

MONISM: THE ONE TRUE LOGIC

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Abstract

Logical pluralism is the claim that different accounts of validity can be equally correct. Beall and Restall have recently defended this position. Validity is a matter of truth-preservation over cases, they say: the conclusion should be true in every case in which the premises are true. Each logic specifies a class of cases, but differs over which cases should be considered. I show that this account of logic is incoherent. Validity indeed is truth-preservation, provided this is properly understood. Once understood, there is one true logic, relevance logic. The source of Beall and Restall's error is a recent habit of using a classical metalanguage to analyse non-classical logics generally, including relevance logic.

1 Logical Pluralism

J.C. Beall and Greg Restall have recently defended a position they call “logical pluralism”, that “there is more than one sense in which arguments may be *deductively valid*, that these senses are equally good, and equally deserving of the name *deductive validity*” ([4] § 1). Their argument for logical pluralism is this:

1. the meaning of the term ‘valid’ is given by (V):

(V) A conclusion, A , *follows from* premises, Σ , if and only if any case in which each premise in Σ is true is also a case in which A is true.

2. A logic specifies the cases which are mentioned in (V)
3. There are at least two different such specifications. ([3] pp. 476-7)

In fact, they describe three such specifications, namely, worlds (yielding classical logic), constructions (yielding constructive logic) and situations (yielding relevant logic). (*op.cit.*) Thus there are at least three logics, all equally good. All three tell us when truth is preserved, as (V) shows: classical logic tells us when logic is preserved in complete and consistent situations, that is, worlds; constructive logic tells us when truth is preserved in possibly incomplete (better, indeterminate or undecidable) situations, that is, constructions; and relevance logic tells us when truth is preserved in possibly inconsistent (and incomplete) situations. Indeed, Beall and Restall later introduce us to a fourth possibility, truth-preservation in all situations

(possibly incomplete, inconsistent and indeterminate), which they rather confusingly also call “relevant consequence” ([4] § 4 fn. 17).

The position described here as logical pluralism is in fact incoherent. To see this, we need to look more closely at (V).

2 Priest’s Challenge

Graham Priest challenges Beall and Restall as follows: suppose there really are two equally good accounts of deductive validity, K_1 and K_2 , that β follows from α according to K_1 but not K_2 , and we know that α is true. Is β true? ([13]. Cf. [4] § 6.) Does the truth of β follow (deductively) from the information presented? Beall and Restall do not mean that β is true according to K_1 but not true according to K_2 . K_1 and K_2 are accounts of validity, not of truth. As Priest notes, Beall and Restall deny that they are relativists about truth. So the question, ‘Is β true?’ is a determinate one. It follows K_1 -ly that β is true, but not K_2 -ly. Should we, or should we not conclude that β is true? The answer seems clear: K_1 trumps K_2 . After all, K_2 does not tell us that β is false; it simply fails to tell us whether it is true. The information in the case is insufficient to determine, according to K_2 , whether β is true. But according to K_1 , the information supplied does tell us that β is true. So if K_1 and K_2 are both good accounts of derivability, K_1 tells us what we want to know: β is true.

It follows that in a very real sense, K_1 and K_2 are not equally good. K_1 answers a crucial question which K_2 does not. For Priest’s question is the central question of logic. As Beall and Restall say, “the chief aim of logic is to account for [logical] consequence,” that is, to tell us when “a conclusion A . . . logically follow[s] from premises Σ .” ([3] pp. 475-6) In none of Beall and Restall’s examples do logics seriously disagree, that is, does one logic say that A follows from Σ and the other that $\ulcorner \sim A \urcorner$ follows (unless, of course, Σ is inconsistent and they both say that both follow). And their pluralism is not unbounded. Although they admit classical, constructive and relevant accounts of validity to be equally good, they do not countenance any and every account of consequence to be logic. ([3] p. 487 fn. 26) (V) builds in reflexivity and transitivity of consequence, since clearly inclusion (of Σ -ways in A -ways) is reflexive and transitive. So, they say, any system, such as Aristotle’s system of syllogisms, which rejects reflexivity ([2] 24b18-20), or Tennant’s ([26] ch. 17) or Smiley’s ([25]), which reject transitivity of consequence, is simply not a system of logic.

