Endogenous Price Flexibility and Optimal Monetary Policy*

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July 2012, Revised July 2013

Abstract

Much of the literature on optimal monetary policy uses models in which the degree of nominal price flexibility is exogenous. There are, however, good reasons to suppose that the degree of price flexibility adjusts endogenously to changes in monetary conditions. This paper extends the standard New Keynesian model to incorporate an endogenous degree of price flexibility. The model shows that endogenising the degree of price flexibility tends to shift optimal monetary policy towards complete inflation stabilisation, even when shocks take the form of cost-push disturbances. This contrasts with the standard result obtained in models with exogenous price flexibility, which show that optimal monetary policy should allow some degree of inflation volatility in order to stabilise the welfare-relevant output gap.

Keywords: welfare, endogenous price flexibility, optimal monetary policy.

JEL: E31, E52

*We are grateful to two anonymous referees for many useful comments on an earlier draft of this paper.

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1 Introduction

Much of the recent literature on optimal monetary policy uses models in which the degree of nominal price flexibility is imposed exogenously (see for example Woodford (2003) and Benigno and Woodford (2005)). There are, however, good theoretical and empirical reasons to suppose that the degree of price flexibility adjusts endogenously to changes in economic conditions, including changes in monetary policy. The ability of monetary policy to affect the real economy is closely linked to the degree of price flexibility, so endogenous changes in price flexibility may have important implications for the welfare effects of monetary policy.

This paper extends the standard New Keynesian DSGE model to incorporate an endogenous degree of price flexibility and uses this model to analyse optimal monetary policy in the face of stochastic shocks. The model is based on an adaptation of the Calvo (1983) price setting structure first proposed in Romer (1990). The key difference compared to the standard Calvo model is that we allow producers to choose the average frequency of price changes. An important advantage of our approach is that it is based on the general workhorse model used in the monetary policy literature. Our model is easy to analyse and offers potentially important results which can be compared directly to standard results from the monetary policy literature. Alternative approaches to modelling endogenous price flexibility, such as models of state-dependent pricing, may be more theoretically appealing, but they represent a more radical and much less tractable departure from the standard model used to analyse welfare maximising monetary policy.

Our model shows that monetary policy, by determining the volatility of macro variables, has an impact on the benefits of price flexibility relative to the costs and thus affects the optimal degree of price flexibility chosen by firms. For example, the greater is the volatility of CPI inflation, the larger will be the benefits of price flexibility, so firms will choose a greater frequency of price adjustment. A monetary rule which allows volatility in inflation will therefore tend to imply more price flexibility in equilibrium.

Having established a framework which captures the connection between monetary policy and price flexibility, we re-examine one of the main results from the literature on welfare maximising monetary policy. This result (analysed in detail by, for instance, Woodford (2003)) is that, in the face of cost-push shocks, it is optimal for monetary policy to allow some volatility in CPI inflation in order to stabilise the output gap. How might this result be changed when the degree of price flexibility is endogenised? Will endogenising the degree of price flexibility make it optimal for the monetary authority to raise or lower the volatility of inflation?

Our model shows that endogenising the degree of price flexibility tends to shift optimal monetary policy towards a reduction in inflation volatility relative to the case of exogenous price flexibility. Indeed, when the degree of price flexibility is endogenous, it appears that optimal policy should almost completely stabilise inflation in the face of cost-push shocks. This is in sharp contrast to the standard result emphasised in Woodford (2003) and Benigno and Woodford (2005). The essential point is that, lower inflation volatility tends to reduce the equilibrium degree of price flexibility and this both enhances the power of monetary policy and reduces the resource cost of price adjustment.

Besides demonstrating these results, this paper makes a technical contribution by demonstrating a relatively simple way to incorporate endogenous price flexibility into an otherwise standard model. Romer (1990), Devereux and Yetman (2002), Yetman (2003) and Kimura and Kurozumi (2010) have previously proposed adaptations of the Calvo model which are similar in nature to the one described below. However, unlike Romer and Devereux and Yetman, we incorporate the modified approach into the standard micro-founded New Keynesian model widely used by the literature on optimal monetary policy. Note also that the main issue examined by Romer (1990) and Devereux and Yetman (2002) is the impact of trend inflation on the equilibrium degree of price flexibility. They do not consider the impact of inflation volatility on equilibrium price flexibility, nor do they analyse welfare maximising monetary policy in the face of stochastic shocks.[1]

Kimura and Kurozumi (2010) do analyse endogenous price flexibility in a standard New Keynesian model and, as in our paper, they do consider the implications of monetary policy responses to stochastic shocks. However, Kimura and Kurozumi (2010) do not explicitly derive the expected profit function from the micro-foundations of the model and they adopt a solution approach which may not be robust in all cases. Our solution approach on the other hand can more reliably deal with degenerate cases. Kimura and Kurozumi (2010) also do not consider welfare maximising monetary policy in the presence of endogenous price flexibility, which is one of the main contributions of this paper.[2]

In two further related papers Devereux and Yetman (2003, 2010) analyse exchange rate pass-through in an open economy model with endogenous price flexibility. These papers are amongst the first to introduce this modification into a micro-founded general equilibrium model. While the research question is different, in technical terms there are parallels with our modelling approach. However, Devereux and Yetman either assume that all shocks are i.i.d. or they make use of stochastic simulation techniques. In contrast to this, we show that, for quite general cases, it is possible to derive a closed-form representation for producers' expected profits. This greatly facilitates the derivation of equilibrium in our model.

There are a number of other approaches to modelling endogenous price flexibility which, compared to our approach, involve a greater departure from the general workhorse

[1] Empirically, it is arguable that trend inflation has a larger effect on price flexibly than does the variability of inflation, so it is natural that the first focus of this literature has been on the analysis of the link between trend inflation and price flexibility. However, while the empirical impact of inflation variability on price flexibility may be smaller, it nevertheless has a potentially significant effect on the trade-off that determines the optimal choice of monetary rule (as shown in this paper).

[2] In an early draft of their paper (Kimura, Kurozumi and Hara, 2008) there is some discussion of the implications of endogenous price flexibility for the optimal choice of the parameters of the Taylor rule. The authors speculate that endogenous price flexibility is likely to imply that a more aggressive anti-inflation stance is optimal. This is consistent with the results we present below.

model used in the monetary policy literature. These include the models of Calmfors and Johansson (2006), Devereux (2006), Kiley (2000) and Levin and Yun (2007). Calmfors and Johansson (2006) and Devereux (2006) consider the interaction between endogenous nominal flexibility and choice of exchange rate regime. In a related paper (Senay and Sutherland, 2006) we also consider the impact of exchange rate regime choice on the equilibrium degree of price flexibility.[3]

The paper proceeds as follows. Section 2 describes the model. Section 3 discusses our solution approach. Section 4 briefly discusses some features of equilibrium. Section 5 shows how welfare maximising monetary policy is affected by endogenising the degree of price flexibility. Section 6 concludes the paper.

2 The Model

The model is a variation of the general equilibrium structure which is standard in the literature on monetary policy. There is a single country which is populated by many homogeneous households that supply labour to firms and consume a basket of all goods. There are many firms, each indexed on the unit interval and each a monopoly producer of a single differentiated product. There is a unit mass of households and a unit mass of firms.

Price setting follows the Calvo (1983) structure. In any given period, firm j is allowed to change the price of good j with probability $(1 - \gamma(j))$.

