Policy Instrument Choice and Non-Coordinated Monetary Policy in Interdependent Economies*

Giovanni Lombardo† and Alan Sutherland‡

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Abstract

Non-coordinated monetary policy is analysed in a stochastic two-country general equilibrium model. Non-coordinated equilibria are compared in two cases: one where policy is set in terms of state-contingent money supply rules and one where policy is set in terms of state-contingent nominal interest rate rules. In general the non-coordinated equilibrium differs between the two types of policy rule but a number of special cases are identified where the equilibria are identical. The endogenous choice of policy instrument is analysed and the Nash equilibrium in the choice of policy instrument is shown to depend on the interest elasticity of money demand.

Keywords: Monetary policy, money supply rules, interest rate rules.

JEL: E52, E58, F42

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†Deutsche Bundesbank, Postfach 100602, D-60006 Frankfurt am Main, Germany. Email: Giovanni.Lombardo@bundesbank.de

‡(corresponding author) Department of Economics, University of St Andrews, St Andrews, Fife, KY16 9AL, UK. Email: ajs10@st-and.ac.uk Web: www.st-and.ac.uk/~ajs10/home.html
1 Introduction

There is currently an active literature analysing a wide range of issues relating to monetary policy in open economies. This has been prompted by the development of tractable microfounded general equilibrium models of open economies where sticky prices give a role for monetary policy. These models provide a natural basis for studying the welfare implications of coordinated and non-coordinated monetary policy. In many respects this new literature has developed a more or less common theoretical framework which includes such features as real money balances in the utility function, imperfectly competitive goods or labour markets, and sticky nominal prices or wages. There are however some differences in approach between authors. One such difference is the way in which the monetary policy instrument is specified. Some authors, such as Obstfeld and Rogoff (1998, 2002), Devereux and Engel (2000) and Sutherland (2002b) adopt the traditional approach of specifying monetary policy in terms of choices for the nominal money stock. But other authors, such as Corsetti and Pesenti (2001b) and Clarida, Gali and Gertler (2002) adopt an alternative approach which is to specify monetary policy in terms of choices for the nominal interest rate. An important question which, hitherto, has not been addressed in the new literature is whether, and to what extent, it matters which instrument is used to implement monetary policy. This is the focus of the current paper. Using a simple model which is consistent with those used in the current literature we investigate the circumstances in which the choice of monetary policy instrument affects the equilibrium outcome.

The appropriate choice of policy instrument has, of course, been the subject of an extensive earlier literature starting with Poole (1970). The issue at stake in this previous literature was the stabilising properties of a fixed money supply target compared to a policy of fixing the nominal interest rate. Typically the conclusion from this earlier literature was that the choice of instrument could have a substantial effect on the volatility of macro variables and that the ‘welfare’ ranking of instruments depended on the mixture of shocks hitting the economy.

An important feature of the question addressed by this earlier literature, which

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1 The literature started with the deterministic analyses of Obstfeld and Rogoff (1995) and Corsetti and Pesenti (2001a) but has been extended to a stochastic framework in Obstfeld and Rogoff (1998, 2002), Devereux and Engel (2000), Corsetti and Pesenti (2001b), Bacchetta and van Wincoop (2000), Sutherland (2002a, 2002b), Benigno and Benigno (2001) and Clarida, Gali and Gertler (2002). These papers have addressed such issues as the welfare gains from policy coordination, the relative welfare performance of fixed and flexible exchange rate regimes, the implications of exchange rate pass-through and the implications of financial market integration. See Lane (2001) for a survey of this new literature. Clearly, many of the issues listed above have been central questions in international macroeconomics for many years. For instance, the analysis of international monetary policy coordination was the subject of an extensive earlier literature (see for instance Hamada (1976), Canzoneri and Henderson (1991) and Oudiz and Sachs (1984)). The distinguishing feature of the new literature (and therefore of this paper) is the emphasis on microeconomic foundations and, in particular, the focus on welfare measures based on aggregate utility.
distinguishes it from the issue we are investigating in this paper, is the focus on non-state-contingent rules for the monetary policy instrument. Thus the earlier literature focused on a policy of fixing the quantity of money as compared to a policy of fixing the nominal interest rate. In this paper, on the other hand, we analyse monetary policy rules which allow the monetary authority to react to shocks. Thus, in our framework (in common with many of the contributions to the recent open economy literature) the monetary authority chooses a feedback rule for the policy instrument which specifies a reaction to the exogenous shocks hitting the economy. The comparison to be made therefore is between a feedback rule for the money supply and a feedback rule for the nominal interest rate.\(^2\)

In this context one obvious reaction is to argue that, if the parameters of the rule are chosen optimally by the monetary authority (as is typically the case in the recent literature), it should not matter which policy instrument is determined by the rule. A welfare maximising monetary authority is only concerned with the real equilibrium that is delivered by the rule. If the policy instrument is the money supply then the monetary authority will choose parameters for the feedback rule which deliver the desired real equilibrium. Likewise if the policy instrument is the interest rate the monetary authority can (and will) choose feedback parameters which deliver exactly the same desired real equilibrium. Thus (in contrast to the literature following Poole (1970)) one may be tempted to assume that the choice of policy instrument is irrelevant when the monetary authority chooses a welfare maximising state-contingent rule.

We show in this paper that, although this logic may be correct in a closed economy (where, by definition, there is only one monetary authority), it does not carry over to an open economy setting. We show that, in general, when one is considering non-coordinated policymaking represented by a Nash game between national policymakers, the real equilibrium (and thus the level of welfare) delivered by the Nash equilibrium in the choice of feedback parameters does depend on the particular way in which monetary policy is specified. It is therefore not correct to assume that the choice of policy instrument is irrelevant. There are however some special cases where the Nash equilibrium is unaffected by the choice of policy instrument. These cases are identified and explained.\(^3\)

\(^2\)In order for state-contingent rules to be feasible it is obviously necessary to assume that the shocks are observable and (given the presence of rational forward-looking price setters) it is necessary to assume that policymakers can credibly commit to their chosen rule. It is precisely because these assumptions were felt to be unrealistic that, in the previous literature, so much attention was focused on analysing non-state-contingent monetary rules. More recently, however, the greater independence of central banks has (to some extent) reduced the perceived credibility problem attached to active monetary policy (even if it hasn’t removed the problems relating to the observability of shocks). This has led to a renewed interest in state-contingent rules for monetary policy. It is therefore important to re-examine the role of policy instrument choice in a context where state-contingent rules are available.

