Suppressing and restoring the Dicke superradiance transition by dephasing

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SUPA®

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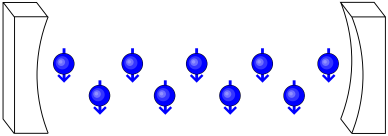
Dicke model and Dicke-Hepp-Lieb transition



$$H = \omega\psi^\dagger\psi + \sum_\alpha \frac{\omega_0}{2}\sigma^z_\alpha + g(\psi + \psi^\dagger)(\sigma^+_\alpha + \sigma^-_\alpha)$$

- Coherent state: $|\Psi\rangle \to e^{\lambda\psi^\dagger + \eta S^+}|\Omega\rangle$
- Small g, min at $\lambda, \eta = 0$

[Hepp, Lieb, Ann. Phys. '73]

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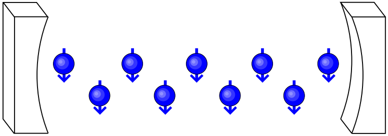

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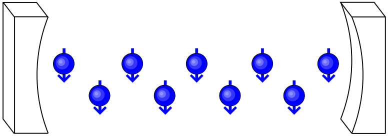

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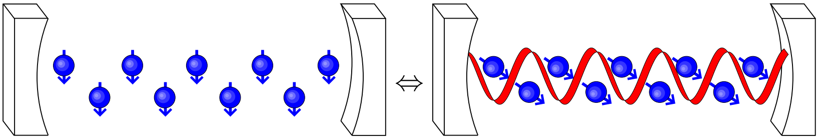

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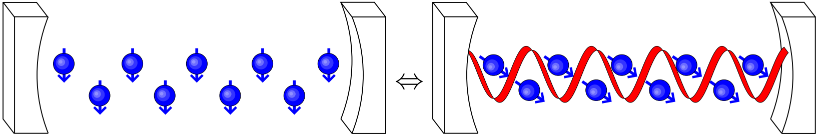

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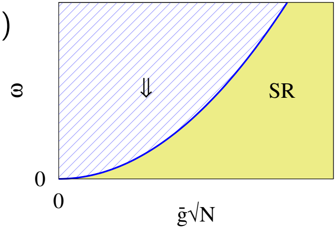

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“Textbook” Laser: Maxwell Bloch equations

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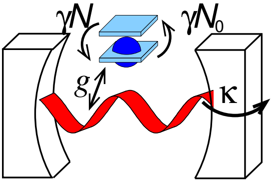

- $|\psi|^2 > 0$ if $2N_0 g^2 > \gamma\kappa$
- Requires inversion

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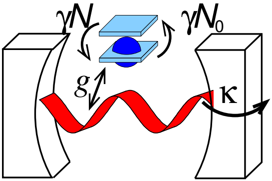

Maxwell-Bloch eqns: $P = -i \langle \sigma^- \rangle, N = 2 \langle \sigma^z \rangle$

$$\partial_t \psi = -i \omega \psi - \frac{\kappa}{2} \psi + \textstyle\sum_\alpha g P_\alpha$$

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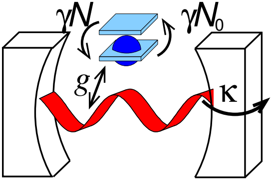

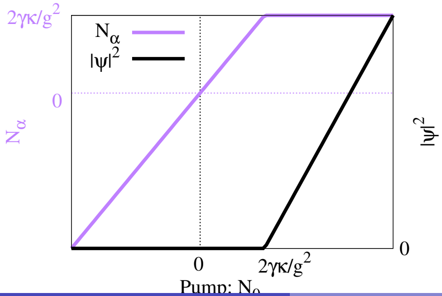

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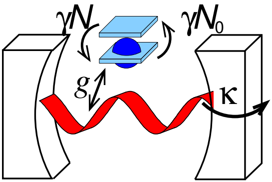

