Supermode-density-wave-polariton condensation, and Meissner-like effect with multimode cavity-QED
Supermode-density-wave-polariton condensation, and Meissner-like effect with multimode cavity-QED

Jonathan Keeling

Trento, January 2017
What can quantum systems do?

Condensed matter physics: two types of question

What physics is needed to explain the material properties we do see
What can quantum systems do?

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What can quantum systems do?

Condensed matter physics: two types of question

What physics is needed to explain the material properties we do see

to

What material properties can be possible from quantum physics?
Once upon a time there was cavity QED . . .

- Precision tests of quantum optics
  - Purcell effect, strong coupling
  - Rabi oscillations, collapse & revival
  - Resonant fluorescence, EIT

- Many atom physics
  - Phase transitions: Lasing, superfluorescence, superradiance
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Synthetic cavity QED: Raman driving

- Tunable coupling via Raman

\[ H_{\text{eff}} = \ldots \frac{\Omega g}{\Delta} (\sigma_n^+ a + \text{H.c.}) \]

- Real systems: loss

\[ \partial_t \rho = -i[H, \rho] + \kappa \mathcal{L}[a, \rho] + \ldots \]

- To balance loss, counter-rotating:

\[ H_{\text{eff}} = \ldots \frac{\Omega g}{\Delta} \sigma_n^x (a + a\dagger) \]

[Dimer et al. PRA '07]
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[Dimer et al. PRA ’07]
Multimode cavity QED

- Full model:

\[
H_{\text{eff}} = \sum_{\mu} \left( \omega_{\mu} - \omega_{P} \right) a_{\mu}^\dagger a_{\mu} + \sum_{N} \frac{\omega_{0}}{2} \sigma_{n}^{z} + \frac{\Omega g_{0}}{\Delta} \sum_{\mu} \Xi_{\mu}(r_{n}) \sigma_{n}^{x}(a + a^\dagger)
\]

Possibilities

- **XY vs Ising**

- **Momentum state vs hyperfine state**

- **Single mode vs multimode**

- **Thermal gas vs BEC vs disorder localised**
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Introduction: Tunable multimode Cavity QED

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   - Spin-non-conserving loss

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   - Experimental setup
   - Supermode density wave polariton condensation

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   - Spin glass, Hopfield memory
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Single mode cavity QED

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Single mode experiments

Ritsch et al. PRL ’02
**Single mode experiments**

![Diagram of single mode experiment](image)

Ritsch *et al.* PRL '02

**Thermal atoms, momentum state**

![Diagram of thermal atoms and momentum state](image)

Vuletic *et al.* PRL '03 (MIT)
**Single mode experiments**

**BEC, momentum state**

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**Thermal atoms, momentum state**

(a) Pump beams

(b) Emitted light

Cavity Mirrors

**Baumann et al. Nature ’10 (ETH)**

**Kinder et al. PRL ’15 (Hamburg)**

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**Jonathan Keeling**

Multimode cavity QED
Single mode experiments

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(a) Pump Beams
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Pump laser
Bragg planes

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Badeen et al. PRL ’14 (Singapore)

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Single mode theory

- Momentum degrees of freedom:
  \[ \psi = \psi_\downarrow + \psi_\uparrow \cos(kx) \cos(kz) \]
- Effective 2LS \((\psi_\downarrow, \psi_\uparrow)\)

\[ H_{\text{eff}} = (\omega_c - \omega_P) a^\dagger a + \sum_n \frac{\omega_0}{2} \sigma_n^z + \frac{\Omega g_0}{\Delta} \sigma_n^x (a + a^\dagger) \]

- Extra “feedback” term \(U\), cavity loss \(\kappa\)
- Single mode – mean-field EOM, \(\alpha = \langle \hat{a} \rangle\), \(S_i = \sum_n \sigma_n^i/2\).

\[
\begin{align*}
\dot{S}^- &= -i(\omega_0 + U|\alpha|^2) S^- + 2i g_{\text{eff}} (\alpha + \alpha^*) S^x \\
\dot{S}^x &= i g_{\text{eff}} (\alpha + \alpha^*) (S^- - S^+) \\
\dot{\alpha} &= -[\kappa + i(-\Delta_c + US^2)] \alpha - i g_{\text{eff}} (S^- + S^+) 
\end{align*}
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  \[
  \dot{S}^- = -i(\omega_0 + U|\alpha|^2)S^- + 2ig_{\text{eff}}(\alpha + \alpha^*)S^z
  \]
  \[
  \dot{S}^z = ig_{\text{eff}}(\alpha + \alpha^*)(S^- - S^+)
  \]
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Classical dynamics

