Supermode-density-wave-polariton condensation, and Meissner-like effect with multimode cavity-QED

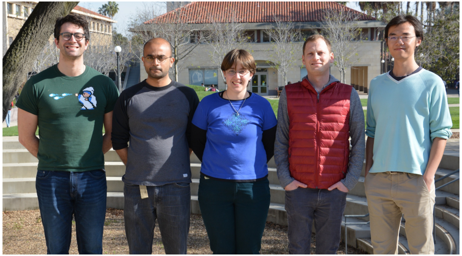
Jonathan Keeling

University of St Andrews
FOUNDED 1413

DOQS, Glasgow, August 2016

Acknowledgments

Experiment (Stanford):
Benjamin Lev



STANFORD UNIVERSITY

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Theory:

Ben Simons (Cambridge), Joe Bhaseen (KCL), James Mayoh (Southampton)

Sarang Gopalakrishnan (CUNY)
Surya Ganguli, Jordan Cotler (Stanford)
Laura Staffini, Kyle Ballantine (St Andrews)

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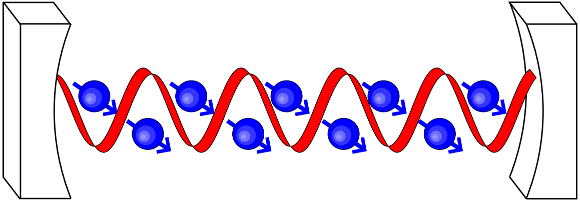
EPSRC
Engineering and Physical Sciences Research Council

1. Introduction: Tunable multimode Cavity QED
   - Many body cavity QED
   - Multimode cavity QED
2. Experimental results: supermode density wave polariton condensation
3. Theoretical results: Meissner-like effect

(Multimode) cavity QED

$$H = \sum_k \omega_k a_k^\dagger a_k + \sum_n \omega_0 \sigma_n^+ \sigma_n^- + \sum_{n,k} g_{k,n}(a_k^\dagger + a_{-k})(\sigma_n^+ + \sigma_n^-)$$

$$\dot{\rho} = -i[H, \rho] + \kappa \sum_k \mathcal{L}[a_k, \rho] + \gamma \sum_i \mathcal{L}[\sigma_n^-, \rho]$$



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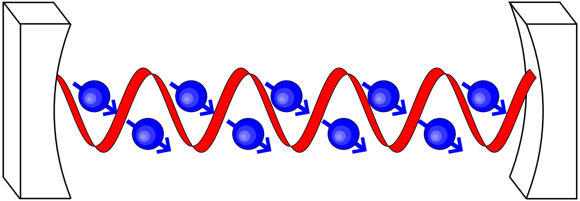
- Compare g (or $g\sqrt{N}$) vs:
  - κ, γ
  - ω_k, ω_0



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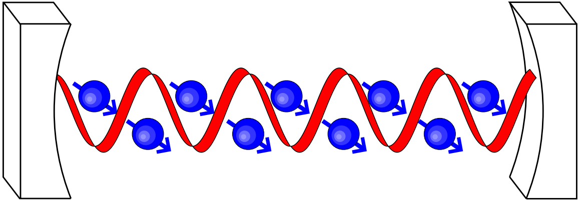
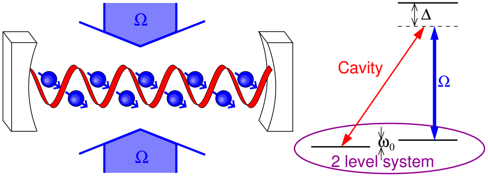


Synthetic cavity QED: Raman driving

- Tunable coupling via Raman



$$H_{\text{eff}} = \ldots \frac{\Omega g}{\Delta} \left(\sigma_n^{+} a + \text{H.c.} \right)$$

- Real systems: loss $\partial_t \rho = -i[H, \rho] + \kappa \mathcal{L}[a, \rho] + \ldots$
- To balance loss, counter-rotating:

$$H_{\text{eff}} = \ldots \frac{\Omega g}{\Delta} \sigma_n^{x} (a + a^\dagger)$$

[Dimer *et al.* PRA '07]

