Exact and approximate eigenstates of vibrationally dressed polaritons

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SUPA

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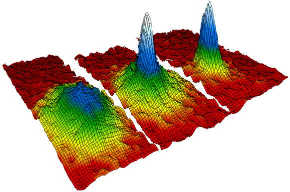
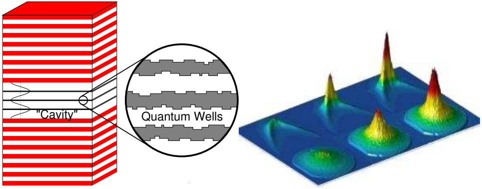
Condensation, Lasing, Superradiance

Atomic BEC $T \sim 10^{-7}$K



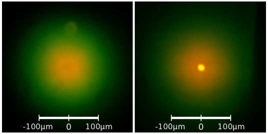
[Anderson *et al.* Science '95]

Polariton Condensate $T \sim 20$K



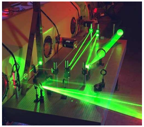
[Kasprzak *et al.* Nature, '06]

Photon Condensate
$T \sim 300$K

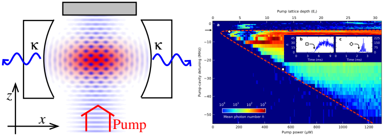


[Klaers *et al.* Nature, '10]

Laser
$T \sim ?, < 0, \infty$



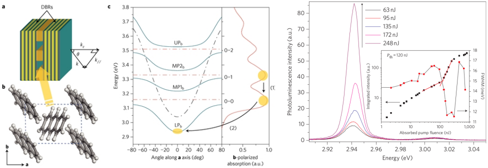
Superradiance transition
$T \sim 0$



[Baumann *et al.* Nature '10]

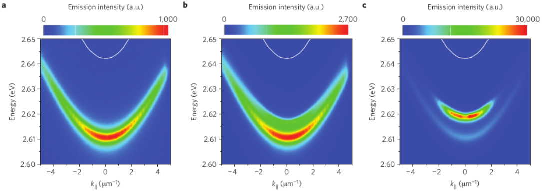
Motivation: polariton condensates

- Anthracene Polariton Lasing
  $T \sim 300\mathrm{K}$



[Kena Cohen and Forrest, Nat. Photon '10]

- Polariton condensates, other materials, e.g. polymers:



[Plumhoff *et al.* Nat. Materials '14, Daskalakis *et al.* ibid '14]

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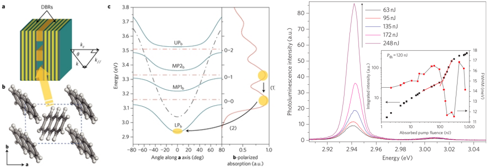

Q1. Vibrational replicas?
Q2. Relevance of disorder?
Q3. Lasing vs condensation?

[Kena Cohen and Forrest, Nat. Photon '10]

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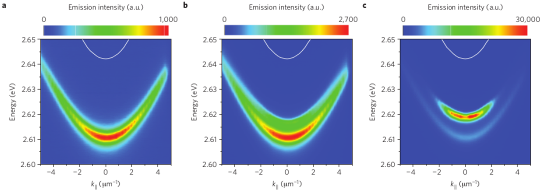

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Paradigms & Models

- Weakly interacting dilute Bose gas
  $$H = \int d^d r \hat{\psi}^\dagger(-\mu - \nabla^2)\hat{\psi} + U\hat{\psi}^\dagger\hat{\psi}^\dagger\hat{\psi}\hat{\psi}$$
  - Single field — assumes strong coupling
  - Continuum model, hard to include molecular physics
- Laser rate equations
  - Emission, absorption — assumes weak coupling, lasing.
- Transfer matrix, exciton susceptibility $\chi(\nu)$
- Microscopic model . . .

