

Modality in Medieval Philosophy

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The Metaphysics of Modality

The synchronic conception of possibility and necessity as describing simultaneous alternatives is closely associated with Leibniz and the notion of possible worlds.¹ The notion seems never to have been noticed in the ancient world, but is found in Arabic discussions, notably in al-Ghazali in the late eleventh century, and in the Latin west from the twelfth century onwards, elaborated in particular by Scotus. In Aristotle, the prevailing conception was what (Hintikka 1973: 103) called the “statistical model of modality”: what is possible is what sometimes happens.² So what never occurs is impossible and what always occurs is necessary. Aristotle expresses this temporal idea of the possible in a famous passage in *De Caelo* I 12 (283b15 ff.), and in *Metaphysics* Θ4 (1047b4-5):

It is evident that it cannot be true to say that this is possible but nevertheless will not be. (Aristotle 2006: 5)

However, Porphyry, in his *Introduction (Isagoge)* to Aristotle’s logic (which he took to be Plato’s), included the notion of inseparable accidents, giving the example of the blackness of crows and Ethiopians.³ Neither, he says, is necessarily black, since we can conceive of a white crow, even if all crows are actually black. Whether this idea is true to Aristotle is uncertain.⁴ Nonetheless, Aristotle seems to have held that once a possibility is

realized any alternatives disappear. So past and present are fixed and necessary, and what never happens is impossible. If the future alone is open—that is, unfixed and possible—then once there is no more future, nothing is possible. Hintikka (1973: 156) quotes Aristotle’s observation in the famous 9th chapter of *De Interpretatione* (18b12-15):

But if it was always true to say of what now is that it was so, or would be so, it could not not be so, or not be going to be so. But if something cannot not happen ... it is necessary for it to happen. (Aristotle 1963: 50-51)

Whether or not this is a fallacy, for Hintikka it serves to explain Aristotle’s endorsement of the statistical model.⁵ The idea is that the instantiation of contrary potentialities cannot be simultaneous but must be successive. Hence if one potency is instantiated forever, the other can never be. Whatever is possible (or potential) must be realized at some time. One might worry that talk of something existing (or having existed) forever would be anathema to Aristotle, who famously rejected the completed infinite, but this is how he talks in *De Caelo* I 12 (281b25): “Hence everything which exists forever is absolutely indestructible” (Aristotle 1939: 115).

Knuuttila (2016) suggests that the conception of a simultaneous alternative present started to take hold in the Latin west from the twelfth century, under the influence of Augustine’s views on divine power and freedom. Augustine and Boethius were the main bridges from the ancient world, which came to an end in Europe in the sixth century, to the revival of learning in the twelfth. Augustine, writing in the fourth and early fifth century, largely uninfluenced by Aristotelianism. Boethius set out to translate Aristotle’s works into Latin (the Roman intellectual world being till then for the most part

linguistically Greek) and comment on it—a project cut short after his barely completing it for Aristotle’s logic alone when put to death in 524 CE. Knuuttila (2008: 507-31, esp. 517 ff.) sees the idea of synchronic alternatives to the present latent in both Augustine and Boethius, ideas which only became explicitly recognized in theological discussion in the twelfth century. Augustine, for example, claimed that some possibilities open to God are never realized, using the slogan “He could but he did not wish it”.⁶ Boethius, however, in his second commentary (190, 5-12) on Aristotle’s discussion of future contingents in *De Interpretatione* ch.9, recognizes only the simultaneous possibilities of the open future:

Things in the past and the present do indeed have a definite and established outcome; for those that have come about cannot not have come about; and as regards those that are happening now, it cannot happen that while they are happening they are not happening. As regards those that have to do with the future and are contingent, however, something can both happen and not happen.