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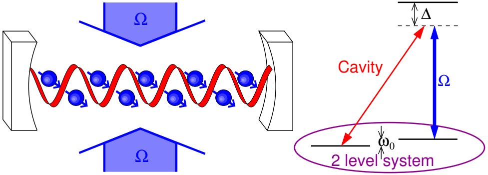

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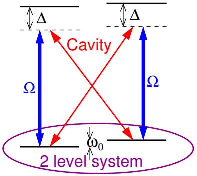

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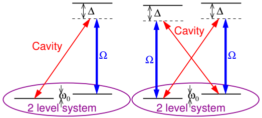
Possibilities

- XY *vs* Ising



- Momentum state *vs* hyperfine state
- Single mode *vs* multimode
- Thermal gas *vs* BEC *vs* disorder localised

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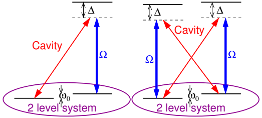

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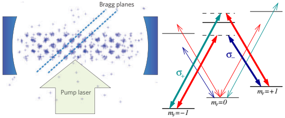

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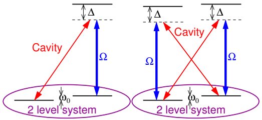

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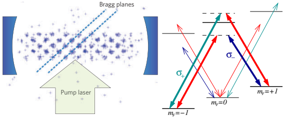

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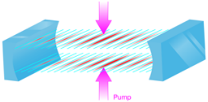
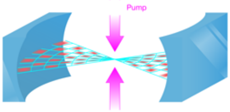


Credit: Alan Stonebreaker, Physics 3, 88 (2010)



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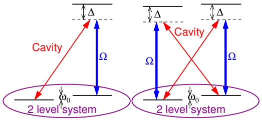

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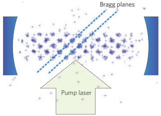

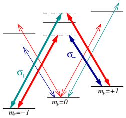

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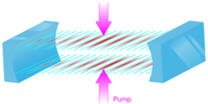

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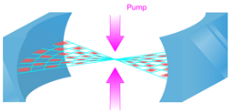

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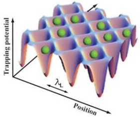
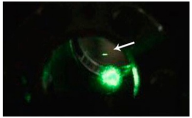
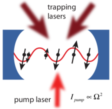
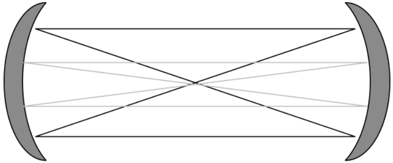




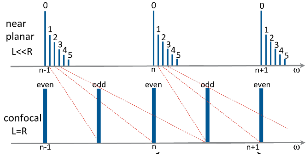


Multimode cavities

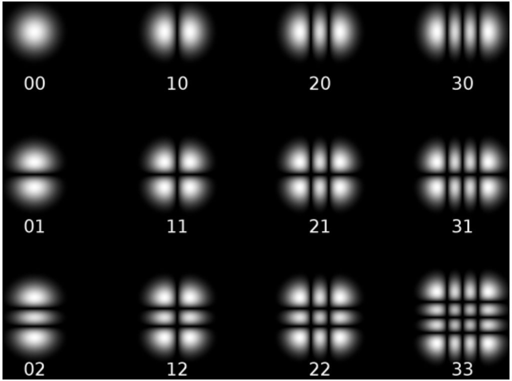
Confocal cavity $L = R$:



- Modes $\Xi_{l,m}(\mathbf{r}) = H_l(x)H_m(y)$, $l + m$ fixed parity



- Extra distinction:
degenerate vs non-degenerate



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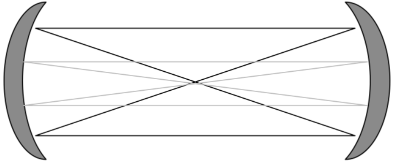

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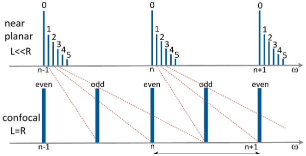