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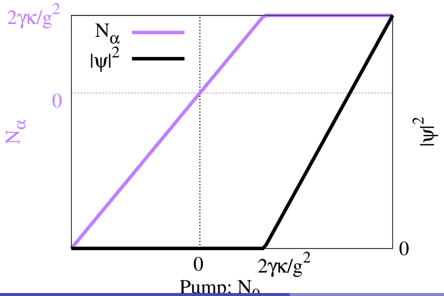

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"Textbook" Laser: Semiclassical equations

- Semiclassical laser theory $n = \langle \psi^\dagger \psi \rangle$

$$\partial_t n = \gamma N_0 \frac{2g^2(n+1)}{\gamma\gamma_t + 4g^2(n+1)} - \kappa n$$

  - MF Transition at $N_0 2g^2/\gamma_t = \kappa$
  - No symmetry breaking

- Spontaneous emission: finite "size" corrections

$$n = \frac{1}{2\beta}\left[\frac{N_0}{N_c} - 1 \pm \sqrt{\left(\frac{N_0}{N_c} - 1\right)^2 + 4\beta\frac{N_0}{N_c}}\right]$$

$\beta \simeq 4g^2/\gamma\gamma_t$

[Haken, RMP, 1975]

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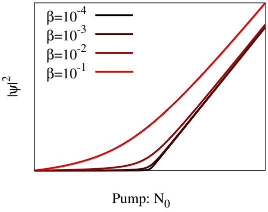

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   - Dicke superradiance vs lasing
   - Driven Dicke model

2. Behaviour with dephasing
   - Mean field theory problem
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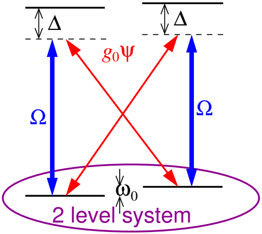
Raman driven Dicke model



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[Dimer *et al.* PRA '07]

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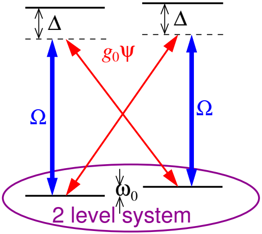

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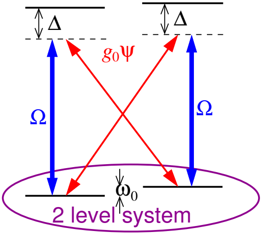

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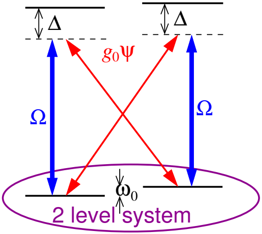

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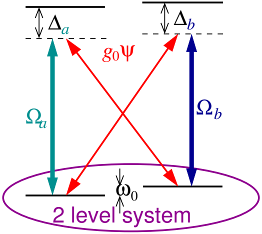

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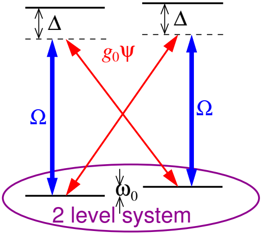

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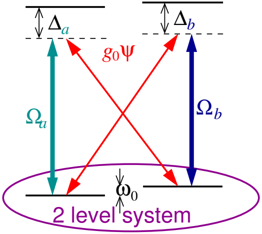

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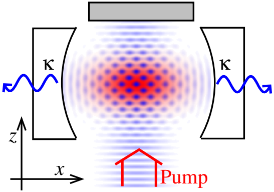
Open Dicke model theory



- Momentum degrees of freedom: $\psi = \psi_{\Downarrow} + \psi_{\Uparrow} \cos(kx)\cos(kz)$
- Effective 2LS $(\psi_{\Downarrow}, \psi_{\Uparrow})$

$$H_{\text{eff}} = \omega \psi^\dagger \psi + \sum_n \frac{\omega_0}{2} \sigma_n^z + g_{\text{eff}} \sigma_n^x (\psi + \psi^\dagger) + U \sigma_n^z \psi^\dagger \psi$$

- Extra “feedback” term U, cavity loss κ
- Single mode – mean-field EOM, $\alpha = \langle \hat{\psi} \rangle$, $S^i = \sum_n \sigma_n^i / 2$.

