Polariton condensation, beyond the weakly interacting Bose gas.

Jonathan Keeling$^1$
N. G. Berloff$^1$, P. R. Eastham$^1$, P. B. Littlewood$^1$, F. M. Marchetti$^2$, M. H. Szymańska$^1$

$^1$University of Cambridge, $^2$University of Oxford

July 24$^{th}$ 2007
Polariton condensation, beyond the weakly interacting Bose gas.

**Polariton condensation vs W.I.D.B.G**

Non-interacting Bose gas

- Three dimensional
- Non-interacting bosons
- Structureless bosons
- Infinite lifetime
- Solved 80 years ago
Polariton condensation, beyond the weakly interacting Bose gas.

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**Polariton condensation vs W.I.D.B.G**

### Non-interacting Bose gas
- Three dimensional
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### Polariton Condensate
- Two dimensions
- Coulomb/saturation interactions
- $T_c$ comparable to $\Omega_R, \mathcal{R}y$
- Pumped, decaying
- Open questions remain

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Polariton condensation, beyond the weakly interacting Bose gas.

Overview

• Polariton internal structure
Polariton condensation, beyond the weakly interacting Bose gas.

Overview

- Polariton internal structure
  - Polariton blueshift
Polariton condensation, beyond the weakly interacting Bose gas.

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  - Density profile
Polariton condensation, beyond the weakly interacting Bose gas.

**Excitons in a disordered Quantum well**
Polariton condensation, beyond the weakly interacting Bose gas.

**Excitons in a disordered Quantum well**

Centre-of-mass wavefunction satisfies:

\[
- \frac{\nabla^2 R}{2m_X} + V(R) \right] \Phi_\alpha(R) = \epsilon_\alpha \Phi_\alpha(R)
\]
Polariton condensation, beyond the weakly interacting Bose gas.

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Want distribution of: Energies,
Oscillator strengths:

\[\epsilon_\alpha, \quad g_{\alpha,p} \propto \psi_{1s}(0)\Phi_{\alpha,p}\]
Polariton condensation, beyond the weakly interacting Bose gas.

**Exciton energies and oscillator strengths**

\[ |g^{2(\epsilon,0)}| \text{ [arb. units]} \]
\[ \text{DoS}(\epsilon) \]

\[ -3 -2 -1 0 1 2 \]
\[ \epsilon - E_x [\text{meV}] \]

[FMM, JK, MHS, PBL cond-mat/0608096]
Polariton condensation, beyond the weakly interacting Bose gas.

**Exciton energies and oscillator strengths**

\[ g^2(\epsilon,0) \]

\[ |g_{\alpha p=0}|^2 \text{ [arb. units]} \]

\[ \text{DoS}(\epsilon) \]

\[ g^2(\epsilon,0) \]

\[ \epsilon - E_x \text{ [meV]} \]

\[ \theta \text{ [degrees]} \]

\[ \epsilon = \frac{p^2}{2m} \]

\[ g^2(\epsilon, p) \]

[ FMM, JK, MHS, PBL cond-mat/0608096 ]

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**Polariton model**

- Couple excitons to photons

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Represent sites as two-level systems (spins):

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\[
H = \sum_k \omega_k \psi_k^\dagger \psi_k
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Represent sites as two-level systems (spins):

\[
H = \sum_{k} \omega_{k} \psi_{k}^{\dagger} \psi_{k} + \sum_{\alpha} \left[ \epsilon_{\alpha} S_{\alpha}^{z} \right]
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Represent sites as two-level systems (spins):

\[
H = \sum_k \omega_k \psi_k^\dagger \psi_k + \sum_\alpha \left[ \epsilon_\alpha S^z_\alpha + \frac{1}{\sqrt{\text{Area}}} \sum_k g_{\alpha,k} \psi_k S^+_\alpha + \text{H.c.} \right]
\]

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Blueshift

\[ \delta = 5.3 \text{meV}, \ k_B T_{\text{eff}} = 17 \text{K} \]

mean-field threshold

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Polariton condensation, beyond the weakly interacting Bose gas.

**Blueshift**

\[ \delta E_{LP} \approx \mathcal{R} y_X a_X^2 n + \Omega_R a_X^2 n \]

**Clean limit:**

\[ \delta = 5.3 \text{meV}, k_B T_{\text{eff}} = 17 \text{K} \]
Polariton condensation, beyond the weakly interacting Bose gas.

**Blueshift**

Clean limit:

\[ \delta E_{\text{LP}} \approx R y_X a_X^2 n + \Omega_R a_X^2 n \]

Here:

\[ \Omega_R a_X^2 \rightarrow \Omega_R \xi^2 \]
Polariton condensation, beyond the weakly interacting Bose gas.

**Blueshift**

![Graph showing energy versus particle density with the equation \( \delta E_{LP} \approx \mathcal{R} y X a_X^2 n + \Omega_R a_X^2 n \) and the condition \( \Omega_R a_X^2 \rightarrow \Omega_R \xi^2 \) for CdTe, \( \times 100 \) larger.]

