Incomplete Pass-Through and the Welfare Effects of Exchange Rate Variability*

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June 2002

Abstract

This paper considers the implications of incomplete exchange rate pass-through for optimal monetary and exchange rate policy. A two-country model is presented which allows an explicit derivation of welfare functions in terms of a weighted sum of the second moments of producer prices and the nominal exchange rate. From a single country perspective the optimal exchange rate variance depends on the degree of pass-through, the size and openness of the economy, the elasticity of labour supply and the volatility of foreign producer prices. The optimal coordinated equilibrium can be supported by requiring national central banks to minimise loss functions which are a weighted sum of the variances of producer prices and the exchange rate, where the weight on the exchange rate variance depends on the degree of pass-through.
Keywords: Monetary policy, pass-through, exchange rate variability.
JEL: E52, E58, F41

*I am grateful for comments and suggestions from Philippe Bacchetta, John Driffill, Michael B Devereux, Jordi Gali, Peter Spencer, Stephen Wright, Jacques Melitz and seminar participants at Birkbeck College, the ECB, Strathclyde University, ESSIM and a CEPR RTN Workshop on International Capital Markets. This research was supported by the ESRC Evolving Macroeconomy Programme grant number L138251046.

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1 Introduction

A number of recent papers have analysed the welfare effects of monetary policy in models of closed and open economies. It has been shown that, in a closed economy, welfare maximising monetary policy should aim to stabilise the consumer price index.[1] While in an open economy it has been shown that the optimal target for monetary policy is the producer price index.[2] The same result emerges when there is home bias in consumption or there are non-traded goods.[3]

The surprising implication of the models considered in all these papers is that exchange rate volatility has no direct impact on welfare. Welfare depends only on the variance of prices - consumer prices in the case of a closed economy or producer prices in the case of an open economy. But Bacchetta and van Wincoop (2000), Devereux and Engel (1998, 2000) and Corsetti and Pesenti (2001b) show that incomplete pass-through from exchange rate changes to local currency prices implies that exchange rate volatility can have a direct impact on welfare. Thus, by implication, when there is incomplete pass-through, optimal monetary policy should take account of exchange rate volatility.[4]

This paper analyses the welfare effects of exchange rate volatility by considering in more detail the links between exchange rate volatility, incomplete pass-through and welfare.[5] A two-country model is developed which allows an explicit derivation of a welfare function.[6] It is shown that welfare can be written in terms of a weighted sum of the second moments of home and foreign producer prices and the exchange rate. The weight on the variance of the exchange rate depends *inter alia* on the degree of pass-through (as implied by the work cited above) and the degree of openness. When there is complete pass-through the weight on the variance of the exchange rate is zero. In this case optimal monetary policy for the home country completely stabilises the price of home produced goods. But when there is incomplete pass-through the optimal monetary policy should take account of exchange rate volatility.[7]

[1] See Aoki (2001), Goodfriend and King (2001), King and Wolman (1999) and Woodford (2001).

[2] See Aoki (2001), Benigno and Benigno (2001a) and Clarida, Gali and Gertler (2001a).

[3] See Gali and Monacelli (2000) and Sutherland (2001a).

[4] Monacelli (1999) analyses a small open economy with imperfect pass-through and shows that the performance of simple monetary rules can be improved by including an exchange rate feedback term.

[5] In addition to the theoretical motivation for studying models of incomplete pass-through there is considerable evidence which suggests that incomplete pass-through is an important empirical feature of pricing behaviour. See for instance Engel (1999) and Engel and Rogers (1996), Goldberg and Knetter (1997) and Knetter (1989, 1993).

[6] The basic framework adopted here is an open economy general equilibrium model (with monopolistic competition and nominal stickiness) in the tradition of Svensson and van Wijnbergen (1987) and Obstfeld and Rogoff (1995, 1998, 2000). See Lane (2001) for a survey of recent developments in this literature.

[7] It should be noted that imperfect pass-through is not the only reason for supposing that the strict price targeting results need to be modified. In a closed economy context the presence

The results of this paper show that there is no simple relationship between exchange rate volatility and welfare.[8] Optimal policy for the home economy may involve stabilising or destabilising the exchange rate depending on the degree of pass-through, the size and openness of the home economy, the elasticity of labour supply and monetary policy in the foreign country. When pass-through is incomplete, labour supply is elastic and foreign monetary policy is being used to stabilise foreign producer prices, then home welfare is decreasing in exchange rate volatility. In these circumstances it is optimal for the home monetary authority to allow some volatility in home producer prices in order to achieve a more stable nominal exchange rate. But when labour supply is inelastic and/or foreign monetary policy is causing volatility in foreign producer prices, then home welfare may be increasing in exchange rate volatility. In these circumstances it may be optimal for the home monetary authority to increase the variance of the nominal exchange rate. In the case of foreign monetary shocks this increased exchange rate volatility arises because it is possible (and welfare improving) for the home monetary authority to use movements in the nominal exchange rate to offset the destabilising effects of foreign producer price movements.

This paper also briefly considers the implications of imperfect pass-through for international policy coordination. In a model with perfect pass-through (and utility which is logarithmic in consumption) Obstfeld and Rogoff (2002) show that both non-cooperative policy making (represented by a Nash equilibrium in monetary policy rules) and optimal coordinated policymaking imply the same rules for monetary policy. Corsetti and Pesenti (2001b), on the other hand, show that, when there is less than perfect pass-through, there are potential welfare gains to monetary policy coordination. A similar result holds in the model of this paper. Rather than demonstrating this result, this paper briefly considers how the coordinated policy outcome in this model can be supported by delegating monetary policy to independent monetary authorities in each county. It is found that the coordinated policy outcome can be supported if monetary authorities are assigned loss functions which depend on the variance of producer prices and the variance of the exchange rate. The weight on

of non-optimal 'cost-push' shocks implies that the optimal policy allows for some flexibility in consumer prices in order to allow some stabilisation of the output gap. This is often referred to as 'flexible inflation targeting' following the terminology suggested by Svensson (1999, 2000). Clarida, Gali and Gertler (2001b) and Benigno and Benigno (2001b) show that the same result holds in an open economy context, but with consumer prices being substituted by producer prices. Sutherland (2002a) also analyses this issue and shows that nominal income targeting can be a good approximation to fully optimal policy when the variance of cost-push shocks is high. A further case where the strict price targeting results must be modified is where the elasticity of substitution between home and foreign goods is greater than unity. Sutherland (2002b) analyses this case and shows that exchange rate volatility can become an important factor in welfare even when there is full pass-through.