$$\dot{S}^- = -i(\omega_0 + U|\alpha|^2)S^- + 2ig_{\text{eff}}(\alpha + \alpha^*)S^z$$

$$\dot{S}^z = ig_{\text{eff}}(\alpha + \alpha^*)(S^- - S^+)$$

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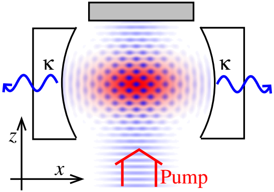

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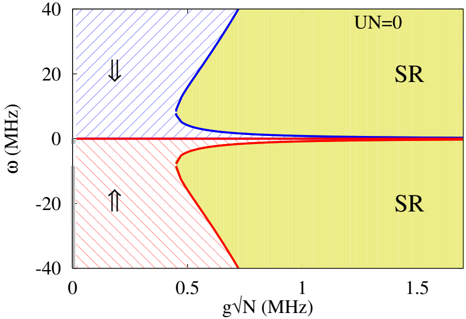
Classical dynamics

Changing *U*:

$U = 0$



[JK *et al.* PRL '10, Bhaseen *et al.* PRA '12]

Classical dynamics

Changing U:

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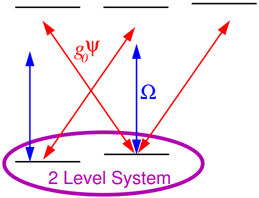
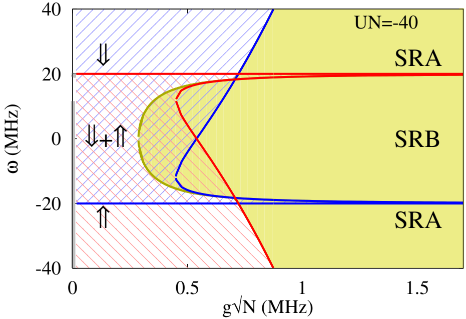


$$U \propto \frac{g_0^2}{\omega_c - \omega_a}$$



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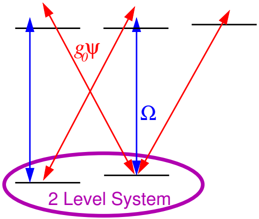

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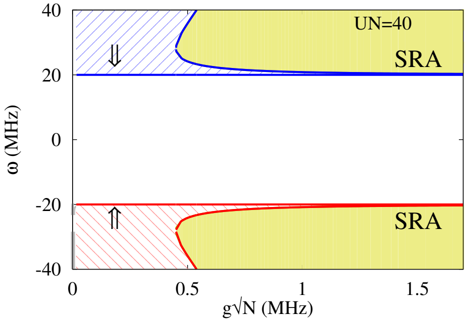

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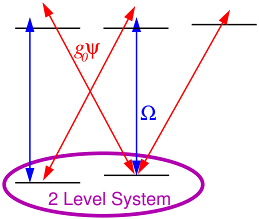

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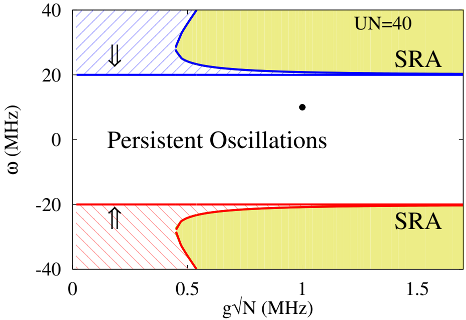

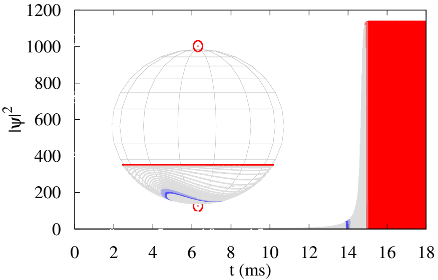

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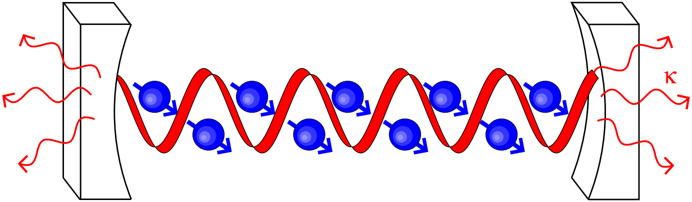
2. Behaviour with dephasing
   - Mean field theory problem
   - Polynomial algorithm for exact solution
   - Cumulant expansion and $N \to \infty$