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Tavis-Cummings & Dicke model

Model capable of lasing & condensation

- Tavis-Cummings / Dicke model

$$H = \omega \hat{a}^\dagger \hat{a} + \sum_{i=1}^{N} \left[\omega_X \sigma_i^+ \sigma_i^- + g \left(\sigma_i^+ (\hat{a} + \hat{a}^\dagger) + \text{H.c.} \right) \right]$$

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  - High temperature: Maxwell-Bloch laser

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  Szymanska et al. PRL 06; Keeling et al. book chapter 1010.3338

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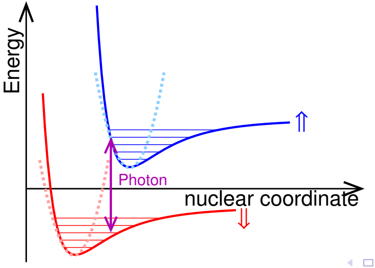

Holstein-Tavis-Cummings & Holstein-Dicke model

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  Kirton & JK, PRL 2013, PRA 2015; PRA 2016 ...
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  Cwik *et al.* EPL 105 2014; Spano, J. Chem. Phys 2015; Galego *et al.* PRX 2015; Cwik *et al.* PRA 2016; Herrera & Spano PRL 2016; Wu *et al.* arXiv:1608.08019; Zeb *et al.* arXiv:1608.08929; Herrera & Spano arXiv:1610.04252;

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Introduction and models

1. Introduction and models
   - Holstein-Dicke model
2. Strong coupling: polariton states
   - Exact solutions
   - Scaling with N
3. Strong coupling: spectrum

Strong coupling: polariton states

One excitation subspace, questions

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- Rotating wave approximation — Holstein Tavis Cummings
- Restrict, $\hat{a}^\dagger \hat{a} + \sum_i \sigma_i^+ \sigma_i^- = 1$.
- Questions:
  - Competition of $g\sqrt{N}$ vs ω_v, $\omega_v \lambda_0^2$
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Exact solution, $N = 2$

Vibrational Wigner function:

$$W(x,p) = \int dy \langle x + y/2|\rho|x - y/2\rangle_i e^{iyp}, \quad \left(\frac{\hat{b}_i + \hat{b}_i^\dagger}{\sqrt{2}}\right)|x\rangle_i = x|x\rangle_i$$

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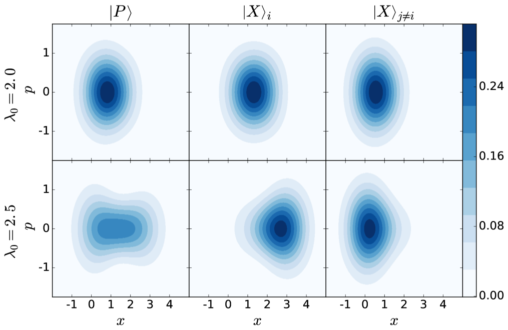

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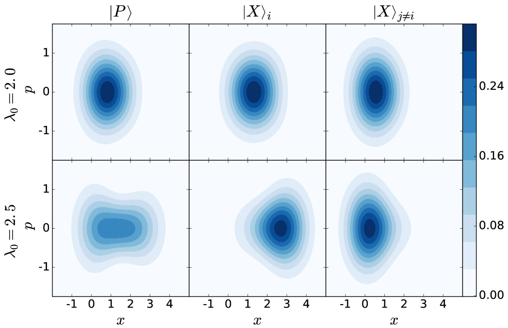

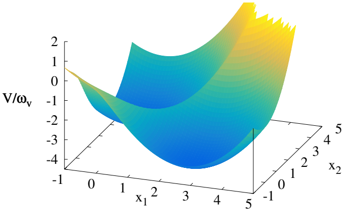

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- Permutation symmetry: $D_{\text{Hilbert}} \sim N^M$, typical $M \sim 5-6$