[8] The model presented in this paper focuses on the effects of labour supply and foreign monetary shocks. The relationship between welfare and exchange rate volatility is found to depend on the source of shocks so, in a more general model, where more sources of shocks are present, the relationship may in fact be more complicated than suggested here.

the variance of the exchange rate in each national loss function is found to depend on the degree of openness and the degree of pass-through. If the home and foreign economies are very open and pass-through is low then the coordinated policy regime may require central banks to give significant weight to exchange rate stabilisation.

The paper proceeds as follows. Section 2 presents the structure of the model. Section 3 derives the solution of the model and the welfare function. Section 4 analyses the implications of the home country welfare function for optimal price and exchange rate volatility for the home economy. Section 5 considers international policy coordination and the optimal coordinated regime. Section 6 concludes the paper.

2 The Model

2.1 Market Structure

The world exists for a single period[9] and consists of two countries, the home economy and the rest of the world (the foreign economy). Each country is populated by agents who consume a basket of goods consisting of all home and foreign produced goods. Each agent is a monopoly producer of a single differentiated product.

There are two categories of agent in each country. The first set of agents supply goods in a market where prices are set in advance of the realisation of shocks and the setting of monetary policy. Agents in this market are contracted to meet demand at the pre-fixed prices. Agents in this group will be referred to as 'fixed-price agents'. The second set of agents supply goods in a market where prices are set after shocks are realised and monetary policy is set. Agents in this group will be referred to as 'flexible-price agents'.[10] The proportion of fixed-price agents in the total population is denoted ψ so ψ is a measure of the degree of price stickiness in the economy. The total population of the home economy is indexed on the unit interval with fixed-price agents indexed $[0,\psi]$ and flexible-price agents indexed $(\psi, 1]$. The population of the foreign country is ω with fixed-price agents indexed $[0, \psi\omega]$ and flexible-price agents indexed $(\psi\omega, \omega]$. Prices and quantities relating to fixed-price agents will be indicated with the subscript '1' while those relating to flexible-price agents will be indicated with the subscript '2'.

[9] The model can easily be recast as a multi-period structure but this adds no significant insights. A true dynamic model, with multi-period nominal contracts and asset stock dynamics would be considerably more complex and would require much more extensive use of numerical methods. Newly developed numerical techniques are available to solve such models and this is likely to be an interesting line of future research (see Kim and Kim (2000), Sims (2000), Schmitt-Grohé and Uribe (2001) and Sutherland (2001b)). However, the approach adopted in this paper yields useful insights which would not be available in a more complex model.

[10] This structure can be thought of as a static version of the Calvo (1983) staggered price setting framework. A fixed/flexible price structure similar to the one used here has previously been used in Aoki (2001) and Woodford (2001). The division of agents into fixed and flexible price groups is taken to be a fixed institutional feature of the economy.

This framework provides the minimal structure necessary to study the effects of price variability on welfare while allowing some degree of price stickiness. The fixed-price agents provide the nominal rigidity that is necessary to give monetary policy a role while the flexible-price agents provide the partial aggregate price flexibility that allows an analysis of the connection between price volatility and welfare.

2.2 Preferences

All agents in the home economy have utility functions of the same form. The utility of agent z of type i is given by

$$U(z) = E\left[\log C(z) + \log \frac{M(z)}{P} - \frac{K}{\mu} y_i^{\mu}(z)\right] \tag{1}$$

where $i = 1$ for a fixed-price agent and $i = 2$ for a flexible-price agent, C is a consumption index defined across all home and foreign goods, M denotes end-of-period nominal money holdings, P is the consumer price index, $y_i(z)$ is sales of good z, E is the expectations operator and K is a log-normal stochastic shock ($E[\log K] = 0$ and $Var[\log K] = \sigma_K^2$).

The consumption index C is defined as

$$C = \frac{C_H^{1-\nu} C_F^{\nu}}{(1-\nu)^{1-\nu}\nu^{\nu}} \tag{2}$$

where

$$\nu = (1-n)\gamma$$

where n is the share of the home population in the world population, i.e. $n = 1/(1+\omega)$ and $0 \leq n \leq 1$, C_H and C_F are indices of consumption of home and foreign produced goods and $0 \leq \gamma \leq 1$. The parameter ν measures the overall share of foreign goods in the consumption basket of home agents. It depends on two factors, the share of foreign goods in the total measure of goods in the world (i.e. $1-n$) and the degree of openness of the home economy, which is measured by γ. $\gamma = 0$ implies a completely closed economy while $\gamma = 1$ implies a completely open economy.[11]

The form of the utility function implies a unit elasticity of substitution between home and foreign goods. This ensures that there is no idiosyncratic income risk between the home country and the rest of the world. The structure of financial markets is therefore irrelevant.[12]

[11] This structure is similar to the modelling of "home bias" in Gali and Monacelli (2000). It is also formally identical to the modelling of non-traded goods in Obstfeld and Rogoff (2000) and Sutherland (2001a). In the latter two papers the relative price of nontraded and home produced traded goods is fixed at unity so consumption of nontraded goods can be thought of as home bias in consumption.

[12] This assumption was introduced into a deterministic open economy model by Corsetti and Pesenti (2001a) and has proved to be a key assumption allowing a tractable solution to stochastic models of the type used in this paper.

