BCS-BEC crossover in a system of microcavity polaritons

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Theory of Condensed Matter, Cambridge

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Overview

- Microcavity polariton condensation: review of experiments.
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- Dicke model for polaritons.
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- Fluctuations in two dimensions; fluctuations with condensate
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- Microcavity polariton condensation: review of experiments.
- Dicke model for polaritons.
- Summary of results from mean field theory.
- **Diversion**: Atomic gases near Feshbach resonance – analogies
- Fluctuations in two dimensions; fluctuations with condensate
- Consider crossover between “B.E.C.” and “B.C.S.-like” transition.
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**Exciton Polaritons**

- Strong coupling of photons to excitons

[Pekar, *JETP* **6** 785 (1958)]
[Hopfield, *Phys. Rev.* **112** 1555 (1958)]
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**Exciton Polaritons**

- Strong coupling of photons to excitons
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Exciton Polaritons

- Strong coupling of photons to excitons
- Anti-crossing – form two new modes
- No condensation – can relax to photon mode.

[Pekar, *JETP* 6 785 (1958)]
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Microcavity polaritons

Quantum well excitons coupled to photons confined in a microcavity.
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Microcavity polaritons

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Quantum Wells

Distributed Bragg Reflector

Cavity mode

Cavity Photon $m_{ph} \sim 10^{-5} m_e$

Upper Polariton

Lower Polariton $m_{pol} = 2m_{ph}$

QW Exciton

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Why polariton condensation:

\[ m_{\text{ph}} \approx 10^{-5} m_e \]

Lower Polariton

QW Exciton

Cavity Photon

Upper Polariton

Lower Polariton

\[ m_{\text{pol}} = 2m_{\text{ph}} \]
Why polariton condensation:

- Polariton mass $10^{-4}m_{\text{electron}}$, high $T_c$. 

\[ m_{\text{pol}} = 2m_{\text{ph}} \]

\[ m_{\text{ph}} \sim 10^{-5} m_{\text{e}} \]
Why polariton condensation:

- Polariton mass $10^{-4}m_{\text{electron}}$, high $T_c$.
- Photon component – Non-classical light.
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Problems?

- Cavity lifetime is short (ps), hard to thermalise.
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Polariton Experiments

Non-linear ground state occupation.

[Deng et al. Science 298 199 (2002)]
(also [Dang et al. PRL. 81 3920 (1998)])
Non-linear ground state occupation.

(also [Dang et al. PRL. 81 3920 (1998)])

Peak in angular distribution.

[Deng et al. PNAS 100 15318]
Polariton Experiments (2)

Second order coherence of photons.

[Deng et al. Science 298 199 (2002)]
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Polariton Experiments (2)

Second order coherence of photons.

Interference fringes:

[Deng et al. Science 298 199 (2002)]

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Localised two level systems

Coupling to light:

\[ \varepsilon = \frac{p^2}{2m} \]

\[ g^2(\varepsilon, p) \]

[Marchetti et al. cond-mat/0509438].
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**Localised two level systems**

**Coupling to light:**

\[ \varepsilon = \frac{p^2}{2m} \]

\[ g^2(\varepsilon, p) \]

-3 -2 -1 0 1 2

Transverse photon mode

2 Level systems

[Marchetti et al. cond-mat/0509438 ].

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Localised two level systems

Coupling to light:

- Effective hard-core exciton-exciton interaction exists.

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Localised two level systems

Coupling to light:

- Effective hard-core exciton-exciton interaction exists.
- Energy difference between levels represents energy of bound exciton state.

\[ \varepsilon = \frac{p^2}{2m} \]

\[ g^2(\varepsilon, p) \]

\[ \varepsilon = \frac{p^2}{2m} - g^2(\varepsilon, p) \]

\[ \varepsilon \] [meV]

\[ 0 \quad 2 \quad 4 \quad 6 \quad 8 \quad 10 \quad 12 \]

\[ -3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \]

Momentum [a.u]

[Marchetti et al. cond-mat/0509438].