In period 0 the monetary authority makes its choice of monetary rule. Immediately following this policy decision, all firms are allowed to make a first choice of output price. Simultaneously, all firms are also allowed to make a once-and-for-all choice of $\gamma(j)$. In

[3] The degree of price flexibility is, in a sense, also endogenous in models of state-dependent pricing. See, for instance, Devereux and Siu (2007), Dotsey, King and Wolman (1999), Dotsey and King (2005), Golosov and Lucas (2007) and Ho and Yetman (2008). These papers typically focus on the implications of state dependent pricing for business cycle dynamics and the propagation of monetary shocks.

each subsequent period, beginning with period 1, stochastic shocks are realised, individual firms receive their Calvo-price-adjustment signal, those firms which are allowed to adjust their prices do so, and finally trade takes place.

Firms face costs of price adjustment. These costs are increasing in the average frequency of price changes. When firms make their decision on the choice of γ they must balance the benefits of greater price flexibility against the costs of price adjustment.

The model economy is subject to stochastic shocks from two sources: productivity and cost-push shocks.

2.1 Preferences

Households have preferences represented by

$$U_t = E_t \left[\sum_{s=t}^{\infty} \beta^{s-t} \left(\frac{C_s^{1-\rho}}{1-\rho} - \frac{\chi}{\mu} H_s^{\mu} \right) \right] \tag{1}$$

where $0 < \beta < 1$, ρ, $\chi > 0$, $\mu > 1$ and H is hours worked, E_t is the expectations operator (conditional on time t information) and C is a consumption index given by

$$C_t = \left[\int_0^1 c_t(i)^{\frac{\phi-1}{\phi}} \, di \right]^{\frac{\phi}{\phi-1}} \tag{2}$$

where $\phi > 1$ and $c(i)$ is consumption of good i. The aggregate consumer price index is

$$P_t = \left[\int_0^1 p_t(i)^{1-\phi} \, di \right]^{\frac{1}{1-\phi}} \tag{3}$$

The budget constraint of the representative household is

$$B_t + C_t = r_t B_{t-1} + H_t \frac{W_t}{P_t} + \Pi_t \tag{4}$$

where B_t is holdings of risk-free real bonds at the end of period t, r_t is the gross real return on bonds, W_t is the nominal wage and Π_t is the household's share in the profits of firms. It is assumed that all households own an equal share in all firms so households are insured against stochastic variation in individual firm profits caused by Calvo price setting.

2.2 Firms, Price Setting and Cost-push Shocks

Each individual firm produces a single differentiated product. The only input into production is labour, which is hired in a homogeneous labour market where firms act as price takers. The production function for product j is given by $y_t(j) = A_t L_t(j)$ where $L(j)$ is labour input. A is a stochastic shock to productivity where $\log A_t = \delta_A \log A_{t-1} + \varepsilon_{A,t}$, $0 < \delta_A < 1$ and ε_A is symmetrically distributed over the interval $[-\varepsilon, \varepsilon]$ with $E[\varepsilon_A] = 0$ and $Var[\varepsilon_A] = \sigma_A^2$.[4]

Firm j maximises the discounted present value of expected profits, i.e. firm j maximises

$$\Pi_t(j) = E_t \left\{ \sum_{s=t}^{\infty} \beta^{s-t} \frac{C_s^{-\rho}}{C_t^{-\rho}} [y_s(j) \frac{p_s(j)}{P_s} - \Lambda_s L_s(j) \frac{W_s}{P_s} - \theta(\gamma(j)) \frac{W_s}{P_s}] \right\} \tag{5}$$

where $\theta(\gamma(j))$ represents the costs of price adjustment, which we assume take the form of additional labour input. The function $\theta(.)$ is clearly crucial in the determination of the optimal degree of price flexibility. The details of this function will be discussed in the next section. Λ is a stochastic shock to labour costs (which may arise for instance from a distortionary employment tax), where $\log \Lambda_t = \delta_\Lambda \log \Lambda_{t-1} + \varepsilon_{\Lambda,t}$, $0 < \delta_\Lambda < 1$ and ε_Λ is symmetrically distributed over the interval $[-\varepsilon, \varepsilon]$ with $E[\varepsilon_\Lambda] = 0$ and $Var[\varepsilon_\Lambda] = \sigma_\Lambda^2$.

In equilibrium, all firms choose the same value of $\gamma(j)$, denoted γ. In any given period, proportion $(1 - \gamma)$ of firms are allowed to reset their prices. All producers who set their price at time t choose the same price, denoted x_t. The first-order condition for the choice of prices implies

$$x_t = \frac{\phi}{(\phi - 1)} E_t \left\{ \frac{\sum_{s=t}^{\infty} (\beta\gamma)^{s-t} \Lambda_s A_s^{-1} Y_s C_s^{-\rho} W_s P_s^{\phi-1}}{\sum_{s=t}^{\infty} (\beta\gamma)^{s-t} Y_s C_s^{-\rho} P_s^{\phi-1}} \right\} \tag{6}$$

where Y_t is aggregate demand for final goods (which in equilibrium equals C_t).

Later it proves useful to consider the price that an individual firm would choose if

[4] Extending the model to allow for diminishing returns to labour is straightforward and has no qualitative impact on the results reported below.

prices could be reset every period. This price is denoted p_t^o and is given by

$$p_t^o = \frac{\phi}{(\phi - 1)} \Lambda_t A_t^{-1} W_t \tag{7}$$

2.3 Costs of Price Adjustment

Price flexibility is made endogenous in this model by allowing all firms to make a once-and-for-all choice of the Calvo-price-adjustment probability in period zero. When making decisions about price flexibility each firm acts as a Nash player. Given that all firms are infinitesimally small, the choice of individual $\gamma(j)$ is made while assuming that the aggregate choice of γ is fixed. The equilibrium γ is assumed to be the Nash equilibrium value (i.e. where the individual choice of $\gamma(j)$ equals the aggregate γ).

Firms make their choice of γ in order to maximise the discounted present value of expected profits. From the point of view of the individual firm, the optimal γ is the one which equates the marginal benefits of price flexibility with the marginal costs of price adjustment. The benefits of price flexibility arise because a low value of γ implies that the individual price can more closely respond to shocks. The costs of price adjustment may take the form of menu costs, information costs and other decision making costs. These costs of price adjustment are captured by the function $\theta(\gamma(j))$ in equation (5). It is further assumed that price adjustment costs are proportional to the expected number of price changes per period, i.e. proportional to $1 - \gamma(j)$. Thus $\theta(\gamma(j))$ is of the following form

$$\theta(\gamma(j)) = \alpha(1 - \gamma(j)) \tag{8}$$

where $\alpha > 0$. Note that the cost of price flexibility is a function of the average rate of price adjustment, and is not linked to actual price changes.[5] We assume that price adjustment costs are second order in the sense that they are proportional to the variance of the

[5] Kimura and Kurozumi (2010) on the other hand assume that a lump-sum cost arises on each occasion prices are adjusted. Our formulation in identical to Kimura and Kurozumi's assumption in terms of the *expected* costs of price flexibility.

innovations to exogenous shocks. Thus, the labour effort expended on price adjustment is irrelevant in determining equilibrium employment in both the non-stochastic steady state of the model and in the first-order approximation.

The assumption that $\theta(.)$ is linear is adopted for simplicity. An alternative (which could easily be accommodated by our solution procedure) would be to assume that $\theta(.)$ is convex, so that price adjustment is subject to increasing marginal costs as the average frequency of price changes rises. We comment further on this below.

The assumption that there is a once-and-for-all decision about γ is required for tractability. A more general model would allow firms to choose γ in each period (or alternatively in those periods in which they have the opportunity to change price). This would allow the equilibrium γ to respond dynamically to the evolution of state variables. However it would also imply that the choice of γ will vary across firms (because in any given period output prices differ across firms so the incentive for price flexibility differs across firms). The solution of the model is this case would be very complex because it would need to account for the evolution of the distribution of γs across the population of firms. By restricting the choice of γ to period zero we avoid this added level of complexity. Given that our main objective is to investigate how the choice of γ responds to the choice of monetary rule, and given that the choice of the monetary rule is itself assumed to be a once-and-for-all decision, it is unlikely that much is lost by restricting the choice of γ in this way. Nevertheless, extending the model to allow for time variation in γ is likely to be an interesting avenue of future research.