Utility from consumption of home and foreign goods is defined as follows

$$C_H = \frac{C_{H,1}^{\psi} C_{H,2}^{(1-\psi)}}{\psi^{\psi}(1-\psi)^{(1-\psi)}}, \quad C_F = \frac{C_{F,1}^{\psi} C_{F,2}^{(1-\psi)}}{\psi^{\psi}(1-\psi)^{(1-\psi)}} \tag{3}$$

where $C_{H,1}$and $C_{H,2}$ are indices of home fixed-price and flexible-price goods defined as follows

$$C_{H,1} = \left[\left(\frac{1}{\psi}\right)^{\frac{1}{\phi}} \int_0^{\psi} c_{H,1}(h)^{\frac{\phi-1}{\phi}} dh \right]^{\frac{\phi}{\phi-1}}, C_{H,2} = \left[\left(\frac{1}{1-\psi}\right)^{\frac{1}{\phi}} \int_{\psi}^{1} c_{H,2}(h)^{\frac{\phi-1}{\phi}} dh \right]^{\frac{\phi}{\phi-1}}$$

and $C_{F,1}$and $C_{F,2}$ are indices of foreign fixed-price and flexible-price goods defined as follows

$$C_{F,1} = \left[\left(\frac{1}{\psi\omega}\right)^{\frac{1}{\phi}} \int_0^{\psi\omega} c_{F,1}(f)^{\frac{\phi-1}{\phi}} df \right]^{\frac{\phi}{\phi-1}}, C_{F,2} = \left[\left(\frac{1}{(1-\psi)\,\omega}\right)^{\frac{1}{\phi}} \int_{\psi\omega}^{\omega} c_{F,2}(f)^{\frac{\phi-1}{\phi}} df \right]^{\frac{\phi}{\phi-1}}$$

where $\phi > 1$, $c_{H,i}(h)$ is consumption of home good h produced by an agent of type i and $c_{F,i}(f)$ is consumption of foreign good f produced by an agent of type i, (where $i = 1$ indicates a good produced by a fixed-price agent and $i = 2$ indicates a good produced by a flexible-price agent). The above functions imply a constant elasticity of substitution between different varieties of good of the same type and a unit elasticity of substitution between fixed-price and flexible-price goods.[13]

2.3 Price Indices

The consumer price index for home agents is

$$P = P_H^{1-\nu} P_F^{\nu} \tag{4}$$

and the price indices of home and foreign produced goods are

$$P_H = P_{H,1}^{\psi} P_{H,2}^{(1-\psi)}, \quad P_F = P_{F,1}^{\psi} P_{F,2}^{(1-\psi)} \tag{5}$$

where $P_{H,1}$ and $P_{H,2}$ are the price indices of home fixed-price and flexible-price goods defined as follows

$$P_{H,1} = \left[\frac{1}{\psi} \int_0^{\psi} p_{H,1}(h)^{1-\phi} dh \right]^{\frac{1}{1-\phi}}, \quad P_{H,2} = \left[\frac{1}{1-\psi} \int_{\psi}^{1} p_{H,2}(h)^{1-\phi} dh \right]^{\frac{1}{1-\phi}}$$

[13] The assumption that the elasticity of substitution between fixed- and flexible-price goods differs from the elasticity of substitution between goods within each type has the slightly odd implication that the degree of price stickiness is in effect embedded in the structure of preferences. It would be possible to relax this assumption (and, for instance, have a common elasticity of ϕ between all goods) but the present assumption allows some useful simplifications of the algebra (because it ensures that all home agents have identical income and consumption levels regardless of which type they are).

and $P_{F,1}$ and $P_{F,2}$ are the price indices of foreign fixed-price and flexible-price goods defined as follows

$$P_{F,1} = \left[\frac{1}{\psi\omega} \int_{0}^{\psi\omega} p_{F,1}(f)^{1-\phi} df \right]^{\frac{1}{1-\phi}}, \quad P_{F,2} = \left[\frac{1}{(1-\psi)\,\omega} \int_{\psi\omega}^{\omega} p_{F,2}(f)^{1-\phi} df \right]^{\frac{1}{1-\phi}}$$

All the above prices are denominated in domestic currency.

2.4 Consumption Choices

The budget constraint of home agent z (where z is of type i) is given by

$$M(z) = M_0 + p_{H,i}(z)\, y_{H,i}(z) + S p^*_{H,i}(z)\, y^*_{H,i}(z) - PC(z) - T \tag{6}$$

where M_0 and $M(z)$ are initial and final money holdings, T is a lump-sum government transfer, S is the nominal exchange rate defined as the domestic price of foreign currency. $y_{H,i}(z)$ and $y^*_{H,i}(z)$ are sales of good z to home and foreign agents. It is assumed that all changes in the money supply enter and leave the economy via transfers thus $T = M_0 - M$. Producers set different prices for home and foreign purchasers. The price charged to home purchasers is $p_{H,i}(z)$ (denominated in home currency) and the price charged to foreign purchasers is $p^*_{F,i}(z)$ (denominated in foreign currency). The nature of the price contracts governing the setting of prices, and the consequent degree of pass-through from exchange rate changes, is discussed in more detail below.

In a symmetric equilibrium the consumption decisions of all home agents are identical. Demands for representative home fixed-price good h_1 and representative home flexible-price good h_2 are given by the following expressions

$$c_{H,1}(h_1) = \frac{1}{\psi} C_{H,1} \left(\frac{p_{H,1}(h_1)}{P_{H,1}} \right)^{-\phi}, \quad c_{H,2}(h_2) = \frac{1}{1-\psi} C_{H,2} \left(\frac{p_{H,2}(h_2)}{P_{H,2}} \right)^{-\phi}$$

where

$$C_{H,1} = \psi C_H \left(\frac{P_{H,1}}{P_H} \right)^{-1}, \quad C_{H,2} = (1-\psi)\, C_H \left(\frac{P_{H,2}}{P_H} \right)^{-1} \tag{7}$$

and

$$C_H = (1-\nu)\, C \left(\frac{P_H}{P} \right)^{-1} \tag{8}$$

Demands for representative foreign fixed-price good f_1 and representative foreign flexible-price good f_2 are given by the following expressions

$$c_{F,1}(f_1) = \frac{1}{\psi\omega} C_{F,1} \left(\frac{p_{F,1}(f_1)}{P_{F,1}} \right)^{-\phi}, \quad c_{F,2}(f_2) = \frac{1}{(1-\psi)\,\omega} C_{F,2} \left(\frac{p_{F,2}(f_2)}{P_{F,2}} \right)^{-\phi}$$