\(^3\)It must be emphasised that the type of interest rate rules analysed in this paper are rules which explicitly make the nominal interest rate a function of the underlying exogenous shocks hitting the economy. These rules are therefore not "Taylor rules" (which make the nominal interest rate a
Having shown that the real equilibrium delivered by the Nash game over the choice of policy feedback parameters differs depending on the choice of policy instrument, a natural further question to analyse is the endogenous choice of the policy instrument itself. This issue has already been addressed within the context of the earlier Poole-style analysis by Turnovsky and d’Orey (1989). They consider a simple Nash game between national policymakers in a two-country model where the policymakers make a choice between the money supply or the nominal interest rate as the monetary policy instrument. In a similar spirit we analyse an extension to our model where policymaking is a two-stage process. In the first stage there is a Nash game over the choice of policy instrument and in the second stage there is a Nash game over the choice of parameters for feedback rules (which determine the setting of the policy instruments chosen in the first stage).\footnote{It is important to emphasise that in Turnovsky and d’Orey (1989), once the equilibrium choice of policy instrument has been determined, the monetary authorities follow a \textit{non-state-contingent rule} for that instrument. This contrasts with our analysis, where the second-stage game results in \textit{state-contingent rules} which determine the setting of the policy instrument selected in the first-stage game.} We show that, when the choice of policy instrument makes a difference to the equilibrium in the Nash game over feedback parameters, there is a unique Nash equilibrium in the game over policy instruments. The Nash equilibrium in the choice of policy instrument can either result in the choice of money supply rules or interest rate rules depending on the elasticity of money demand with respect to the nominal interest rate.

The paper proceeds as follows. Section 2 describes a simple two-country model which is used as a framework for analysing the implications of policy instrument choice. Section 3 analyses the comparison between interest rate rules and money supply rules. In this section the choice of policy instrument is assumed to be exogenous. Section 4 considers the endogenous choice of policy instrument. Section 5 discusses the implications of some generalisation of our model. Section 6 concludes the paper.

\section{The Model}

The model has many of the features that have become standard in the recent open economy macro literature. One of the main aims of this recent literature has been to analyse models based on consistent microeconomic foundations. But, within this general aim, an important objective has been to develop simple structures which are, as far as possible, tractable and easily solved. This implies, amongst other things, a tendency to focus on explicit functional forms for utility and production functions. In particular a number of authors have shown that certain choices of utility function can eliminate many troublesome features which would otherwise make progress very difficult. We follow this general modelling approach in this paper. We choose a model which is as simple as possible but which is general enough to illustrate the function of endogenous variables such as output and inflation).
central point at issue. It is also general enough to encompass the models used in a number of important contributions to the recent literature.

There are two countries (which will be referred to as the home country and the foreign country) and each country is populated by many infinitely-lived agents. Each agent is a monopoly producer of a single differentiated product and all agents set prices in advance of the realisation of shocks and are contracted to meet demand at the pre-fixed prices. The assumption of pre-fixed prices implies that monetary policy has some power to affect real variables, such as output and consumption. Pre-fixed prices also imply that ex post, after the realisation of shocks, the economy may not be in a welfare maximising equilibrium. There is thus a welfare role for active monetary policy.

Agents derive utility from consuming a basket of all home and foreign goods. They also derive utility from holding real money balances. This latter assumption ensures a well defined demand for money. The only input into production is the agent’s own work effort and the production function is assumed to be linear in work effort. Agents derive disutility from work effort.

A number of authors (such as Devereux and Engel (2000), Corsetti and Pesenti (2001b), Bacchetta and van Wincoop (2000), Sutherland (2002a)) have recently shown that the degree of exchange rate pass-through can have important implications for the welfare role of monetary policy in open economies. In order to capture this feature of the recent debate it is assumed that price contracts for the cross-border supply of goods specify a degree of indexation to nominal exchange rate changes. This allows the degree of exchange rate pass-through to be parameterised.

The detailed structure of the home country is described below. The foreign country has an identical structure. Where appropriate, foreign real variables and foreign currency prices are indicated with an asterisk.

2.1 Preferences

There is a continuum of agents of unit mass in each country. All agents in the home economy have utility functions of the same form. Utility is assumed to be additively separable across consumption of goods, holdings of real balances and work effort. The utility of home agent \( h \) is given by

\[
U_t(h) = E_t \sum_{s=t}^{\infty} \beta^{s-t} \left[ \log C_s(h) + \frac{\chi_s}{1 - \varepsilon} \left( \frac{M_s(h)}{P_s} \right)^{1-\varepsilon} - K_s y_s(h) \right]
\]

where \( C \) is a consumption index defined across all home and foreign goods, \( M \) denotes end-of-period nominal money holdings, \( P \) is the consumer price index, \( y(h) \)

5 The degree of exchange rate ‘pass-through’ is the extent to which a change in the nominal exchange rate results in a change in the nominal prices charged to consumers for imported goods.

6 Note that the structure of the production function implies that there is a one-to-one correspondence between an agent’s work effort and the amount of output produced by that agent. It is therefore possible to express the disutility of work effort directly in terms of output.
is the output of good $h$, $E_t$ is the expectations operator conditional on information at time $t$. The discount factor is given by $\beta$ (where $0 < \beta < 1$). The parameter $\varepsilon$ (where $\varepsilon > 0$) determines the elasticity of demand for real balances.\(^7\)

$K$ is an i.i.d. random shock to labour supply preferences and $\chi$ is an i.i.d. random shock to money demand preferences.\(^8\) Labour supply and money demand shocks are distributed such that $E[\ln K] = E[\ln \chi] = 0$ and $Var[\ln K] = \sigma_K^2$ and $Var[\ln \chi] = \sigma_\chi^2$.

The consumption index $C$ for home agents is defined as follows
\[
C_t = 2C_{H,t}^{1/2}C_{F,t}^{1/2}
\] (2)

$C_H$ and $C_F$ are indices of home and foreign produced goods defined as follows
\[
C_{H,t} = \left[ \int_0^1 c_{H,t}(i) \frac{\phi}{\phi - 1} \, di \right]^{\frac{1}{\phi - 1}}, \quad C_{F,t} = \left[ \int_0^1 c_{F,t}(j) \frac{\phi - 1}{\phi} \, dj \right]^{\frac{1}{\phi - 1}}
\] (3)

where $\phi > 1$ is the elasticity of substitution between individual goods, $c_H(i)$ is consumption of home good $i$ and $c_F(j)$ is consumption of foreign good $j$.