Beall and Restall’s actual response to Priest’s challenge is to say that we are entitled to infer β from α according to K_1 , but not according to K_2 . ([4] § 6) But this is no answer. That simply repeats the description of the case. Suppose K_1 is classical logic and K_2 is relevance logic (as Beall and Restall do). We are given that the inference from α to β is classically valid and not relevantly valid. We are also told that α is true. Does this information tell us whether β is true? Apparently so, for classical validity is validity: “classical logic *is* logic . . . If the premises of a classically valid argument are

true, so is the conclusion.” ([3] p. 490) So β is true, and not relatively true, but true *simpliciter*. The fact that β does not follow relevantly from α is irrelevant. Classical logic dominates, and β is true.

Two puzzles arise from this. First, relevance logic actually says more than that β does not follow relevantly from α . It says that one is not entitled to infer β from α . Relevance logic is an account of consequence. Beall and Restall describe this as saying that one is not relevantly entitled to infer β from α , whereas one is classically entitled to do so. But that classical entitlement, we saw, allowed us to infer β from α . So, given that α is true (and that β follows classically from α), we can infer that β is true—and not just classically infer it. If α is true then β is true. By their account, classical validity (or whatever is the stronger validity, K_1) dominates. This makes a mockery of relevance considerations. Relevance logic was not put forward as a mere alternative to classical logic. Ackermann, for example, believed that strict implication, which expresses classical validity, was wrong: “Thus one would reject the validity of the formula $A \rightarrow (B \rightarrow A)$.” ([1] p. 113) So too for intuitionistic reasoning. Brouwer wrote:

“An a priori character was so consistently ascribed to the laws of theoretical logic that until recently these laws, including the principle of excluded middle, were applied without reservation even in the mathematics of infinite systems and we did not allow ourselves to be disturbed by the consideration that the results obtained in this way are in general not open, either practically or theoretically, to any empirical corroboration. On this basis extensive incorrect theories were constructed.” ([6] p. 336)

Can a relevance logician, or an intuitionist maintain, in the face of Beall and Restall’s pluralism, that one should not infer that β is true? We will return to this question in § 3.

Secondly, Beall and Restall offer a hostage to fortune here. Although none of their supposedly equally good logics disagrees in inferring contradictory statements from the same (consistent) premises, this would appear to be a possibility. Classical logic, K_1 , dominates K_2 , so does not disagree with it. Suppose it disagrees with K_3 , in that while β follows K_1 -ly from α , $\lceil \sim\beta \rceil$ follows K_3 -ly from α , while α is consistent—that is, there is some world, indeed this one, in which α is true. Should we infer that β is true, or that $\lceil \sim\beta \rceil$ is true? We have seen that, according to Beall and Restall’s pluralism, classically valid (that is, K_1 -valid) arguments are valid. So β is true. But if K_3 -valid arguments are also valid, β is false. Unless Beall and Restall accept the truth of a contradiction, they must find some reason for rejecting K_3 as not logic, like the syllogism and non-transitive systems. Such reasons had better not be *ad hoc*. One good reason (or at least, not *ad hoc*) would be if K_3 did not admit a semantics of cases, and so did not fit their guiding principle (V). But that needs argument.

An example is given by Abelian logic [11], whose characteristic axiom is $((A \rightarrow B) \rightarrow B) \rightarrow A$. This is not a classical tautology, but Abelian logic is consistent (and Post-complete), lacking certain classical validities in

compensation. Hence in classical logic,

$$\sim A, B \vdash \sim((A \rightarrow B) \rightarrow B) \rightarrow A,$$

that is, A false and B true is a counterexample. But in Abelian logic,

$$\sim A, B \vdash ((A \rightarrow B) \rightarrow B) \rightarrow A,$$

since the conclusion is (Abelianly) logically true. Suppose now that we discover that A is false and B is true. Should we infer that $((A \rightarrow B) \rightarrow B) \rightarrow A$ is true, or false? Classical and Abelian logic give conflicting answers. Here pluralism meets its limit.

Beall and Restall might try to dismiss Abelian logic on the grounds that it does not admit a semantics of cases, and so does not fall under (V). But one should note Routley's proof [24] that every logic admits a two-valued worlds semantics. If he is right, every logic falls under (V). Thus Abelian logic really is a counterexample to Beall and Restall's pluralism.