A more general model could also allow price adjustment costs to be convex in the *size* of price changes (as in Rotemberg (1982)). This would effectively turn the model into a hybrid of the Calvo (1983) and Rotemberg (1982) models. In technical terms this would again create problems of tractability because each individual firm's price would effectively become a state variable of the model. For these reasons we confine attention to a simple cost function of the form given in (8). However, as it is known that the

Calvo and Rotemberg models have a number of different implications for monetary policy (see Ascari and Rossi (2012)) the interaction between the endogenous determination of the frequency of price changes and the macro dynamics implied by the Rotemberg cost assumption are likely to be an interesting topic of future research.

2.4 Monetary Policy

Monetary policy is modelled in the form of a targeting rule. The monetary authority is assumed to choose the monetary instrument (which is the nominal interest rate) in order to ensure that the following targeting relationship holds

$$\log \frac{P_t}{P_{t-1}} + \psi \log \frac{Y_t}{Y_t^*} = 0 \tag{9}$$

where Y_t^* is monetary authority's target output level. We assume that Y_t^* is chosen to be the welfare maximising output level (which is specified in more detail below). Thus the monetary authority follows a inflation targeting policy where ψ measures the degree to which inflation is allowed to vary in response to changes in the welfare-relevant output gap. The analysis below focuses on the welfare implications of the choice of ψ. A rule of this form is of particular interest because it is known to be optimal (within the class of "non-inertial rules") in a model where the degree of price flexibility is exogenously specified (see for instance the discussion of non-inertial policy rules in Benigno and Woodford (2005)). Note that it is not necessary to specify explicitly the form of the interest rate rule which delivers the targeted outcome defined by (9).

3 Model Solution

It is not possible to derive an exact solution to the model described above. The model is therefore approximated around a non-stochastic equilibrium. In what follows a bar indicates the value of a variable at the non-stochastic steady state and a hat indicates the log deviation from the non-stochastic equilibrium.

Our objective is to solve for the Nash equilibrium value of γ. In order to do this it is necessary to consider the optimal choice of $\gamma(j)$ at the level of the individual firm. Before going into detail, however, we outline our general solution approach.

First note that, for a given value of aggregate γ it is possible to solve for the behaviour of the aggregate economy. The behaviour of the aggregate economy is, by definition, unaffected by the choices of an individual firm (because each firm is assumed to be infinitesimally small). It is therefore possible to analyse the choices of firm j while taking the behaviour of the aggregate economy as given. This allows us to solve for the expected profits of firm j for given values of γ and $\gamma(j)$. It is then possible to identify the profit maximising value of $\gamma(j)$ for any given value of γ. In other words, it is possible to plot the "best response function" of firm j to aggregate γ. Using the best response function it is straightforward to identify Nash equilibria in the choice of γ. A Nash equilibrium is simply one where the profit maximising choice of $\gamma(j)$ is equal to aggregate γ.

Below we outline each stage of this solution process. We start with the aggregate economy. We then derive an explicit solution for firm j's expected profits as a function of γ and $\gamma(j)$. We use a grid search technique to analyse the expected profit function and to plot the best response function. This allows us to identify all possible Nash equilibria for each parameter combination.

3.1 The Aggregate Economy

For a given value of aggregate γ, the aggregate economy behaves exactly like the standard New Keynesian model analysed by Benigno and Woodford (2005) (and many others).

As is standard, it is possible the derive the New Keynesian Phillips curve, which, using first-order approximations of (3), (6) and (7), can be written in the form[6]

$$\pi_t = \kappa(\hat{Y}_t - \hat{Y}_t^N) + \beta E_t[\pi_{t+1}] + O\left(\varepsilon^2\right) \tag{10}$$

[6]The term $O\left(\varepsilon^n\right)$ gathers all terms of order n and higher in the size of the shocks.

where $\pi_t = \hat{P}_t - \hat{P}_{t-1}$, $\kappa = (1-\gamma)(1-\beta\gamma)\lambda/\gamma$, $\lambda = \mu + \rho - 1$ and $\hat{Y}_t^N$ is the natural rate of output, which is given by $\hat{Y}_t^N = (-\hat{\Lambda}_t + \mu \hat{A}_t)/\lambda$.

The New Keynesian Phillips curve provides one of the relationships which ties down equilibrium in the macro economy. The second key relationship is provided by the policy rule (9), which can be written in the form

$$\pi_t + \psi(\hat{Y}_t - \hat{Y}_t^*) = 0 + O\left(\varepsilon^2\right) \tag{11}$$

where $\hat{Y}_t^*$ is defined to be the welfare maximising output level, which, following Benigno and Woodford (2005), we assume is given by $\hat{Y}_t^* = c_\Lambda \hat{\Lambda}_t + c_A \hat{A}_t$ where

$$c_\Lambda = \frac{-\mu\Phi}{\lambda[\lambda + (1-\rho)\Phi]}, \qquad c_A = \frac{1}{\lambda}$$

and where $\Phi = 1 - (\phi - 1)/\phi$.

Using (7), (10) and (11) it is simple to show that the solution for $\hat{p}_t^o - \hat{P}_t$ and π_t can be written in the form

$$\hat{p}_t^o - \hat{P}_t = p_\Lambda \hat{\Lambda}_t + p_A \hat{A}_t + O\left(\varepsilon^2\right) \tag{12}$$

and

$$\pi_t = \pi_\Lambda \hat{\Lambda}_t + \pi_A \hat{A}_t + O\left(\varepsilon^2\right) \tag{13}$$

where p_Λ, p_A, π_Λ and π_A are defined in the Appendix.

3.2 Expected Profits of the Representative Firm

In order to derive the equilibrium value of γ it is necessary to consider the impact of the choice of γ on the expected profits of a representative individual firm. The expected profits of firm j at time zero (i.e. at the time $\gamma(j)$ is chosen) are given by

$$\Pi_t(j) = E_t\left\{\sum_{s=t}^{\infty} \beta^{s-t} \frac{C_s^{-\rho}}{C_t^{-\rho}} [y_s(j) \frac{p_s(j)}{P_s} - \Lambda_s L_s(j) \frac{W_s}{P_s} - \theta(\gamma(j)) \frac{W_s}{P_s}]\right\} \tag{14}$$

The Appendix shows that a second-order approximation of (14) takes the form

$$\begin{aligned}\frac{\tilde{\Pi}_0(j)-\bar{\Pi}}{\bar{C}} &= \frac{(\phi-1)}{2}E_0\sum_{t=1}^{\infty}\beta^{t-1}\left[-(\hat{p}_t(j)-\hat{P}_t)^2+2(\hat{p}_t^o-\hat{P}_t)(\hat{p}_t(j)-\hat{P}_t)\right] \\ &\quad -\frac{\phi-1}{\phi}\frac{\alpha/\bar{C}}{1-\beta}(1-\gamma(j))+tij+O\left(\varepsilon^3\right) \qquad (15)\end{aligned}$$

where tij represents terms independent of firm j.