Changing $U$:

$U = 0$

$U < 0$

$U > 0$

[JK et al. PRL ’10, Bhaseen et al. PRA ’12]
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2 Level System

$\Omega$

$\psi_g$

$\psi$

$\rightarrow$

$\downarrow$

$\uparrow$

SRA

$\downarrow$

$\uparrow$

SRA

UN=40

$\omega$ (MHz)

$g\sqrt{N}$ (MHz)

0 0.5 1 1.5

0 200 400 600 800 1000 1200

$|\psi|$ 2

0 5 10 15

0 400 800 1200

[JK et al. PRL ’10, Bhaseen et al. PRA ’12]
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Persistent Oscillations

[JK et al. PRL '10, Bhaseen et al. PRA '12]
Effect of particle losses

- Adding other loss terms

\[ \partial_t \rho = -i[H, \rho] + \kappa \mathcal{L}[\hat{a}] + \sum_i \Gamma_{\downarrow} \mathcal{L}[\sigma_i^-] + \Gamma_{\phi} \mathcal{L}[\sigma_i^z] \]

\[ \mathcal{L}[X] = X_\rho X^\dagger - (X^\dagger X_\rho + \rho X^\dagger X)/2 \]

- \( \Gamma_{\downarrow}, \Gamma_{\phi} \) break S conservation.

[Dalla Torre et al., PRA (Rapid) 2016, Kirton & JK, arXiv:1611.03342]
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Effect of particle losses

- Wigner function $W(\hat{a} = \hat{x} + i\hat{p})$

- Finite $N$: no symmetry breaking

- $\Gamma_{\phi}$ only: MFT $\rightarrow$ no SR

- Asymptotic scaling

[Kirton & JK, arXiv:1611.03342]
Effect of particle losses

- **Wigner function** $W(\hat{a} = \hat{x} + i\hat{p})$

![Graph showing Wigner functions for different cases: (a) $\kappa$, (b) $\kappa, \Gamma_\downarrow$, (c) $\kappa, \Gamma_\phi$, and (d) $\kappa, \Gamma_\downarrow, \Gamma_\phi$.]

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Multimode cavity QED experiments

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Multimode cavities

Confocal cavity $L = R$:

- **Modes**
  \[ \Xi_{l,m}(r) = H_l(x)H_m(y), \]
  $l + m$ fixed parity

- Extra distinction: degenerate vs non-degenerate
Multimode cavities

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Adjustable length multimode cavity

[Kollár, Papgeorge, Baumann, Armen & Lev, NJP ’15]
Superradiance in multimode cavity: Even family

![Graph showing cavity transmission vs frequency with peaks at multiples of 2 MHz.]
Superradiance in multimode cavity: Even family

![Graph with frequency and cavity transmission](image)

- **a**: Lower frequency side
- **b**: Higher frequency side
Superradiance in multimode cavity: Even family

![Graph showing cavity transmission vs frequency (MHz)]

- **Frequency (MHz)**
  - l+m = 0
  - 0
  - 50
  - 100
  - 150

- **Cavity transmission (arb. units)**
  - 1
  - 10

- **Images**
  - a
  - b
  - c
  - d
  - e
  - f
  - g
Supermodes vs polariton condensation

Supermode density-wave polariton:
- Hybrid cavity photon and atomic density wave
- Atoms remix cavity modes → superposition
- Condensation of polaritons remixes again
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Bare modes:

Super modes:

Graph showing counts and admixture fraction vs frequency (MHz).
Superradiance in multimode cavity: Odd family

- Dependence on cloud position

Near-degeneracy of (1, 0), (0, 1) modes broken by matter-light coupling.

Atomic time-of-flight — structure factor

Jonathan Keeling
Multimode cavity QED
Trento, 2017
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Disordered atoms

- Multimode cavity, Hyperfine states,

\[ H_{\text{eff}} = -\sum_{\mu} \Delta_{\mu} a_{\mu}^{\dagger} a_{\mu} + \sum_{n} \frac{\omega_{0}^{2}}{2} \sigma_{n}^{z} + \frac{\Omega g_{0}}{\Delta} \sum_{\mu} \Xi_{\mu}(r_{n}) \sigma_{n}^{x}(a_{\mu} + a_{\mu}^{\dagger}) \]