(Boethius 1998: 148)

Knuuttila (2008: 517) concedes, “Boethius did not develop a theory of simultaneous synchronic possibilities which remain intact even when diachronic possibilities have vanished”. He claims that it was not fully recognized in the Latin west until Scotus spelled it out in the early fourteenth century. Scotus wrote:

I do not call something contingent because it is not always or necessarily the case, but because the opposite of it could be actual at the very moment when it occurs.⁷

In the Arabic world, however, the concept of synchronic alternatives was recognized in the eleventh century, if not earlier. Kukkonen (2000) finds it in al-Ghazali's discussion of the eternity of the world in his *The Incoherence of the Philosophers*. The clash between Aristotle's arguments for the eternity of the world and religious doctrines of the creation arose in both the Islamic and the Christian worlds. But they surfaced earlier in the Islamic tradition since their program of translation of Aristotle's works occurred in the ninth century, whereas Boethius' similar exercise was not resumed in the Latin west until the twelfth, only completed in the thirteenth. Al-Ghazali's claim of "incoherence" (*tahafut*, stumble or bankruptcy⁸) was directed specifically at Avicenna, who had repeated and elaborated Aristotle's arguments in a number of works written in the early eleventh century. In 1095, al-Ghazali set out to show the error in this reasoning. Take the third proof that al-Ghazali discusses (cited in Averroes 1952: 57): suppose the world were created at some instant. Then it was possible that it be created, and had always been possible. So it was possible that it was created earlier. Indeed, for any time at which it might have been created, it might have been created earlier. But then we have an actual infinity of earlier possibilities, which is impossible. Hence it must have existed forever, and so is eternal.

The argument goes back, once again, to Aristotle's *De Caelo* I 12 (283a11-18):

Why generated now, when for an infinite time [the world] had not existed? If there is no reason at all, and the possible points of time are infinite in number, then clearly there existed for an infinite time something susceptible to generation ... If then we suppose its powers to be realized, both opposites will be present to it simultaneously. (Aristotle 1939: 123)

The nub of the argument is that if the world was created at some time, it could have been created earlier (given God's free choice), but possibility is actuality at some time, so it must have been created earlier. Contradiction. Thus, it has existed forever. The argument depends crucially on the temporal or statistical interpretation of possibility, that there is a single series of possibilities. Al-Ghazali sensed this, and his rebuttal turns on rejecting this conception and envisaging alternative synchronous possibilities. Just as the world could have been larger than it is, it could have been created at a different time. These possibilities do not aggregate into a single eternal past, but represent separate options between which God can choose. Al-Ghazali preserves the revealed truth of creation by rethinking the nature of counterfactual possibility.

Returning to Scotus, this synchronous conception of alternative possibilities is what Knuuttila (1993: 143) claims justifies the title "possible world semantics". His argument turns on Scotus' talk of compossibility and impossibility. "According to Scotus, that is logically possible the formulation of which does not contain a contradiction, and possibilities are partitioned into groups of possible states of affairs on the basis of relations of compossibility" (Knuuttila 1993: 138). These possibilities are objective and independent: God chooses between them rather than creating them from nothing:

This logical possibility could remain separately in power by its own nature even if there were, *per impossibile*, no omnipotence to which it would be an object.

(Scotus 1963 dist. 36: 296)

Wyatt (2000) contests this attribution to Scotus of the recognition of a domain of possible worlds constituting a semantics against which to interpret modal sentences.

Nonetheless, she concedes (2000: 210) that Scotus maintains that not only the future is open: so too is the present. Aristotle's claim that "Everything which is, when it is, is necessary" (*De Interpretatione* ch.9, 19a23) is ambiguous. Aristotle commits a fallacy of composition and division, says Scotus (1963 dist. 39: 419), in claiming that the present is fixed and necessary. This fallacy was, ironically, one that Aristotle himself described in *De Sophisticis Elenchis*.⁹

"A man can walk while sitting and can write while not writing". For the meaning is not the same if one divides the words and if one composes them in saying "It is possible to walk-while-writing". (Aristotle 1928: 166a23)

Aristotle's statement in *De Interpretatione* ch.9 is similarly ambiguous. The fallacy is to move from the necessity of the inference (*necessitas consequentiae*) to that of the consequent (*necessitas consequentis*). Necessarily, if it is, then it is; but that does not warrant the inference that if it is, then it necessarily is:

So no true sense of this proposition ["Everything which is, when it is, is necessary"] means that being something—at the moment when it is—is necessary, but only that it is necessary relative to when it is, since at the moment when it is, it is still simply contingent, and consequently at that moment its opposite could still be. (Scotus 1963 dist. 39: 423¹⁰)

So although I cannot be actually doing anything inconsistent with what I am doing, I could be.