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The Dicke Model Hamiltonian

\[ H = \sum_{\alpha=1}^{\alpha=nA} \epsilon \left( b_{\alpha}^\dagger b_{\alpha} - a_{\alpha}^\dagger a_{\alpha} \right) \]
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The Dicke Model Hamiltonian

\[ H = \sum_{\alpha=1}^{\alpha=nA} \epsilon (b_\alpha^\dagger b_\alpha - a_\alpha^\dagger a_\alpha) + \sum_{k=l/\sqrt{A}} \hbar \omega_k \psi_k^\dagger \psi_k \]
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+ \frac{g}{\sqrt{A}} \sum_{\alpha,k} \left( e^{2\pi i k \cdot r_n} \psi^\dagger_k b^{\dagger}_\alpha a_\alpha + e^{-2\pi i k \cdot r_n} \psi^\dagger_k a^{\dagger}_\alpha b_\alpha \right).
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- T.L.S. areal density $n$
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- T.L.S. areal density \( n \)
- Photon dispersion in cavity:
  \[ \omega_k = \sqrt{\omega_0^2 + \gamma^2 k^2} \approx \omega_0 + \hbar k^2 / 2m \]
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- T.L.S. areal density \( n \)
- Photon dispersion in cavity:
  \[
  \omega_k = \sqrt{\omega_0^2 + c^2 k^2} \approx \omega_0 + \hbar k^2 / 2m
  \]

Assume thermal equilibrium with fixed number of excitations, \( \tilde{H} = H - \mu N \)

\[
N = \sum_{\alpha=1}^{\alpha=nA} \frac{1}{2} (b_\alpha^\dagger b_\alpha - a_\alpha^\dagger a_\alpha + 1) + \sum_{k=l/\sqrt{A}} \psi_k^\dagger \psi_k.
\]

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Mean field theory

At zero temperature, BCS-like ansatz is exact minimum

$$|\Psi\rangle = e^{\lambda(\psi_0^\dagger + \sum_{\alpha} X_\alpha b_\alpha^\dagger a_\alpha)} \prod_{\alpha} a_{\alpha}^\dagger |0\rangle$$


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At finite T, Integrate out TLS, and minimise w.r.t \psi.


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At finite T, Integrate out TLS, and minimise w.r.t $\psi$. Gap equation:

$$\tilde{\omega}_0 \psi_0 = g^2 n \frac{\tanh(\beta E)}{E} \psi_0$$

$$E = \sqrt{\tilde{c}^2 + g^2 n |\psi_0|^2}$$


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Gap equation:

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Excitation density:

\[ E = \sqrt{\tilde{\epsilon}^2 + g^2 n |\psi_0|^2} \quad \rho_{ex} = -\frac{\tilde{\epsilon}}{2E} \tanh(\beta E) + |\psi_0|^2 \]


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**Fluctuation spectrum**

Consider fluctuations about mean field — Poles of greens function for photon response.

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For small $k$, linear dispersion mode,

$$\xi_1 = \pm ck + O(k^2)$$
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Fluctuation spectrum

Consider fluctuations about mean field — Poles of greens function for photon response.

For small $k$, linear dispersion mode,

$$\xi_1 = \pm ck + O(k^2)$$

At large $k$, recover bare exciton/photon spectra.
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**Diversion:** Feshbach resonance

Condensation in system of bosons coupled to fermion pairs — analogies to Feshbach resonance.
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**Diversion: Feshbach resonance**

Condensation in system of bosons coupled to fermion pairs — analogies to Feshbach resonance.

- Interaction between fermions depends on spin states.
- At resonance, “strong-coupling” of atoms and molecule:
- Detuning gives crossover from BCS of atoms to BEC of molecules.
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**Diversion:** Analogies and differences

Comparison of physical systems:

**Feshbach resonance** $\iff$ **Microcavity Polaritons**
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**Diversion: Analogies and differences**

Comparison of physical systems:

- Feshbach resonance  $\iff$  Microcavity Polaritons
- Closed channel molecules  $\iff$  Microcavity Photons
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**Diversion: Analogies and differences**

Comparison of physical systems:

- **Feshbach resonance** $\iff$ **Microcavity Polaritons**
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**Important differences**

- Polaritons: Measure only emitted photons.
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  - Atoms $\iff$ Electron/Holes
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**Important differences**

- Polaritons: Measure only emitted photons.
- Cannot dynamically change exciton-photon detuning.
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**Diversion: Holland-Timmermans model**

One model of Feshbach resonance, very similar to Dicke model:
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\[
H - \mu N = \sum_{k,\sigma} (\epsilon_k - \mu) c_{k,\sigma}^\dagger c_{k,\sigma}
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+ g \sum_{k,q} \left( b_{q}^{\dagger} c_{-k+q/2,\downarrow} c_{k+q/2,\uparrow} + c_{k+q/2,\uparrow}^{\dagger} c_{-k+q/2,\downarrow} b_{q} \right)
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- \frac{U}{2} \sum_{k,k',q} c_{k+q,\uparrow}^\dagger c_{k',-q,\downarrow}^\dagger c_{k,\downarrow} c_{k',\uparrow}.
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\[- \frac{U}{2} \sum_{k,k',q} c_{k+q,\uparrow}^{\dagger} c_{k',-q,\downarrow}^{\dagger} c_{k,\downarrow} c_{k',\uparrow}^{\dagger} . \]