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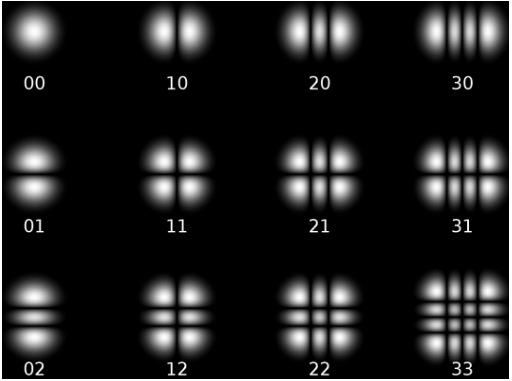
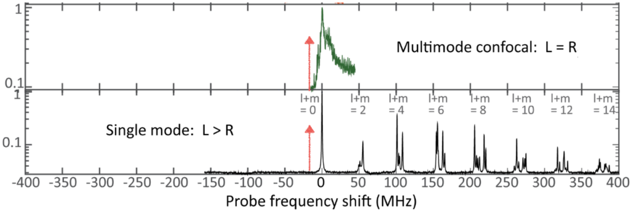
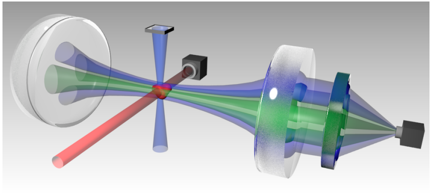




Multimode cavity QED



Hyperfine states:

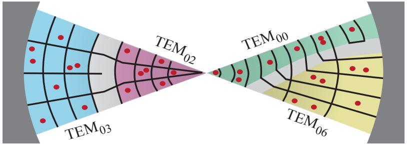
- Full model:

$$H_{\text{eff}} = \sum_{\mu} \underbrace{(\omega_\mu - \omega_P)}_{-\Delta_\mu} a^\dagger_\mu a_\mu + \sum_N \frac{\omega_0}{2}\sigma^z_n + \underbrace{\frac{\Omega g_0}{\Delta}}_{g_{\text{eff}}} \sum_\mu \Xi_\mu(\mathbf{r}_n)\sigma^x_n (a + a^\dagger)$$

[Gopalakrishnan, Lev, Goldbart. Nat. Phys '09, PRA '10]

- Can reach $\delta\Delta_\mu < g_{\text{eff}}$

Degenerate multimode: Liquid crystal physics



- **Spatial states** of atoms
  $\psi(\mathbf{r}) = \psi_{\Downarrow}(\mathbf{r}) + \psi_{\Uparrow}(\mathbf{r}) \cos(kx) \cos(kz)$
- Coupled dynamics of $\alpha(\mathbf{r}) = \sum_\mu \langle \hat{a}_\mu \rangle \Xi_\mu(\mathbf{r})$, and $\psi_{0,1}(\mathbf{r})$
- Degenerate limit, transverse pump:

$$i\partial_t \Psi_{\mathbf{k}} = \left[\Delta + \lambda(|\mathbf{k}| - q)^2\right] \Psi_{\mathbf{k}} + U_{\text{contact}} \sum_{\mathbf{k}',\mathbf{q}} \Psi^*_{\mathbf{k}'+\mathbf{q}} \Psi_{\mathbf{k}'} \Psi_{\mathbf{k}-\mathbf{q}}$$

- Smectic Brazovskii transition

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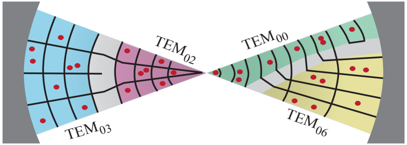

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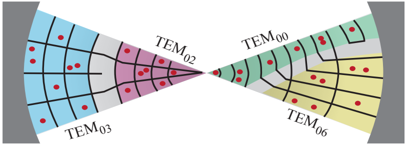

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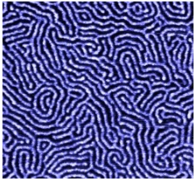