where

$$C_{F,1} = \psi C_F \left(\frac{P_{F,1}}{P_F} \right)^{-1}, \quad C_{F,2} = (1-\psi) C_F \left(\frac{P_{F,2}}{P_F} \right)^{-1} \tag{9}$$

and

$$C_F = \nu C \left(\frac{P_F}{P} \right)^{-1} \tag{10}$$

2.5 The Foreign Economy

The foreign economy is assumed to have a structure similar to that of the home country and foreign agents are assumed to have utility functions similar to (1). Foreign labour supply preferences are subject to log-normal shocks, denoted K^* ($E[\log K^*] = 0$ and $Var[\log K^*] = \sigma^2_{K^*}$). Foreign demands for representative home fixed-price good h_1 and representative home flexible-price good h_2 are given by the following

$$c^*_{H,1}(h_1) = \frac{1}{\psi} C^*_{H,1} \left(\frac{p^*_{H,1}(h_1)}{P^*_{H,1}} \right)^{-\phi}, \quad c^*_{H,2}(h_2) = \frac{1}{1-\psi} C^*_{H,2} \left(\frac{p^*_{H,2}(h_2)}{P^*_{H,2}} \right)^{-\phi}$$

where

$$C^*_{H,1} = \psi C^*_H \left(\frac{P^*_{H,1}}{P^*_H} \right)^{-1}, \quad C^*_{H,2} = (1-\psi) C^*_H \left(\frac{P^*_{H,2}}{P^*_H} \right)^{-1} \tag{11}$$

and

$$C^*_H = \nu^* C^* \left(\frac{P^*_H}{P^*} \right)^{-1} \tag{12}$$

where

$$\nu^* = n\gamma$$

where P^* is the consumer price index in the rest of the world and C^* is *per capita* consumption in the rest of the world. $p^*_{H,1}(h_1)$ and $p^*_{H,2}(h_2)$ are the foreign currency prices of home goods h_1 and h_2. $P^*_{H,1}$ and $P^*_{H,2}$ are the foreign currency price indices of home fixed-price and flexible-price goods and P^*_H is the foreign currency price index of all home goods. The relationship between the home and foreign currency prices of home goods is discussed in detail below. The parameter ν^* measures the overall share of home goods in the foreign consumption basket. It depends on the share of home goods in the total measure of goods in the world, n, and the degree of openness, γ.

Foreign demands for foreign goods are

$$c^*_{F,1}(f_1) = \frac{1}{\psi\omega} C^*_{F,1} \left(\frac{p^*_{F,1}(f_1)}{P^*_{F,1}} \right)^{-\phi}, \quad c^*_{F,2}(f_2) = \frac{1}{(1-\psi)\omega} C^*_{F,2} \left(\frac{p^*_{F,2}(f_2)}{P^*_{F,2}} \right)^{-\phi}$$

where

$$C^*_{F,1} = \psi C^*_F \left(\frac{P^*_{F,1}}{P^*_F}\right)^{-1}, \quad C^*_{F,2} = (1-\psi)\, C^*_F \left(\frac{P^*_{F,2}}{P^*_F}\right)^{-1} \tag{13}$$

and

$$C^*_F = (1-\nu^*)\, C^* \left(\frac{P^*_F}{P^*}\right)^{-1} \tag{14}$$

These consumption levels are *per capita* of the foreign population. The total demands from foreign agents are obtained by multiplying each expression by ω.

2.6 The Balance of Payments

Using the above relationships it is simple to verify that fixed-price and flexible-price agents have the same consumption levels. It is also possible to verify that financial markets are irrelevant. To see this latter point note that current account balance implies

$$\omega S P^*_H C^*_H = P_F C_F \tag{15}$$

which implies

$$\left(\frac{C^*}{C}\right)^{-1} = \frac{SP^*}{P} \tag{16}$$

This equation shows that, when the current account is in balance, the ratio of marginal utilities across the two countries is equal to the ratio of aggregate prices (i.e. the real exchange rate). This implies that there can be no Pareto improving reallocation of consumption across countries. Financial markets are therefore redundant.

2.7 Price Contracts and the Degree of Pass-Through

Agents are required to enter into separate price contracts for sales in home and foreign markets. The price contract signed by home fixed-price agents for sales to foreign consumers is assumed to enforce a fixed degree of indexation to unanticipated exchange rate changes. Home fixed-price agent z selling to foreign consumers chooses a price $\breve{p}_{H,1}(z)$ denominated in home currency. The actual foreign currency price charged is determined by the following formula (which is part of the contract)

$$p^*_{H,1}(z) = \frac{\breve{p}_{H,1}(z)}{S} \left(\frac{S}{S_E}\right)^{1-\eta_1} \tag{17}$$

where S_E is the *ex ante* expected exchange rate and $0 \leq \eta_1 \leq 1$. This structure allows a full range of degrees of pass-through. $\eta_1 = 1$ implies producer currency pricing and full pass-through. $\eta_1 = 0$ implies local currency pricing and zero pass-through.[14]

[14] Implicitly the assumption of separate price contracts for home and foreign sales allows agents to price discriminate between home and foreign markets. In a deterministic context there would be no

Foreign fixed-price producers also set separate prices for home and foreign consumers. The contract for foreign sales to home consumers also enforces a fixed degree of indexation to unanticipated exchange rate changes. Foreign fixed-price agent z selling to home consumers sets a price $\breve{p}^*_{F,1}(z)$ in terms of foreign currency. The actual home currency price charged is given by the following formula

$$p_{F,1}(z) = S\breve{p}^*_{F,1}(z)\left(\frac{S}{S_E}\right)^{-(1-\eta_2)} \tag{18}$$

where $0 \leq \eta_2 \leq 1$. This structure allows a full range of degrees of pass-through. $\eta_2 = 1$ implies producer currency pricing and full pass-through. $\eta_2 = 0$ implies local currency pricing and zero pass-through.