- Increasing N, suppress $W_{|P\rangle}(x \neq 0)$
- Distinct behaviour vs λ_0
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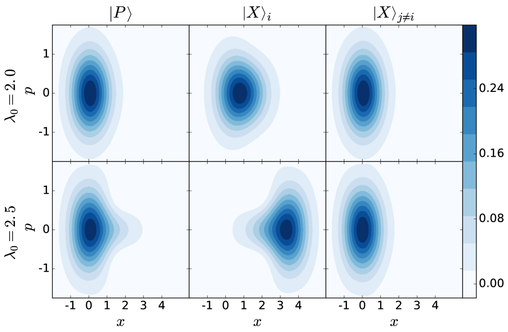

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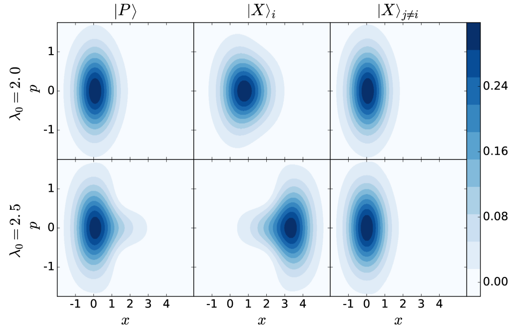

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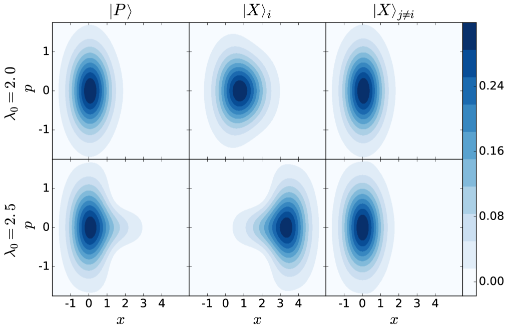

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Extending to arbitrary N, polaron ansatz

- Polaron transform, $\mathcal{D}_i(\lambda) = \exp\left(\lambda(\hat{b}_i^\dagger - \hat{b}_i)\right)$
- Single molecule ansatz:

$$|\Psi\rangle = [\alpha\mathcal{D}(\lambda_\uparrow)|\uparrow\rangle + \beta\mathcal{D}(\lambda_\downarrow)|\downarrow\rangle]|0\rangle_V$$

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[Herrera and Spano PRL 2016]

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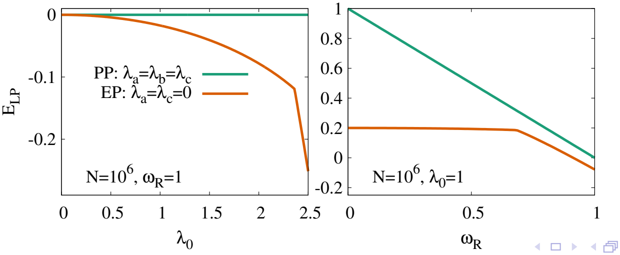

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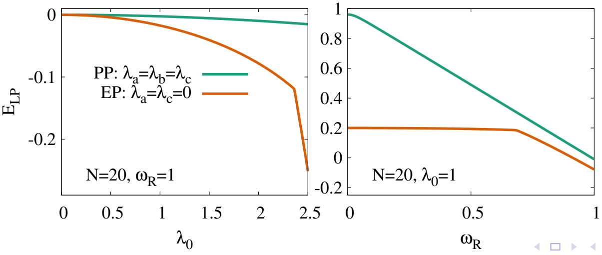

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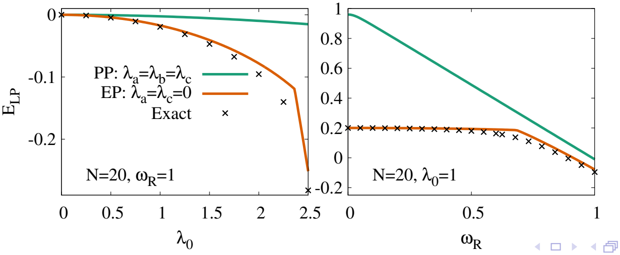