Individual dephasing



- Extra loss terms

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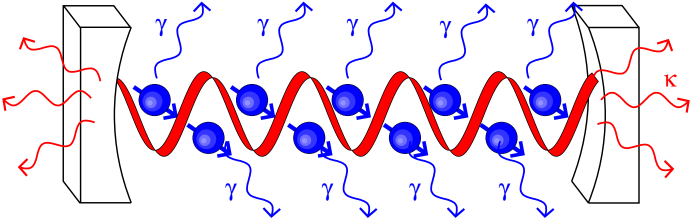

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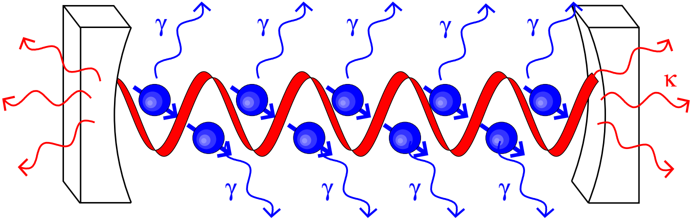

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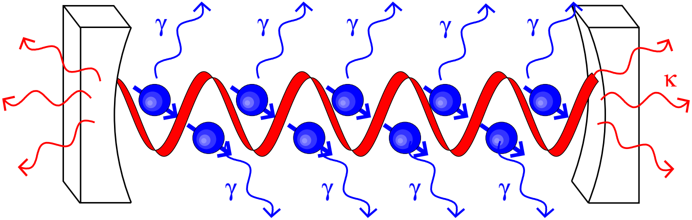

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- Steady state *and* Linear stability

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- No fixed $\mathbf{S} = \sum_i \sigma_i$: $\text{Dim}[\mathcal{H}] = 2^N$ vs N
- **But:** Permutation symmetry of ρ remains
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[Kirton & JK, arXiv:1611.03342]

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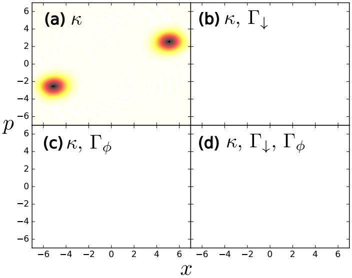

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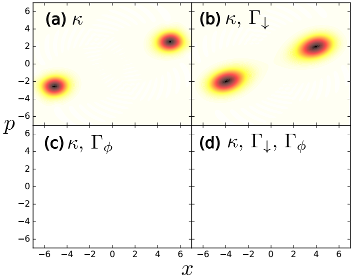

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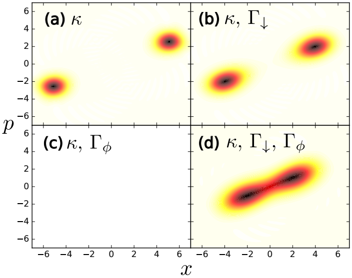

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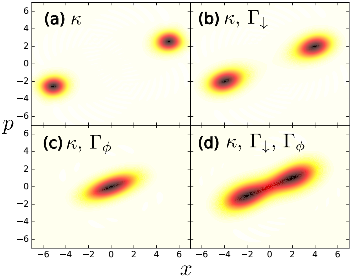

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Asymptotic behaviour

- Proof of transition: Finite size scaling
  - Superradiant: $\langle \psi^\dagger \psi \rangle \propto N$
  - Normal: $\langle \psi^\dagger \psi \rangle \propto \sqrt{N}$
- Very suggestive — large N?