The Cobb-Douglas form of (2) obviously implies a unit elasticity of substitution between home and foreign goods in consumption. This is a structure which has been adopted by many authors in the recent open-economy literature (see for instance Obstfeld and Rogoff (1998, 2002), Devereux and Engel (2000), Corsetti and Pesenti (2001b), Bacchetta and van Wincoop (2000), Sutherland (2002a, 2002b) and Clarida, Gali and Gertler (2002)) because it implies that the current account is automatically in balance in all states of the world.\(^9\) It is therefore not necessary to model international asset markets or to consider asset stock dynamics. This considerably simplifies the analysis of the model.\(^10\)

The utility-based consumer price index for home agents is
\[
P_t = P_{H,t}^{1/2}P_{F,t}^{1/2}
\] (4)

where $P_H$ and $P_F$ are the price indices for home and foreign goods respectively defined as
\[
P_{H,t} = \left[ \int_0^1 p_{H,t}(i)^{1-\phi} \, di \right]^{\frac{1}{1-\phi}}, \quad P_{F,t} = \left[ \int_0^1 p_{F,t}(j)^{1-\phi} \, dj \right]^{\frac{1}{1-\phi}}
\] (5)

\(^7\)The assumption that utility is logarithmic in consumption and linear in work effort are obviously quite special. These assumptions are very useful in simplifying the solution of the model. The implications for our results of more general functional forms are discussed in Section 5.

\(^8\)The assumption of zero autocorrelation in the shocks has the advantage that it eliminates some irrelevant constant terms from the model solution without affecting the important features of results.

\(^9\)When there is less than full exchange rate pass-through this result only holds in the case where utility is logarithmic in consumption. When there is full exchange rate pass-through the result also holds when utility has a CRRA functional form for consumption. We discuss the implications of some more general forms of utility function in Section 5.

\(^10\)This assumption may however have an important impact on the welfare performance of non-coordinated policymaking (as discussed in Sutherland (2002b)). The potential implications for the results reported in this paper are discussed briefly in Section 5.
All the above prices are denominated in domestic currency.

2.2 Consumption Choices

The intertemporal dimension of home agents’ consumption choices gives rise to the familiar consumption Euler equation

$$\frac{1}{C_t} = \beta R_t P_t E_t \left[ \frac{1}{C_{t+1} P_{t+1}} \right]$$

where $R$ is the gross return on a home currency riskless bond. A similar condition holds for foreign agents where $R^*$ is the gross return on a foreign currency riskless bond.

The intratemporal dimension of the consumption choice problem gives rise to the following individual home demands for home good $i$ and foreign good $j$

$$c_{H,t} (i) = C_{H,t} \left( \frac{p_{H,t} (i)}{P_{H,t}} \right)^{-\phi}, \quad c_{F,t} (j) = C_{F,t} \left( \frac{p_{F,t} (j)}{P_{F,t}} \right)^{-\phi} \quad (6)$$

where

$$C_{H,t} = \frac{1}{2} C_t \left( \frac{P_{H,t}}{P_t} \right)^{-1}, \quad C_{F,t} = \frac{1}{2} C_t \left( \frac{P_{F,t}}{P_t} \right)^{-1} \quad (7)$$

Foreign demands for home and foreign goods have an identical structure to the home demands.

Each country has a population of unit mass and (in a symmetric equilibrium) all agents within each country choose the same consumption bundle so the total demands for goods are equivalent to individual demands. Thus for home agent $h$ $y_{H,t} (h) = c_{H,t} (h)$ and $y^*_{H,t} (h) = c^*_{H,t} (h)$ where $c^*_{H,t} (h)$ is the foreign individual demand for home good $h$ and $y_{H} (h)$ and $y^*_{H} (h)$ are sales of good $h$ to home and foreign agents.

It is simple to show that home and foreign consumption choices imply the following

$$\frac{S_t P^*_{t}}{P_t} = \frac{C_t}{C^*} \quad (8)$$

where $P^*$ is the aggregate consumer price index for foreign consumers and $S$ is the nominal exchange rate defined as the domestic home currency price of foreign currency. This relationship implies that complete international sharing of consumption risk is achieved without the need to assume the presence of markets in state-contingent assets.

2.3 The Money Market

The first-order condition for the choice of money holdings is

$$\chi_t \left( \frac{M_t}{P_t} \right)^{-\varepsilon} = \left( \frac{R_t - 1}{R_t} \right) C_t^{-1} \quad (9)$$
It is assumed that all changes in the money supply enter and leave the economy via lump-sum transfers, so the government’s budget constraint is

\[ M_t - M_{t-1} + T_t = 0 \] (10)

where \( T \) are lump-sum transfers from home agents. The timing and structure of monetary policy decisions are discussed in more detail below.

### 2.4 Price Setting and the Degree of Pass-Through

To allow for the possibility of incomplete exchange rate pass-through it is assumed that producers enter into separate price contracts for sales in home and foreign markets. The price contract signed by home agents for sales to foreign consumers is assumed to enforce a fixed degree of indexation to unanticipated exchange rate changes. Home agent \( h \) selling to foreign consumers chooses a price \( \hat{p}_H(h) \) denominated in home currency. The actual foreign currency price charged (denoted \( p_{H,*}(h) \)) is determined by the following formula (which is part of the contract)

\[ p_{H,*}(h) = \frac{\hat{p}_H(h)}{S_t} \left( \frac{S_t}{S_{E,t}} \right)^{1-\eta} \] (11)

where \( S_E \) is the \textit{ex ante} expected exchange rate and \( 0 \leq \eta \leq 1 \). This structure allows a full range of degrees of pass-through. \( \eta = 1 \) implies ‘producer currency pricing’ and full pass-through. \( \eta = 0 \) implies ‘local currency pricing’ and zero pass-through.\(^{11}\) A relationship similar to (11) enforces a fixed degree of pass-through for foreign goods sold in the home country. The degree of pass-through is assumed to be identical for both home and foreign producers.

