From Lasers to Bose-Einstein condensates
How superfluids, superconductors, polaritons and lasers fit together

Jonathan Keeling

Stokes Society, November 2008
The two-slit experiment
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Why no such interference for macroscopic objects?

- Wavelength would be very small $\lambda \sim 1/\sqrt{m}$
- Internal degrees of freedom remember "which way"
- Different initial conditions wash out path.
Macroscopic objects

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The two-slit experiment with condensates
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All atoms in single quantum state — like a classical wave.
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Overview

1. Particles and waves
   - The two-slit experiment with atoms
   - History of quantum condensates

2. Signatures of macroscopic occupation
   - Superfluidity
   - Superconductivity

3. Why low temperature

4. What about Lasers

5. Polaritons
   - What are excitons, polaritons,
   - What do they do
   - Why (else) are they interesting
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What is superfluidity

\[ \text{Pressure} \quad \text{Super-fluid} \quad \text{Solid} \quad \text{Liquid} \quad \text{Gas} \quad \text{Temperature} \quad 1 \text{atm} \quad 3 \text{K} \]
What is superfluidity

\[ \frac{dH}{dt} \]

Viscous

Superfluid
Solid
Liquid
Gas

Temperature
1 atm
3 K

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What is superfluidity

Superfluid
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Stokes Society 8 / 28
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1. Macroscopic occupation of single wavefunction
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\textbf{But:} single wavefunction \( \Psi \)
\[ \rho \vec{v} = \Psi^\dagger i\hbar \nabla \Psi = |\Psi|^2 \nabla \phi \]
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In moving frame, \( E(\vec{p}) \rightarrow E(\vec{p}) + \vec{p} \cdot \vec{v} + \frac{1}{2} Mv^2 \)
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Normal state:
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Energy vs. Momentum diagram
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![Energy vs. Momentum Graph]
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Superconductor

Why superconductivity:
- Fermions and macroscopic occupation

- Probabilities: $\psi \rightarrow P = |\psi|^2$

- $\psi(r_1, r_2) = \pm \psi(r_2, r_1)$

- + Bosons (Helium, Polaritons, Photons)
- - Fermions (Electrons)
- Cannot occupy same state
Superconductor

Why superconductivity:
- Fermions and macroscopic occupation
- How does this change conductivity

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- $\Psi(r_1, r_2) = \pm \Psi(r_2, r_1)$
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\[ \psi_0 \left( \frac{r_1 + r_2}{2} \right) \Phi (r_1 - r_2) \]

Macroscopically occupy pair state:

\[ \prod_{i \neq j} \Phi (r_i - r_j) \]
Scattering and conductivity

Normally conductivity disrupted by disorder:

Disorder

\[ \text{Disorder} \]

\[ k_1 \]

\[ k_2 \]

\[ k_1 \]

\[ k_2 \]

\[ k_y \]

\[ k_x \]

\[ \text{Normal:} \]

Energy

Momentum

DOS

\[ \text{Superconducting:} \]

Energy

Momentum

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Why only at low temperatures

Temperature populates excitations – Depletes condensate

- Superconductors — electron pairs break apart
  - Mercury (first experiment) 4K.
  - Record (at $P = 1$ atm) 138K.

- Helium, cold atoms — populate low momentum excitations

- Chemical potential at bottom of band:

$$k_B T_c = \frac{\hbar^2}{2m} \left( \frac{n}{2.612} \right)^{2/3}$$

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- Atoms: $n = 10^{10}\text{ cm}^{-3}$ (this room, $\sim 10^{20}\text{ cm}^{-3}$) $T_c \approx 10^{-6}\text{ K}$
- Helium 2.17K — liquid, so density unchangable.
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What about Lasers

- Other condensates show:
  - Particle-like $\rightarrow$ wave-like
- Light is normally wave-like.

- But Thermal radiation $\rightarrow$ many frequencies; fringe patterns wash out.
- Laser provides light at single frequency
- Macroscopic occupation of mode.
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Origin of coherence

\[ \frac{\partial}{\partial t} \langle n \rangle = 2r g^2 \gamma^2 - \kappa \langle n \rangle + 2r g^2 \gamma^2 - 8r (g^2 \gamma^2) \langle (n+1)^2 \rangle. \]
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Origin of coherence

Balance of gain and loss:
\[ \partial_t \langle n \rangle = \left[ 2r g - \kappa \right] \langle n \rangle + 2r g - \frac{8r (g^2 \gamma^2)}{2 \langle (n+1)^2 \rangle} . \]
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Laser spectrum

- Modes defined by cavity
- At threshold, all emission → single mode.
- Linewidth and threshold controlled by gain/loss.
- Weak nonlinearity — spectrum unchanged.

Cavity modes

Atomic emission (gain)

\( \omega \)
Laser spectrum

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Below threshold
Spontaneous emission
Laser spectrum

Above threshold
Lasing

Cavity modes

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Excitons: quasiparticles in semiconductors
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Electrostatic attraction:
- Bound state.
Excitons: quasiparticles in semiconductors

Electrostatic attraction: Bound state.

Semiconductor 1
Semiconductor 2

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Excitons: quasiparticles in semiconductors

Electrostatic attraction: Bound state.

Extra holes:
Filled states
Empty states
Extra electrons:
Height

Semiconductor 1
Semiconductor 2

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Excitons: quasiparticles in semiconductors

Energy

Momentum

Holes

Electrostatic attraction:
Bound state.

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Excitons: quasiparticles in semiconductors

Electrostatic attraction: Bound state.