Beall and Restall's logical pluralism tries to be eclectic and all-embracing (up to a point which excludes Aristotle, Smiley and Tennant), but it falls down on two counts: first, it does not respect the core motivation of the non-classical logics, which first prompted them as rivals to the classical orthodoxy; and it threatens to plunge into inconsistency, if explicitly incompatible logics both turn out to accord with the governing principle, (V).

Let us turn to examine (V) more closely.

3 Truth-Preservation

Beall and Restall describe (V) as a principle of truth-preservation. It states that validity requires truth to be preserved in all cases. Different specification of the cases then yields different logics consonant with (V). Any system not consonant with (V) is not logic; and any system consonant with (V) is equally good as a logic.

This is puzzling, for as Beall and Restall point out, there are, for example, "too many modal logics to hold each of them as the logic of broad metaphysical necessity" ([3] p. 489). What is required, they say, is to specify what a logic is meant to do, and then there is scope for disagreement. If we want to capture metaphysical necessity, one modal logic is the right logic. Perhaps if we want to capture moral obligation, a different modal logic will better capture the interpretation we want for the operators, and so too for formalising the logic of knowledge and the logic of provability.

But this difference of logic is orthogonal to Beall and Restall's thesis of logical pluralism, as Restall observes ([22] p. 431). The former is Carnapian tolerance, which tolerates logical disagreement as due to difference of language. There is no real disagreement, and nothing the logical monist might object to. Clearly, if $\ulcorner \Box p \urcorner$ expresses ' p is obligatory', we reject $\Box p \vdash p$, for unfortunately not everyone fulfils their obligations. Again, if $\ulcorner \Box p \urcorner$ expresses ' p is provable', we reject $\Box p \vdash p$, since not all systems of proof are consistent. But if $\ulcorner \Box p \urcorner$ expresses logical necessity, we insist on $\Box p \vdash p$, for what is

necessary must happen. (As a Gifford Lecturer at St Andrews once put it, referring to personal experience: “if you can’t breathe, you don’t.”) These alternative logics are supplementary logics, in Haack’s sense ([9], p. 2), and do not illustrate any real sense in which one and the same inference can be both valid (according to one logic) and invalid (according to another).

The same point applies to another example which Beall and Restall mention, the distinction between formal and material consequence. Take their example, ‘ a is red, so a is coloured’. There is nothing here for a logical monist to jib at. Every instance of a formally valid argument is valid. But not every instance of a formally invalid argument is invalid. Formally invalid arguments can have valid instances, some of which will be formally valid in virtue of instantiating a different valid form, but others valid not in virtue of form at all. (V) allows validity to arise from many causes, and does not distinguish formal validity from other sorts of validity.

Again, the distinction between first-order and higher-order validity need not disturb a logical monist. Many valid arguments are first-order valid; some are not. Some of the latter are second-order valid, but others are not. To repeat, every instance of a valid form is valid; but invalid forms can have valid instances. Allowing higher-order expressive power, and increasing the range of logical constants (e.g., to include modal and bimodal, i.e., temporal, connectives) both increase the range of formal validity. But these are all part of the one canon of validity for the monist. As (V) puts it: if any case in which the premises are true is one in which the conclusion is true, the argument is valid, and vice versa.

Beall and Restall believe that (V) covers relevant consequence, as well as other logics. This is, however, tendentious. Relevant consequence is paraconsistent, in rejecting the inference from a contradiction to any proposition whatever: formally, $\lceil A \ \& \ \sim A \rceil$ does not imply arbitrary B . (Let us call this Ex Falso Quodlibet, EFQ for short.) Beall and Restall distinguish three types of paraconsistent logician ([4] § 2): first, there is the regular dialetheist, who believes there are true contradictions—the actual world is inconsistent, as shown, for example, by the paradoxes. One man’s *modus tollens* is another’s *modus ponens*, so the fact that, say, Naive Set Theory leads to contradiction does not refute Naive Set Theory, but gives the regular dialetheist reason to believe that the ensuing contradictions are true. The light dialetheist is more cautious: the actual world might be inconsistent, but the jury is still out on whether that is the right conclusion to draw from the paradoxes. Both types of dialetheist are paraconsistentists, since even if some contradictions might be true, not every proposition could be true.