We will evaluate and use (15) to analyse the optimal choice of $\gamma(j)$ for firm j. It is first useful to consider the general form of (15) in order to understand the underlying links between $\gamma(j)$ and firm j's profits. The choice of $\gamma(j)$ most obviously affects profits via the costs of price adjustment. However, $\gamma(j)$ also affects profits via its impact on the evolution of firm j's price, $\hat{p}_t(j)$. Equation (15) shows that there are two routes by which the evolution of $\hat{p}_t(j)$ affects profits. The first route arises via the term $E_0[(\hat{p}_t(j)-\hat{P}_t)^2]$. This term captures the variance of $\hat{p}_t(j)$ relative to the general price level. A higher variance of $(\hat{p}_t(j)-\hat{P}_t)$ implies lower expected profits. Note that the variance of $(\hat{p}_t(j)-\hat{P}_t)$ depends on the difference between the flexibility of $\hat{p}_t(j)$ and the flexibility of $\hat{P}_t$. This is captured by the difference between $\gamma(j)$ and aggregate γ. This term is therefore likely to create a tendency for the individual firm to prefer a value of $\gamma(j)$ close to aggregate γ. The second route by which the evolution of $\hat{p}_t(j)$ affects expected profits arises via the term $E_0[(\hat{p}_t^o-\hat{P}_t)(\hat{p}_t(j)-\hat{P}_t)]$. This term is the covariance between the actual price charged by firm j and the optimal price if firm j had complete freedom to choose a new price every period. Not surprisingly, an increase in the covariance has a positive effect on expected profits. Also not surprisingly, this covariance is negatively related to $\gamma(j)$, i.e. the more rigid is $\hat{p}_t(j)$ the less correlated it can be with $\hat{p}_t^o$.[7]

The effect of $\gamma(j)$ on expected profits is a combination of these three effects, i.e. it is

[7] Note that Romer (1990), Devereux and Yetman (2002) and Kimura and Kurozumi (2010) start from a postulated profit function of the form $K\left[\hat{p}_t(j)-\hat{p}_t^o\right]^2$. This can be expanded to yield $K\left[\hat{p}_t^2(j)-2\hat{p}_t^o\hat{p}_t(j)+\hat{p}_t^{o2}\right]$. If it is noted that $\hat{p}_t^{o2}$ is independent of the actions of producer j this can be written as $K\left[\hat{p}_t^2(j)-2\hat{p}_t^o\hat{p}_t(j)\right]+tij$ which has the same form as the first term in (15). Note also that these other authors define the profit function using a recursive equation which (after taking uncon-

a combination of the effect on the costs of price adjustment, the variance of $(\hat{p}_t(j) - \hat{P}_t)$ and the covariance between $\hat{p}_t(j)$ and $\hat{p}_t^o$. An increase in $\gamma(j)$ will reduce the costs of price adjustment and will thus increase expected profits, but it will also tend to reduce the covariance between $\hat{p}_t(j)$ and $\hat{p}_t^o$ which tends to reduce expected profits. The effect of an increase in $\gamma(j)$ on the variance of $(\hat{p}_t(j) - \hat{P}_t)$ will depend on whether $\gamma(j)$ is greater than or less than aggregate γ. The optimal $\gamma(j)$ will obviously depend on the net outcome of the interaction of all these three factors.

In order to analyse equation (15) in more detail it is necessary to derive equations which describe the expected evolution of firm j's price, i.e. $\hat{p}_t(j)$. In particular it is necessary to derive second-order accurate solutions for $E_0[(\hat{p}_t^o - \hat{P}_t)(\hat{p}_t(j) - \hat{P}_t)]$ and $E_0[(\hat{p}_t(j) - \hat{P}_t)^2]$. Since $\hat{p}_t(j)$ depends on $\hat{x}_t(j)$ (i.e. the optimal price chosen when firm j is allowed to reset the price of good j) it is first necessary to solve for the first-order accurate behaviour of $\hat{x}_t(j)$. The Appendix shows that $\hat{x}_t(j) - \hat{P}_t$ can be written in the following form

$$\hat{x}_t(j) - \hat{P}_t = x_\Lambda \hat{\Lambda}_t + x_A \hat{A}_t + O\left(\varepsilon^2\right) \tag{16}$$

and further shows how this expression can be used to solve for $E_0[(\hat{p}_t^o - \hat{P}_t)(\hat{p}_t(j) - \hat{P}_t)]$ and $E_0[(\hat{p}_t(j) - \hat{P}_t)^2]$. The definitions of x_Λ and x_A are given in the Appendix.

Using equations (12), (13), (16) and the expressions for $E_0[(\hat{p}_t^o - \hat{P}_t)(\hat{p}_t(j) - \hat{P}_t)]$ and $E_0[(\hat{p}_t(j) - \hat{P}_t)^2]$ derived in the Appendix we can derive a closed-form solution for the expected profits of firm j. For the sake of simplicity we focus on the case where the only source of shocks is the cost-push disturbance, Λ. Extending the expression to incorporate other shocks is straightforward. The resulting expression is

$$\begin{aligned}\frac{\tilde{\Pi}_0(j) - \bar{\Pi}}{\bar{C}} &= -\frac{(\phi - 1)\Delta\sigma_\Lambda^2}{2(1-\beta)(1-\beta\delta_\Lambda^2)(1-\beta\delta_\Lambda\gamma(j))(1-\beta\gamma(j))} \\ &\quad -\frac{\phi - 1}{\phi}\frac{\alpha/\bar{C}}{1-\beta}(1-\gamma(j)) + tij + O\left(\varepsilon^3\right)\end{aligned} \tag{17}$$

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ditional expectations) yields an expression based on expected profits for a given output price discounted at rate $\delta\gamma$. A similar form of expression can be obtained (after much rearrangement) from (15) by taking unconditional expectations.

where

$$\Delta = (1 - \beta\delta_\Lambda\gamma(j))(1 - \gamma(j))x_\Lambda^2 + (1 + \beta\delta_\Lambda\gamma(j))\gamma(j)\pi_\Lambda^2$$
$$-2\beta\delta_\Lambda\gamma(j)(1 - \gamma(j))x_\Lambda\pi_\Lambda - 2(1 - \beta\gamma(j))[(1 - \gamma(j))x_\Lambda p_\Lambda - \gamma(j)\pi_\Lambda p_\Lambda]$$

In principle it would be possible to analyse the optimal choice of $\gamma(j)$ by examining the derivatives of (17). The resulting expressions are however very complex. Furthermore, because the choice of $\gamma(j)$ must lie in the $[0, 1]$ range (since it is a probability) the optimal choice of $\gamma(j)$ may be a corner solution rather than an interior solution. It is therefore easier to analyse (17) by means of a numerical grid search technique. This is the most reliable way to identify the global maximising value of $\gamma(j)$ within the $[0, 1]$ range for each γ in the $[0, 1]$ range. We use this grid search technique to plot the best response function for each parameter combination. In turn, this allows us to identify all possible Nash equilibria for each parameter combination.[8]

4 The Equilibrium Degree of Price Flexibility

This section presents numerical results which illustrate the general nature and properties of equilibrium in the model described above. We analyse the model using the benchmark set of parameter values in Table 1. Most of the values chosen are quite standard and require no explanation. The only parameter which requires some discussion is α, i.e. the coefficient determining the costs of price adjustment. The function $\theta(\gamma(j))$ in principle captures a wide range of costs associated with price adjustment. Not all these costs are directly measurable, so there is no simple empirical basis on which to select a value for α. As a starting point, for the purposes of illustration, the value of α is set at 0.0006 in the

[8] Devereux and Yetman (2003) derive a closed-form solution for their model which plays a similar role to (17). However, their expression is only valid for the case of *i.i.d.* shocks. But note that our model differs from the Devereux and Yetman model in many respects, so our expression (17) does not encompass the equivalent expression in Devereux and Yetman (2003).

Discount factor	$\beta = 0.99$
Elasticity of substitution for individual goods	$\phi = 8.00$
Risk aversion	$\rho = 1$
Price adjustment costs	$\alpha = 0.0006$
Cost-push shocks	$\delta_\Lambda = 0.9$, $\sigma_\Lambda = 0.01$
Policy parameter	$\psi = 0.1$
Labour supply elasticity	$1/(\mu - 1) = 2.0$

Table 1: Parameter Values

benchmark case. In equilibrium (given the benchmark values for other parameters) this implies prices are adjusted at an average rate of once every four quarters (i.e. $\gamma = 0.75$) so aggregate price adjustment costs are 0.015% of GDP in equilibrium. This total aggregate cost is not implausibly high, given the potentially wide range of costs incorporated in $\theta(\gamma(j))$, but it is acknowledged that a more satisfactory basis needs to be found for calibrating α. In order to test the sensitivity of the main results we consider a wide range of alternative values for α.