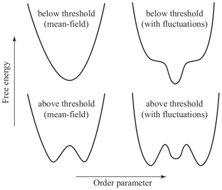

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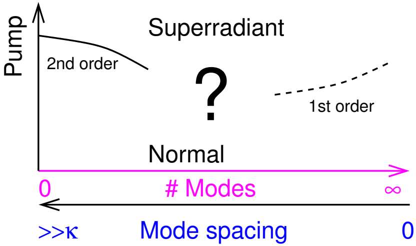
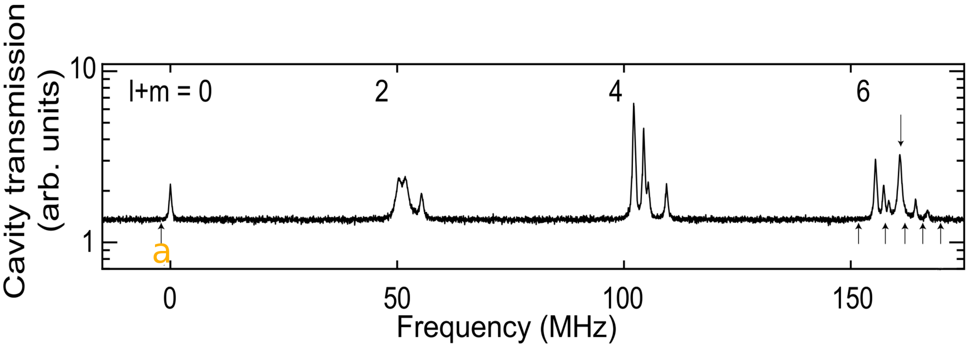
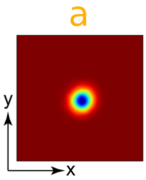


Experimental results: supermode density wave polariton condensation

Superradiance in multimode cavity: Even family





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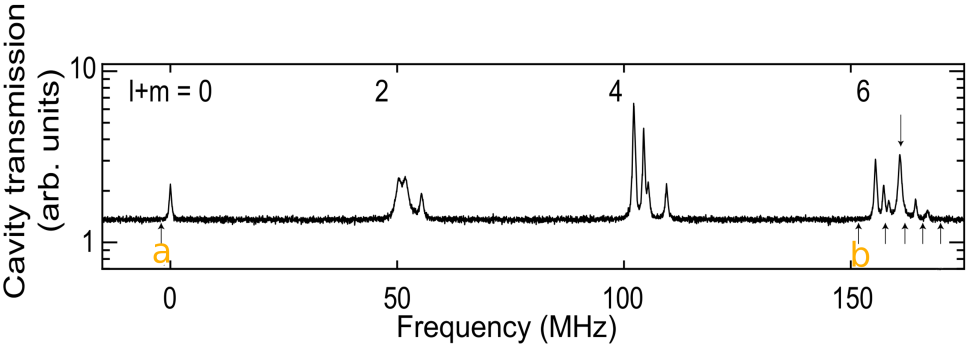

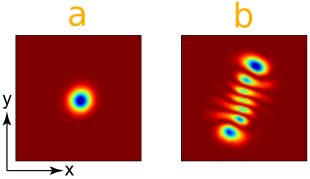

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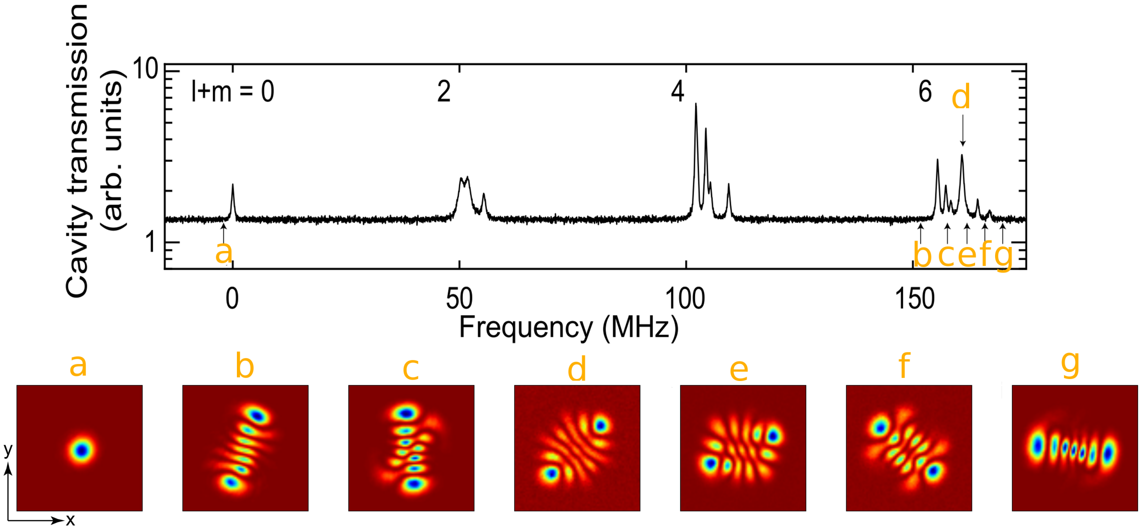
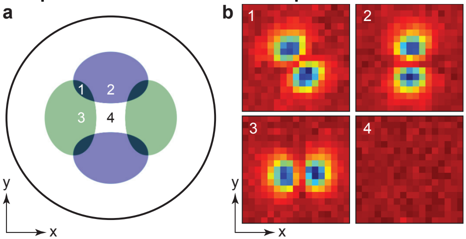