In contrast, non-dialetheic paraconsistentists, so-called, do not think contradictions could be true. Beall and Restall describe them as concerned with ways the world could not be—with impossible worlds. (See [4] §§1-2) For EFQ to be invalid, according to (V), there must be cases where $\lceil A \ \& \ \sim A \rceil$ is true and B false. But $\lceil A \ \& \ \sim A \rceil$ cannot be true—any case where $\lceil A \ \& \ \sim A \rceil$ is true is impossible. So the non-dialetheic paraconsistentist seems committed to saying that there are ways the world could not be, and that such cases must be considered in deciding on the validity of an

argument. This is an incoherent position, for if such cases cannot arise, it is hard to see why they need to be considered.

The dialethic paraconsistentist is not in such a bind. For the dialetheist, the actual case is, or at least could be, inconsistent. So there is a real possibility that the premise of EFQ is true, and no guarantee that if it is, the conclusion is true too. So (V) shows that EFQ is invalid.

But if one does not think that $\lceil A \ \& \ \sim A \rceil$ could be true, how can EFQ fail to conform to (V)? Beall and Restall dub this the “Peircean objection” ([4] § 2). There cannot be a case in which $\lceil A \ \& \ \sim A \rceil$ is true and B false, for cases in which $\lceil A \ \& \ \sim A \rceil$ is true are impossible. One cannot be led astray by EFQ, whereby a case where $\lceil A \ \& \ \sim A \rceil$ was true would transform itself into one where everything was true, for there can no more be a case where $\lceil A \ \& \ \sim A \rceil$ is true than there can be a case where arbitrary B is true—such cases are impossible.

Beall and Restall’s response to the Peircean objection is to claim that there is more than one way of being led astray—that is, that “even if the whole purpose of Logic is to avoid being led astray, there seems to be more than one logic that may arise given this purpose.” ([4] § 2) That is, there is more than one way of going astray. Relevant validity, endorsed by the non-dialethic paraconsistentist, fleshes out this purpose in one way; constructive and classical validity in yet others. Proponents of the latter pair are safe: they endorse EFQ, and do not think there is any case where $\lceil A \ \& \ \sim A \rceil$ is true and B false. The first of the three is in a tighter corner: by Beall and Restall’s lights (but see § 4 below), the non-dialethic paraconsistentist thinks there are such cases, but they are impossible. Consequently, Beall and Restall ascribe to the non-dialethic paraconsistent the thought: “One would be led astray if one’s conclusion didn’t conform to the canons of relevance. Better put: One would be led astray if one’s conclusion failed to *follow relevantly* from one’s premises.” ([4] § 2)

This is to abandon truth-preservation as the criterion of validity. What is now required of validity is not just preservation of truth, but preservation of relevance, too. (V) may look like a statement of truth-preservation as the criterion of validity, but interpreted as Beall and Restall take it, it is not. For an impossible case is not a case in which anything can be true. If it is a case at all, it is a case in which it is impossible for anything to be true. Thus if the non-dialethic paraconsistentist is committed to interpreting (V) as ranging over impossible cases, (V) describes at best case-preservation, not truth-preservation.

Moreover, to attribute to the (non-dialethic) relevantist the view that validity requires more than just preservation of truth, namely, preservation of relevance, falls prey to a variant of Priest’s objection. (I posed this question in [18] and [19].) For again, suppose the argument from α to β preserves truth, and that α is true. Should we conclude that β is true? According to (V), any case in which α is true is one in which β is true, and by hypothesis, α is true. So it seems clear that β is true. According to the non-dialethic relevantist, however, we can be led astray here. How? By failing to keep to the “canons of relevance”, we are told. Yet what is

the sanction of violating these canons? Not that β is not true. For β is true in every case in which α is true, and α is true. What, however, of the impossible cases? In these, α could be true and β not. But these cases are impossible. So the only cases in which β is not true are impossible. So β is true. Apparently, however, according to Beall and Restall’s non-dialetheic relevantist, we should not infer that β is true. That is absurd.

