Notes on the growth model example (Alan Sutherland, November 2003)

The following equations and notation correspond to the growth model discussed in Sutherland (2002) (see the paper for a full description of the model and the equations).

$$-\gamma\hat{c}_t = (\alpha-1)\hat{k}_{t+1} - \gamma E_t\left[\hat{c}_{t+1}\right] + E_t\left[\hat{A}_{t+1}\right] + \lambda_{c,t}$$

$$\phi\hat{c}_t + \theta\hat{k}_{t+1} = \hat{A}_t + \alpha\hat{k}_t + \lambda_{k,t}$$

$$\hat{A}_{t+1} = \rho\hat{A}_t + \varepsilon_{t+1}$$

where

$$\lambda_{c,t} = E_t\left[\frac{1}{2}\left(\gamma^2\hat{c}_{t+1}^2 - 2\gamma\hat{c}_{t+1}\hat{A}_{t+1} + \hat{A}_{t+1}^2\right)\right] - \frac{1}{2}\left(\gamma^2 E_t\left[\hat{c}_{t+1}\right]^2 - 2\gamma E_t\left[\hat{c}_{t+1}\right]E_t\left[\hat{A}_{t+1}\right] + E_t\left[\hat{A}_{t+1}\right]^2\right)$$

$$\lambda_{k,t} = \frac{1}{2}\left(\hat{A}_t^2 + \alpha\hat{k}_t^2 + 2\alpha\hat{A}_t\hat{k}_t - \phi\hat{c}_t^2 - \theta\hat{k}_{t+1}^2\right)$$

Notice that the second-order term $\lambda_{c,t}$ contains terms in the one-period-ahead forecasts of *A* and *c*. In order to generate second moments of one-period-ahead forecasts it is necessary to augment the vector of variables with dummy variables $\hat{A}_t^e$ and $\hat{c}_t^e$ and to augment the set of equations with the definitions of $\hat{A}_t^e$ and $\hat{c}_t^e$:

$\hat{A}_t^e = E_t\left[\hat{A}_{t+1}\right]$ and $\hat{c}_t^e = E_t\left[\hat{c}_{t+1}\right]$.

The second-order term $\lambda_{c,t}$ can then be rewritten in the form

$$\lambda_{c,t} = E_t\left[\frac{1}{2}\left(\gamma^2\hat{c}_{t+1}^2 - 2\gamma\hat{c}_{t+1}\hat{A}_{t+1} + \hat{A}_{t+1}^2\right)\right] - \frac{1}{2}\left(\gamma^2\hat{c}_t^{e2} - 2\gamma\hat{c}_t^e\hat{A}_t^e + \hat{A}_t^{e2}\right)$$

The variables of the model are grouped into the following vectors:

$$Z_{1,t} = \left[\hat{k}_t\right] \qquad Z_{2,t} = \left[\hat{A}_t\right] \qquad Z_{3,t} = \begin{bmatrix}\hat{c}_t \\ \hat{A}_t^e \\ \hat{c}_t^e\end{bmatrix} \qquad \Lambda_t = \begin{bmatrix}\lambda_{c,t} \\ \lambda_{k,t}\end{bmatrix}$$

The mat-file `growth.mat` contains example input matrices for the following parameter set:

θ=0.285, ϕ=0.715, γ=2, β=0.95, α=0.3, ρ=0 and $\sigma_\varepsilon^2 = 1$

The script `growth_numeric.m` loads the input matrices and calls the solution functions. The output is stored in `SOL`. The output should correspond to the example solution reported in Table 3 of Sutherland (2002).

The text file `growth_model.dat` contains the equations of the model in symbolic form and `growth_parameters.dat` contains the set of parameter names and values. The script `growth_symbolic.m` loads the symbolic version of the model, generates the numerical input matrices and calls the solution functions. The output is stored in `SOL`.

References

Sutherland, Alan (2002) "A Simple Second-Order Solution Method for Dynamic General Equilibrium Models" CEPR Discussion Paper No 3554.