2.8 Optimal Price Setting

First consider the prices set by flexible-price producers. Flexible-price producers in both countries are able to set prices after shocks have been realised and monetary policy has been set. This, coupled with the assumption of equal elasticity of demand in the two countries, implies that flexible-price firms have no incentive to price discriminate between home and foreign consumers. It therefore follows that $p_{H,2}(z) = Sp^*_{H,2}(z)$. This is true for all flexible-price producers so $P_{H,2} = SP^*_{H,2}$. The first order condition for the choice of price is derived in the Appendix and is given by the following

$$P_{H,2} = \frac{\phi}{\phi - 1} KY_2^{\mu-1} PC \tag{19}$$

where

$$Y_2 = \frac{1}{(1-\psi)}\left(C_{H,2} + \omega C^*_{H,2}\right) = C\left(\frac{P_{H,2}}{P}\right)^{-1} \tag{20}$$

where Y_2 is the output of flexible-price agents expressed *per capita* of the population of flexible-price agents. (The output expression has been simplified by using $P_{H,2} = SP^*_{H,2}$ and $PC = SP^*C^*$.) The same price is chosen by all flexible-price producers so the argument z is omitted from these expressions. This condition holds *ex post* for all realisations of the shock variables. Notice that the optimal price depends on the product of three factors. The first term, $\phi/(\phi - 1)$, is the mark-up. The middle

incentive to price discriminate because elasticities of demand are equal at home and abroad. But, in a stochastic context, given the different stochastic characteristics of home and foreign demand, price discrimination in fact becomes optimal for fixed-price agents. However, this incentive to price discriminate is incidental to the analysis of this paper. The important point is that there are separate contracts. The structure of contracts is taken to be a fixed institutional feature of the economy. This structure follows, and is formally identical to, the structure first used by Corsetti and Pesenti (2001b) to model incomplete pass-through. Recently a number of authors have begun to consider the implications of endogenous choice of the degree of pass-through (see Bacchetta and van Wincoop (2001), Devereux and Engel (2001) and Corsetti and Pesenti (2002)).

term, $KY_2^{\mu-1}$, is the marginal disutility of work effort. And the last term, PC, is made up of factors which affect the demand for goods.

Now consider the price setting problem for fixed-price agents. There are two first order conditions (derived in the Appendix) for the choice of prices for the home fixed-price producer. One for the price charged to home consumers as follows

$$P_{H,1} = E\left[V_{H,1}\right] \text{ where } V_{H,1} = \left(\frac{Y_1}{Y_2}\right)^{\mu-1} P_{H,2} \tag{21}$$

And one for the price charged to foreign consumers as follows

$$P_{H,1}^* = S^{-\eta_1} E\left[V_{H,1}^*\right] \text{ where } V_{H,1}^* = \left(\frac{Y_1}{Y_2}\right)^{\mu-1} P_{H,2} S^{\eta_1 - 1} \tag{22}$$

where

$$Y_1 = \frac{1}{\psi}\left(C_{H,1} + \omega C_{H,1}^*\right) \tag{23}$$

where Y_1 is the total output of fixed-price agents expressed *per capita* of the population of fixed-price agents. All fixed-price producers make the same choice of price so the argument z is omitted from these conditions. Notice that fixed-price producers set their home price equal to the expected value of the price set by flexible-price producers adjusted by a factor which reflects the different level of work effort (and hence different marginal disutility of labour) of fixed-price agents. The price charged to foreign consumers is adjusted by a further factor which reflects the effects of the exchange rate on demand when there is incomplete pass-through.

The first order conditions for foreign price setting have a similar structure. Foreign flexible-price producers have no incentive to price discriminate and their prices are given by

$$P_{F,2}^* = \frac{\phi}{\phi - 1} K^* Y_2^{*\mu-1} P^* C^* \tag{24}$$

where

$$Y_2^* = \frac{1}{1-\psi}\left(\frac{1}{\omega} C_{F,2} + C_{F,2}^*\right) = C^* \left(\frac{P_{F,2}^*}{P^*}\right)^{-1} \tag{25}$$

and the prices set by foreign fixed-price agents are given by

$$P_{F,1}^* = E\left[V_{F,1}^*\right] \text{ where } V_{F,1}^* = \left(\frac{Y_1^*}{Y_2^*}\right)^{\mu-1} P_{F,2}^* \tag{26}$$

and

$$P_{F,1} = S^{\eta_2} E\left[V_{F,1}\right] \text{ where } V_{F,1} = \left(\frac{Y_1^*}{Y_2^*}\right)^{\mu-1} P_{F,2}^* S^{1-\eta_2} \tag{27}$$

where

$$Y_1^* = \frac{1}{\psi}\left(\frac{1}{\omega} C_{F,1} + C_{F,1}^*\right) \tag{28}$$

The fact that fixed-price agents must set prices before shocks are realised clearly implies that risk premia will be built into contract prices. This is apparent because of the expectational terms in the above price setting conditions for fixed-price agents. The risk premia will depend on the variances of the V terms in the above pricing conditions. The log-normal structure of the model will allow an explicit derivation of expressions for these risk premia.

2.9 Money Demand

The first order condition for the choice of money holdings is

$$\frac{M}{P} = C \tag{29}$$

The money supply is fixed by the monetary authority.

3 Model Solution and Welfare

One of the main advantages of the model just described is that it provides a very natural and tractable measure of welfare which can be derived from the aggregate utility of agents. Following Obstfeld and Rogoff (1998, 2000) it is assumed that the utility of real balances is small enough to be neglected. It is therefore possible to measure *ex ante* aggregate home-country welfare using the following

$$\Omega = E\left[\psi\left(\log C - \frac{K}{\mu}Y_1^{\mu}\right) + (1-\psi)\left(\log C - \frac{K}{\mu}Y_2^{\mu}\right)\right] \tag{30}$$

The Appendix shows that the following relationships hold

$$E\left[KY_1^{\mu}\right] = \frac{\phi-1}{\phi}, \quad E\left[KY_2^{\mu}\right] = \frac{\phi-1}{\phi} \tag{31}$$

so the welfare measure can be written as

$$\Omega = E\log C - \frac{\phi-1}{\phi\mu} \tag{32}$$

It proves useful to consider the solution of the model in terms of the *ex ante* expected log deviation of variables from the deterministic equilibrium. Define the deterministic equilibrium of the model as the solution which results when $K = K^* = 1$ with $\sigma_K^2 = \sigma_{K^*}^2 = 0$ and for any variable X define $\hat{X} = \log\left(X/\bar{X}\right)$ where $\bar{X}$ is the value of variable X in the deterministic equilibrium. Notice that welfare can be expressed in terms of the deviation from the deterministic equilibrium as follows

$$\Omega_D = \Omega - \bar{\Omega} = E\left[\hat{C}\right] \tag{33}$$

This is the measure of welfare used throughout the remainder of the paper.