Individual agents are each monopoly producers of a single differentiated good. They therefore set prices as a mark-up over marginal costs where the mark-up is given by \( \Phi = \phi/(\phi - 1) \). Prices in period \( t \) are set before period-\( t \) shocks are realised. Period-\( t \) prices therefore depend on expectations of period \( t \) conditional on period-\( (t-1) \) information. There are two first-order conditions for the choice of prices for the home producer.\(^{12}\) One for the price charged to home consumers as follows

\[ P_{H,t} = \Phi E_{t-1} [K_t P_t C_t] \] (12)

\(^{11}\)Implicitly the assumption of separate price contracts for home and foreign sales allows agents to price discriminate between home and foreign markets. In a deterministic context there would be no incentive to price discriminate because elasticities of demand are equal at home and abroad. But, in a stochastic context, given the different stochastic characteristics of home and foreign demand, price discrimination in fact becomes optimal for fixed-price agents. However, this incentive to price discriminate is incidental to the analysis of this paper. The important point is that there are separate contracts. The structure of contracts is taken to be a fixed institutional feature of the economy. This structure follows, and is formally identical to, the structure first used by Corsetti and Pesenti (2001b) to model incomplete pass-through.

\(^{12}\)A detailed derivation of these first-order conditions can be found in Sutherland (2002a).
And one for the price charged to foreign consumers as follows
\[ P_{H,t}^* = \Phi E_{t-1} [K_t P_t^* C_t^* S_t^0] S_t^\eta \]  

The first-order conditions for foreign price setting have a similar structure.

2.5 Policy Rules and Welfare

As described in Section 1, we assume that policy can be set in terms of a state-contingent rule where the policymaker in each country is free to choose the parameters of the rule in an attempt to maximise welfare. Goods prices are pre-fixed for only one period so monetary policy in any given period can only affect real outcomes in that period. It is therefore sufficient to specify monetary policy in terms of feedback rules relating the policy instrument to the shock variables. Thus money supply rules are of the following form

\[ \hat{M}_t = \delta_{M,K} \varepsilon_{K,t} + \delta_{M,K^*} \varepsilon_{K^*,t} + \delta_{M,\chi} \varepsilon_{\chi,t} + \delta_{M,\chi^*} \varepsilon_{\chi^*,t} \]  (14)

\[ \hat{M}_t^* = \delta_{M,K} \varepsilon_{K,t} + \delta_{M,K^*} \varepsilon_{K^*,t} + \delta_{M,\chi} \varepsilon_{\chi,t} + \delta_{M,\chi^*} \varepsilon_{\chi^*,t} \]  (15)

where a hat over a variable indicates a log deviation of that variable from a non-stochastic steady state.

In the case of interest rate rules it is necessary to include an additional term in each rule in order to ensure determinacy of the price level. This additional term must depend on a nominal magnitude, such as \( P \) and \( P^* \). Thus interest rate rules are of the following form

\[ \hat{R}_t = \delta_{R,K} \varepsilon_{K,t} + \delta_{R,K^*} \varepsilon_{K^*,t} + \delta_{R,\chi} \varepsilon_{\chi,t} + \delta_{R,\chi^*} \varepsilon_{\chi^*,t} + \delta P \hat{P}_t \]  (16)

\[ \hat{R}_t^* = \delta_{R,K} \varepsilon_{K,t} + \delta_{R,K^*} \varepsilon_{K^*,t} + \delta_{R,\chi} \varepsilon_{\chi,t} + \delta_{R,\chi^*} \varepsilon_{\chi^*,t} + \delta P^* \hat{P}_t^* \]  (17)

The coefficients \( \delta P \) and \( \delta P^* \) can be made arbitrarily small and thus have no substantive impact on the behaviour of the model other than to ensure determinacy.

Policymakers choose the parameters of their policy rule for period \( t \) before period-\( t \) prices have been set or shocks have been realised. Policymakers therefore make their decisions based on the information available at the end of period \( t - 1 \). It is assumed that policymakers can credibly commit to their choice of feedback parameters.

Because monetary policy in period \( t \) only affects real outcomes in period \( t \) it is possible to measure the welfare impact of period-\( t \) policy choices solely with reference to aggregate expected flow utility in period \( t \). Following Obstfeld and Rogoff (1998, 2002) it is assumed that the utility of real balances is small enough to be neglected. Therefore, at the time period-\( t \) policy choices are made, aggregate expected flow welfare of home agents is measured using the following

\[ \Omega_t = E_{t-1} [\log C_t - K_t Y_t] \]  (18)
where \( Y = C_H + C_H^* \) is the total output of the home economy. Using (7), (11), (12) and (13) it is possible to show that \( E_{t-1} [K_t Y_t] = 1/\Phi \) thus welfare can be measured by

\[
\tilde{\Omega}_t = E_{t-1} \left[ \hat{C}_t \right]
\]

where, as above, a hat indicates a log deviation from a non-stochastic steady state and \( \tilde{\Omega} \) is the absolute deviation from welfare in the non-stochastic steady state.

3 Equilibrium Choice of Policy Rules with Policy Instruments Specified Exogenously

In this section we assume that the choice of policy instrument is specified exogenously. We compare the equilibrium that results when the policymakers in both countries use money supply rules with the case where the policymakers in both countries use interest rate rules. The endogenous choice of policy instrument is considered in the next section. In this section policymakers only make choices over the feedback parameters in the appropriate policy rule.

The optimal coordinated choice of feedback parameters is considered first. In this case monetary policy is set by a single world monetary authority which chooses the feedback parameters for both home and foreign monetary rules in order to maximise aggregate world welfare, i.e. \( \hat{\Omega}_{W,t} = \left( \hat{\Omega}_t + \hat{\Omega}^*_t \right) / 2 \). The implications of money supply rules and interest rate rules for non-coordinated policy\(^{13}\) are then described and some special cases are identified and discussed. Some numerical solutions are also presented.

3.1 Optimal Coordinated Policy

The optimal coordinated coefficients for the money supply and interest rate rules are reported in Table 1. The first and second moments of output and consumption for the coordinated policy case are reported in Table 2.\(^{14}\)

\(^{13}\)Non-coordinated policy is defined as the case where national monetary authorities act as Nash players attempting to maximise national welfare.

\(^{14}\)The variances of the i.i.d. innovations are set to unity.

