Notes on the closed economy example (Alan Sutherland, September 2003)

The following equations and notation correspond to the closed-economy model discussed in Sutherland (2002) (see the paper for a full description of the model and the equations).

$$\hat{P}_t - \gamma \hat{P}_{t-1} = (1-\gamma)\hat{X}_t + \lambda_{P,t} \qquad \lambda_{P,t} = \frac{1-\phi}{2}\left[(1-\gamma)\hat{X}_t^2 - \hat{P}_t^2 + \gamma \hat{P}_{t-1}^2\right]$$

$$\hat{K}_{t+1} = \delta \hat{K}_t + \varepsilon_{t+1}$$

$$\hat{Q}_t - \beta\gamma E_t\left[\hat{Q}_{t+1}\right] = (1-\beta\gamma)\hat{q}_t + \lambda_{Q,t} \qquad \lambda_{Q,t} = \frac{1}{2}\left\{(1-\beta\gamma)\hat{q}_t^2 - \hat{Q}_t^2 + \beta\gamma E_t\left[\hat{Q}_{t+1}^2\right]\right\}$$

$$\hat{B}_t - \beta\gamma E_t\left[\hat{B}_{t+1}\right] = (1-\beta\gamma)\hat{b}_t + \lambda_{B,t} \qquad \lambda_{B,t} = \frac{1}{2}\left\{(1-\beta\gamma)\hat{b}_t^2 - \hat{B}_t^2 + \beta\gamma E_t\left[\hat{B}_{t+1}^2\right]\right\}$$

$$\psi\left(\hat{K}_t - E_0\left[\hat{K}_t\right]\right) = \hat{C}_t + \hat{P}_t$$

$$\hat{q}_t = (\phi - 1)\hat{P}_t$$

$$\hat{b}_t = \hat{K}_t + \mu\hat{C}_t + \phi\mu\hat{P}_t$$

$$(1-\phi(1-\mu))\hat{X}_t = \hat{B}_t - \hat{Q}_t$$

$$\hat{Y}_t = \hat{C}_t$$

Welfare in this model can be represented by the following equations:

$$\tilde{W}_t = \hat{C}_t - \frac{\phi-1}{\phi}\hat{Y}_t - \frac{\phi-1}{2}\left(1+\phi(\mu-1)\right)\left(H_t - \lambda_{\Pi,t}\right) + \lambda_{W,t} \qquad \lambda_{W,t} = \frac{1-\phi}{2\phi}\mu\left(\hat{Y}_t + \frac{1}{\mu}\hat{K}_t\right)^2$$

$$H_t = \gamma H_{t-1} + (1-\gamma)\lambda_{H,t} \qquad \lambda_{\Pi,t} = \hat{P}_t^2$$

$$\tilde{\Omega}_t - \beta E_t\left[\tilde{\Omega}_{t+1}\right] = (1-\beta)\tilde{W}_t \qquad \lambda_{H,t} = \hat{X}_t^2$$

where $\tilde{W}_t$ is flow welfare in period *t* and $\tilde{\Omega}_t$ is the present discounted value of welfare.

The variables of the model are grouped into the following vectors:

$$Z_{1,t} = \begin{bmatrix} \hat{P}_{t-1} \\ H_{t-1} \end{bmatrix} \quad Z_{2,t} = \begin{bmatrix} \hat{K}_t \end{bmatrix} \quad Z_{3,t} = \begin{bmatrix} \hat{Q}_t \\ \hat{B}_t \\ \tilde{\Omega}_t \\ \hat{C}_t \\ \hat{q}_t \\ \hat{b}_t \\ \hat{X}_t \\ \hat{Y}_t \\ \tilde{W}_t \end{bmatrix} \quad \Lambda_t = \begin{bmatrix} \lambda_{P,t} \\ \lambda_{Q,t} \\ \lambda_{B,t} \\ \lambda_{H,t} \\ \lambda_{\Pi,t} \\ \lambda_{W,t} \end{bmatrix}$$

The mat-file `closed.mat` contains example input matrices for the following parameter set:

γ=0.75, β=0.95, ϕ=2, μ=2, δ=0.5, ψ=-0.1 and $\sigma_\varepsilon^2 = 1$

The script `closed_numeric.m` loads the input matrices and calls the solution functions. The output is stored in `SOL`. The output should correspond to the example solution reported in Table 2 of Sutherland (2002).

The text file `closed_model.dat` contains the equations of the model in symbolic form and `closed_parameters.dat` contains the set of parameter names and values. The script `closed_symbolic.m` loads the symbolic version of the model, generates the numerical input matrices and calls the solution functions. The output is stored in `SOL`.

References

Sutherland, Alan (2002) "A Simple Second-Order Solution Method for Dynamic General Equilibrium Models" CEPR Discussion Paper No 3554.