Scotus considers a further argument for the necessity of the present which appeals to a particular rule in obligational disputations—a genre peculiar to the late middle ages,

developed throughout the thirteenth century and prevalent in numerous philosophical works in the fourteenth. An obligational disputation starts with a *casus*, a hypothetical situation, and a particular proposition, the *positum*, usually false in the given *casus*. The opponent puts a series of propositions to the respondent, who must respond by granting or denying them respectively, if they are relevant in either following from or being inconsistent with the *positum* and other propositions previously granted (or the negations of those denied); if not relevant, granting and denying them if they are known to hold or not hold in the *casus*, while doubting them if their truth is unknown.¹¹ The argument Scotus considers appeals to the rule that if the *positum* is false, it must be denied that now is the present moment (since the respondent must consistently maintain the truth of the *positum*).¹² Scotus' rejection of the rule again shows that he rejects the necessity of the present:

The proof is not valid, for although what is posited should be maintained as true, yet it can be maintained for that instant without denying that the instant is one for which it is false, because the inference does not hold, "It is false for the instant, therefore it is impossible". (Scotus 1963: 423)

Knuuttila (1993: 150) argues that the series of propositions granted by the respondent starts to describe a possible world. However, this series of responses is often brief, hardly answering to Wyatt's demand (2000: 212) that reason be given to suppose that Scotus envisages anything like a complete description of how things are—i.e., a possible world. Rather, the *casus* which is nearly always present in an obligational disputation makes a better argument for Knuuttila's claim. The *casus* is envisaged as an entire situation, alternative to how things actually are, since the *positum* is often in fact true but taken to

be false in the *casus*. The anonymous author of the *Ars Obligatoria* in (Kretzmann and Stump 1985), in giving three reasons why “*casūs* are to be posited”, observes that

the third reason is so that we may admit something false [but] possible while finding out what follows in order that we may know what to do and how to respond when things are in fact as the false *casus* indicates. (Kretzmann and Stump 1985: 251-2)

The false *casus* makes a much better candidate for an alternative possible world, representing all that the respondent knows to be the case on some false assumption, and used as it is to evaluate the truth of irrelevant propositions proposed by the opponent.

Even if Wyatt is right to question whether Scotus’ observations amount to a “detailed semantic theory” (Knuuttila 2008), George Hughes (1985: 97) found it implicit in Buridan’s system of modal logic, developed in the 1330s:

I want to suggest here ... that we might understand what [Buridan] says in terms of modern “possible world semantics”. Possible world theorists are quite accustomed to talking about possible worlds in which there are more horses than there are in the actual world. And then, if Buridan assures us that by “Every horse can sleep” he means “Everything that is or can be a horse can sleep”, we could understand this to mean that for everything that is a horse in any possible world, there is a (perhaps other) possible world in which it is asleep. It seems to me, in fact, that in his modal logic he is implicitly working with a kind of possible worlds semantics throughout.

The crucial point is that nothing that is not a horse could be a horse. “Horse” is a substance term. The only possible and non-actual horses are pure possibilities.

Hughes concluded his article with an invitation “to try to give a Kripke-style possible worlds semantics for Buridan’s modal system” (108). That has recently been done in (Johnston 2015). But, as Hodges and Johnston (2017: 20) say, to infer from the fact that this semantics fits his logical system that it correctly reflects Buridan’s intentions is dangerous. Rather, what that fact can do is add weight to what Hughes inferred from Buridan’s own metaphysical views.

The Logic of Modality

Buridan was one of the few logicians in the Middle Ages who developed his own system of modal logic, distinct from and independent of Aristotle’s system. Most others felt constrained to follow Aristotle’s lead, even if they gave it their own semantic basis, and struggled in the attempt. Indeed, Buridan’s system was so successful that it dominated discussion of modal logic for the next two centuries, replacing Aristotle’s hegemony.¹³ Another major medieval logician who developed his own system was Peter Abelard, writing in the early twelfth century, before Aristotle’s treatise *Prior Analytics* was recovered in the Latin west through Boethius’ translation. Thus Abelard could develop his ideas independently of Aristotle, though many of these were eclipsed when Aristotle’s account became known. A third major thinker who developed a modal system independent of Aristotle’s was Avicenna, writing in what is now Iran, on the edge of the Islamic world, in the early eleventh century, though he was well aware of Aristotle’s modal logic, and of its shortcomings.