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<td>( \delta_{M,K}^* ) = ( \delta_{M,K}^* )</td>
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<td>( -\frac{1-\eta+(1+\beta+\varepsilon-\beta\eta)^2}{2\eta(1-\beta)(1-2\eta+2\eta^2)} )</td>
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Table 1: Coordinated policy - policy rule coefficients
In the case of coordinated policymaking it is simple to see that the equilibrium outcome is identical regardless of whether policy is set in terms of interest rate rules or money supply rules. The crucial feature of coordinated policymaking which leads to this result is the fact that there is a single world policymaker. This is a case, therefore, where the closed-economy logic (described in the Section 1) also extends to an open economy setting. The coordinated policymaker has control of monetary policy in both countries. For every pair of state-contingent money supply rules which can be chosen there is a corresponding pair of state-contingent nominal interest rate rules which yield the same equilibrium. The coordinated policymaker can therefore specify policy in terms of the optimal pair of money supply rules or the corresponding optimal pair of interest rate rules - both specifications of policy will yield the same real equilibrium and the same level of aggregate world welfare.

3.2 Non-coordinated Policy

The case of non-coordinated policy is now considered. The Nash equilibrium coefficients for the money supply and interest rate rules are reported in Table 3 (where $\Delta = (1 - \beta)(\epsilon - 1)$) and the implied first and second moments of output and consumption are reported in Table 4.\textsuperscript{15}

It is apparent from Table 4 that the Nash equilibrium yielded by money supply rules is in general not identical to the Nash equilibrium yielded by interest rate rules. As argued in Section 1, this may, at first sight, seem to be a surprising result. Initial intuition would suggest that the logic which applies to the coordinated policy case should, in a modified form, also apply to the Nash policy case. So, for instance, one might be tempted to argue that for each home-country money-supply rule there is a corresponding home-country interest-rate rule which yields the same equilibrium for the home economy, so optimal monetary policy for the home-country policymaker can equivalently be specified in terms of a money-supply rule or an interest-rate rule (with the same being true for foreign monetary policy).

So why then do the results in Table 4 indicate that the Nash equilibrium differs depending on the policy instrument? The answer is that the logic described in the previous paragraph omits a crucial difference between the coordinated and Nash

\begin{table}[h]
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\begin{tabular}{|c|c|}
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 & M and R rules \\
\hline
$\hat{\Omega}_{W,t} = E_{t-1} [\hat{C}_t]$ & $-\frac{(1-\eta)^4}{4(1-2\eta+2\eta^2)^2}$ \\
\hline
$Var_{t-1} [\hat{C}_t]$ & $\frac{(1+\eta+\eta^2)^2+(1-3\eta+3\eta^2)^2}{4(1-2\eta+2\eta^2)^2}$ \\
\hline
$E_{t-1} [\hat{Y}_t]$ & $-\frac{(1-\eta)^4}{4(1-2\eta+2\eta^2)^2}$ \\
\hline
$Var_{t-1} [\hat{Y}_t]$ & $\frac{(1-2\eta+3\eta^2)^2+(1-\eta)^4}{4(1-2\eta+2\eta^2)^2}$ \\
\hline
\end{tabular}
\caption{Table 2: Coordinated policy - first and second moments of output and consumption}
\end{table}

\textsuperscript{15}Again, the variances of the i.i.d. innovations are set to unity.
cases. In the coordinated case the world policymaker is simultaneously choosing monetary policy for both countries. But in the Nash case each policymaker is choosing policy for a single country. Thus, when setting policy, the Nash policymaker in one country has to take account of the way policy is set in the other country. And, crucially, a given change in monetary policy in one country will have a different impact on economic outcomes depending on whether the other economy is following an interest-rate rule or a money-supply rule. Thus, for instance, if the foreign economy is following a money-supply rule, an easing of home monetary policy will affect money demand in the foreign country and therefore affect the foreign interest rate. But if the foreign economy is following an interest-rate rule there will be no impact on the foreign interest rate. In the first case the change in the foreign interest rate will either amplify or offset the impact of the change in home monetary policy. This amplifying or offsetting effect is not present in the case of a foreign interest rate rule.

To make this point more clearly it is useful to consider a simple deterministic policy experiment. First notice that (after goods prices have been fixed for period $t$) home and foreign consumer prices can be written as follows

$$\hat{P}_t = \frac{1}{2} \eta \hat{S}_t \quad \hat{P}^*_t = -\frac{1}{2} \eta \hat{S}_t$$

Also notice that the home and foreign consumption Euler equations imply (after omitting irrelevant constant terms)

$$\hat{C}_t = -\hat{R}_t - \hat{P}_t \quad \hat{C}^*_t = -\hat{R}^*_t - \hat{P}^*_t$$

while a log-linear approximation of the foreign money market relationship yields

$$\varepsilon (\hat{M}^*_t - \hat{P}^*_t) = \hat{C}^*_t - \frac{\beta}{1 - \beta} \hat{R}^*_t$$

If the foreign economy is following an interest rate rule then, in a deterministic experiment, this implies $\hat{R}^*_t = 0$. While a foreign money supply rule implies $\hat{M}^*_t = 0$.

The above five equations (together with equation (8)) can be solved to yield the following expressions for home and foreign consumption for the case of a foreign money supply rule ($\hat{M}^*_t = 0$)

$$\hat{C}_t = -\frac{2 - \eta + \Delta \eta}{2 + \Delta \eta} \hat{R}_t \quad \hat{C}^*_t = -\frac{1 + \Delta}{2 + \Delta \eta} \hat{R}_t$$

<table>
<thead>
<tr>
<th>M rules</th>
<th>R rules</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta_{M,K} = \delta_{M^<em>,K^</em>}$</td>
<td>$\delta_{R,K} = \delta_{R^<em>,K^</em>}$</td>
</tr>
<tr>
<td>$\delta_{M,K}^* = \delta_{M^*,K}$</td>
<td>$\delta_{R,K}^* = \delta_{R^*,K}$</td>
</tr>
<tr>
<td>$\delta_{M,X} = \delta_{M^<em>,X^</em>}$</td>
<td>$\delta_{R,X} = \delta_{R^<em>,X^</em>}$</td>
</tr>
<tr>
<td>$\delta_{M,X^<em>} = \delta_{M^</em>,X}$</td>
<td>$\delta_{R,X^<em>} = \delta_{R^</em>,X}$</td>
</tr>
</tbody>
</table>

Table 3: Nash equilibrium - policy rule coefficients
(where again $\Delta = (1 - \beta)(\varepsilon - 1)$ and the following expressions for the case of a foreign interest rate rule ($\hat{R}_t = 0$)

$$\hat{C}_t = -\frac{2 - \eta}{2} \hat{R}_t \quad \hat{C}_t^* = -\frac{\eta}{2} \hat{R}_t$$