Gives energy dependant fermion-fermion scattering.
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\]

Gives energy dependant fermion-fermion scattering. Unnecessary for current experiments. e.g. [Simonucci et al. Europhys. Lett. 69 713 (2005)]
Comparing mean field theories

General form

$$\frac{1}{U_{\text{eff}}} = \int \nu_s(\epsilon) \frac{\tanh(\beta(\epsilon - \mu))}{\epsilon - \mu} d\epsilon$$

BCS superconductor  Holland-Timmermans  Dicke model
Comparing mean field theories

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BCS superconductor   Holland-Timmermans   Dicke model

DOS

Occupation

Energy
Comparing mean field theories

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BCS superconductor  Holland-Timmermans  Dicke model

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\frac{1}{U} = \nu(\mu) \ln \left( \frac{\Omega}{T} \right)
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Comparing mean field theories

General form

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BCS superconductor          Holland-Timmermans          Dicke model

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\[ \frac{1}{U_{\text{bg}}} + \frac{g^2}{\Delta - \mu} = \nu(\mu) \ln \left( \frac{\Omega}{T} \right) \]
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Comparing mean field theories

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BCS superconductor  Holland-Timmermans  Dicke model

\[ \frac{1}{U} = \nu(\mu) \ln \left( \frac{\Omega}{T} \right), \quad \frac{1}{U_{\text{bg}}} + \frac{g^2}{\Delta - \mu} = \nu(\mu) \ln \left( \frac{\Omega}{T} \right), \quad \frac{\omega - \mu}{g^2} = \frac{\tanh(\beta(\epsilon_{\text{ex}} - \mu))}{\epsilon_{\text{ex}} - \mu} \]

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**Fluctuation corrections**

- Consider crossover to BEC with changing density.
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- Treatment similar to \([\text{Nozières & Schmitt-Rink } J.L.T.P \textbf{59} 195 (1985)](\)
Fluctuation corrections

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- However:
  - Two dimensional system — consider Kosterlitz-Thouless
Fluctuation corrections

- Consider crossover to BEC with changing density.

- Treatment similar to [Nozières & Schmitt-Rink J.L.T.P 59 195 (1985)]

- However:
  - Two dimensional system — consider Kosterlitz-Thouless
  - Boson field dynamic, with chemical potential — similar to Holland-Timmermans model, e.g. [Ohashi & Griffin, PRA. 67 063612 (2003)]
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Fluctuations in 2d

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Fluctuations in 2d

\[ \rho_s = \frac{2mk_B T}{\hbar^2} \]

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Fluctuations in 2d

Need \( \rho_{sf} = \rho_{total} - \rho_{normal} \).

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Fluctuations in 2d

Need \( \rho_{sf} = \rho_{\text{total}} - \rho_{\text{normal}} \).

\( \rho_{\text{normal}} \) found by current response:

\[ J_i(q) = \chi_{ij}(q)F_j(q). \]

\[ \rho_s = \# \frac{2mk_BT}{\hbar^2} \]
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**Fluctuations in 2d**

Need $\rho_{sf} = \rho_{total} - \rho_{normal}$. 

$\rho_{normal}$ found by current response:

$$J_i(q) = \chi_{ij}(q)F_j(q).$$

$$\chi_{ij} = \chi_L \frac{q_i q_j}{q^2} + \chi_T \left( \delta_{ij} - \frac{q_i q_j}{q^2} \right)$$

Thus $\rho_{normal} = m\chi_T(q \to 0)$

\[ \rho_s = \# \frac{2mk_B T}{\hbar^2} \]
BCS-BEC crossover in a system of microcavity polaritons

**Fluctuations in 2d**

The figure shows a graph with temperature on the x-axis and superfluid density on the y-axis. The graph has a dashed line indicating a phase transition at a certain temperature.

Need $\rho_{sf} = \rho_{total} - \rho_{normal}$.  

$\rho_{normal}$ found by current response: 

$J_i(q) = \chi_{ij}(q)F_j(q)$. 

$$\chi_{ij} = \chi_L \frac{q_i q_j}{q^2} + \chi_T \left( \delta_{ij} - \frac{q_i q_j}{q^2} \right)$$

Thus $\rho_{normal} = m \chi_T(q \to 0)$

Thus, need to find: $\rho_{total}$ in presence of condensate.
BCS-BEC crossover in a system of microcavity polaritons

**Fluctuations in presence of condensate**

Density is total derivative of free energy:

\[ \rho = - \frac{\partial F}{\partial \mu} - \frac{d\psi_0}{d\mu} \frac{\partial F}{\partial \psi_0} \]
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\[ F_{\text{fluct}} = -k_B T \ln \left\langle e^{-\beta (H_{\text{fluct}}[\psi_0] - \mu \rho_{\text{uncondensed}})} \right\rangle \]
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\]

Condensate depletion changes critical chemical potential.

