Supermode-density-wave-polariton condensation, and Meissner-like effect with multimode cavity-QED

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FOUNDED 1413

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Acknowledgments

Experiment (Stanford): Benjamin Lev

Theory:

Ben Simons (Cambridge), Joe Bhaveen (KCL), James Mayoh (Southampton)

Sarang Gopalakrishnan (CUNY)
Surya Ganguli, Jordan Cotler (Stanford)
Laura Staffini, Kyle Ballantine (St Andrews)

The Leverhulme Trust
Introduction: Tunable multimode Cavity QED
- Many body cavity QED
- Multimode cavity QED

Experimental results: supermode density wave polariton condensation

Theoretical results: Meissner-like effect
(Multimode) cavity QED

\[
H = \sum_k \omega_k a_k^\dagger a_k + \sum_n \omega_0 \sigma_n^+ \sigma_n^- + \sum_{n,k} g_{k,n} (a_k^\dagger + a_{-k})(\sigma_n^+ + \sigma_n^-)
\]

\[
\dot{\rho} = -i[H, \rho] + \kappa \sum_k \mathcal{L}[a_k, \rho] + \gamma \sum_i \mathcal{L}[\sigma_i^-, \rho]
\]

Compare \( g \) (or \( g \sqrt{N} \)) vs:

- \( \kappa \)
- \( \gamma \)
- \( \omega_k, \omega_0 \)
(Multimode) cavity QED

\[ H = \sum_k \omega_k a_k^{\dagger} a_k + \sum_n \omega_0 \sigma_n^{+} \sigma_n^{-} + \sum_{n,k} g_{k,n} (a_k^{\dagger} + a_{-k}) (\sigma_n^{+} + \sigma_n^{-}) \]

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- Compare \( g \) (or \( g\sqrt{N} \)) vs:
  - \( \kappa, \gamma \)
  - bandwidth
  - \( \omega_k, \omega_0 \)
Synthetic cavity QED: Raman driving

- Tunable coupling via Raman

Real systems: loss $\frac{\partial}{\partial t} \rho = -i[H, \rho] + \kappa \mathcal{L} [a, \rho] + \ldots$

To balance loss, counter-rotating:

$$H_{\text{eff}} = \ldots \frac{\Omega g}{\Delta} (\sigma_n^+ a + \text{H.c.})$$

[Dimer et al. PRA '07]
Synthetic cavity QED: Raman driving

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\[
H_{\text{eff}} = \ldots \frac{\Omega g}{\Delta} \sigma^x_n (a + a^\dagger)
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[Dimer et al. PRA ’07]
Possibilities

- XY vs Ising

- Momentum state vs hyperfine state

- Single mode vs multimode

- Thermal gas vs BEC vs disorder localised
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Possibilities

- **XY vs Ising**
  ![XY vs Ising Diagram]

- **Momentum state vs hyperfine state**
  ![Momentum state vs hyperfine state Diagram]

- **Single mode vs multimode**
  ![Single mode vs multimode Diagram]

- **Thermal gas vs BEC vs disorder localised**

Credit: Alan Stonebreaker, Physics 3, BB [201.0]
Possibilities

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Credit: Allan Stonebreaker, Physics 3, 88 (2010)
Multimode cavities

Confocal cavity $L = R$:

Modes $\Xi_{l,m}(\mathbf{r}) = H_l(x)H_m(y)$, $l + m$ fixed parity

Extra distinction: degenerate vs non-degenerate
Multimode cavities

Confocal cavity $L = R$:

- Modes $\Xi_{l,m}(r) = H_l(x)H_m(y)$, $l + m$ fixed parity

- Extra distinction: degenerate vs non-degenerate
Multimode cavity QED

Hyperfine states:

- Full model:

\[
H_{\text{eff}} = \sum_{\mu} (\omega_{\mu} - \omega_P) a_{\mu}^\dagger a_{\mu} + \sum_{N} \frac{\omega_0}{2} \sigma^z_n + \frac{\Omega g_0}{\Delta} \sum_{\mu} \Xi_{\mu}(r_n) \sigma^x_n(a + a^\dagger)
\]