(23)

It is readily apparent from these expressions that the impact on consumption demand of a given change in the home nominal interest rate differs between the two cases. Thus the optimal choice of home monetary policy (and therefore the Nash equilibrium) can differ depending on the rule followed by the foreign policymaker.\(^\text{16}\)

### 3.3 Some Special Cases

The expressions in Table 4 and (22) and (23) show that, in general, non-coordinated money supply rules can yield a different equilibrium from non-coordinated interest rate rules. But it is also clear that there are some special cases where the two types of rule produce the same Nash equilibrium. It is easily seen from Table 4 and (22) and (23) that three such special cases arise when $\varepsilon = 1$ or $\eta = 0$ or $\beta = 1$. Close inspection of the expressions in Table 4 show that a further special case arises when $\eta = 1$.\(^\text{17}\)

The reasons why these special cases arise can be understood by considering the example illustrated by equations (20) to (23). This example showed how monetary policy in the home country could have an impact on the interest rate in the foreign country when the foreign country is following a money supply rule. It can be seen

\(^{16}\text{In this example home monetary policy is expressed in terms of the nominal interest rate. It is simple to check that the logic is the same if home monetary policy is expressed in terms of the money supply.}\)

\(^{17}\text{Two of these special cases ($\eta = 0$ and $\eta = 1$) correspond to the cases of zero pass-through (or local currency pricing) and full pass-through (or producer currency pricing). Devereux and Engel (2000), who analyse a model very similar to the one used in this paper, focus on the extreme cases of full-pass-through and zero pass-through. They are therefore working with two of the special cases identified here. Corsetti and Pesenti (2001b), who also analyse a model very similar to the one used here, allow for intermediate degrees of pass-through but focus on the case where $\varepsilon = 1$. (i.e. when utility is logarithmic in real balances). They are thus also working with one of the special cases identified here.}\)

\[(\text{Table 4: Nash equilibrium - first and second moments of output and consumption)]
from (21) that this mechanism operates through the presence of \( P^* \) and \( C^* \) in the foreign money market expression. Home-country monetary policy only affects the foreign interest rate when home-country monetary policy affects \( P^* \) and \( C^* \) and when these variables affect foreign money market equilibrium. The first three special cases identified above are cases where these connections are broken.

When \( \varepsilon = 1 \) it can be seen from (20) and (21) that the direct effect of any change in \( P^* \) on foreign money market equilibrium is precisely offset by the effect of \( P^* \) on foreign consumption operating through the consumption Euler equation. When \( \eta = 0 \) (i.e. when the degree of exchange rate pass-through is zero) it can be seen from (20) that \( P^* \) is completely fixed in the short run and hence \( C^* \) depends only on the foreign nominal interest rate. In both these cases home monetary policy has no effect on foreign money market equilibrium. In the \( \beta = 1 \) case it can be seen from (21) that the interest elasticity of money demand becomes infinite so again the foreign nominal interest rate can not be affected by home monetary policy.

The forth special case (which arises when \( \eta = 1 \)) has a more subtle explanation than the other three. In this case equations (22) and (23) can be written as follows

\[
\hat{C}_t = -\frac{1 + \Delta R}{2 + \Delta} \hat{R}_t \quad \hat{C}^*_t = -\frac{1 + \Delta}{2 + \Delta} \hat{R}_t
\]  

and

\[
\hat{C}_t = -\frac{1}{2} \hat{R}_t \quad \hat{C}^*_t = -\frac{1}{2} \hat{R}_t
\]

(24)

So, unlike the other three special cases just discussed, in this case the impact of a change in the home interest rate on consumption demand differs depending on the monetary rule followed by the foreign country. But nevertheless the results in Table 4 show that the Nash equilibrium is the same regardless of which type of rule is followed. This is because (as shown by Obstfeld and Rogoff (2002)) when there is full pass-through there is in fact no strategic interaction between the two economies so the Nash outcome is identical to the coordinated outcome (as can be confirmed by comparing the expressions shown in Tables 2 and 4). There can therefore be no difference between the two types of policy rule.

The absence of strategic interaction can be demonstrated very easily. When \( \eta \) equals unity there is full pass-through and purchasing power parity holds. It therefore follows from equation (8) that home and foreign consumption levels are equal, and thus, from (19), it follows that home and foreign welfare levels are equal. Nash and coordinated policymaking must therefore yield the same equilibrium.

It is immediately apparent from this argument, and from (24) and (25), that the equivalence of interest rate rules and money supply rules in the \( \eta = 1 \) case will break down when there is some degree of strategic interaction between policymakers. This will be true, for instance, when there are non-traded goods (combined with preferences which are non-logarithmic in consumption and non-linear in labour supply) or when the elasticity of substitution between home and foreign goods is not equal to unity. The \( \eta = 1 \) case is therefore a more fragile special case than the
other three special cases discussed above (which do not depend on the absence of strategic interaction). We return to this point below in Section 5.

3.4 Numerical Results

The quantitative implications for welfare of the above results are illustrated in Figure 1. This figure plots the difference between the level of world welfare in the two Nash equilibria (with positive values indicating higher welfare with money supply rules).\(^{18}\) The figure plots the welfare difference against values of \(\eta\) and for three values of \(\epsilon\). Other parameter values are \(\beta = 0.95\) and \(\sigma^2_{\epsilon_K} = \sigma^2_{\epsilon_K^*} = 0.1\) and \(\xi_K = \xi_{K^*} = 0\). Figure 1 shows that the welfare difference between the two types of policy rules is never very large even for quite extreme values of \(\epsilon\). For instance the welfare gap at its widest is only -0.01 percent of steady state consumption. However, it is important to note that (as has been emphasised in much of the recent literature) the welfare difference between coordinated and non-coordinated policy is very small in a model of the form used here. In itself this is likely to suggest that the welfare difference between different forms of non-coordinated policy are likely to be even smaller. Some recent contributions have analysed cases where the welfare gains from coordination can be relatively large. For instance, Benigno and Benigno (2001) and Sutherland (2002b) analyse the implications of allowing the elasticity of substitution between home and foreign goods to differ from unity. In this case the welfare difference between policy instruments is also likely to be much larger. In Section 5 we briefly discuss the implications of allowing the elasticity of substitution between home and foreign goods to differ from unity.\(^{19}\)

4 Endogenous Choice of Policy Instruments

In the previous section the choice of policy instrument is assumed to be exogenously given. This section analyses the endogenous choice of policy instrument.