Superradiance in multimode cavity: Odd family

- Atomic time-of-flight — structure factor

- Dependence on cloud position



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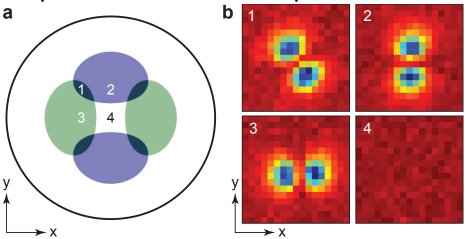

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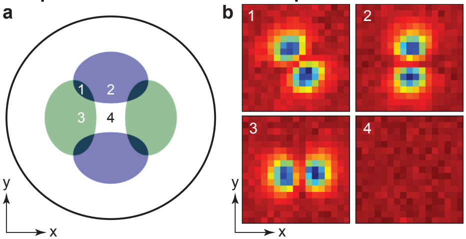

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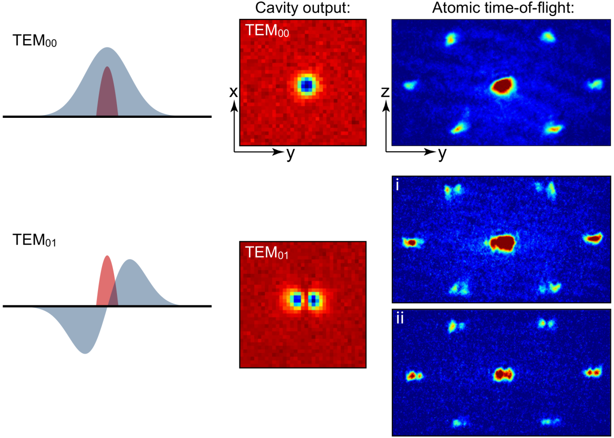

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Supermode density-wave polariton condensation

Supermode density-wave polariton:

- Hybrid cavity photon and atomic density wave
- Atoms remix cavity modes $\rightarrow$ superposition
- Condensation of polaritons remixes again

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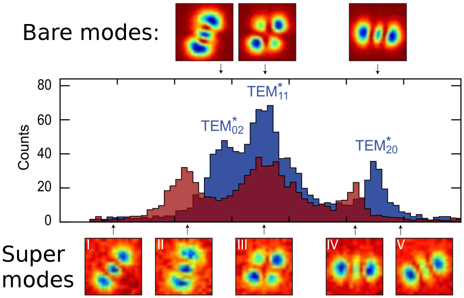