Figure 1 illustrates some of the main features of equilibrium in the benchmark case and a number of simple variations around that benchmark case. Figure 1 plots the optimal value of $\gamma(j)$ against values of aggregate γ. In other words it shows the "best response function" of firm j against aggregate γ. The benchmark case is illustrated with the plot marked with circles. This plot intersects the 45^0 just once. This point of intersection represents a single Nash equilibrium where $\gamma(j) = \gamma \approx 0.75$.

The position of the best response function, and its shape, are contingent on the values of other parameters of the model. A change in the value of any parameter implies a change in the best response function and a change in the equilibrium value of γ. Figure 1 shows four other examples of best response functions - each based on a different value of ψ, the parameter of the monetary policy rule. The higher is ψ the more monetary policy is aimed at stabilising the output gap, while a lower value of ψ implies that monetary policy is aimed at stabilising inflation. The general pattern that emerges from the cases illustrated in Figure 1 is that an decrease in ψ shifts the best response function upwards, and thus leads to an increase in the equilibrium value of γ, while an increase in ψ shifts the best response function downwards and thus leads to a reduction in the equilibrium value of γ. In other words, the more policy focuses on stabilisation of inflation, the lower is equilibrium price flexibility.

The intuition behind this result is relatively straightforward. If the monetary authority is stabilising aggregate inflation it is by definition stabilising the desired price, $\hat{p}_t^o$. If the desired price is very stable then the incentive to incur the costs of price flexibility are much reduced, hence firms choose a high value of γ. At the other extreme, a high value of ψ (i.e. a monetary rule which allows fluctuations in inflation in order to achieve some stabilisation of the output gap) will necessarily cause fluctuations in the desired price, $\hat{p}_t^o$. This raises the incentive for firms to incur the costs of price flexibility and therefore choose a lower value of γ.

The results illustrated in Figure 1 correspond to the results presented by Kimura and Kurozumi (2010). They model monetary policy in terms of a Taylor rule, rather than as a targeting rule of the form used in this paper, but the same relationship between the anti-inflation stance of monetary policy and the equilibrium value of γ emerges. As the feedback coefficient on inflation in the Taylor rule is increased the equilibrium value of γ increases, so output prices become less flexible.

The cases illustrated in Figure 1 are quite regular in the sense that for each value of ψ there is a single intersection between the best response function and the 45^0 line, so there is a unique Nash equilibrium. Furthermore that equilibrium is strictly within the 0-1 range of values for γ. A more complex picture emerges, however, as the policy parameter becomes very small (i.e. as policy approaches strict inflation stabilisation). Figure 2 illustrates an example of this case. This figure focuses on values of ψ very close to zero and it shows that, for instance when $\psi = 0.002$, there is a discontinuity in the best response function which implies that there is no intersection with the 45^0 line. There is thus no simple Nash equilibrium for this (or smaller) values of ψ.[9]

It will become apparent below that the non-existence of a simple Nash equilibrium for low values of ψ is relevant in the analysis of the welfare maximising choice of ψ, so it is worth considering this result in more detail. Consider the best response function for $\psi = 0.002$ shown in Figure 2. The best response function is horizontal at unity for values of γ less than approximately 0.987. There is then a discrete downward jump at $\gamma \simeq 0.987$ after which the best response function becomes downward sloping. The discontinuity arises because the profit function is not single-peaked. Thus, for values of γ in the neighbourhood of the discontinuity, the profit function has two local maxima, one at $\gamma(j) = 1$ and one at a lower value of $\gamma(j)$. For values of γ to the left of the discontinuity profits are higher at $\gamma(j) = 1$. But for values of γ to the right of the discontinuity profits are higher at the internal turning point in the profit function. At the discontinuity the two maxima yield the same value of profits.

The fact that the profit function has two local maxima suggests that, even though there is no simple Nash equilibrium with a unique equilibrium value of γ, there may be a more complex Nash equilibrium where the set of firms divides into two groups, each group setting a different value of γ (corresponding to the two maxima). It is considerably more complex to derive solutions for such equilibria so we do not investigate them further in this paper. Below, when we consider the welfare maximising choice of ψ, we therefore confine attention to the range of values of ψ for which simple Nash equilibria exist (i.e. where there is single equilibrium value γ).

A number of other potentially important special cases are illustrated in Figure 3, which

[9] In fact the discontinuity appears for values of ψ less than approximately 0.0035.

shows the effect of varying α, the parameter that determines the cost of price adjustment. This figure shows that as α increases, the equilibrium value of γ tends to increase. The explanation for this is obvious. As price adjustment becomes more costly firms optimally choose to make output prices less flexible.[10] Note however that for very high values of α the best response function intersects the 45^0 twice. There are thus two Nash equilibria. And for the highest values of α the best response function does not intersect with the 45^0 line within the zero-one interval. In this latter case the Nash equilibrium is a corner solution at $\gamma = 1$, i.e. completely fixed prices. In other words, when the costs of price adjustment are sufficiently high, firms optimally choose to avoid price adjustment entirely.

Figures 2 and 3 illustrate a number of cases where the Nash equilibrium of this simple model either does not exist, is based on a corner solution or is not unique. These cases illustrate the benefits of working with the explicit functional form for the profit function given by (17). By applying a numerical grid search technique to this functional form it is possible reliably to identify the profit maximising value of $\gamma(j)$ even when there are multiple turning points in the function or when the maximum occurs at corner. By contrast, methods which rely on evaluating the first derivative of the profit function, such as used by Kimura and Kurozumi (2010), potentially yield spurious results for some parameter configurations.

5 Welfare and Optimal Policy

We now consider the welfare implications of endogenous price flexibility. In particular we consider the implications for the welfare maximising choice of the policy parameter,

[10] Notice from (17) that the profit function consists of two main terms: one that measures the expected profits foregone because of price inflexibility and one that measures the expected costs of price changes. Increasing α obviously increases the weight on the second term relative to the first. Notice that, other things being equal, this is equivalent to reducing σ^2 (the variance of the innovations in the cost-push shocks). Figure 3 can therefore also be used to understand the effects of varying σ^2.

ψ. For the purposes of this exercise aggregate welfare in period 0 (i.e. at the time the monetary policy parameter, ψ, is set) is defined as

$$\Omega = E_0 \sum_{t=1}^{\infty} \beta^{t-1} \left\{ \frac{C_t^{1-\rho}}{1-\rho} - \frac{\chi}{\mu} H_t^{\mu} \right\} \tag{18}$$

A second-order approximation of Ω can be written as follows

$$\begin{aligned} \left(\Omega - \bar{\Omega}\right) \bar{C}^{\rho-1} = E_0 \sum_{t=1}^{\infty} \beta^{t-1} \Bigg\{ \hat{C}_t + \frac{1}{2}(1-\rho)\hat{C}_t^2 \\ - \frac{\phi-1}{\phi} \left[\hat{Y}_t + \frac{1}{2}\mu(\hat{Y}_t - \hat{A}_t)^2 + \frac{1}{2}\frac{\phi\gamma}{(1-\gamma)(1-\beta\gamma)}\pi_t^2 \right] \Bigg\} \\ - \underbrace{\frac{\phi-1}{\phi}\frac{\alpha/\bar{C}}{1-\beta}(1-\gamma)}_{\text{welfare cost of price adjustment}} + tip + O\left(\varepsilon^3\right) \end{aligned} \tag{19}$$

Equation (19) shows that the welfare expression in this model takes the same form as in the conventional model (with exogenous γ) but with an added term that represents the welfare costs of price adjustment.