- Cumulant expansion: $\langle \psi^\dagger \psi \rangle$, $\langle \psi \psi \rangle$, $\langle \sigma^z \rangle$, $\langle \psi \sigma^{x,y} \rangle$, $\langle \sigma_\alpha^{x,y} \sigma_{\beta \neq \alpha}^{x,y} \rangle$.
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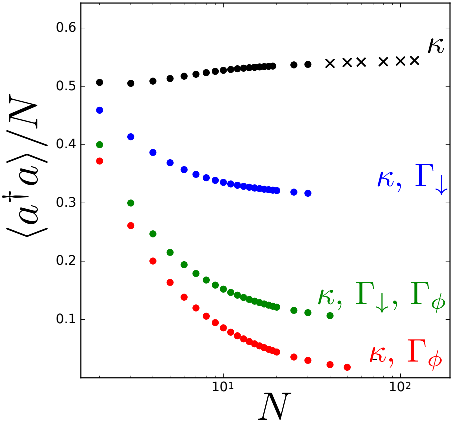

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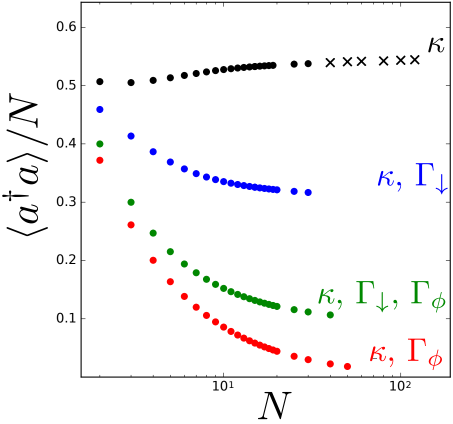

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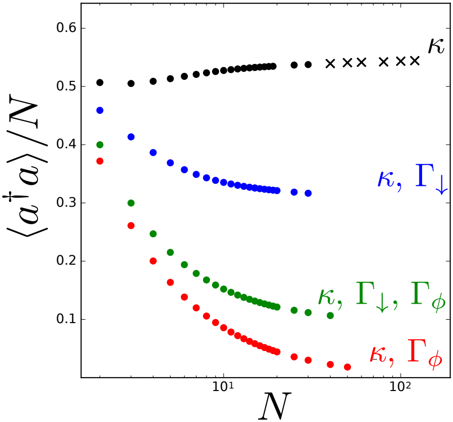

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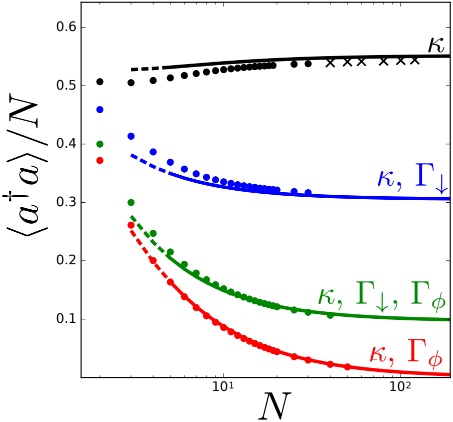

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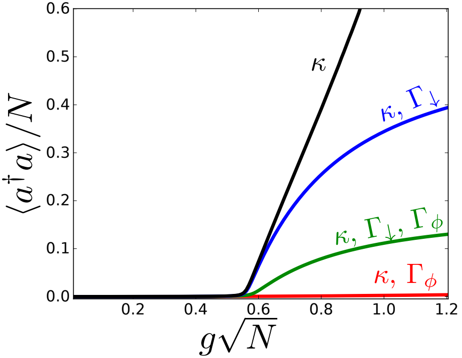

Further applications

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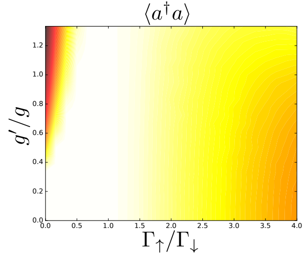

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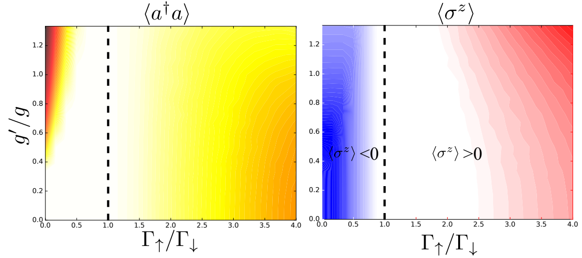