[Gopalakrishnan, Lev, Goldbart. Nat. Phys '09, PRA '10]

- Can reach \( \delta \Delta_{\mu} < g_{\text{eff}} \)
Degenerate multimode: Liquid crystal physics

- **Spatial states** of atoms
  \[ \psi(r) = \psi_\downarrow(r) + \psi_\uparrow(r) \cos(kx) \cos(kz) \]
- Coupled dynamics of
  \[ \alpha(r) = \sum_\mu \langle \hat{a}_\mu \rangle \Xi_\mu(r) \]
  and \[ \psi_{0,1}(r) \]

- Degenerate limit, transverse pump:
  \[ i \hbar \frac{\partial}{\partial t} \Psi_k = \left[ \Delta + \lambda(|k| - q)^2 \right] \Psi_k + U_{\text{contact}} \sum_{k',q} \Psi^*_k \Psi_{k'+q} \Psi_{k+q} \]

- Smectic Brazovski transition

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[Jonathan Keeling] Multimode cavity QED
Experimental results: supermode density wave polariton condensation

1 Introduction: Tunable multimode Cavity QED
   - Many body cavity QED
   - Multimode cavity QED

2 Experimental results: supermode density wave polariton condensation

3 Theoretical results: Meissner-like effect
Superradiance in multimode cavity: Even family
Superradiance in multimode cavity: Even family

![Graph showing cavity transmission vs frequency](image)

- **a**
- **b**

![Images of polarizations](image)
Superradiance in multimode cavity: Even family

![Graph and images showing superradiance in multimode cavity](image)

- **a**: Cavity transmission (arb. units)
- **b** to **g**: Frequency (MHz)
Superradiance in multimode cavity: Odd family

- Dependence on cloud position

- Near-degeneracy of $(1, 0), (0, 1)$ modes broken by matter-light coupling.
Superradiance in multimode cavity: Odd family

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Atomic time-of-flight — structure factor

Near-degeneracy of $(1, 0), (0, 1)$ modes broken by matter-light coupling.
Supermode density-wave polariton condensation

Supermode density-wave polariton:
- Hybrid cavity photon and atomic density wave
- Atoms remix cavity modes → superposition
- Condensation of polaritons remixes again
Supermode density-wave polariton condensation

Supermode density-wave polariton:
- Hybrid cavity photon and atomic density wave
- Atoms remix cavity modes → superposition

Condensation of polaritons remixes again

Bare modes:

Super modes:

Counts
Supermode density-wave polariton condensation

Supermode density-wave polariton:
- Hybrid cavity photon and atomic density wave
- Atoms remix cavity modes $\rightarrow$ superposition
- Condensation of polaritons remixes again

Bare modes:

Super modes

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<td>iv</td>
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![Graph showing counts and admixture fraction](image)

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Multimode cavity QED

DOQS, 2016 13
Theoretical results: Meissner-like effect

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Meissner-like physics: idea

- Multimode cQED → local matter-light coupling
- Variable profile synthetic gauge field?
- Raman-based scheme

▶ Follow Spielman, PRA ’09.

\[ H = (\psi_A, \psi_B) \left( E_a + (\nabla - Q\hat{x})^2 \right) \left( \frac{\Omega}{E_b + (\nabla + Q\hat{x})^2} \right) \left( \psi_A \psi_B \right) \]

▶ New feature: \( E_A, E_B \) from cavity-light Stark shift
▶ Ground state \( \psi \): \( E_g(k) = (k - Q\hat{x}|\phi|^2)^2 + \ldots \)
▶ Reciprocity: matter affects field

Meissner-like physics: idea

- Multimode cQED → local matter-light coupling
- Variable profile synthetic gauge field?
- Raman-based scheme

Follow Spielman, PRA '09.

\[ H = \left( \psi_A \hspace{1em} \psi_B \right) \left( \begin{array}{cc} E_A + (\nabla - Q\hat{x})^2 & O \\ O & E_B + (\nabla + Q\hat{x})^2 \end{array} \right) \left( \begin{array}{c} \psi_A \\ \psi_B \end{array} \right) \]

