Non-Linear Optics

Lecture 3: Achieving Phase Matching

Learning goals

By the end of this lecture you should:

• Show that we can use refractive index ellipsoids to define particular directions for phase matching.
• Understand other techniques to achieve phase matching (eg. temperature.)
• Define critical and non-critical phase matching.
• Understand what is meant by type I and type II phase matching.
• Understand the concepts of quasi-phase matching (QPM.)

From last time:

Phase mismatch:

$\Delta k = k_{2\omega} - 2k_{\omega} = \frac{4\pi}{\lambda} [n_{2\omega} - n_{\omega}]$ where $\lambda$ is the fundamental wavelength

Has profound implications for the efficiency with which non-linear light is generated

Phase matching $\Rightarrow \Delta k = 0 \Rightarrow$ Efficient SHG!

Coherence length $\Rightarrow$ the effective interaction length for SHG when $\Delta k \neq 0 = \pi / \Delta k$

In the last lecture we showed that Phase Matching was of key importance for non-linear optics. Remember that phase matching occurs when a constant phase relationship is maintained between the generated and propagating waves. In general, this does not occur, so a repeated build up and decline of radiation is observed with a characteristic length given by the coherence length $l_0$. In order to make phase matching occur, we need to arrange a circumstance where $n_{2\omega} = n_{\omega}$. In order to do this we can use birefringent materials.

Birefringence is a property of certain materials where different polarisations have different refractive indices. You may be familiar with the concept of double refraction in calcite – this is an example of
birefringence where the two rays formed have different polarisations – these are the ordinary, o-ray and the extraordinary, e-ray.

We’ll consider the case of the negative uniaxial crystal – $n_o > n_e$. There is a special direction where $n_o = n_e$ – this is called the optical axis. Making a plot of $n_e$ and $n_o$ as a function of angle generates the refractive index ellipsoids.

So, the crystal has different refractive indices for the ordinary and extraordinary ray. Now let’s expand this picture to include the second harmonic index ellipses: $n_o^2$ and $n_e^2$:

We can see that there’s an angle, $\theta_0$ where the ordinary polarised fundamental and the extraordinary polarised second harmonic have the same refractive index. This implies that when the radiation propagates in the crystal at angle $\theta_0$ relative to the optical axis, perfect phase
matching is possible. This technique is sometimes known as *birefringent phase matching – BPM*.

So in this case:

$$n_0(\theta) + n_0(\theta) = 2n_e^2(\theta)$$ – This is known as *Type I phase matching*.

There are a range of other non-linear processes that can take place and requiring phasematching and sometimes the following condition is satisfied:

$$n_0(\theta) + n_0(\theta) = 2n_e^2(\theta)$$ – This is known as *Type II phase matching*.

In order to tune for different wavelengths of operation, we can simple change the angle $\theta_0$ – this gives rise to so-called *angle tuning*. This technique is very sensitive dependent on the angle of propagation – get the angle only a little bit wrong and no phase matching occurs. This type of phase matching is also therefore known as *CRITICAL PHASE MATCHING*.

**Problems:**

1. Limited range of uni- and bi-axial crystals.
2. Critical dependence on angle.
3. Walk-off occurs between generating beam and second harmonic limiting interaction length.
4. $\chi^2$ varies with angle (it’s a tensor remember!) giving variable efficiency with wavelength.

*Non-critical phase matching (NCPM)*

In order to circumvent some of these problems, another approach, non-critical phasematching can sometimes be adopted. In this case, the beams propagate down one of the axes of the non-linear crystal and the temperature of the crystal is adjusted. The propagation direction is normally at 90° to the optical axis, so this techniques is also sometimes known as 90° phase matching. Often, the $n_e$ is much more temperature sensitive than $n_0$ meaning a phase matching condition can be set up.

*Quasi Phase Matching (QPM)*
But what happens if your material is not birefringent (eg. GaAs) or has a large non-linear coefficient in a direction in which it’s impossible to do BPM or NCPM (eg. LiNbO$_3$)? In this case we can physically engineer the material itself to perform phase matching. From before:

Now what would happen if we were to change the sign of the phase every coherence length? Instead of seeing a periodic reduction in non-linear signal, it would continue to grow – this is the basis of quasi-phase matching. Imagine that we do this in a periodic way with a period given by $\Lambda_g$, where $\Lambda_g$ is the period of the grating ($= 2l_c$.)

In the frequency domain, we can represent the grating wave vector:

$$k_g = \frac{2\pi}{\Lambda_g}$$

And the phase matching relationship becomes:

$$\Delta k = 2k_\omega - k_\omega k_g$$

Which allows us to write:

\[
\frac{2\pi}{\Lambda_g} = \frac{4\pi}{\lambda} [n_\omega - n_{2\omega}]
\]

Therefore:

\[
\Lambda_g = \frac{\lambda_\omega}{2(n_\omega - n_{2\omega})}
\]

Example: LiNbO$_3$

For BPM, we are forced to use a direction which has a non-linear coefficient = 4.3pm/V, using QPM allows us to access a coefficient = 27pm/V giving a large enhancement in non-linear optical efficiency.

Let’s calculate the grating period required – use a Sellmeier Equation:

\[
(n^2) = 4.9048 + \frac{0.11768}{\lambda^2} - 0.04750 \frac{0.027169}{\lambda^2}
\]

Note that for this equation $\lambda$ is in \(\mu\)m!

$\lambda=1.064\mu$m $\Rightarrow$ $n = 2.2364$

$\lambda=0.532\mu$m $\Rightarrow$ $n = 2.3231$

$\Rightarrow$ Grating period = 6.1\mu m.

Fabrication of QPM structures is complex and has developed in the last 15 years. The simplest approach is to take very thin slices of material and re-orientate them every coherence length. This has done with GaAs where up to 50 layers of GaAs slices have been formed.

Where we want to use a ‘standard’ non-linear crystal, a different approach is taken. Here we use crystals that are ‘ferroelectric.’ Ferroelectric crystals can be given a permanent electrical polarization by the application of electricity. This process is known as ‘periodic poling.’ An example of such a crystal is LiNbO$_3$ which has a high non-linear coefficient in a direction that is impossible to access using BPM. To do this, we pattern electrodes onto the top surface of the crystal and apply a sophisticated electric field. This causes a permanent change giving us a ‘domain engineered’ structure. The electric field required for this in LN is of the order of 21kV/mm !
**Advantages of QPM**

Allows phase matching in non-birefringent materials.
Can access directions with high non-linear coefficients.
Permits non-critical phasematching.
Can be tunable for a range of applications.
Gives direct control over the non-linear materials.

**Disadvantages of QPM.**

Limited range of materials for e-field poling.
Cost and complexity of fabrication.
Limited widths of grating period (<4μm is almost impossible.)