The setting of monetary policy now has two elements. The first is the choice of policy instrument - either the money supply or the nominal interest rate. The second is the choice of parameters for feedback rules. It is convenient to think of these decisions being made as part of a two-stage game where, in the first stage, the policymaker in each country chooses a policy instrument and, in the second stage, feedback parameters are chosen. In the first-stage game each policymaker faces a binary choice over instruments (either the money supply or the nominal interest

\(^{18}\)The welfare difference between equilibria is expressed in terms of the change in consumption (as a percentage of consumption in a non-stochastic steady state) that would produce the equivalent change in aggregate utility.

\(^{19}\)The welfare gains from coordination (and thus the welfare difference between policy instruments) may also be larger when there is greater cross-country asymmetry in economic structures. Furthermore, as emphasised by Canzoneri, Cumby and Diba (2001), asymmetric shocks between traded and non-traded sectors may increase the welfare gains from coordination.
rate. There are thus four possible outcomes from the first stage game. The four possible outcomes are summarised in Table 5.

The payoffs for each outcome from the first-stage game are the equilibrium welfare levels yielded by the relevant equilibrium in the second stage game. Thus, for instance, when both countries choose to follow money-supply rules, the payoff to each policymaker is the welfare level yielded by a Nash equilibrium in the choice of feedback parameters for money supply rules. Alternatively, when both countries choose to follow interest rate rules, the payoff to each policymaker is the welfare level yielded by a Nash equilibrium in the choice of feedback parameters for interest rate rules. Each cell in Table 5 shows the payoffs to the home and foreign policymakers for each of the four possible outcomes from the first stage game (including the payoffs which result when one country chooses to follow a money supply rule and the other country chooses to follow an interest rate rule).

<table>
<thead>
<tr>
<th></th>
<th>Home country</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>M-rule</td>
</tr>
<tr>
<td>Foreign country</td>
<td></td>
</tr>
<tr>
<td>M-rule</td>
<td>$\Omega_t = \Omega_t^* = W_1$</td>
</tr>
<tr>
<td>R-rule</td>
<td>$\Omega_t = W_2, \Omega_t^* = W_3$</td>
</tr>
</tbody>
</table>

\[
W_1 = -\frac{(1-\eta)^2[4-4(1-\Delta)\eta+5-(1+\Delta)(3-\Delta)\eta^2]}{4[2-(3-\Delta)\eta+(2-\Delta)\eta^2]^2} \\
W_2 = -\frac{(2-\eta)^2(1-\eta)^2[4-4(1-\Delta)\eta+5-(1+\Delta)(3-\Delta)\eta^2]}{2(1-\eta)^2[2-(1-\Delta)\eta]^2[2-2\eta+\eta^2]^2} \\
W_3 = -\frac{(2-\eta)^2(1-\eta)^2[4-4(1-\Delta)\eta+5-(1+\Delta)(3-\Delta)\eta^2]}{[8-4(4-\Delta)\eta+2(7-3\Delta)\eta^2-(4-3\Delta)\eta^3]^2} \\
W_4 = -\frac{(1-\eta)^2[2-2\eta+\eta^2]}{2(2-3\eta+2\eta^2)^2}
\]

Table 5: Welfare payoffs for game over policy instrument choice
It is simple to check that the payoffs to all four outcomes are identical in any of the four special cases discussed in the previous section. Thus if \( \eta = 0 \) or \( \eta = 1 \) or \( \beta = 1 \) or \( \varepsilon = 1 \) the two policymakers are indifferent about their choice of policy instrument and therefore any one of the four outcomes to the first stage game can be regarded as a Nash equilibrium. The outcome of the general case, where \( 0 < \eta < 1 \), \( 0 < \beta < 1 \) and \( \varepsilon \neq 1 \) is summarised in the following proposition.

**Proposition 1** When \( 0 < \eta < 1 \), \( 0 < \beta < 1 \) and \( \varepsilon < 1 \) there is a single Nash equilibrium to the first-stage game where both policymakers choose the money supply to be the policy instrument. When \( 0 < \eta < 1 \), \( 0 < \beta < 1 \) and \( \varepsilon > 1 \) there is a single Nash equilibrium to the first-stage game where both policymakers choose the nominal interest rate to be the policy instrument.

**Proof.** It is sufficient to show that each player has a dominant strategy in the two cases. The choice of the money supply as the policy instrument is the dominant strategy for both policymakers when \( W_1 > W_3 \) and \( W_2 > W_4 \). Using the definitions for \( W_1, W_2, W_3 \) and \( W_4 \) it is simple to show that this is true when \( \varepsilon < 1 \). The choice of the interest rate as the policy instrument is the dominant strategy for both policymakers when \( W_1 < W_3 \) and \( W_2 < W_4 \). It is simple to show that this is true when \( \varepsilon > 1 \).

Proposition 1 shows that, provided \( \varepsilon \neq 1 \), there is a unique Nash equilibrium to the first-stage game where both policymakers choose the same policy instrument.\(^20\) Notice that the case where the choice of money supply rules is a Nash equilibrium to the first stage game (i.e. \( \varepsilon < 1 \)) corresponds to the case where money supply rules yield higher world welfare in the second-stage game and the case where the choice of interest rate rules is a Nash equilibrium to the first stage game (i.e. \( \varepsilon > 1 \)) corresponds to the case where interest rate rules yield higher world welfare in the second-stage game.

## 5 Robustness

In the previous sections we have discussed the instrument-choice problem under a set of simplifying assumptions. In particular, we have assumed log preferences in consumption and linear preferences in labour. Furthermore, we have assumed a unit elasticity of substitution between home and foreign goods. In this section we briefly discuss the implications for our results of relaxing these assumptions.

### CRRA preferences

At this point, it should be clear that the instrument-choice problem arises only when there is strategic interdependence in monetary policy making and the policy makers...

\(^20\)The symmetry of the Nash equilibrium in the choice of policy instrument is likely to be a consequence of the symmetric structure of the model. In an asymmetric world Nash equilibria may exist where the two countries choose different policy instruments.
do not coordinate their decisions. Imposing a degree of relative risk aversion in consumption different from unity (i.e. adopting a general CRRA functional form) does not eliminate strategic interdependence, nor does the assumption of non-linear preferences in labour supply. The special cases discussed in Section 3.3 also continue to apply with this more general structure of preferences. The results derived in Section 3 are therefore robust to generalisations of this type.