Aristotle's modal system is described in chs.8-22 of *Prior Analytics*, chapters obviously inserted later, presumably by Aristotle himself, into an existing work: the start of ch.23 clearly follows straight on from the end of ch.7 (and ch.3 is a later insert, discussing modal conversions). Aristotle takes his theory of non-modal (so-called "assertoric" or "categorical") syllogisms and considers which syllogistic moods remain valid when one or more premises and the conclusion are modified by the modes "necessary" and "possible". A complication is that he distinguishes two senses of "possible", so-called "one-way" and "two-way" possibility, what we now call respectively "possible" (the dual of "necessary", that is, what is not necessarily not so) and "contingent" (what is neither necessary nor impossible). Moreover, he seems to allow "possible" in the sense of "contingent" to modify either the premises or the conclusion (or both), but only considers "possible" as the dual of "necessary" to modify the conclusion, not the premises.

In *Prior Analytics* ch.8, Aristotle observes that if the premises of any assertoric, two-premise syllogism are modified by "necessary", then a conclusion modified by "necessary" may be validly inferred in all cases where the corresponding assertoric conclusion follows from the unmodified premises. In chs.9-11, he considers mixed premises, one of necessity, one assertoric, in each figure, summarizing the results in ch.12. In ch.13 he discusses possibility, noting the validity of complementary conversion, namely, that any affirmative statement modified by "contingent" is equivalent to the corresponding negative statement so modified; e.g., "Every *A* is contingently *B*" converts to "No *A* is contingently *B*", and "Some *A* is contingently *B*" converts to "Not every *A* is

contingently *B*”.¹⁴ In subsequent chapters he considers what follows modally or categorically from mixtures of premises of necessity, of contingency, and assertoric.

Even Aristotle’s immediate successors in the Lyceum, e.g., Theophrastus, head of the school after Aristotle’s death, found Aristotle’s account of the modal syllogism problematic in ways that his non-modal system was not. Was there an understanding of the modal terms which supported the claims of validity he had made, or should those claims be revised? There seemed to be many inconsistencies, but the most immediate concerned his claim in ch.9 that a conclusion of necessity follows in the first figure from mixed premises, one of necessity, the other assertoric, if the premise modified by “necessary” is the major premise (that is, the premise containing the predicate of the conclusion), but not if it is the minor premise. Although the point applies to all four valid first-figure moods, the problem has come to be called the “problem of the two Barbaras”, that is, inferring a universal affirmative of necessity from modally mixed universal affirmatives. Theophrastus went so far as to deny any such validity, advocating the so-called “ad peiorem” rule, that the conclusion can never be modally stronger than the weaker premise.¹⁵

Much ink was spilled over the next two thousand years trying to make sense of this,¹⁶ but the boldest thinkers broke free of Aristotle’s yoke and developed a modal system *ab initio*. First, we need to clarify what is covered by “mode” in “modal logic”. Aristotle, as we saw, deals only with the alethic modes “necessary”, “possible” and “contingent”. There are also temporal modes, “past”, “present” and “future”, and the epistemic modes, such as “know”, “believe”, “doubt”. Buridan considers the epistemic modes in Q40 of the

first book of his *Questions on Aristotle's Prior Analytics*. He starts by distinguishing compound from divided uses of modals:¹⁷

I call “compound” ones like “I know that a man is seated”, “That a man is a stone is believed, or apparent” and so on; but I call “divided” those like “A man I know to be an animal”, “An ass appears to me to be a man”, “God is believed to be triune” and so on. (Buridan n.d.)