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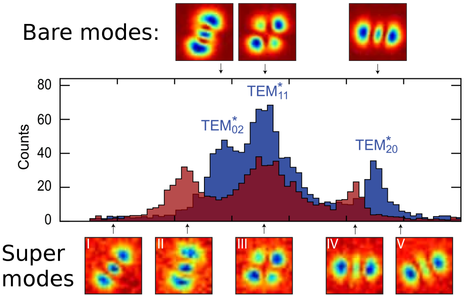
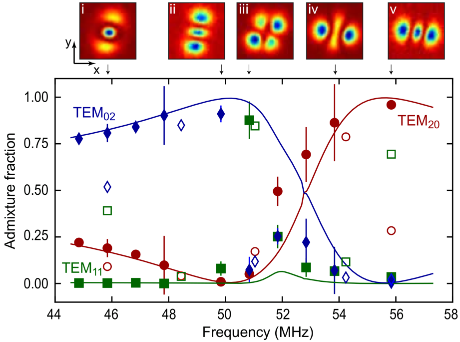
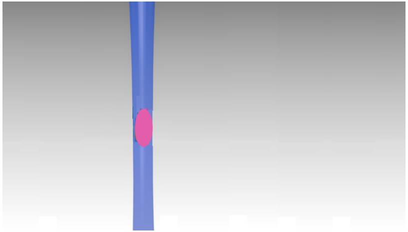




Theoretical results: Meissner-like effect

Meissner-like physics: idea

- Multimode cQED $\rightarrow$ local matter-light coupling
- Variable profile synthetic gauge field?
- Raman-based scheme



- Follow Spielman, PRA '09.

$$H = \begin{pmatrix} \psi_A & \psi_B \end{pmatrix} \begin{pmatrix} E_a + (\nabla - Q\hat{x})^2 & \Omega \\ \Omega & E_b + (\nabla + Q\hat{x})^2 \end{pmatrix} \begin{pmatrix} \psi_A \\ \psi_B \end{pmatrix}$$

- New feature: E_A, E_B from cavity-light Stark shift
- Ground state ψ_-, $\quad E_-(\mathbf{k}) = (\mathbf{k} - Q\hat{x}|\varphi|^2)^2 + \ldots$
- Reciprocity: matter affects field

See poster by Kyle Ballantine [Ballantine *et al.* arXiv:1608.07246]

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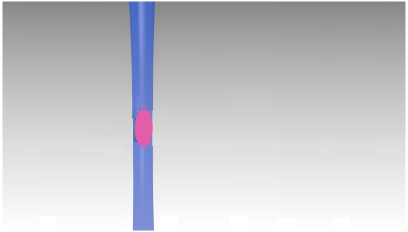
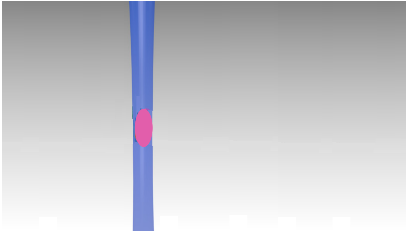


See poster by Kyle Ballantine [Ballantine *et al.* arXiv:1608.07246]

Meissner-like physics: idea



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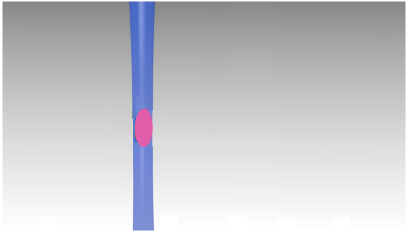

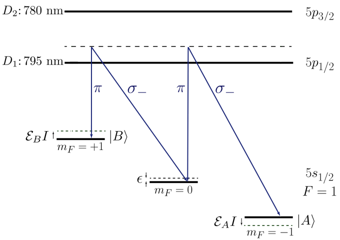

- Follow Spielman, PRA ’09.

$$H = \begin{pmatrix}\psi_A & \psi_B\end{pmatrix} \begin{pmatrix} E_a + (\nabla - Q\hat{x})^2 & \Omega \\ \Omega & E_b + (\nabla + Q\hat{x})^2 \end{pmatrix} \begin{pmatrix}\psi_A \\ \psi_B\end{pmatrix}$$

- New feature: E_A, E_B from cavity-light Stark shift
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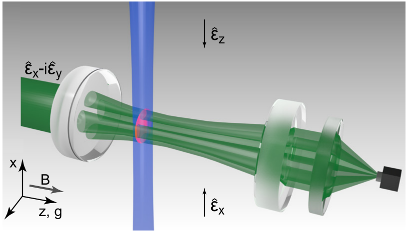