4 Classical Semantics

How have Beall and Restall argued themselves into this absurd position? The answer is that they have misunderstood Meyer’s Sermon to the Gentiles ([10]). Semantics is invariably carried out in a classical metalanguage. Modal logic for years—decades—had no semantics, and felt inferior for that reason. Kripke eventually supplied a semantics, developed in a non-modal, extensional metalanguage. Intuitionistic logic had no semantics, at least, no formal semantics, and some dismissed it for that reason. Without a semantics, one could not understand what justified intuitionistic methods. Beth and Kripke provided a semantics, framed in a classical metatheory. Relevance logic lacked a semantics through its first decade, and suffered the same criticism. Meyer, Routley and others provided the semantics in the classical metatheory of their critics. As Meyer put it, they set out “to preach to the Gentiles in their own tongue.” ([10] p. 1)

There is a common assumption in all these cases, namely, that classical logic is right, or at least, right for doing semantics. It allows classical logicians to understand what modal, constructive and relevance logicians are doing. Except that it doesn’t. It provides a classical model, or classical interpretation, of modal, constructive and relevant reasoning. Modal logicians are interpreted as talking about truth-values (extensional properties) of propositions at other possible worlds (sets), rather than about modal properties of those propositions. Intuitionist logicians are interpreted as talking about possible constructions in states of information, and the provability of propositions, rather than about those propositions’ (epistemically constrained) truth and falsity. Relevance logicians are interpreted as concerned with truth-preservation in arcane situations, situations which in the interesting cases—that is, the cases where their account of validity differs from that in classical logic—turn out to be impossible. To say that such cases are impossible should mean that there are no such cases, yet Beall and Restall saddle the non-dialetheic paraconsistentist with holding that there are such cases, only they are impossible. But if they are impossible, then it is impossible that there are such cases. If they are impossible, then there is no situation, however arcane, in which they hold.

This may seem a cheap point. After all, Beall and Restall write:

“These situations are . . . ‘impossible’. Not in the sense that they do not exist (one may well be a realist about these impossible situations) but in the sense that they can never be *actualised*. They are never part of any possible world.” ([4] §4)

Priest ([12]) in fact distinguishes three notions of impossible world. First, they may be (what Beall and Restall would call) “cases” where A and $\lceil \sim A \rceil$ are both true, for some A ; or they may be cases where classical logic does not hold; or they may be cases where one’s preferred logic does not hold. For the non-dialetheic paraconsistentist, of course, there can be no cases of the first kind; and no one should think there can be cases of the third. Any non-classical logician should believe there can be cases of the second kind—indeed, that the actual case is one.

But what of Beall and Restall’s distinction between whether these cases exist, and whether they can be actualised? Restall ([20]) offers us a modelling of such cases as sets of possible worlds. (Cresswell, [8] p. 42, called them “heavens”.) It is again part of the pluralist project: “we can enjoy the fruits of both paraconsistent and classical logic.” (p. 594) However, this is not realism about impossible situations; these impossibilities exist, as sets, but they are not real (as situations).

Varzi ([27]) offers us a moderate realism: just as there are ways things could be, namely, maximal consistent states of affairs, so there are ways things could not be, namely, maximal inconsistent states of affairs (p. 598). For ‘There is no way that a can be F ’ is equivalent to ‘There is a way a cannot be, namely, F ’— a “couldn’t be *that way!*” (*loc.cit.*) But of course there is a way a couldn’t be F —*every way* is a way the impossible cannot be. This book, for example, couldn’t be black and white and red all over, indeed, everything is like that. So too, every way is one in which a couldn’t be F , if a can’t be F . But that does not magically yield impossible worlds. Far from showing that impossible worlds are real, Varzi’s argument reinforces the conviction that they are unreal and that there are no such things.

Yagisawa ([28]) argues for the admission of impossible worlds within a Lewisian modal realism by a kind of Cantor-paradox argument. Consider the collection of all possible worlds—the Lewis universe. Suppose it had been different in some way—more worlds, or different accessibility relations, or whatever. Such a supposition is an impossibility. Hence, he says, the Lewisian universe is an island within a much larger realm of impossibilities. Such an argument shows the danger of conceiving of possible worlds in such a literal way as Lewis’. Talk of possible worlds is a *façon de parler*, and like all *façons de parler*, its extensionalist merits must be balanced against the risk of being misled by it. There are not really any possible worlds, and there certainly are no impossible worlds—they’re impossible. What there is, is what there is, the actual world. This world has certain actual properties (how it is) and certain modal properties (how it might be, how it must be, and how it could not be).