- Random atom positions – queched disorder

Effective XY/Ising spin glass

\[ H_{\text{eff}} = \sum_{n,m} J_{n,m} \begin{cases} \sigma_{n}^{x} \sigma_{m}^{x} & \text{Ising} \\ \sigma_{n}^{+} \sigma_{m}^{-} & \text{XY} \end{cases} \]

\[ J_{nm} = \sum_{\mu} \frac{\Omega^{2} g_{0}^{2} \Xi_{\mu}(r_{n}) \Xi_{\mu}(r_{m})}{\Delta^{2} \Delta_{\mu}} \]

[Gopalakrishnan, Lev and Goldbart. PRL ’11, Phil. Mag. ’12]
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Tunable spin glass

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- Tunable complexity
- Explore RSB/Droplet order
- Open system spin-glass.
  [Strack & Sachdev PRL '11]

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- Tunable complexity
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[Strack & Sachdev PRL ’11]

\[ \begin{align*}
\text{Tunable spin glass} & \quad \text{Ferromagnet} \\
\text{Ferromagnet} & \quad \text{Spin glass} \\
\text{Paramagnet} & \quad \text{Sherrington Kirkpatrick} \\
\text{Sherrington Kirkpatrick} & \quad \text{Edwards Anderson} \\
\text{Edwards Anderson} & \quad \text{(short range)}
\end{align*} \]

[Jonathan Keeling, Lev and Goldbart. PRL ’11, Phil. Mag. ’12]
Tunable spin glass

\[ H_{\text{eff}} = \sum_{n,m} J_{n,m} \sigma_n^x \sigma_m^x \quad \text{and} \quad J_{nm} = \sum_{\mu} \frac{\Omega^2 g_0^2 \Xi_{\mu}(r_n) \Xi_{\mu}(r_m)}{\Delta^2 \Delta_{\mu}} \]

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Hopfield memory

- Between Mean-Field and Spin-Glass
  - Multiple fixed points
  - Recover corrupted image

Low dimensional cartoon:

- Neurons → Spins
- Synapses → Modes
- Plasticity → Atom movement
- Need \(|s_n| = 1\) (hard spins)

[\text{Gopalakrishnan, Lev and Goldbart. PRL ‘11, Phil. Mag. ’12}]
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Cavity QED and synthetic gauge fields

- [Spielman, PRA '09] scheme, hyperfine states $A, B$

\[ H = (\psi_A \ \psi_B) \begin{pmatrix} E_a + (\nabla - Q\hat{x})^2 & \Omega/2 \\ \Omega/2 & E_B + (\nabla + Q\hat{x})^2 \end{pmatrix} \begin{pmatrix} \psi_A \\ \psi_B \end{pmatrix} \]

Feedback
- Why?
  - Meissner effect, Anderson-Higgs mechanism, confinement-deconfinement transition.
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Ground state:

\[
E_-(k) \approx \frac{(k - \frac{(E_B - E_A)Q\hat{x}}{\Omega})^2}{2m^*}
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  - How?
    - Multimode cavity QED
Meissner-like physics: idea

- Follow Spielman scheme

\[ \left( \frac{E_A + (\nabla - Q\hat{x})^2}{\Omega/2} E_B + (\nabla + Q\hat{x})^2 \right) \]

- \( E_A, E_B \propto |\phi|^2 \) from cavity Stark shift
- Ground state: \( E_\gamma (k) \propto (k - Q\hat{x}|\phi|^2)^2 \)

- Multimode cQED \( \rightarrow \) local matter-light coupling
- Variable profile synthetic gauge field?
- Reciprocity: matter affects field

[Ballantine et al. arXiv:1608.07246]
Meissner-like physics: idea

- Follow Spielman scheme

\[
\begin{pmatrix}
E_A + (\nabla - Q\hat{x})^2 & \Omega/2 \\
\Omega/2 & E_B + (\nabla + Q\hat{x})^2
\end{pmatrix}
\]

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- Ground state \(E_-(k) \propto (k - Q\hat{x})|\varphi|^2\)

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[Ballantine et al. arXiv:1608.07246]
Meissner-like physics: setup

Atoms:
\[ i \frac{\partial}{\partial t} \begin{pmatrix} \psi_A \\ \psi_B \end{pmatrix} = \left[ -\frac{\nabla^2}{2m} + \left( -\mathcal{E}_\Delta |\varphi|^2 + i \frac{q}{m} \partial_x \right) \frac{\Omega}{2} \right. \left. \mathcal{E}_\Delta |\varphi|^2 - i \frac{q}{m} \partial_x \right) - \frac{\Omega}{2} \right] \begin{pmatrix} \psi_A \\ \psi_B \end{pmatrix} \]