Only trivial inferences follow with compound modals, he says: for one can know (or believe, or doubt) the premises of a (valid) syllogism without knowing (etc.) the conclusion. But since knowledge entails truth, if one knows the premises, the conclusion will certainly be true. Moreover:

If you have arranged the premises into a syllogism, and you have considered the conclusion, and you know the syllogism is a necessary consequence, it follows that you will immediately know the conclusion. (*loc.cit.*)

Turning to the divided sense, Buridan claims that any valid first- or third-figure syllogism remains valid if the major premise and conclusion are modally qualified in the same way, and the minor is left assertoric:

A modal conclusion follows by the *dictum de omni et nullo*, e.g., “Every *B* is supposed to be *A*, every *C* is *B*, so every *C* is supposed to be *A*. (*loc.cit.*)

Recall that Aristotle had made the same claim where the major and conclusion are modified by “necessary”. Buridan denies this for the alethic modal cases with universal major, for he takes universal propositions of necessity to be amplified to the possible. Again, like the distinction between compound and divided senses, the notion of

ampliation was suggested by Aristotle himself, but was developed and taken much further by the medievals. At *Prior Analytics* 32b25 ff. Aristotle had observed that

“this possibly belongs to that” may be understood in two ways—either of what that belongs to, or of what that may belong to. (Aristotle 2009: 19)

The medievals were divided on how far to extend this idea. Buridan thought the subjects of universal affirmatives and negatives both of necessity and of possibility (and of contingency) were amplified by the modal predicate to the possible. Thus “Every *A* is necessarily *B*” means that every actual or possible *A* is necessarily *B*—and since he followed Aristotle¹⁸ in accepting that the actual was *ipso facto* possible (in the sense of “not impossible”), that amounts to the claim that every possible *A* is necessarily *B*; and similarly, “No *A* is possibly *B*” (that is, “Every *A* is necessarily not *B*”) means that no possible *A* is possibly *B*. Thereby, Buridan ensured that “No *A* is possibly *B*” is the contradictory of “Some *A* is possibly *B*” and that “Every *A* is necessarily *B*” is the contradictory of “Not every *A* is necessarily *B*”.¹⁹

Others disagreed. For example, William Ockham followed Aristotle more closely:

According to Aristotle’s opinion, the argument “Every white thing is of necessity a swan, no ass is a swan, so every ass is of necessity not white” is not valid, because if the only white thing is a swan, the premises are true and the conclusion false. For “Every white thing is of necessity a swan” would be true, because each singular would be true, so it is true. And the major is definitely true, and the conclusion is manifestly false. So the argument is not valid. (Ockham 1974 III-1 32: 448)

Thus Ockham takes the truth of “Every *A* is of necessity not *B*”, that is, “No *A* is possibly *B*”, to follow from the truth of all singular propositions “This cannot be *B*”, where “this” denotes an actual *A*. This seems to have been Aristotle’s view too,²⁰ and also that of Abelard for propositions taken *de re* (Thom 2003: 53). Thom (2003: 66, 68) claims Avicenna held that not only are the subjects of modal propositions amplified to the possible, but also both subject and predicate of assertorics. However, this seems to be based on a misreading, for El-Rouayheb (2016: 75) notes:

Avicenna was understood [by his successors] to have rejected ampliation to the possible and to have upheld the view that the subject term should be understood to include only that of which it is true in actuality (past, present, or future).²¹

Avicenna was, perhaps, the most original and ambitious logician of the middle ages, not least in developing what Hodges (in preparation: ch.9) has called his two-dimensional logic, quantifying over times as well as objects. Hodges and Johnston (2017: 15) claim that the resulting logic contains a part that is equivalent to his modal logic, and also to Buridan’s logic of divided modals, and construct a possible-worlds semantics that shows this equivalence.²² This might suggest that Avicenna accepted the statistical model of modality, equating worlds with times. But Hodges and Johnston (2017: 15) cite Avicenna as denying this identification:

Being permanent is not the same as being necessary ... But it is not for the logician as logician to know the truth about this.²³

That is, the logical laws of times and worlds are the same, as shown by the model theory, but their metaphysics is different.