Clean limit:

\[ \delta E_{LP} \approx \mathcal{R} y X a_X^2 n + \Omega_R a_X^2 n \]

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For CdTe, \( \times 100 \) larger

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Polariton condensation, beyond the weakly interacting Bose gas.

Blueshift

Clean limit:

\[ \delta E_{LP} \simeq \mathcal{R} y_X a_X^2 n + \Omega_R a_X^2 n \]

Here:

\[ \Omega_R a_X^2 \rightarrow \Omega_R \xi^2 \]

For CdTe, \( \times 100 \) larger

Upper polariton:

\[ \delta E_{UP} \simeq \mathcal{R} y_X a_X^2 n - \Omega_R a_X^2 n \]
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Phase diagram

\[ n \text{ [cm}^{-2} \text{]} \]

\[ k_B T \]

\[ n \text{ [cm}^{-2} \text{]} \]

\[ 1 \times 10^9 \]

\[ 2 \times 10^9 \]

Polariton condensation, beyond the weakly interacting Bose gas.

**Phase diagram**

![Diagram showing phase transitions and energy levels](image)

Polariton condensation, beyond the weakly interacting Bose gas.

**Phase diagram**

![Phase diagram](image)

[J. Keeling, FOPS, July 2007]

Polariton condensation, beyond the weakly interacting Bose gas.

**Pumping and decay; energy scales**

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\[
g \quad \text{Pumping Bath} \\
\gamma = \pi \Gamma N_p \\
\kappa = \pi \zeta N^\kappa
\]

\[
\text{System} \\
\text{Excitons} \\
\text{Cavity mode} \\
\text{Bulk photon modes}
\]
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**Linewidth on approaching transition**

Approaching transition, susceptibility diverges

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Gap equation/Ginzburg-Landau:

\[ 0 = \mathcal{G}^{-1}(\omega, k) \]

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**Linewidth on approaching transition**

Approaching transition, susceptibility diverges

Gap equation/Ginzburg-Landau:

\[ 0 = \mathcal{G}^{-1}(\omega, k) \sim (\omega - \omega_k^*) \]

Polariton condensation, beyond the weakly interacting Bose gas.

**Linewidth on approaching transition**

Approaching transition, susceptibility diverges

Gap equation/Ginzburg-Landau:

\[ 0 = G^{-1}(\omega, k) \simeq (\omega - \omega^*_k) - i\alpha \]

Polariton condensation, beyond the weakly interacting Bose gas.

**Linewidth on approaching transition**

Approaching transition, susceptibility diverges
Gap equation/Ginzburg-Landau:

\[ 0 = \mathcal{G}^{-1}(\omega, k) \simeq (\omega - \omega_k^*) - i\alpha(\mu_{\text{eff}} - \omega) \]

Polariton condensation, beyond the weakly interacting Bose gas.

**Linewidth on approaching transition**

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Linewidth vanishes at transition

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Linewidth vanishes at transition

![Graph showing linewidth vs. bath chemical potential](image)

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**Linewidth on approaching transition**

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[MHS, JK, PBL *Phys. Rev. B* 75 195331 (2007)]
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**Excitations of decaying condensate; power spectrum**

When condensed, poles become:

\[ \omega = c|k| \]
Polariton condensation, beyond the weakly interacting Bose gas.

**Excitations of decaying condensate; power spectrum**

When condensed, poles become:

$$\omega = -ix \pm \sqrt{c^2k^2 - x^2}$$
Polariton condensation, beyond the weakly interacting Bose gas.

**Excitations of decaying condensate; power spectrum**

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\[
\langle \psi^\dagger(r, t) \psi(0, 0) \rangle \simeq \rho_0 \exp \left[ - \right]
\]

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**Excitations of decaying condensate; power spectrum**

When condensed, poles become:

$$\omega = -ix \pm \sqrt{c^2 k^2 - x^2}$$

$\langle \psi^\dagger(r, t) \psi(0, 0) \rangle \simeq \rho_0 \exp \left[ - \left\{ \eta \ln(r/\xi) \right\} \right]$, $r \to \infty, t \simeq 0$
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**Excitations of decaying condensate; power spectrum**

When condensed, poles become:

\[ \omega = -ix \pm \sqrt{c^2k^2 - x^2} \]

\[
\langle \psi^\dagger(r, t)\psi(0, 0) \rangle \simeq \rho_0 \exp \left[ - \begin{cases} 
\eta \ln(r/\xi) & r \to \infty, t \simeq 0 \\
\frac{n}{2} \ln(c^2t/x\xi^2) & r \simeq 0, t \to \infty
\end{cases} \right]
\]

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Polariton condensation, beyond the weakly interacting Bose gas.