Light:
\[ i \frac{\partial}{\partial t} \varphi = \left[ \frac{\partial}{\partial r} \left( -l^2 \nabla^2 + \frac{l^2}{r^2} \right) - \Delta_0 - i\kappa - N\mathcal{E}_\Delta (|\psi_A|^2 - |\psi_B|^2) \right] \varphi \]

[Ballantine et al. arXiv:1608.07246]
Meissner-like physics: setup

- **Atoms:**
  \[ i \partial_t \begin{pmatrix} \psi_A \\ \psi_B \end{pmatrix} = \left[ -\frac{\nabla^2}{2m} + \left( -\mathcal{E}_\Delta |\varphi|^2 + \frac{i q_m \partial_x}{\Omega/2} \mathcal{E}_\Delta |\varphi|^2 - i \frac{q_m \partial_x}{\Omega/2} \right) \right] \begin{pmatrix} \psi_A \\ \psi_B \end{pmatrix}. \]

- **Light:**
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- **Light:**
  \[ i \partial_t \varphi = \left[ \frac{\delta}{2} \left( -l^2 \nabla^2 + \frac{r^2}{l^2} \right) - \Delta_0 - i \kappa - N \mathcal{E}_\Delta (|\psi_A|^2 - |\psi_B|^2) \right] \varphi + f(r). \]

[Ballantine et al. arXiv:1608.07246]
Consider \( f(\mathbf{r}) \) such that
\[
|\varphi|^2 \propto y.
\]
Without feedback (\( \mathcal{E}_\Delta = 0 \)) for field

With feedback

[Ballantine et al. arXiv:1608.07246]
Meissner-like physics: numerical simulations

Consider $f(r)$ such that $|\varphi|^2 \propto y$.

- Without feedback ($\mathcal{E}_\Delta = 0$) for field
- With feedback
  - Field expelled
  - Cloud shrinks

Atoms

Cavity light

Synthetic field

[Ballantine et al. arXiv:1608.07246]
Meissner-like physics: numerical simulations

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[Ballantine et al. arXiv:1608.07246]
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Summary

- Open Dicke model, $\kappa, \Gamma_\phi, \Gamma_\downarrow$ [Kirton & JK, arXiv:1611.03342]

- Many possibilities of multimode cavity QED

- Supermode polariton condensation [Kollár et al. arXiv:1606.04127]

1 Introduction: Tunable multimode Cavity QED

2 Single mode cavity QED
   • Spin-non-conserving loss

3 Multimode cavity QED experiments
   • Experimental setup
   • Supermode density wave polariton condensation

4 Theoretical possibilities
   • Spin glass, Hopfield memory
   • Meissner-like effect
Training Hopfield
How to train your atoms

- Input/output by cavity modes, \( Q_\mu = \langle \hat{a}_\mu \rangle \)

\[
H_{\text{eff}} = -\sum_\mu \Delta_\mu a_\mu^\dagger a_\mu + \sum_n \frac{\omega_0}{2} \sigma^z_n + E_P \sum_{\mu,n} \Xi_\mu(r_n) \sigma^x_n (a_\mu + a_\mu^\dagger) \\
+ \sum_\mu f_\mu a_\mu^\dagger + \text{H.c.}
\]

- Effective problem:

\[
H_{\text{eff}} = -E_P \sum_\mu (f_\mu + Q_\mu)^2, \quad Q_\mu = \sum_n \Xi_\mu(r_n) \sigma^x_n
\]
How to train your atoms

- Input/output by cavity modes, $Q_\mu = \langle \hat{a}_\mu \rangle$

$$H_{\text{eff}} = -\sum_\mu \Delta_\mu a_\mu^\dagger a_\mu + \sum_n \frac{\omega_0}{2} \sigma_n^Z + E_P \sum_{\mu,n} \Xi_\mu (r_n) \sigma_n^x (a_\mu + a_\mu^\dagger)$$

$$+ \sum_\mu f_\mu a_\mu^\dagger + \text{H.c.}$$

Effective problem:

$$H_{\text{eff}} = -E_P \sum_\mu (f_\mu + Q_\mu)^2, \quad Q_\mu = \sum_n \Xi_\mu (r_n) \sigma_n^x$$
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How to train your atoms

- Before training, many fixed points

Train by moving atoms:

- Recover corrupted image

\[ \sigma^x_i \]

\[ Q_\mu \]
How to train your atoms

- Before training, many fixed points

\[ \sigma^X_i \]

\[ Q_\mu \]

- Train by moving atoms:

\[ f(x) = \sum_{m} \phi_{m} \]

\[ \text{Optimisation Step} \]

- Recover corrupted image
How to train your atoms

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