J. Keeling, MIT CMT Informal Seminar, 2005
Simple example: Weakly interacting Bose gas

\[ H - \mu N = \sum_k \left( \epsilon_k - \mu \right) a_k^{\dagger} a_k + \frac{g}{2} \sum_{k,k',q} a_{k+q}^{\dagger} a_{k'}^{\dagger} - q a_k a_{k'}. \]
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Normal state exists for \( \mu > 0 \): Need self energy.
BCS-BEC crossover in a system of microcavity polaritons

The phase diagram

Calculate density where \( \rho_{\text{superfluid}} = 0 \).

J. Keeling, MIT CMT Informal Seminar, 2005
BCS-BEC crossover in a system of microcavity polaritons

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\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{phase_diagram.png}
\caption{Phase diagram of the BCS-BEC crossover in a system of microcavity polaritons.}
\end{figure}
Calculate density where $\rho_{\text{superfluid}} = 0$. 

![Diagram showing phase diagram]

- **BCS-like regime**
- **BEC of polaritons**
- **BEC of photons**
BCS-BEC crossover in a system of microcavity polaritons

The phase diagram

Calculate density where $\rho_{\text{superfluid}} = 0$. 

![Phase diagram with density and temperature axes showing the crossover between BCS-like regime and BEC of polaritons.](image)
BCS-BEC crossover in a system of microcavity polaritons

The phase diagram

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Crossover when:

$$T_{\text{deg}} = \frac{\rho}{m}$$
BCS-BEC crossover in a system of microcavity polaritons

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Current experiments in BCS-like regime: $\rho_{\text{crossover}}/n \approx mg/\sqrt{n} \approx 10^{-3}$, experiments around $\rho/n \approx 0.01$. 
Conclusions

- Including fluctuations, B.E.C. transition at low density, internal structure matters at higher densities.

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BCS-BEC crossover in a system of microcavity polaritons

Supplementary material
Experimental signatures: $N(k)$

From spectrum find:

$$N(k) = \left\langle \psi_{k}^\dagger(\tau + \eta)\psi_{k}(\tau) \right\rangle$$
BCS-BEC crossover in a system of microcavity polaritons

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Can calculate if:

- Uncondensed,
- or low \( T \), phase fluctuations.

![Graph showing N(k) vs Density and Temperature](image_url)
Experimental signatures: $N(k)$

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Can calculate if:

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Universal form:

$$N(p) \propto \rho_0 \frac{\xi_T^\eta}{p^{2-\eta}}, \quad \eta = \frac{m}{2\pi\beta\rho_0\hbar^2}$$
Inhomogeneous broadening — spectral weight

With inhomogeneous broadening of exciton energies, lines become broadened.
Can plot $\Im G(i\omega = z + i\eta)$, absorption coefficient.
Inhomogeneous broadening — spectral weight

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Can plot $\Im G(i\omega = z + i\eta)$, absorption coefficient. Figures for broadening, $0.3g\sqrt{n}$ other parameters as previously.
BCS-BEC crossover in a system of microcavity polaritons

**Inhomogeneous broadening — spectral weight**

With inhomogeneous broadening of exciton energies, lines become broadened.

Can plot $\Im G(i\omega = z + i\eta)$, absorption coefficient. Figures for broadening, $0.3g\sqrt{n}$ other parameters as previously.

Note Goldstone mode is not broadened.
Inhomogeneous broadening: What the spectrum means

Absorption probability is:

\[ P_{\text{absorb}}(x) = \sum_{n,m} |\langle m | \psi^\dagger | n \rangle|^2 e^{\beta (F - E_n)} \delta(x - E_{mn}) = (1 + n_B(x)) \rho_L(x). \]
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Where \( \rho_L \), the difference is given by:

\[ \rho_L(x) = \lim_{\eta \to 0} \Im G(i\omega = x + i\eta) = P_{\text{absorb}}(x) - P_{\text{emit}}(x). \]
BCS-BEC crossover in a system of microcavity polaritons

Inhomogeneous broadening — Emission probability

Alternative plots: $P_{\text{emit}}$ Figures for broadening, $0.1g\sqrt{n}$ other parameters as previously.