's who endorsed Buridan's earlier solution to the paradoxes mentioned above. The example is well known from discussions of Buridan:

> Every sentence is affirmative
> So no sentence is negative.

The inference is valid, but not truth-preserving, since the premise can be true, but if it were the conclusion would not be, for it would not exist. Rather, says Albert, "in order that an inference be valid it is required and sufficient that it be impossible that things be as the premise signifies without things being as the conclusion signifies."[2] Things can be as a sentence signifies without that sentence being true. In the case of the Liar sentence, L, for example, things are as L signifies so far as L signifies that L is false; but they are not as L signifies in so far as L signifies that L is true; so L is (simply) false.

Albert raises a puzzle about L and its like, as does Bradwardine, namely, what is the contradictory of an insoluble sentence.[3] For example, supposing that L signifies that L is not true, Bradwardine has proved that L is contradictory, since it signifies both that it is true and that it is false. So, he says (¶7.3), "if [L] has a contradictory, it is necessary, by Aristotle's rule, to prefix negation to the whole of [L], and just as [L] signifies conjunctively [that L is false and that L is true], so its opposite signifies disjunctively, namely, that L is not false or L is not true." Accordingly, the contradictory of L is not 'L is not false' but 'L is not false or L is not true', which is not self-referential, and is indeed a tautology—for it is the contradictory of a contradiction.

For this reason, Miller is mistaken when he says (p. 3) that "the negation of U, ¬U, is equally a liar." We standardly write ¬y to denote the contradictory of y, but quite how ¬y is formed from y is not always clear.

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[2] Alberto de Sajonia (1988, §§1673-5).

[3] See Alberto de Sajonia (1988, §§1643-5, 1655-7, 1702-3, 1721-3) and Bradwardine (2010, §ad 7.3).

For example, the contradictory negation of 'Snow is white' is 'Snow is not white', but the contradictory of 'Every man is running' is not 'Every man is not running' but 'Not every man is running', and of 'Everyone sings or everyone cries' is not 'Not everyone sings or everyone cries', nor 'Not everyone sings or not everyone cries' or even 'Everyone does not sing or everyone does not cry', but 'Not everyone sings and not everyone cries'. Nor is 'This sentence does not contain five words' the contradictory of 'This sentence contains five words', since they are both true. So we cannot assume that the negation of 'b has B' is 'b does not have B', for that depends, given Bradwardine's second thesis, what 'b has B' signifies. Since '"b has a false transform" has a false transform' (that is, U) signifies not only that 'b has a false transform' has a false transform but also that 'b has a false transform' has a true transform (as we will see shortly), to express $\neg\mathsf{U}$ we need to form a sentence which strictly contradicts U, that is, contradicts what U signifies: the result is '"b has a false transform" either has a true transform or has a false transform' (assuming bivalence, as Bradwardine does). The contradictory of a Liar cannot itself be a Liar, for a Liar is contradictory (on Bradwardine's account) and so its contradictory must be a tautology.

U is not directly self-referential, as Miller point out (p. 3). No more is Daphnis' utterance (p. 2) which signifies that Chloe's is false, though it is indirectly so in virtue of the fact that Chloe's utterance refers to Daphnis' (and says that it is true). U refers to the utterance 'b has a false transform', whose transform U is. Nonetheless, U, and Daphnis' utterance and Chloe's, all signify of themselves that they are false, and so by Bradwardine's second thesis, also signify that they are true. In particular, U signifies that the transform of that sentence of which it is the transform is false, so by Bradwardine's second postulate (that signification is closed under consequence), it signifies that U is false. But W, namely, '"b has a false transform" does not have a false transform' is simply false, for 'b has a false transform' does have a false transform, namely, U. So W is not interdeducible with $\neg\mathsf{U}$, for $\neg\mathsf{U}$ is a tautology and W is false.

Finally, Miller is right to note that Bradwardine's theory is not materially adequate in his sense. It does not entail all equivalences of the form $\mathbb{T}(X) \leftrightarrow p$ (where what replaces 'p' is a sentence a structural-descriptive name of which replaces 'X') or even $\mathbb{T}(X) \leftrightarrow (X \in \mathcal{L}) \wedge p$. Nor should it. Some instances of those schemata are false, in particular $\mathbb{T}(\mathsf{L}) \leftrightarrow \neg\mathbb{T}(\mathsf{L})$ and $\mathbb{T}(\mathsf{U}) \leftrightarrow b$ has a false transform. What Tarski's adequacy condition, in Miller's formulation, overlooks is that we cannot always use an expression of the form X to express X's truth-condition. Tarski put it better when he said that an adequate theory of truth must entail "all sentences which are obtained from the expression '$x \in$ Tr if and only if p' by substituting for the symbol 'x' a structural-descriptive name of any sentence of the language in question and for the symbol 'p' the expression which forms the trans-

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lation of this sentence [in the theory]."[4]   Thus the correct account of the truth-conditions of L and U is given by

$$\mathbb{T}(\mathsf{L}) \leftrightarrow \neg\mathbb{T}(\mathsf{L}) \wedge \mathbb{T}(\mathsf{L})$$

and

$\mathbb{T}(\mathsf{U}) \leftrightarrow b$ has a false transform and has a true transform.

So L and U are not true, but false.

To conclude: Miller is right to focus on Bradwardine's theory of signification, and in particular on the postulate (E). Without it, Bradwardine's second thesis, that every sentence signifying that it itself is false, is false (and not true), has no bite. It is the existence of sentences like L and U which threaten to produce paradox. But Bradwardine's theory of signification, incorporating postulates (K) or (K′), as well as (E), does neutralize that threat. Where Miller is mistaken is to claim that both L and its contradictory are "liars", and similarly for U. Bradwardine has shown that liars are implicitly contradictory (and so false), and the contradictory of a contradiction is a tautology. So ¬L and ¬U are tautologies. Here, and in the proper statement of Tarski's material adequacy condition, and in the proper account of logical consequence, particular care is needed. According to Bradwardine's theory of signification, a sentence signifies many things. Everything a sentence signifies must obtain for it to be true. That is, a sentence is true if and only if things are wholly as it says they are. So the right-hand side of the T-scheme (the material adequacy condition) must spell out fully what is required for truth. Similarly, in the statement of logical consequence, of t by s, everything t signifies must follow from something s signifies. Once these principles are properly stated, the confusions which lead to paradox are removed.

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[4](Tarski, 1956a, pp. 187-8). Cf. Read (2007, p. 259). Where I have put 'in the theory', Tarski actually wrote 'into the metalanguage'.

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