The fact that welfare depends on the *ex ante* expected value of consumption implies that it is necessary to solve the model in *ex ante* expected terms. It will be shown below that the *ex ante* expected solution of the model in turn depends on the second moments of variables. In order to obtain expressions for second moments it is necessary also to solve for the *ex post* realisation of variables. The solution process therefore proceeds in two stages. In the first stage an expression is derived for the *ex ante* expected value of consumption in terms of second moments. In the second stage the *ex post* solution is derived for all relevant variables and a full solution for the welfare function is derived.

3.1 Ex Ante Solution

As pointed out above, it is useful to consider the *ex ante* solution of the model in terms of the *ex ante* expected log deviation of variables from the deterministic solution. It is useful to consider the model is these terms because the effect of stochastic shocks on the *ex ante* solution is to create deviations from the deterministic solution. These deviations depend on the second moments of the variables of the model. By writing the model in terms of deviations from the deterministic solution it is possible to isolate the equations where the second-moment effects enter the model.

Most of the equations of the model are linear in logs and can easily be translated into equations in terms of deviations from the deterministic solution. Second-moment terms do not enter directly into any of these equations. There are just six equations where second-moment terms do enter. However, before considering these equations in detail, it is first useful to note that the solutions for a number of variables can be very easily obtained. The home and foreign money demand and supply equations imply

$$E\left[\hat{P}\right]+E\left[\hat{C}\right]=0 \text{ and } E\left[\hat{P}^{*}\right]+E\left[\hat{C}^{*}\right]=0 \tag{34}$$

(where it is assumed that the home and foreign monetary authorities adopt monetary rules which imply $E\left[\hat{M}\right]=E\left[\hat{M}^{*}\right]=0$). In combination with the expression for current account balance these relationships imply

$$E\left[\hat{S}\right]=0 \tag{35}$$

The equations for home and foreign flexible-price price setting and output imply

$$E\left[\hat{P}_{H,2}\right]=E\left[\hat{Y}_{2}\right]=0 \tag{36}$$

$$E\left[\hat{P}_{F,2}\right]=E\left[\hat{Y}_{2}^{*}\right]=0 \tag{37}$$

From the above results and the definition of the various price indices it follows that

$$\Omega_{D}=E\left[\hat{C}\right]=-E\left[\hat{P}\right]=E\left[-\nu\psi\hat{P}_{F,1}-(1-\nu)\psi\hat{P}_{H,1}\right] \tag{38}$$

Thus *ex ante* consumption, and therefore welfare (expressed as the deviation from the deterministic equilibrium), depend only on the prices set by home and foreign fixed-price producers.

In order to solve for $E\left[P_{F,1}\right]$ and $E\left[P_{H,1}\right]$ it is necessary to consider the equations of the model which directly contain second-moment terms. There are six such equations. Four of these equations are the price-setting conditions for the home and foreign fixed-price agents, i.e. (21), (22), (26) and (27). These four equations contain second-moment terms because they contain terms in the expectations of the levels of log-normal variables. The standard properties of log-normal variables (together with the results already derived for $E\left[\hat{P}\right]$, $E\left[\hat{C}\right]$, $E\left[\hat{S}\right]$ etc.) mean it is possible to rewrite these equations in the following form

$$E\left[\hat{P}_{H,1}\right] = (\mu - 1)E\left[\hat{Y}_1\right] + \lambda_1 \text{ where } \lambda_1 = \frac{1}{2}\sigma^2_{V_{H,1}} \tag{39}$$

$$E\left[\hat{P}^*_{H,1}\right] = (\mu - 1)E\left[\hat{Y}_1\right] + \lambda_2 \text{ where } \lambda_2 = \frac{1}{2}\sigma^2_{V^*_{H,1}} \tag{40}$$

$$E\left[\hat{P}^*_{F,1}\right] = (\mu - 1)E\left[\hat{Y}^*_1\right] + \lambda_3 \text{ where } \lambda_3 = \frac{1}{2}\sigma^2_{V^*_{F,1}} \tag{41}$$

$$E\left[\hat{P}_{F,1}\right] = (\mu - 1)E\left[\hat{Y}^*_1\right] + \lambda_4 \text{ where } \lambda_4 = \frac{1}{2}\sigma^2_{V_{F,1}} \tag{42}$$

where for any variable X, $\sigma^2_X = Var\left[\hat{X}\right]$.

The final two equations which contain second-moment terms are the equations for home and foreign fixed-price output, i.e. equations (23) and (28). Notice that these equations are not linear in logs. It is therefore necessary to use a second-order approximations as follows

$$E\left[\hat{Y}_1\right] = (1 - \nu)E\left[\hat{C}_{H,1}\right] + \nu E\left[\hat{C}^*_{H,1}\right] + \lambda_5 + O\left(\|\xi\|^3\right) \tag{43}$$

$$E\left[\hat{Y}^*_1\right] = \nu^* E\left[\hat{C}_{F,1}\right] + (1 - \nu^*)\, E\left[\hat{C}^*_{F,1}\right] + \lambda_6 + O\left(\|\xi\|^3\right) \tag{44}$$

where

$$\lambda_5 = (1 - \nu)\nu\left(\sigma^2_{c_{H,1}} + \sigma^2_{c^*_{H,1}} - 2\sigma_{c_{H,1},c^*_{H,1}}\right)/2 \tag{45}$$

$$\lambda_6 = (1 - \nu^*)\nu^*\left(\sigma^2_{c_{F,1}} + \sigma^2_{c^*_{F,1}} - 2\sigma_{c_{F,1},c^*_{F,1}}\right)/2 \tag{46}$$

and $O\left(\|\xi\|^3\right)$ is a residual which contains terms which are of order three and above in deviations from the non-stochastic steady state. The appendix provides a detailed derivation of these approximations.[15]

[15] In general for variables X and Y the following notation is used: $\sigma^2_X = Var\left[\hat{X}\right]$, $\sigma^2_Y = Var\left[\hat{Y}\right]$ and $\sigma_{X,Y} = Cov\left[\hat{X}, \hat{Y}\right]$.