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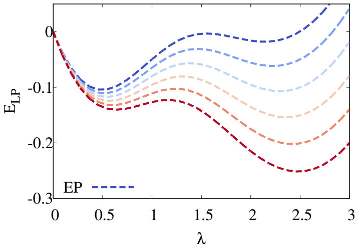

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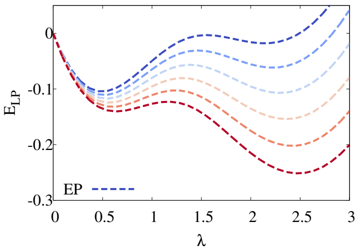

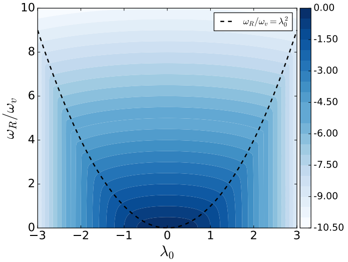

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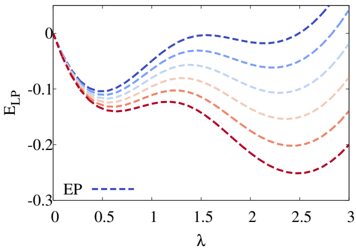

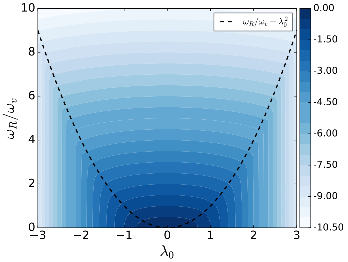

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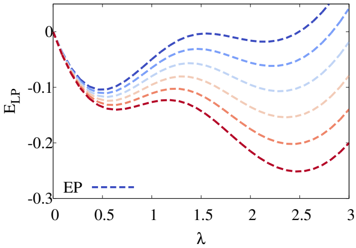

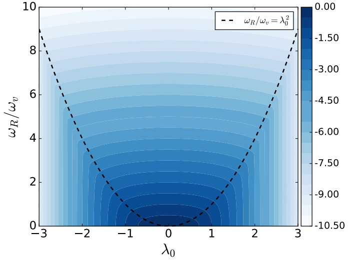

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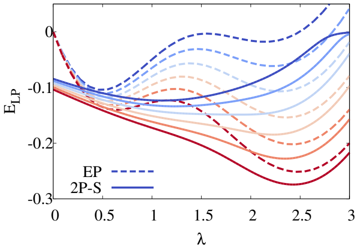

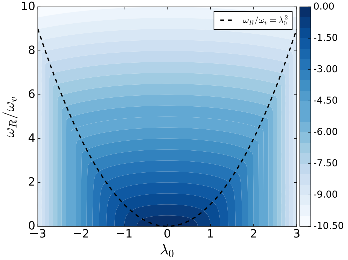

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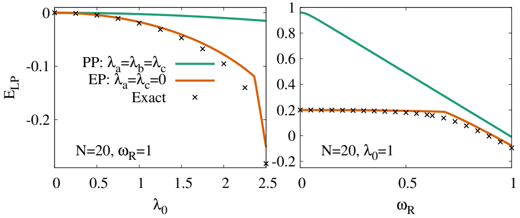
Simplified two-polaron physics

- Accurate energy & wavefunction



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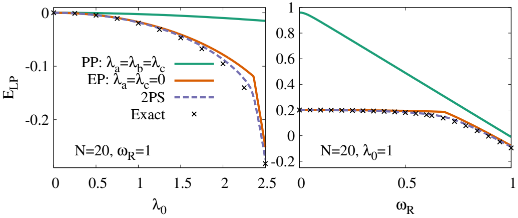