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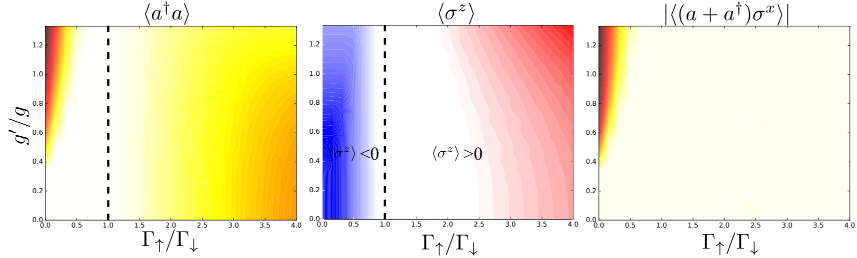

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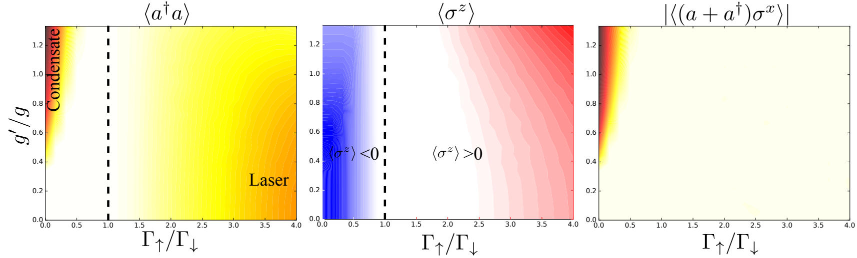
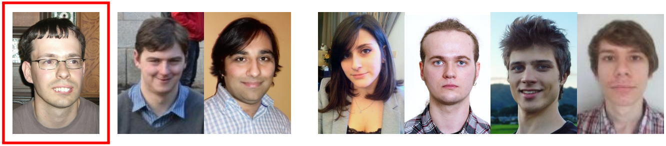


Acknowledgements

GROUP:



FUNDING:

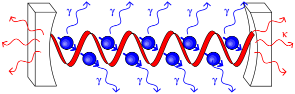
EPSRC
Engineering and Physical Sciences
Research Council

topnes
Topological Protection and Non-Equilibrium States in Strongly Correlated Electron Systems

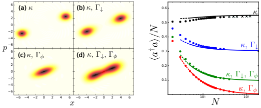
Hybrid
Polaritonics

Summary

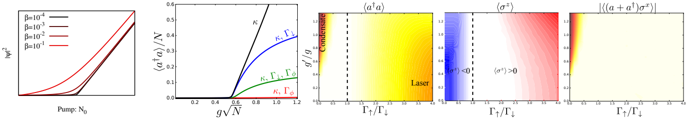
- Open Dicke model, $\kappa, \Gamma_\phi, \Gamma_\downarrow$



- Exact numerics: permutation symmetry of ρ



- Cumulant expansion — connection to laser rate equation



[Kirton & JK, arXiv:1611.03342]

Green's function as common language

- Green's function: Response to weak perturbation

$$\left[D^R(\nu)\right]^{-1} = \nu - \omega + i\frac{\kappa}{2} + \frac{g^2 N_0}{\nu - \omega_0 + i\Gamma}$$

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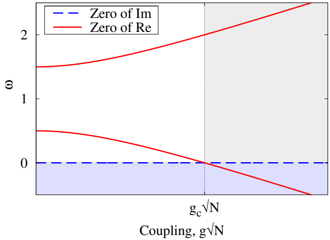

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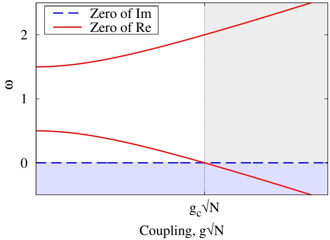

Laser