Hence talk of inconsistent situations (Priest’s first kind of impossible world) is a metaphorical way of talking of inconsistency, of how the actual situation cannot be. It may assist the classical logician to model counterexamples to relevantly invalid reasoning. But it should not be allowed to mislead him into supposing that the non-dialetheic paraconsistentist believes there can somehow be (unactualisable but real) impossible situations or cases.

What the classical perspective is insensitive to, is the real motivation for questioning whether, e.g., EFQ is valid, just as it is insensitive to the real nature of modality or of constructivism. The background assumption is that classical logic is one correct way of doing logic. To accommodate the constructivist and relevantist concerns, it is necessary to admit other ways of doing logic as correct. Thus is logical pluralism born. It is born out of combining a non-classical theory with a classical metatheory. If classical logic is right, how can we understand what the non-classical logician is doing? Having understood those non-classical criticisms, there must be at least two ways of doing logic.

Copeland [7] responded to Meyer and Routley’s classical semantics by dismissing it as purely technical, a mathematical method of obtaining meta-theoretical results, but of no semantic import. In particular, Copeland objected to the Routleys’ clause for negation:

$$(T^*) \quad \ulcorner \sim A \urcorner \text{ is true at } w \text{ iff } A \text{ is not true at } w^*$$

Either this has nothing to do with semantics, but enables one to manipulate the uninterpreted symbol ‘ \sim ’ in pure semantics for relevance logic; or it does explain the meaning of ‘ \sim ’, in which case, classical and relevance logic are discussing different connectives. If (T*) gives the meaning of ‘ \sim ’, then its meaning is different from the negation in classical logic and, as Prior put it, classical and relevance logicians are “simply talking past one another”. ([14] p. 75) Indeed, as Quine famously quipped, “when he tries to query the doctrine, [the deviant logician] only changes the subject”. ([15] p. 81) If Routley semantics is applied semantics, then (T*) shows that ‘ \sim ’ is not ‘not’; if the relevance logician really denies classical laws about negation, then it cannot be logic which the semantic techniques are explaining, but some other strange game—pure semantics.

Restall challenges this argument by showing how the so-called classical negation clause:

$$(T \sim) \quad \ulcorner \sim A \urcorner \text{ is true at } w \text{ iff } A \text{ is not true at } w$$

is a special case of (T*) when w is a world (i.e., consistent and complete). ([21] p. 61) For w^* is the maximal point consistent with w (relative to the ordering that $w \subseteq w'$ iff whatever is true at w is true at w'), which is just w if w is consistent: “for if xCx [i.e., x is self-compatible, i.e., consistent] then if $x \models \sim A$ we cannot have $x \models A$ ”. (*loc.cit.*) But this assumes that the metalanguage is consistent, in this case, classical. If the metalanguage matches the object-language (where we may have both $x \models A$ and $x \models \sim A$) then we may have both $x \models A$ and $x \not\models A$. If a dialetheist (about the object-language) accepts a classical (i.e., consistent) metalanguage then of course he is a pluralist—indeed, schizophrenic.

What is a non-dialetheic relevantist to make of all this? Certainly, the suggestion that both $x \models A$ and $x \not\models A$ is absurd. The non-dialetheic relevantist shares an aversion to dialetheism with the classicist. But then, as we have seen, there are no worlds in which both A and $\ulcorner \sim A \urcorner$ are true (if

‘ \sim ’ means ‘not’): such worlds would be impossible, and so there are no such worlds. Talk of “worlds” (and “truth” etc.) is just a *façon de parler*, and the semantics (so-called) is just pure semantics, as Copeland observed.

It is a mistake to describe $(T \sim)$ as the classical clause for negation. It is only classical if the interpretation of ‘not’ is classical; and it is only correct if the interpretation of ‘ \sim ’ and ‘not’ is the same. If one allows object- and metalanguage to drift apart, then a split personality and logical pluralism are just around the corner. The right response is to insist on doing one’s semantics in the logic in which one believes. If Beall and Restall insist on doing semantics classically, then they are classical logicians for whom non-classical “logics” are, if not just an intellectual amusement, then an exercise in applying logic to some more particular activity—e.g., database management (see [21] p. 69) or warrant transfer (see [23]). In contrast, if one believes that, e.g., double negation elimination, or EFQ are invalid (as constructivist and relevantist do, respectively), then one should reject the canons of classical logic even, or especially, when applied to the semantic study of one’s chosen account of validity.