There are thus six equations where second moments enter the model and six second-moment terms, λ_1, λ_2, λ_3, λ_4, λ_5 and λ_6. The interpretation of these terms will be discussed below. First it is useful to complete the derivation of the *ex ante* solution in order to obtain an expression for welfare in terms of these second-moment terms.

Using equations (39) to (44) and the equations for home and foreign demand for home fixed-price goods it is possible to write welfare in the following form

$$\begin{aligned}\Omega_D = & -\frac{\psi}{\mu}(1-\nu)\left[1+\nu(\mu-1)\right]\lambda_1 \\ & -\frac{\psi}{\mu}\nu\left[1+(\mu-1)(1-v^*)\right]\lambda_4 \\ & +\frac{\psi}{\mu}(\mu-1)\nu\left[(1-\nu)\lambda_2+(1-\nu^*)\lambda_3\right] \\ & -\frac{\psi}{\mu}(\mu-1)\left[(1-\nu)\lambda_5+\nu\lambda_6\right]+O\left(\|\xi\|^3\right) \end{aligned} \tag{47}$$

Thus welfare can be expressed in terms of the six second-moment terms λ_1, λ_2, λ_3, λ_4, λ_5 and λ_6.

Given the role played by these second-moment terms it is useful at this point to consider their economic interpretation. Four of the second-moment terms, namely λ_1, λ_2, λ_3 and λ_4, have a straightforward economic interpretation in terms of risk premia. Fixed-price agents have to set prices in advance of the realisation of shocks and will thus build risk premia into their contract prices.[16] The risk premia will depend on the stochastic nature of demand and marginal costs. Thus the risk premia will differ depending on whether demand is from home agents or foreign agents and whether the producer is home or foreign. The risk premia will also depend on the degree of pass-through. The degree of pass-through affects the extent to which exchange rate changes affect demand, so pass-through affects the form and extent of risk faced by producers.

Notice that λ_1 and λ_4 have a negative effect on welfare. λ_1 is the risk premium in the price charged to home consumers by home producers so a higher value of λ_1 increases the prices faced by home consumers and therefore reduces their welfare. λ_4 is the risk premium in prices charged by foreign producers for sales to home consumers (i.e. the risk premium in import prices). Again higher prices reduce the welfare of home consumers.

In contrast λ_2 has a positive effect on welfare. λ_2 is the risk premium in prices set by home producers for sales to foreign consumers (i.e. the risk premium in export prices). Higher prices charged to foreign consumers reduce foreign demand for home goods. This allows home consumers to consume more home goods for a given level

[16] The fact that a monopolist will set a higher price in a stochastic environment than in a deterministic environment has previously been noted and analysed by, for instance, Sorensen (1992) and Rankin (1998).

of work effort. In effect the reduction of foreign demand has a “crowding in” effect on home consumption of home goods. The extent to which λ_2 makes a positive contribution to welfare depends on the slope of the disutility of labour function (which is determined by μ). The larger is μ the more reluctant home agents are to vary work effort and the stronger is the crowding in/out effect of changes in foreign demand.[17]

The last two second-moment terms, λ_5 and λ_6, have a different interpretation from the other four. These terms enter the model via the equations for the output of home and foreign fixed-price agents. These terms are effectively capturing the inefficiency that arises when the prices charged by fixed-price agents to home and foreign consumers diverge *ex post* due to exchange rate changes and incomplete pass-through.[18] This represents a distortion which implies that fixed-price agents have to work harder to supply a given level of utility to home and foreign consumers. This effect is captured by the second-order terms appearing in the approximated fixed-price output equation.[19] λ_5 and λ_6 have a negative effect on home welfare.

Before considering the *ex post* solution of the model it is useful to note that, by writing the welfare function in the form shown in (47), it is possible to see a direct link between the welfare function in this paper and the welfare functions derived in Rotemberg and Woodford (1999) and Woodford (2001). These authors emphasise that the welfare cost of inflation volatility arises because changes in the absolute price level cause changes in relative output prices between producers with flexible prices and producers with fixed prices. These changes in relative prices produce a welfare reducing distortion in relative outputs across producers. In the context of the model of this paper such distortions would take the form of changes in the output of fixed-price agents relative to the output of flexible-price agents. But it is apparent from the price setting equations (21), (22), (26) and (27) and from the definitions of the risk premia in (39), (40), (41) and (42) that a major determinant of the risk premia in this model is volatility in the relative outputs of fixed-price and flexible-price agents. In fact, when there is complete pass-through, the only determinants of the risk premia are relative outputs. Furthermore, welfare is entirely determined by the risk premia. Thus the risk premia in prices and their role in determining welfare are

[17] The positive welfare effect of reducing exports is partly a consequence of the unit elasticity of substitution between home and foreign goods in the utility function. This assumption implies that the elasticity of demand for home goods is unity, so total national export revenue is independent of the volume of exports. In such a situation, any factor (such as the risk premium in export prices) which reduces export volume is obviously welfare enhancing for home agents. In a more general model, where the elasticity of demand for home goods is greater than unity, there would be a welfare maximising export volume and the welfare effects of the risk premium in export prices would depend on whether export volume is above or below the welfare maximising level.

[18] *Ex ante* fixed-price agents may rationally set different prices for home and foreign consumers (because of the differing stochastic characteristics of the two sources of demand). However, these prices are also *ex post* subject to different degrees of indexation to exchange rate changes and this may cause a non-optimal divergence of the two prices.