Energy
Momentum
Holes

Semiconductor 1
Semiconductor 2
Semiconductor 1

Filled states
Empty states

Height
Energy

(extra holes)
(extra electrons)
Microcavity Polaritons

![Diagram of Microcavity Polaritons]

Ph \quad Ex

Quantum Wells - Cavity

Cavity photons:

\[ \omega_k = \sqrt{\omega_0^2 + c^2 k^2} \approx \omega_0 + \frac{k^2}{2m^*} \sim 10^{-4} m_e \]

Energy - Momentum

\( n = 1 \)
\( n = 2 \)
\( n = 3 \)
\( n = 4 \)

Bulk Energy - Momentum
Microcavity Polaritons

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Why polaritons

\[ n \text{ [cm}^{-2}\text{]} \]

\[ k_B T \]

Cavity

\[ \theta \]

\[ \Phi \]

\[ \text{Ex} \]

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Why polaritons

![Graph showing condensed and non-condensed states of polaritons]

$n \text{ [cm}^{-2}\text{]}$ vs. $k_B T$

Cavity

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Why polaritons

![Graph showing condensed and non-condensed states with respective plots of $n$ vs. $k_B T$.]
Polariton experiments: Momentum/Energy distribution

![Graphs and images illustrating momentum and energy distribution of polaritons.](image-url)

- **Graph a**: Temperature dependence of polariton emission at 5 K.
- **Graph b**: Occupancy distribution with varying energy levels and in-plane wavevector.

**Equations and Notations**:
- $T = 5\, \text{K}$
- $\theta$: Emission angle (degree)
- Energy (meV) and Occupancy
- In-plane wavevector ($10^4\, \text{cm}^{-1}$)

**Legend**:
- $P/P_{\text{thr}}$ with various values indicating different experimental conditions.

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23 / 28
Polariton experiments: Coherence

Basic idea:

- Tunable Splitter
- Beam Splitter
- CCD
- Retroreflector
- Sample
- Tunable Delay

Coherence map:

![Graph showing normalized signal vs. phase (Rad)]

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Polariton experiments: Coherence

Basic idea:

Coherence map:
Non-equilibrium condensation

- Condensate ↔ Laser.

- What kinds of coherence out of equilibrium?
- What happens to superfluidity?
Non-equilibrium condensation

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Non-equilibrium condensate in a trap

\[ \frac{3\gamma_{\text{net}}}{2\Gamma} \]
Non-equilibrium condensate in a trap

\[ \frac{3 \gamma_{\text{net}}}{2 \Gamma} \]

\[ 0 \quad 2 \quad 4 \quad 6 \quad 8 \quad \text{Radius} \]

\[ 0 \quad 5 \quad 10 \quad 15 \quad 20 \quad 25 \quad 30 \quad \text{Density} \]
Non-equilibrium condensate in a trap

\[ \frac{3\gamma_{\text{net}}}{2\Gamma} \]

Unstable growth

\[ 0 \quad 2 \quad 4 \quad 6 \quad 8 \] Radius

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Non-equilibrium condensate in a trap

\[ \frac{3\gamma_{\text{net}}}{2\Gamma} \]

Stabilised

\[ 0 \quad 2 \quad 4 \quad 6 \quad 8 \quad \text{Radius} \]

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Non-equilibrium condensate in a trap

\[ \frac{3\gamma_{\text{net}}}{2\Gamma} \]

Density

Radius

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Time evolution:
Why change of excitations?

Macroscopic occupation of $\Psi$:

\[ \{N \text{ in } \Psi\} \rightarrow \{(N - 2) \text{ in } \Psi, +\vec{k}, -\vec{k}\} \]

![Energy vs. Momentum Diagram]
Why change of excitations?

Macroscopic occupation of $\Psi$:

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Number of excitations not fixed
Why change of excitations?

Macroscopic occupation of $\Psi$:

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Number of **excitations** not fixed
Non-equilibrium theory; fluctuations

Approach transition, Gap Equation/Hugenholtz-Pines relation:

\[
\mu_s + i\kappa = \chi(\psi_0 = 0, \mu_s) \Leftrightarrow G^{-1}(\omega = \mu_S, k = 0) = 0
\]
Non-equilibrium theory; fluctuations

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Before transition:

\[
G^{-1}(\omega, k) = (\omega - \omega_k^*) + i\alpha
\]

[Szymańska et al., PRL ’06; PRB ’07]
Non-equilibrium theory; fluctuations

Approach transition, Gap Equation/Hugenholtz-Pines relation:

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$$G^{-1}(\omega, k) = (\omega - \omega_k^*) + i\alpha(\omega - \mu_{\text{eff}})$$

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[Szymańska et al., PRL ’06; PRB ’07]
Fluctuations above transition

When condensed

\[ G^{-1}(\omega, k) = \omega^2 - c^2 k^2 \]

Poles:

\[ \omega^* = c |k| \]

[Szymańska et al., PRL ’06; PRB ’07]
Fluctuations above transition

When condensed

\[ G^{-1}(\omega, k) = (\omega + i\lambda)^2 + \chi^2 - c^2 k^2 \]

Poles:

\[ \omega^* = -i\lambda \pm \sqrt{c^2 k^2 - \chi^2} \]

[Szymańska et al., PRL '06; PRB '07]
Fluctuations above transition

When condensed

\[ G^{-1}(\omega, k) = (\omega + ix)^2 + x^2 - c^2 k^2 \]

Poles:

\[ \omega^* = -ix \pm \sqrt{c^2 k^2 - x^2} \]

Correlations (in 2D):

\[ \langle \psi^\dagger(r, t)\psi(0, 0) \rangle \simeq |\psi_0|^2 \exp \left[ -\eta \begin{cases} \ln(r/\xi) & r \to \infty, t \simeq 0 \\ \frac{1}{2} \ln(c^2 t/x\xi^2) & r \simeq, t \to \infty \end{cases} \right] \]

[Szymańska et al., PRL '06; PRB '07]