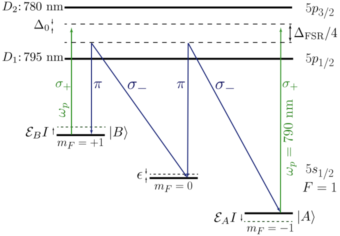

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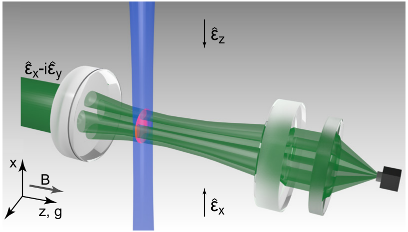

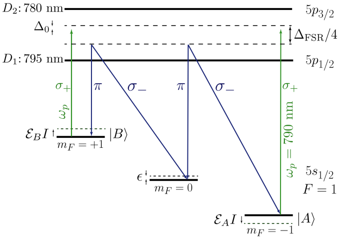

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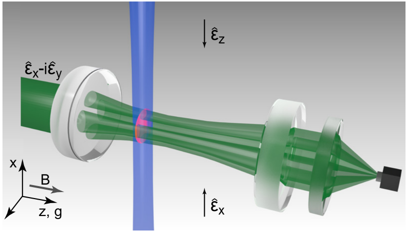

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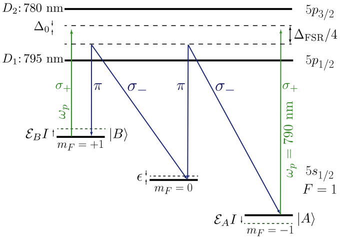

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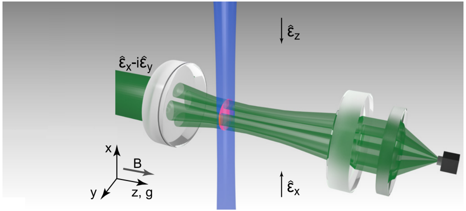
[Ballantine *et al.* arXiv:1608.07246]

Meissner-like physics: setup



- Atoms: $$i\partial_t \begin{pmatrix} \psi_A \\ \psi_B \end{pmatrix} = \left[-\frac{\nabla^2}{2m} + \begin{pmatrix} -\mathcal{E}_\Delta|\varphi|^2 + i\frac{q}{m}\partial_x & \Omega/2 \\ \Omega/2 & \mathcal{E}_\Delta|\varphi|^2 - i\frac{q}{m}\partial_x \end{pmatrix} + \ldots \right] \begin{pmatrix} \psi_A \\ \psi_B \end{pmatrix}.$$
- Light: $$i\partial_t \varphi = \left[\frac{\delta}{2}\left(-l^2\nabla^2 + \frac{r^2}{l^2} \right) - \Delta_0 - i\kappa - N\mathcal{E}_\Delta(|\psi_A|^2 - |\psi_B|^2) \right] \varphi + f(\mathbf{r}).$$

See poster by Kyle Ballantine [Ballantine *et al.* arXiv:1608.07246]

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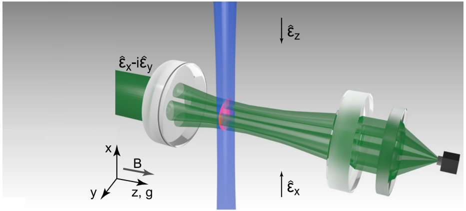

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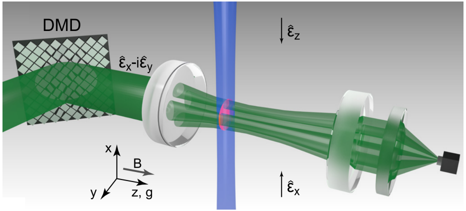

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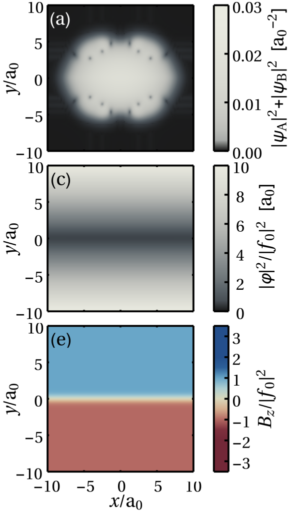
Meissner-like physics: numerical simulations