- New feature: \( E_A, E_B \) from cavity-light Stark shift
- Ground state \( \psi_-, \quad E_-(k) = (k - Q\hat{x}|\phi|^2)^2 + \ldots \)
- Reciprocity: matter affects field

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Follow Spielman, PRA '09.

\[ H = (\psi_A \psi_B) \left( \frac{E_A + (\nabla - Q\hat{x})^2}{\Omega} \psi_A \right) \left( \frac{E_B + (\nabla + Q\hat{x})^2}{\Omega} \psi_B \right) \]

- New feature: \( E_A, E_B \) from cavity-light Stark shift
- Ground state \( \psi_{\text{gs}} \):
  \[ E_{\text{gs}}(k) = (k - Q\hat{x}|\psi_{\text{gs}}|^2)^2 + \ldots \]
- Reciprocity: matter affects field

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Follow Spielman, PRA ’09.

\[ H = \left( \begin{array}{cc} \psi_A & \psi_B \end{array} \right) \left( \begin{array}{cc} E_a + (\nabla - Q\hat{x})^2 & \Omega \\ \Omega & E_b + (\nabla + Q\hat{x})^2 \end{array} \right) \left( \begin{array}{c} \psi_A \\ \psi_B \end{array} \right) \]

- New feature: \( E_A, E_B \) from cavity-light Stark shift
- Ground state \( \psi^\prime \), \( \psi \cdot \psi^\prime = (k - Q\hat{x}|\phi|^2)^2 + \ldots \)
- Reciprocity: matter affects field

Meissner-like physics: idea

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- Follow Spielman, PRA '09.

\[ H = \begin{pmatrix} \psi_A & \psi_B \end{pmatrix} \begin{pmatrix} E_a + (\nabla - Q\hat{x})^2 & \Omega \\ \Omega & E_b + (\nabla + Q\hat{x})^2 \end{pmatrix} \begin{pmatrix} \psi_A \\ \psi_B \end{pmatrix} \]

- New feature: $E_A, E_B$ from cavity-light Stark shift

- Ground state $\psi_{-1} = E_{-1}(k) = (k - Q\hat{x})^{5s_1/2} + \cdots$

- Reciprocity: matter affects field

See poster by Kyle Ballantine

[Ballantine et al. arXiv:1608.07246]
Meissner-like physics: idea

- Multimode cQED $\rightarrow$ local matter-light coupling
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$$H = \begin{pmatrix} \psi_A & \psi_B \end{pmatrix} \begin{pmatrix} E_a + (\nabla - Q\hat{x})^2 & \Omega \\ \Omega & E_b + (\nabla + Q\hat{x})^2 \end{pmatrix} \begin{pmatrix} \psi_A \\ \psi_B \end{pmatrix}$$

- New feature: $E_A, E_B$ from cavity-light Stark shift
- Ground state $\psi_-, E_-(\mathbf{k}) = (\mathbf{k} - Q\hat{x}|\phi|^2)^2 + \ldots$

Reciprocity: matter affects field

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$$ H = \begin{pmatrix} \psi_A \\ \psi_B \end{pmatrix} \left( E_a + (\nabla - Q\hat{x})^2 \right)^\Omega E_b + (\nabla + Q\hat{x})^2 \right) \begin{pmatrix} \psi_A \\ \psi_B \end{pmatrix} $$

- New feature: $E_A, E_B$ from cavity-light Stark shift
- Ground state $\psi_-$, $E_-(\mathbf{k}) = (\mathbf{k} - Q\hat{x})|\varphi|^2 + \ldots$
- Reciprocity: matter affects field