5.1 The Benchmark Case

Figure 4 plots welfare against a range of values for ψ, while keeping other parameters at their benchmark values. In this figure the plot marked with circles shows the behaviour of welfare for the case of endogenous price flexibility. As a point of comparison, the figure also shows welfare for the case where price flexibility is exogenously fixed at a given level. (This latter plot is marked with squares.) In this example we set $\gamma = 0.75$. The exogenous γ case corresponds to the standard model widely analysed in the literature on optimal monetary policy (see e.g. Benigno and Woodford (2005)) and it is therefore a natural point of comparison.

The welfare plot for the exogenous γ case shows that the welfare maximising value of ψ is approximately 0.013. This implies that it is optimal for monetary policy to allow some volatility in inflation in order to achieve some stabilisation of the welfare-relevant output gap. This corresponds to the result emphasised by Benigno and Woodford (2005). In essence, cost-push shocks are distortionary and imply that the flexible price equilibrium is sub-optimal. In a sticky price environment it is therefore *not* optimal for monetary policy to reproduce the flexible price equilibrium. Sticky prices give monetary policy some degree of leverage which can be used to stabilise output around the welfare maximising level. This requires some volatility in inflation.

Now consider the plot of welfare in the case were price flexibility is endogenous (i.e. the plot marked with circles). It is immediately clear from this plot that, when price flexibility is endogenous, welfare appears to increase monotonically as ψ decreases towards zero. In other words, when price flexibility is endogenous, it appears to be optimal to engage in aggressive inflation stabilisation. This is in contrast to the case of exogenous price flexibility, where it is optimal to allow some variability in inflation.

As noted above (in relation to the discussion of Figure 2) in Figure 4 we confine attention to the range of values of ψ for which a simple Nash equilibrium exists, i.e. where there is a single equilibrium value of ψ. This implies that we do not consider values of ψ below 0.0035. It is apparent from Figure 4 that this lower limit on the value of ψ corresponds to the welfare maximising choice of ψ (within the range that we consider). Because ψ does not equal zero at this point, we describe this as *aggressive* inflation stabilisation rather than *strict* inflation targeting.

The contrast between the exogenous and endogenous flexibility cases illustrated in Figure 4 arises for two reasons. Firstly, as ψ is reduced, the rise in the equilibrium value of γ reduces the resource cost of price flexibility (i.e. less frequent price changes implies lower costs). Secondly, the rise in the equilibrium value of γ implies that monetary policy becomes a more effective policy tool (because the real effects of monetary policy depend on the presence of nominal rigidities and these become more significant as γ increases). Therefore, as γ increases, monetary policy becomes more effective in dealing with the distortions caused by cost-push shocks. This is reflected in a higher level of welfare as ψ

decreases and γ increases.

It is simple to measure the relative contributions of these two welfare effects by plotting the welfare measure excluding the welfare costs of price adjustment (i.e. excluding the term indicated in (19)) and comparing it to the full welfare measure (which includes the costs of price adjustment). This comparison is also shown in Figure 4. The plot marked with asterisks in Figure 4 is the value of welfare *excluding* the costs of price adjustment. This can be compared to the plot marked with the circles, which shows welfare *including* the costs of price adjustment. It appears that the decline in the welfare costs of price adjustment (i.e. the vertical distance between the two plots) contributes about half of the rise in total welfare as ψ is reduced. The other half of the rise in welfare (i.e. the rise in welfare illustrated in the upper plot in Figure 4) comes from the effect of more rigid prices on the ability of monetary policy to deal with cost push shocks .

The result illustrated in Figures 4 represents a significant departure compared to the literature based on exogenous price flexibility. That literature has emphasised that complete inflation stabilisation is *not* optimal in the face of cost-push shocks. The comparison between the exogenous and endogenous price flexibility cases shown in Figure 4 implies that this basic result may be overturned when price flexibility is endogenised.

Of course, Figure 4 is based on just one set of parameter values. Nevertheless, experiments (not reported) testing the sensitivity of this basic qualitative result indicate that it is robust across a wide range of values for key parameters, such as μ, ϕ and ρ. The basic comparison between the exogenous and endogenous price flexibility cases remains the same across these parameter variations. Namely, it is optimal to allow some degree of inflation variability in the exogenous price flexibility case but it is welfare enhancing to pursue aggressive inflation targeting in the endogenous price flexibility case.

5.2 Discussion and Generalisations

As already noted, our analysis of the welfare effects of ψ in the endogenous γ case is restricted to values of ψ for which a simple Nash equilibrium exists. This places a lower limit on the value ψ. It is apparent from the example illustrated in Figure 4 that welfare reaches its maximum at exactly this lower limit. Our analysis therefore leaves open the question of what happens to welfare for values of ψ below this lower limit. This question can only be answered by considering more complex Nash equilibria than those analysed here. Such equilibria are likely to imply a division of the set of firms into different groups, each with a different value of γ. A preliminary investigation of an example of such an equilibrium has been carried out for the benchmark case, which suggests that welfare continues to increase as ψ is reduced below the value considered in Figure 4. But a full analysis of this type of equilibrium is well beyond the scope of this paper.

There are a number of other generalisations of our analysis which are more straightforward. For instance, as noted above, a more general approach to the modelling of price adjustment costs would be to allow convexity in $\theta(.)$, so that price adjustment is subject to increasing marginal costs as the average frequency of price changes rises. Note that this alternative assumption would tend to strengthen the incentive for firms to set a high value of γ. This suggests that our main result (i.e. that the optimal choice of ψ is very close to zero and that the resulting equilibrium choice of γ by firms is very close to unity) is robust to convexity in the price adjustment costs function $\theta(.)$.

The cost function could further be generalised to allow for the effects of aggregate variables on the cost of price adjustment. For instance, the cost of price adjustment could be assumed to fall as aggregate prices become more flexible. An effect such as this could easily be incorporated into our analysis.

It would also be straightforward to generalise our analysis to encompass more sources of shocks. The results described above focus entirely on the implications of cost-push shocks. Woodford (2003) and Benigno and Woodford (2005), using models with exogenous price flexibility, show that government spending shocks are very similar to cost-push shocks in terms of their implications for optimal monetary policy. This similarity carries over into our model. Extending our analysis to consider productivity shocks (i.e. shocks to A in our model) would also add little to the results we have emphasised above. Woodford (2003) and Benigno and Woodford (2005), using models of exogenous price flexibility, show that optimal monetary policy should completely stabilise inflation in the face of productivity shocks. In that case, our model of endogenous price flexibility simply predicts that prices will be completely rigid. This has no impact on the nature of optimal monetary policy in the face of productivity shocks.

A potentially important further extension of the model would be to introduce a cost of inflation indexation. It is well-known that the standard Calvo model generates inertia in the *price level* but leaves the *rate of inflation* completely flexible. Likewise, our model makes the degree of price-level inertia endogenous while retaining the assumption that the inflation rate is completely flexible. It has become standard in many applications of the Calvo model to include an partial degree of inflation indexation which creates an exogenous degree of inertia in the inflation rate. Just as the model used in this paper endogenises the degree of price-level inertia, it would be possible to endogenise the degree of inflation inertia by allowing firms to choose the rate of indexation subject to some cost of indexation. This would be another interesting line of future research.

6 Conclusion

This paper takes a standard sticky-price general equilibrium model and incorporates a simple mechanism which endogenises the degree of nominal price flexibility. The analysis shows that the equilibrium degree of price flexibility is sensitive to changes in monetary policy. For instance, the more the more weight monetary policy places on inflation stabilisation, the more inflexible prices become. We show that endogenising the degree of price flexibility tends to shift optimal monetary policy towards complete inflation stabilisation, even when shocks take the form of cost-push disturbances. This contrasts with the standard result obtained in models with exogenous price flexibility, which show that optimal monetary policy should allow some degree of inflation volatility in order to stabilise the welfare-relevant output gap.