Williamson (2013: 45), following Movahed (2016: 34), claims that Avicenna anticipated and endorsed the Barcan formula and its converse, that “Something is possibly *F*” is equivalent to “Possibly, something is *F*”.²⁴ This would force a constant domain across worlds. Movahed cites Avicenna as saying:

But to say that *some people are possibly not writers* is modally the same as saying that *possibly some people are not writers*, and although one implies the other the meaning of the one may be opposite to the other. (*al-Ibara*, 116)

At first glance, this is puzzling: Avicenna seems to be saying of the two expressions that they are at the same time equivalent and opposite and that one follows from the other. But Bäck’s translation makes things a little clearer:

As for our saying (that) some man can not-be a scribe [literally, some man it is ~~not~~ possible that he is not a scribe—footnote] it may be modally equivalent to our saying (that) it is possible that some man is not a scribe, while it might differ from it even though it is a consequence of it, so that the intention in one of them is that some man is being characterized by the possibility of the denial of scribehood of him, and in the other that it is possible for the speaker’s saying (that) some man is a scribe to hold. (Avicenna 2013: 142)

What Avicenna is saying is that “Some man is possibly not a writer” (or scribe) is ambiguous, having two different senses, the composite or *de dicto* (“It is possible that some man is not a writer”) and the divided reading, *de re* (“Some man can fail to be a writer”), the former entailing the latter. This is exactly what Ruth Barcan (1961: 307) intended by her eponymous formula, suggesting that Quine’s *bête noire*, the third grade

of modal involvement (Quine 1953: 66) was in fact a consequence of the second, acceptable if construed as equivalent to the first. So Avicenna does endorse the Barcan formula, though the passage does not endorse its converse, indeed, it says that the two are different and not equivalent.

The Barcan claim is orthogonal to the question of the statistical interpretation, though one might suspect them to be in tension: its being possible, for example, that someone fly to Mars, even if realized, might not be realized by anyone now alive. But whether Avicenna anticipated the idea of possible worlds, as well as the Barcan formula, is an open question. To paraphrase Williamson (2013: 45):

We leave to [the future] the hard work of making sense of this tantalizing glimpse in its historical and systematic context.

[5461 words]

Notes

¹ See, e.g., von Wright 1984: 74, 96.

² Hintikka took the expression from Bekker 1952: 16, for whom possibilities are cases (Fälle), like rows in a truth-table. Cf. von Wright 1984: 73 n.1.

³ Porphyry 2003: 12 (§5).

⁴ See, e.g., van Rijen 1989: ch.4, and Aristotle's *Posterior Analytics* I 6 and *Rhetoric* III 17, 1418a3-5.

⁵ See also Knuuttila 1993: 9-12.

⁶ "Potuit sed noluit" (Knuuttila 1993: 69; 2008: 518).

⁷ Scotus, *De Primi Principio* IV.4, cited in Knuuttila 1993: 143 and 2008: 550.

⁸ See Adamson 2015: episode 144; Averroes 1952: xiii.

⁹ But it is a fallacy many have attributed to Aristotle in connection with his modal logic—see below.

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- ¹⁰ See also Scotus 1994: 132-37.
- ¹¹ For more on the game of obligations, see Dutilh Novaes and Uckelman 2016.
- ¹² The rule is found in, e.g., William of Sherwood's *Obligations* (Green 1963 vol.2: 8; cf. vol.1: 62)—though the authorship is in doubt—and in Walter Burleigh's *Obligations* (Green 1963 vol.2: 59) of 1302: “When a false contingent proposition concerning the present has been posited, one must deny that it is the present instant” (Kretzmann and Stump 1988: 394).
- ¹³ See Knuuttila 2016.
- ¹⁴ See, e.g., Buridan (2015: 48, 104-5).
- ¹⁵ See, e.g., Alexander of Aphrodisias (2013, 124,3-17: 59 and 118-19).
- ¹⁶ And still is: see, e.g., Malink 2013.
- ¹⁷ One case of the compounded/divided distinction is the *de dicto/de re* distinction. The terminology is medieval: see Knuuttila (2008: 533-4) and Lambert of Auxerre (2015: 353 n.99).
- ¹⁸ *Prior Analytics* I 13.
- ¹⁹ See Buridan (2001: 44-45), Buridan (2005: 100)
- ²⁰ See *Prior Analytics* I 10, 30b20, and Malink 2013: 275.
- ²¹ See also the texts from Avicenna cited in Hodges and Johnston (2017: 25-31).
- ²² This proof is in Hodges (in preparation: §12.3).
- ²³ They refer to *Qiyas* 48.14-16.
- ²⁴ Buridan, being committed, as we saw, to pure possibilita, rejected the Barcan formula. See Buridan (2015: 35-36).

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