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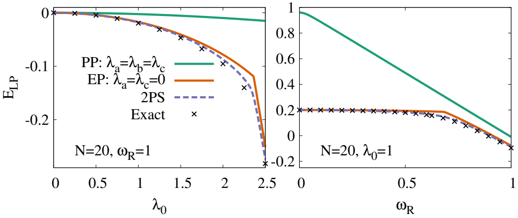

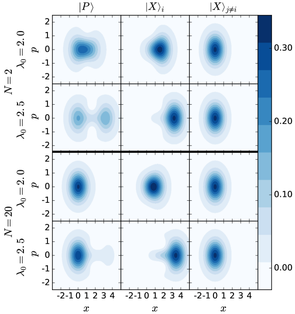

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Strong coupling: spectrum

Calculating spectra: Input-Output formalism

- Obsevable features: absorption spectrum, $A(\nu) = 1 - T(\nu) - R(\nu)$

- Scattering matrix gives:

$$A(\nu) = -\kappa_t \left[2\,\mathrm{Im}[D^R(\nu)] + (\kappa_t + \kappa_b)|D^R(\nu)|^2 \right]$$

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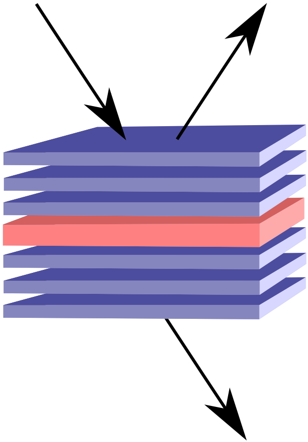

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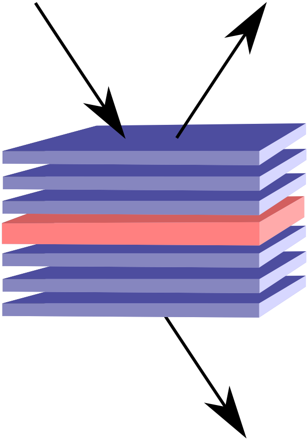

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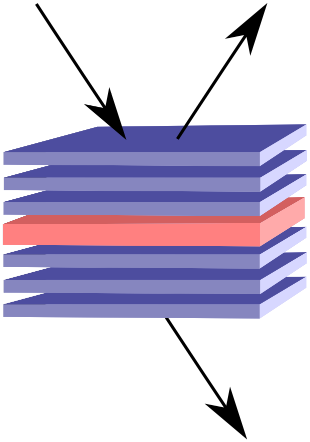

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Tavis-Cummings-Holstein vs Coupled Oscillators

- Coupled oscillator model:

$$H = \omega_P \hat{a}^\dagger \hat{a} + \sum_i \left[\frac{\omega_R}{\sqrt{N}} \left(\hat{a} \sum_n f_n(\lambda_0) \sigma_i^{n0} + \text{H.c.} \right) + \omega_n \sigma_i^{nn} \right]$$

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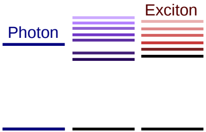

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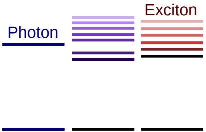

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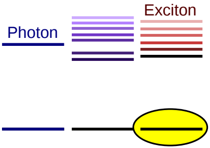

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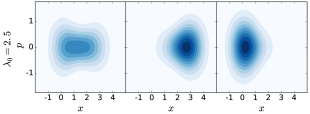

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- Direct calculation

$$D^R(t) = -i \left\langle 0 \middle| \left[\hat{a}(t), \hat{a}^\dagger(0) \right] \middle| 0 \right\rangle \theta(t)$$

- Time-evolve $|\psi_0\rangle = \hat{a}^\dagger|0\rangle$
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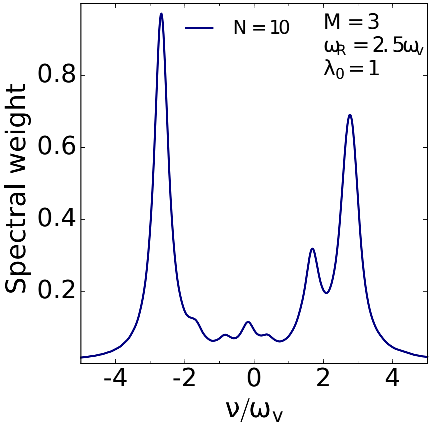