The model we develop and analyse in this paper takes a somewhat stylised shortcut to the representation of endogenous price flexibility. A more theoretically appealing approach would be to develop a structural model of state-dependent price setting. While such models are being developed and analysed in the recent literature, they are still some way from being tractable enough to allow a detailed analysis of optimal monetary policy. In lieu of further progress with the development of state-dependent pricing models, the results we present in this paper offer a potentially useful benchmark for judging the impact of endogenous price flexibility on the welfare effects of monetary policy.

Appendix

Optimal Prices and Aggregate Inflation

Expressions (12) and (13) are derived using (3), (7) and (11). The coefficients p_Λ, p_A, π_Λ and π_A are given by the following

$$p_\Lambda = \Gamma_\Lambda(1-\beta\delta_\Lambda), \quad p_A = \Gamma_A(1-\beta\delta_A), \quad \pi_\Lambda = \Gamma_\Lambda\frac{\kappa}{\lambda}, \quad \pi_A = \Gamma_A\frac{\kappa}{\lambda}$$

where

$$\Gamma_\Lambda = \frac{[1+c_\Lambda\lambda]\,\psi}{\kappa+\psi(1-\beta\delta_\Lambda)}, \quad \Gamma_A = -\frac{[1+c_A\lambda]\,\psi}{\kappa+\psi(1-\beta\delta_A)}$$

Approximation of Firm j's Expected Profit Function

A second-order approximation of (14) is derived as follows. First substitute for $y_t(j)$ and rearrange to yield

$$\Pi_t(j) = \frac{1}{C_t^{-\rho}}E_t\left\{\sum_{s=t}^{\infty}\beta^{s-t}\left[\alpha_{1,s}\left(\frac{p_s(j)}{P_s}\right)^{1-\phi} - \alpha_{2,s}\left(\frac{p_s(j)}{P_s}\right)^{-\phi} - C_s^{-\rho}\alpha(1-\gamma(j))\frac{W_s}{P_s}\right]\right\}$$

where

$$\alpha_{1,s} = C_s^{-\rho}Y_s \text{ and } \alpha_{2,s} = C_s^{-\rho}\Lambda_s\frac{W_s}{P_s}A_s^{-1}Y_s$$

This form of the profit function isolates terms which depend on $\gamma(j)$. A second order approximation implies

$$\begin{aligned}\tilde{\Pi}_0(j)-\bar{\Pi} &= -\frac{(\phi-1)}{2}\bar{C}E_0\sum_{t=1}^{\infty}\beta^{t-1}\left[(\hat{p}_t(j)-\hat{P}_t)^2 - 2(\hat{\alpha}_{2,t}-\hat{\alpha}_{1,t})(\hat{p}_t(j)-\hat{P}_t)\right] \\ &\quad -\frac{\phi-1}{\phi}\frac{\alpha}{1-\beta}(1-\gamma(j)) + tij + O\left(\varepsilon^3\right)\end{aligned}$$

where tij represents terms independent of firm j. This may be written in the form of (15) by noting that

$$\hat{\alpha}_{2,s}-\hat{\alpha}_{1,s} = \hat{\Lambda}_s + \hat{W}_s - \hat{P}_s - \hat{A}_s + O\left(\varepsilon^2\right) = (\hat{p}_t^o - \hat{P}_t) + O\left(\varepsilon^2\right)$$

The Expected Dynamics of Firm j's Price

The first-order condition for price setting for firm j implies

$$\hat{x}_t(j) - \hat{P}_t = (1 - \beta\gamma(j))(\hat{p}_t^o - \hat{P}_t) + \beta\gamma(j)E_t[\hat{x}_{t+1}(j) - \hat{P}_{t+1}] + \beta\gamma(j)E_t[\pi_{t+1}] + O\left(\varepsilon^2\right)$$

When combined with (12) and (13) it is simple to show that $\hat{x}_t(j) - \hat{P}_t$ can be written in the form

$$\hat{x}_t(j) - \hat{P}_t = x_\Lambda \hat{\Lambda}_t + x_A \hat{A}_t + O\left(\varepsilon^2\right) \tag{20}$$

where x_Λ and x_A are given by

$$x_\Lambda = \Gamma_\Lambda \frac{(1 - \beta\delta_\Lambda)(1 - \beta\gamma(j))\lambda + \kappa\beta\delta_\Lambda\gamma(j)}{(1 - \beta\delta_\Lambda\gamma(j))\lambda}, \quad x_A = \Gamma_A \frac{(1 - \beta\delta_A)(1 - \beta\gamma(j))\lambda + \kappa\beta\delta_A\gamma(j)}{(1 - \beta\delta_A\gamma(j))\lambda}$$

To complete the solution for $E_0[(\hat{p}_t^o - \hat{P}_t)(\hat{p}_t(j) - \hat{P}_t)]$ and $E_0[(\hat{p}_t(j) - \hat{P}_t)^2]$ it is useful to decompose the expectations operator E_0 into E_0^D, expectations across aggregate disturbances, and E_0^C, expectations across the Calvo pricing signal for firm j, where $E_0[.] = E_0^D[E_0^C[.]]$. (Note that $E_0^C[.]$ is conditional on particular realised values of the aggregate disturbances, $\hat{\Lambda}_t$ and $\hat{A}_t$.) Since aggregate disturbances and aggregate variables are independent from the Calvo pricing signal for firm j, we may write

$$E_0[(\hat{p}_t^o - \hat{P}_t)(\hat{p}_t(j) - \hat{P}_t)] = E_0^D[(\hat{p}_t^o - \hat{P}_t)E_0^C[\hat{p}_t(j) - \hat{P}_t]]$$

It is thus necessary to obtain a first-order accurate solution for $E_0^C[\hat{p}_t(j) - \hat{P}_t]$.

The Calvo pricing process implies that $E_0^C[\hat{p}_t(j) - \hat{P}_t]$ evolves according to

$$E_0^C[\hat{p}_t(j) - \hat{P}_t] = \gamma(j)E_0^C[\hat{p}_{t-1}(j) - \hat{P}_{t-1}] + (1 - \gamma(j))(\hat{x}_t(j) - \hat{P}_t) - \gamma(j)\pi_t + O\left(\varepsilon^2\right)$$

This can be solved and combined with (12) and (16) to yield a second-order accurate expression for $E_0[(\hat{p}_t^o - \hat{P}_t)(\hat{p}_t(j) - \hat{P}_t)]$.

An equation for the evolution of $E_0[(\hat{p}_t(j) - \hat{P}_t)^2]$ can be derived in a similar way. The Calvo pricing process implies that

$$E_0[(\hat{p}_t(j) - \hat{P}_t)^2] = \gamma(j)E_0[(\hat{p}_{t-1}(j) - \hat{P}_t)^2] + (1 - \gamma(j))E_0[(\hat{x}_t(j) - \hat{P}_t)^2] + O\left(\varepsilon^3\right) \tag{21}$$

Using the relationships

$$E_0[(\hat{p}_{t-1}(j) - \hat{P}_t)^2] = E_0[(\hat{p}_{t-1}(j) - \hat{P}_{t-1})^2] + E_0[\pi_t^2] - 2E_0[\pi_t(\hat{p}_{t-1}(j) - \hat{P}_{t-1})]$$

$$E_0[\pi_t(\hat{p}_{t-1}(j) - \hat{P}_{t-1})] = E_0^D[\pi_t E_0^C(\hat{p}_{t-1}(j) - \hat{P}_{t-1})]$$

equation (21) can be written as

$$\begin{aligned} E_0[(\hat{p}_t(j) - \hat{P}_t)^2] &= \gamma(j)E_0[(\hat{p}_{t-1}(j) - \hat{P}_{t-1})^2] + (1-\gamma(j))E_0[(\hat{x}_t(j) - \hat{P}_t)^2] \\ &\quad + \gamma(j)E_0[\pi_t^2] - 2\gamma(j)E_0^D[\pi_t E_0^C(\hat{p}_{t-1}(j) - \hat{P}_{t-1})] + O\left(\varepsilon^3\right) \end{aligned}$$

which can be solved in combination with (12) and (16) to yield a second-order accurate expression for $E_0[(\hat{p}_t(j) - \hat{P}_t)^2]$.