This robust response is an ingredient of what I once dubbed “logic on the Scottish plan” ([17] § 7.8), in contrast to versions of the semantics of relevance logic which were familiarly known as “logic on the American plan” and “logic on the Australian plan”, which, e.g., adopts (T^*) as the clause for negation in order to work in the Gentiles’ own tongue, classical logic. Under the Scottish plan, the truth-conditions of the connectives are homophonic, as in $(T \sim)$ read properly. Adopting such clauses in a classical metatheory, relevance logic will appear incomplete: (classically) valid inferences concerning ‘not’ will not be validated by the (relevant object-)theory. But what a strange approach to take, if one believes relevance logic is the correct logic. Why use an alien logic for one’s metatheory—and if one does, why trust the result?

Articulating a relevant metatheory requires thought and reflection. In particular, one needs to consider what the relevant account of truth-preservation (validity) is. Suppose we formalise (V):

$$(V \Rightarrow) \quad \Sigma \vdash A \text{ iff } (\forall w)((\forall B \in \Sigma)w \models B \Rightarrow w \models A)$$

Beall and Restall think to obtain different accounts of ‘ \vdash ’ by varying the range of ‘ w ’—cases may be worlds, constructions or situations, for example. But the range of ‘ w ’ should be universal, and unless one is a dialetheist, impossible worlds do not fall under the range of ‘ w ’, for there are no such worlds. Rather, different theories of consequence result from varying the interpretation of ‘ \Rightarrow ’. In classical logic, there is really only one possibility for ‘ \Rightarrow ’, namely, material implication. In relevance logic, there are two. For relevance logic distinguishes material from relevant implication—or better, classical logic conflates them, illicitly warranting their equivalence. Which is the right account of validity?

The right one is the one I dubbed “the Relevant Account of Validity” ([17] § 6.5). For the essential feature of validity is that it should warrant one in proceeding from the truth of the premises to that of the conclusion.

But material detachment is invalid:

$$(V \supset) \quad \Sigma \vdash_{\supset} A \text{ iff } (\forall w)((\forall B \in \Sigma)w \models B \supset w \models A)$$

Learning that $\alpha \vdash_{\supset} \beta$ and that α is true does not warrant belief that β is true. That would be a use of Disjunctive Syllogism for ‘ \vee ’ ($A \vee B, \sim A \vdash B$), which is well-known to lead to the validity of EFQ in four easy moves (the so-called Lewis argument: see, e.g., [17] § 2.6). What does warrant one in moving from the truth of α to that of β is learning that α relevantly implies β :

$$(V \rightarrow) \quad \Sigma \vdash_{\rightarrow} A \text{ iff } (\forall w)((\forall B \in \Sigma)w \models B \rightarrow w \models A)$$

Accordingly, $(V \rightarrow)$ is the correct account of truth-preservation, and the correct account of validity. There is one true logic, relevance logic, and it consists in rejecting classical logic, including classical semantics.

5 Conclusion

Beall and Restall’s logical pluralism is incoherent. It claims that an inference can be both valid according to one account of logic and invalid according to another, and yet that this is not disagreement about validity but about logical purpose. But there is only one purpose of logic: to distinguish the valid inferences from the invalid ones. Among Beall and Restall’s “equally good” logics, one dominates: classical logic. This is because they view all their logics from the perspective of classical semantics. Hence their other logics disagree with classical logic only in failing to recognise certain classical inferences as valid.

Other logics might claim in contrast that classically invalid inferences are valid. Then Beall and Restall’s eclecticism would collapse into inconsistency. Even without that possibility, Beall and Restall’s pluralism ignores the non-classical rejection of classical inference, interpreting it only as an incompleteness, not recognising these validities rather than excluding them as really invalid.

There is one true logic, and it does take (V) as its criterion of validity. But it results from understanding the true nature of truth-preservation, that the conclusion be true whenever the premises are true. (V) needs to be interpreted, and developed, in a relevant metalanguage in which the relevance of the premises to the conclusion is an integral part of truth-preservation: if the conclusion really does follow from the premises then those premises must be, logically, relevant to the conclusion.

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