Atoms

Cavity light

Synthetic field



- Consider $f(\mathbf{r})$ such that $|\varphi|^2 \propto y$.
- Without feedback ($\mathcal{E}_\Delta = 0$) for field
- With feedback

See poster by Kyle Ballantine [Ballantine *et al.* arXiv:1608.07246]

Meissner-like physics: numerical simulations

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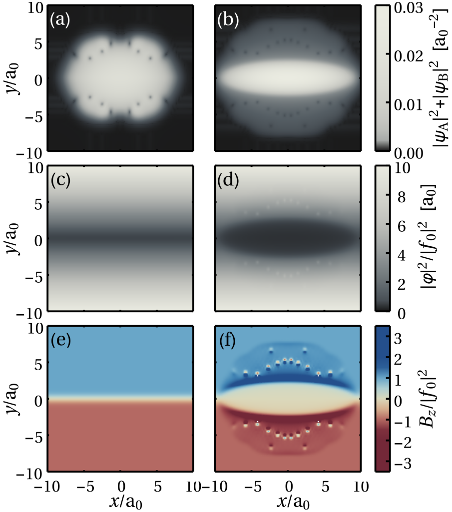

- Consider $f(\mathbf{r})$ such that $|\varphi|^2 \propto y$.
- Without feedback ($\mathcal{E}_\triangle = 0$) for field
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  - Field expelled
  - Cloud shrinks

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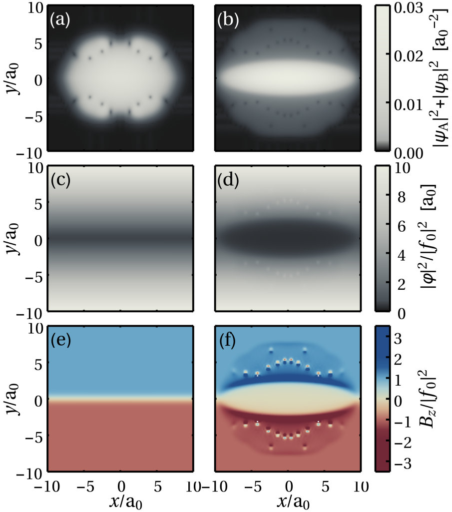

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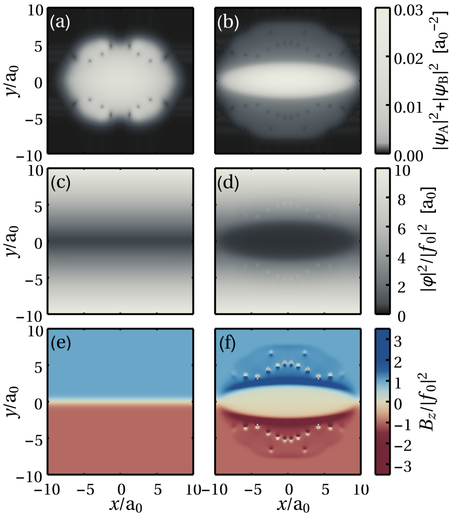

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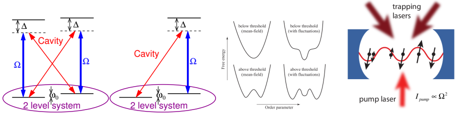
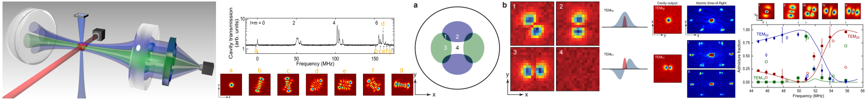
Summary

- Many possibilities of multimode cavity QED
  - Spin glass (XY/Ising); Soft-matter physics with spatial DoF, ...



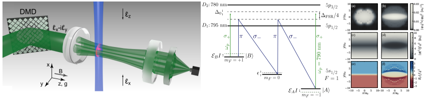
[Gopalakrishnan, Lev and Goldbart. PRL '11, Phil. Mag. '12, Nat. Phys '09, PRA '10]

- Working multimode cavity [Kollár, *et al.* NJP '15]
- Supermode density-wave polariton condensation



[Kollár *et al.* arXiv:1606.04127]

- Meissner like effect



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