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Figure 1: Monetary Policy and the Best Response Function

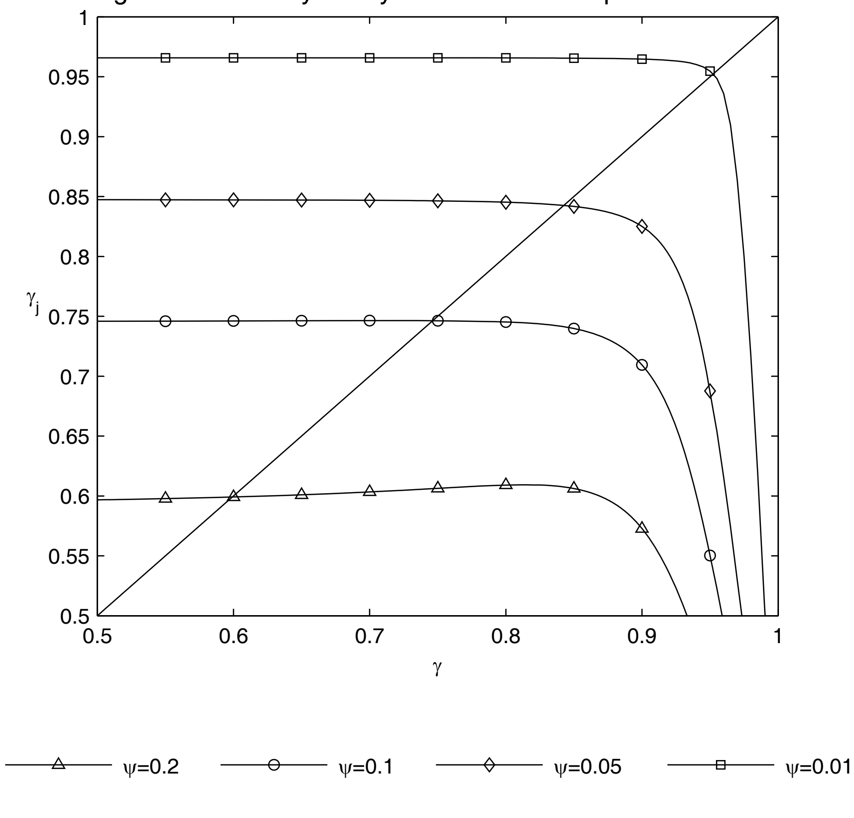

Figure 2: Discontinuities in the Best Response Function

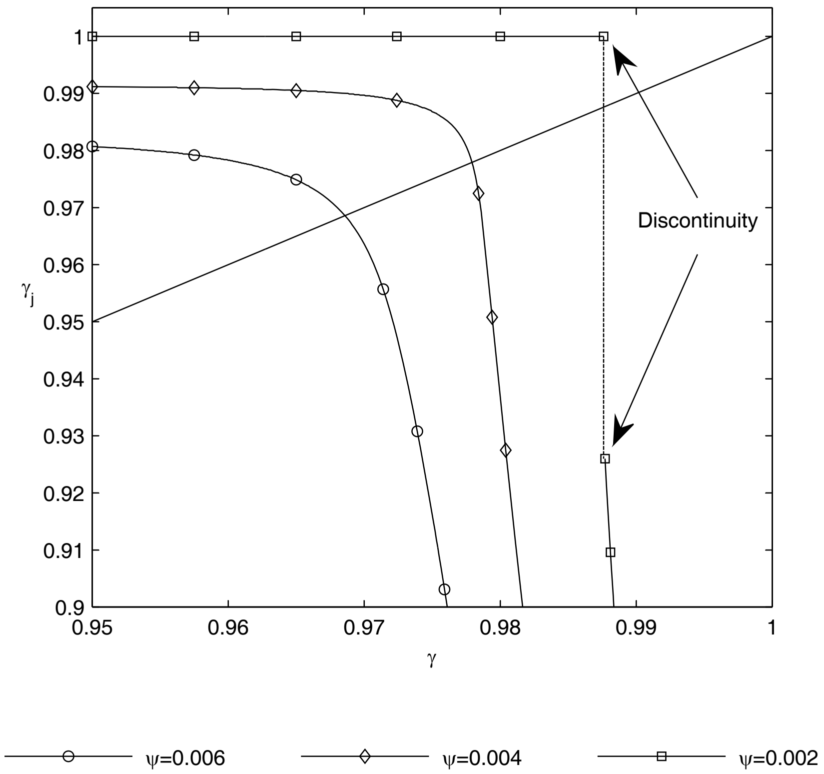

Figure 3: Price Adjustment Costs and the Best Response Function

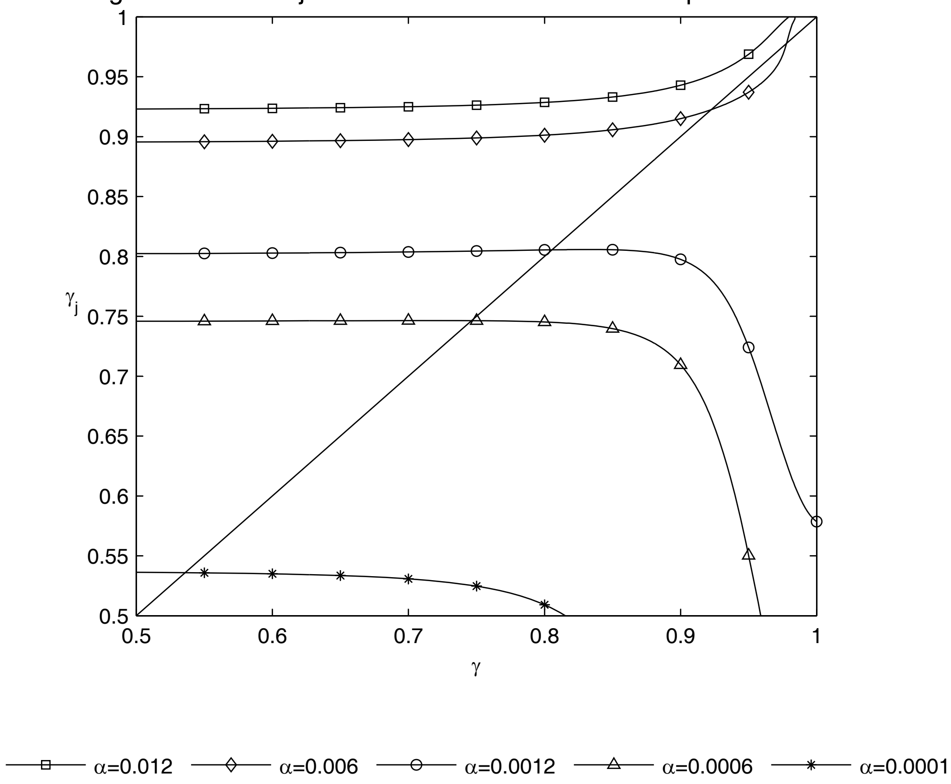

Figure 4: Welfare and Monetary Policy