**Phase boundary; effect of dephasing**

Mean-field theory of pumped decaying system; self-consistent distribution:

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Mean-field theory of pumped decaying system; self-consistent distribution:

![Graph showing the phase boundary and effect of dephasing]

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Mean-field theory of pumped decaying system; self-consistent distribution:

![Graph showing phase boundary and effect of dephasing](image)

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Effect of pumping on density profile

Gross-Pitaevskii equation:

[JK, NGB arXiv:0706.3686v1 [cond-mat.other]]
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**Effect of pumping on density profile**

Gross-Pitaevskii equation:

\[ i\hbar \partial_t \psi = \left[ -\frac{\hbar^2 \nabla^2}{2m} + V(r) + U|\psi|^2 \right] \psi \]

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i\hbar \partial_t \psi = \left[ -\frac{\hbar^2 \nabla^2}{2m} + V(r) + U|\psi|^2 + i(\gamma - \Gamma|\psi|^2) \right] \psi
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Gross-Pitaevskii equation:

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i\hbar \partial_t \psi = \left[ -\frac{\hbar^2 \nabla^2}{2m} + V(r) + U|\psi|^2 + i(\gamma - \Gamma|\psi|^2 - \kappa) \right] \psi = \mu_{\text{eff}} \psi
\]

[J. Keeling, FOPS, July 2007]
Polariton condensation, beyond the weakly interacting Bose gas.

**Effect of pumping on density profile**

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Consider \( V(r) = m\omega^2 r^2/2 \)

[JK, NGB arXiv:0706.3686v1 [cond-mat.other]]
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Consider \( V(r) = m\omega^2 r^2 / 2 \)

\[\frac{\gamma - \kappa}{\hbar \omega} = 2.2\]

\[\frac{\gamma - \kappa}{\hbar \omega} = 0.7\]

[JK, NGB arXiv:0706.3686v1 [cond-mat.other]]

J. Keeling, FOPS, July 2007
Summary

Differences between polariton condensate and non-interacting Bose gas allow investigation of **interesting physics**

- Internal structure/disorder
  - Critical temperature comparable to $\Omega R$, $\mathcal{R}y$
  - Exciton disorder affects effective interactions

- Non-equilibrium
  - Diffusive spectrum / Lineshape
  - Phase boundary
  - Persistent supercurrents and density profile
Polariton condensation, beyond the weakly interacting Bose gas.

Supplementary Slides
Polariton condensation, beyond the weakly interacting Bose gas.

**Density scales/exciton localisation**

\[ n \text{ [cm}^{-2}\text{]} \]

\[ k_B T \]

\[ 1 \times 10^9 \text{ } \text{to} \text{ } 2 \times 10^9 \]

\[ \varepsilon = \frac{p^2}{2m} g^2(\varepsilon, p) \]

\[ \text{DoS}(\varepsilon) \]

\[ p = 6.3 \times 10^5 \text{ cm}^{-1} \]

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**Limit of validity**

Model neglects:

- Inter-site Coulomb
- (High energy) multiple occupancy

Limits to low density: How low?
Limited to states below band-edge

![Graph showing occupation and density of states](image-url)
Polariton condensation, beyond the weakly interacting Bose gas.

**Lineshape in the normal state**

![Diagram showing lineshape in the normal state with various plotted curves for different parameters such as luminescence, absorption, spectral weight, and distribution, $F_S$.](image-url)
Polariton condensation, beyond the weakly interacting Bose gas.

**Finite size, condensate linewidth**

Correlations, $\langle \psi^\dagger(t, r) \psi(0, r) \rangle \propto \exp(-f(t, r, r))$, with:

$$f(t, r, r) = -\sum_n^{n_{\text{max}}} \int \frac{d\omega}{2\pi} \frac{C|\varphi_n(r)|^2(1 - e^{i\omega t})}{[\omega^2 - (n\Delta)^2]^2 + 4\omega^2x^2},$$

$$\Delta \ll \sqrt{x/t} \ll E_{\text{max}} \quad f(t, r, r) \sim 1 + \ln(E_{\text{max}}\sqrt{t/x})$$

$$\sqrt{x/t} \ll \Delta \ll E_{\text{max}} \quad f(t, r, r) \sim (\frac{\pi C}{2x})(\frac{t}{2x})$$

$$\sqrt{x/t} \ll E_{\text{max}} \ll \Delta \quad f(t, r, r) \sim 1$$

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**Pumped, Decaying GPE**

Rescaling:

\[
\tilde{\mu}\psi = \left[-\nabla^2 + r^2 + |\psi|^2 + i(\alpha - \sigma|\psi|^2)\right]\psi
\]

Then, writing:

\[
\psi = \sqrt{\rho}e^{i\phi}
\]

\[
\nabla \cdot [\rho \nabla \phi] = (\alpha - \sigma \rho)
\]

\[
\mu = |\nabla \phi|^2 + r^2 + \rho - \frac{\nabla^2 \sqrt{\rho}}{\sqrt{\rho}}
\]

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