Nevertheless, one might wonder whether the first-stage game (the choice of instruments) still yields the same unique Nash equilibrium. Direct calculation of the payoff functions under general CRRA preferences shows that the ordering of the payoffs changes only in a very special way (Cf Table 5). For certain combinations of values of the consumption and labour risk-aversion coefficients only the pairs \((W_1, W_2)\) and \((W_3, W_4)\) cross more than once (at \(\varepsilon = 1\) and at \(\varepsilon > 1\)). This implies that Proposition 1 still holds under general CRRA preferences in consumption and labour.

**Full exchange-rate pass-through**

Turnovsky and d’Orey (1989) discuss the instrument-choice problem in a setting where the “law of one price” (and purchasing power parity - PPP) holds. In contrast, we have seen in our model that under PPP (i.e. the case where there is full exchange rate pass-through, \(\eta = 1\)), there is no strategic interdependence in monetary policy and, hence, the instrument-choice problem ceases to exists. Strategic interdependence does arise, however, in the PPP case in a more general version of our model where the elasticity of substitution between domestic and imported goods is greater than unity. For reasons of space we do not develop this model here. We simply report the implications of such a model. The more general model shows that, even in the full pass-through case, the equilibrium allocations differ between money supply rules and interest rate rules when the elasticity of substitution between home and foreign goods is greater than unity (and also \(\varepsilon\) and \(\beta\) are not equal to unity). Furthermore, the payoff functions for the first-stage game cross (more than once) only in pairs (i.e. \((W_1, W_2)\) and \((W_3, W_4)\)) leaving the Nash equilibrium in the choice of policy instrument unchanged. Therefore, we can claim that Proposition 1 can be generalized to the full-pass-through case when the elasticity of substitution between home and foreign goods is greater than unity.

An important feature of this more general model is that (as shown in Sutherland (2002b)) the welfare gains from policy coordination are potentially much higher when the elasticity of substitution between home and foreign goods is greater than

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21 With a degree of relative risk aversion different from one the consumption risk-sharing condition does not hold automatically. In this case we follow Devereux and Engel (2000) by assuming the existence of a full set of contingent assets that household can purchase in both countries so to insure themselves against consumption risk.

22 Turnovsky and d’Orey (1989) use a Lucas-supply function to describe output. They assume that the elasticity of output with respect to the relative price can differ from unity.

23 Sutherland (2002b) uses a model of this form to analyse the welfare gains from policy coordination. Sutherland does not, however, analyse the instrument-choice problem.
unity. The welfare differences between the equilibria for the money supply rules and interest rate rules are thus also potentially much larger in this case.

6 Conclusion

This paper has compared state-contingent money supply rules with state-contingent interest rate rules in a two-country sticky-price general equilibrium model. It has been shown that, in general, the equilibrium outcome of non-coordinated policy-making can differ depending on whether monetary policy is specified in terms of money supply rules or interest rate rules. A number of special cases are identified where the two types of rule yield the same equilibrium. The endogenous choice of policy instrument was analysed as part of a two-stage game and it was shown that a unique Nash equilibrium exists in the choice of monetary instrument. This paper has concentrated on deriving explicit results for a fairly restrictive model (which is nevertheless representative of the recent literature). The results are, however, robust to a number of important generalisations of the model.

References


Appendix

The full set of equations needed for the solution of the model is repeated here.

\[ P_t = P_{H,t}^{1/2} P_{F,t}^{1/2}, \quad P_t^* = P_{H,t}^{*1/2} P_{F,t}^{*1/2} \] (A1)

\[ \frac{S_t P_t^*}{P_t} = \frac{C_t}{C_t^*} \] (A2)

\[ \frac{1}{C_t} = \beta R_t E_t \left[ \frac{1}{C_{t+1} P_{t+1}} \right], \quad \frac{1}{C_t^*} = \beta R_t^* E_t \left[ \frac{1}{C_{t+1}^* P_{t+1}^*} \right] \] (A3)

\[ C_{H,t} = \frac{1}{2} C_t \left( \frac{P_{H,t}}{P_t} \right)^{-1}, \quad C_{F,t} = \frac{1}{2} C_t \left( \frac{P_{F,t}}{P_t} \right)^{-1} \] (A4)

\[ C_{H,t}^* = \frac{1}{2} C_t^* \left( \frac{P_{H,t}^*}{P_t^*} \right)^{-1}, \quad C_{F,t}^* = \frac{1}{2} C_t^* \left( \frac{P_{F,t}^*}{P_t^*} \right)^{-1} \] (A5)

\[ \chi_t \left( \frac{M_t}{P_t} \right)^{-\varepsilon} = \left( \frac{R_t - 1}{R_t^*} \right) C_t^{-1}, \quad \chi_t^* \left( \frac{M_t^*}{P_t^*} \right)^{-\varepsilon} = \left( \frac{R_t^* - 1}{R_t} \right) C_t^{*-1} \] (A6)

\[ P_{H,t} = \Phi E_{t-1} \left[ K_t P_t C_t \right], \quad P_{H,t}^* = \Phi E_{t-1} \left[ K_t^* P_t^* C_t^* S_t^\eta \right] S_t^{*-\eta} \] (A7)

\[ P_{F,t}^* = \Phi E_{t-1} \left[ K_t^* P_t^* C_t^* \right], \quad P_{F,t} = \Phi E_{t-1} \left[ K_t P_t C_t S_t^\eta \right] S_t^{\eta} \] (A8)

\[ Y = C_H + C_H^*, \quad Y^* = C_F + C_F^* \] (A9)

\[ \ln K_t = \xi_K \ln K_{t-1} + \epsilon_{K,t}, \quad \ln K_t^* = \xi_K^* \ln K_{t-1}^* + \epsilon_{K^*,t} \] (A10)

\[ \ln \chi_t = \xi_{\chi} \ln \chi_{t-1} + \epsilon_{\chi,t}, \quad \ln \chi_t^* = \xi_{\chi^*} \ln \chi_{t-1}^* + \epsilon_{\chi^*,t} \] (A11)

The model is closed with the addition of the appropriate set of policy rules. All the equations of the model are linear in logs with the exception of the money-market equations (A6) and the aggregate output equations (A9). These equations can be replaced by second order approximations and the model can be solved on the assumption that all the variables of the model are approximately log-normal.