See poster by Kyle Ballantine

[Ballantine et al. arXiv:1608.07246]
Meissner-like physics: setup

Atoms: 
\[ i \partial_t \begin{pmatrix} \psi_A \\ \psi_B \end{pmatrix} = \left[ -\frac{\nabla^2}{2m} + \left( -\mathcal{E}_\Delta |\varphi|^2 + \frac{i q}{m} \partial_x \right) \frac{\Omega}{2} \right. \left. - \mathcal{E}_\Delta |\varphi|^2 - i \frac{q}{m} \partial_x \right] \begin{pmatrix} \psi_A \\ \psi_B \end{pmatrix} + \ldots \]

Light: 
\[ i \partial_t \varphi = \left[ \frac{\hbar}{2} \left( -\hbar^2 \nabla^2 + \frac{\hbar^2}{4} \right) - \Delta_0 - i \kappa - N\mathcal{E}_\Delta (|\psi_A|^2 - |\psi_B|^2) \right] \varphi + f(t). \]

Meissner-like physics: setup

Atoms: \[ i \frac{\partial}{\partial t} \begin{pmatrix} \psi_A \\ \psi_B \end{pmatrix} = \left[ -\frac{\nabla^2}{2m} + \left( \frac{-\mathcal{E}_\Delta |\varphi|^2 + i \frac{q}{m} \partial_x}{\Omega/2} \right) \mathcal{E}_\Delta |\varphi|^2 - i \frac{q}{m} \partial_x \right] \begin{pmatrix} \psi_A \\ \psi_B \end{pmatrix} + \cdots \]

Light: \[ i \frac{\partial}{\partial t} \varphi = \left[ \frac{\delta}{2} \left( -\frac{l^2}{r^2} + \frac{r^2}{l^2} \right) - \Delta_0 - i \kappa - N \mathcal{E}_\Delta (|\psi_A|^2 - |\psi_B|^2) \right] \varphi + f(r). \]

Meissner-like physics: setup

Atoms: \[ i \frac{\partial}{\partial t} \begin{pmatrix} \psi_A \\ \psi_B \end{pmatrix} = \begin{bmatrix} -\nabla^2/2m + \left( -\varepsilon_\Delta |\varphi|^2 + i \frac{q_0}{m} \partial_x \right) \frac{\Omega}{2} & \varepsilon_\Delta |\varphi|^2 - i \frac{q_0}{m} \partial_x \\ \frac{\Omega}{2} & \end{bmatrix} \begin{pmatrix} \psi_A \\ \psi_B \end{pmatrix} + \ldots \] 

Light: \[ i \frac{\partial}{\partial t} \varphi = \begin{bmatrix} \frac{\delta}{2} \left( -l^2 \nabla^2 + \frac{r^2}{l^2} \right) & -\Delta_0 - i \kappa - N\varepsilon_\Delta (|\psi_A|^2 - |\psi_B|^2) \end{bmatrix} \varphi + f(r). \]

Meissner-like physics: numerical simulations

Atoms

Cavity light

Synthetic field

Consider $f(r)$ such that $|\phi|^2 \propto y$.

Without feedback ($\varepsilon_\Delta = 0$) for field

With feedback

Meissner-like physics: numerical simulations

Atoms

Cavity light

Synthetic field

Consider $f(r)$ such that $|\varphi|^2 \propto y$.

Without feedback ($\mathcal{E}_\Delta = 0$) for field

With feedback

- Field expelled
- Cloud shrinks

Meissner-like physics: numerical simulations

- Consider $f(r)$ such that $\varphi^2 \propto y$.
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Meissner-like physics: numerical simulations

Atoms

Cavity light

Synthetic field

- Consider $f(r)$ such that $|\varphi|^2 \propto y$.
- Without feedback ($\mathcal{E}_\Delta = 0$) for field
- With feedback
  - Field expelled
  - Cloud shrinks

Summary

- Many possibilities of multimode cavity QED
  - Spin glass (XY/Ising); Soft-matter physics with spatial DoF, . . .

  [Gopalakrishnan, Lev and Goldbart. PRL ’11, Phil. Mag. ’12, Nat. Phys ’09, PRA ’10]

- Working multimode cavity [Kollár, et al. NJP ’15]

- Supermode density-wave polariton condensation

  [Kollár et al. arXiv:1606.04127]

- Meissner like effect