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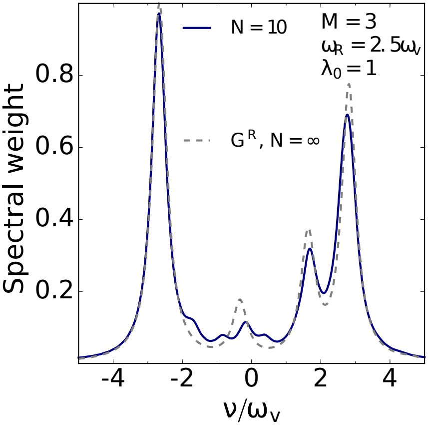

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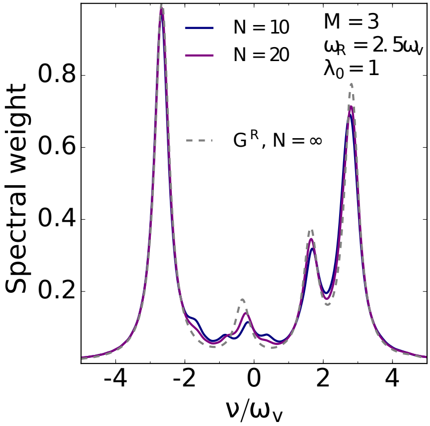

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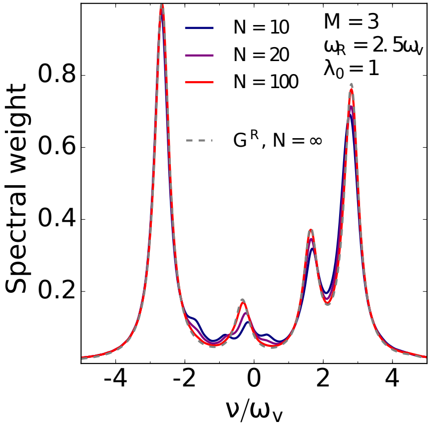

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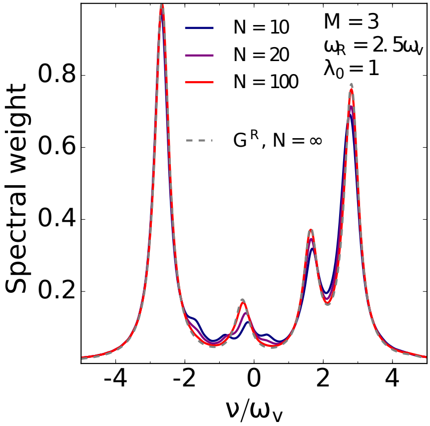

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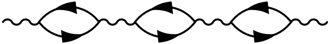
- Why? Multiple excitation $\sim 1/N$, 

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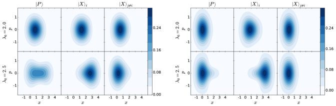
Topological Protection and Non-Equilibrium States in Strongly Correlated Electron Systems

Hybrid Polaritonics

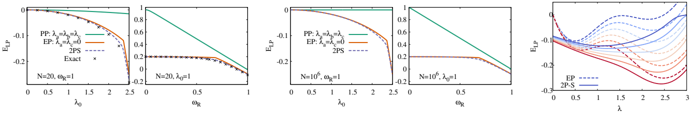
The Leverhulme Trust

Summary

- Holstein-Dicke and Holstein-Tavis-Cummings models
- Single polariton state
  - Exact solution



  - Polaron ansatz



[Zeb, Kirton, JK, `arXiv:1608.08929`]

- Validity of mean-field Green's functions