[19] Notice that when there is complete pass-through the output equations become log-linear and λ_5 and λ_6 are zero.

capturing exactly the same effects as those emphasised by Rotemberg and Woodford (1999) and Woodford (2001). It is interesting to note that the application of the Rotemberg and Woodford method (with some amendments) to the model of this paper produces exactly the same results as presented here. The results presented here are therefore independent of the emphasis on risk premia.

3.2 Ex Post Solution

In order to solve for the second moments of the model it is necessary to obtain the *ex post* solution to the model. Note that for any variable X, $E[\hat{X}]$ depends only on second-order terms. So to obtain a second-order accurate expression for $Var[\hat{X}]$ it is sufficient to consider first-order accurate solutions to $\hat{X}$. It is thus possible to consider the model in terms of the realised *ex post* log deviation of variables from their values in the deterministic steady state. The aim is to express the welfare function in terms of the second moments of $\hat{P}_{H,2}$, $\hat{P}^*_{F,2}$ and $\hat{S}$ so here variables are solved in terms of $\hat{P}_{H,2}$, $\hat{P}^*_{F,2}$ and $\hat{S}$.

Current account balance implies

$$\hat{P} + \hat{C} = \hat{S} + \hat{P}^* + \hat{C}^* \tag{48}$$

Home and foreign flexible-price outputs are given by

$$\hat{Y}_2 = \hat{P} + \hat{C} - \hat{P}_{H,2} \text{ and } \hat{Y}_2^* = \hat{P}^* + \hat{C}^* - \hat{P}^*_{F,2} \tag{49}$$

Home and foreign fixed-price demand schedules imply

$$\hat{C}_{H,1} = \hat{P} + \hat{C} \text{ and } \hat{C}^*_{H,1} = \hat{P}^* + \hat{C}^* + \eta_1 \hat{S} \tag{50}$$

$$\hat{C}^*_{F,1} = \hat{P}^* + \hat{C}^* \text{ and } \hat{C}_{F,1} = \hat{P} + \hat{C} - \eta_2 \hat{S} \tag{51}$$

so the output levels of home and foreign fixed-price agents are

$$\hat{Y}_1 = \hat{P} + \hat{C} - \nu\left(1 - \eta_1\right)\hat{S} + O\left(\|\xi\|^2\right) \tag{52}$$

$$\hat{Y}_1^* = \hat{P}^* + \hat{C}^* + \nu^*\left(1 - \eta_2\right)\hat{S} + O\left(\|\xi\|^2\right) \tag{53}$$

where $O\left(\|\xi\|^2\right)$ is a residual which contains terms of order two and above in deviations from the non-stochastic steady state. It is now possible to derive the following expressions for $\hat{V}_{H,1}$, $\hat{V}^*_{H,1}$, $\hat{V}^*_{F,1}$ and $\hat{V}_{F,1}$

$$\hat{V}_{H,1} = \mu\hat{P}_{H,2} - (\mu - 1)\,\nu\left(1 - \eta_1\right)\hat{S} + O\left(\|\xi\|^2\right) \tag{54}$$

$$\hat{V}^*_{H,1} = \mu\hat{P}_{H,2} - \left[(\mu - 1)\,\nu + 1\right]\left(1 - \eta_1\right)\hat{S} + O\left(\|\xi\|^2\right) \tag{55}$$

$$\hat{V}^*_{F,1} = \mu\hat{P}^*_{F,2} + (\mu - 1)\,\nu^*\left(1 - \eta_2\right)\hat{S} + O\left(\|\xi\|^2\right) \tag{56}$$

$$\hat{V}_{F,1} = \mu\hat{P}^*_{F,2} + \left[(\mu - 1)\,\nu^* + 1\right]\left(1 - \eta_2\right)\hat{S} + O\left(\|\xi\|^2\right) \tag{57}$$

and to derive the following expression for the λs

$$\begin{aligned}\lambda_1 &= \frac{1}{2}\Big\{(\mu-1)^2\nu^2(1-\eta_1)^2\sigma_S^2+\mu^2\sigma_{P_{H,2}}^2 \\ &\qquad -2(\mu-1)\nu(1-\eta_1)\mu\sigma_{S,P_{H,2}}\Big\}+O\left(\|\xi\|^3\right)\end{aligned} \tag{58}$$

$$\begin{aligned}\lambda_2 &= \frac{1}{2}\Big\{[(\mu-1)\nu+1]^2(1-\eta_1)^2\sigma_S^2+\mu^2\sigma_{P_{H,2}}^2 \\ &\qquad -2[(\mu-1)\nu+1](1-\eta_1)\mu\sigma_{S,P_{H,2}}\Big\}+O\left(\|\xi\|^3\right)\end{aligned} \tag{59}$$

$$\begin{aligned}\lambda_3 &= \frac{1}{2}\Big\{(\mu-1)^2\nu^{*2}(1-\eta_2)^2\sigma_S^2+\mu^2\sigma_{P_{F,2}^*}^2 \\ &\qquad +2(\mu-1)\nu^*(1-\eta_2)\mu\sigma_{S,P_{F,2}^*}\Big\}+O\left(\|\xi\|^3\right)\end{aligned} \tag{60}$$

$$\begin{aligned}\lambda_4 &= \frac{1}{2}\Big\{[(\mu-1)\nu^*+1]^2(1-\eta_2)^2\sigma_S^2+\mu^2\sigma_{P_{F,2}^*}^2 \\ &\qquad +2[(\mu-1)\nu^*+1](1-\eta_2)\mu\sigma_{S,P_{F,2}^*}\Big\}+O\left(\|\xi\|^3\right)\end{aligned} \tag{61}$$

$$\lambda_5 = \frac{1}{2}(1-\nu)\nu(1-\eta_1)^2\sigma_S^2+O\left(\|\xi\|^3\right) \tag{62}$$

$$\lambda_6 = \frac{1}{2}(1-\nu^*)\nu^*(1-\eta_2)^2\sigma_S^2+O\left(\|\xi\|^3\right) \tag{63}$$

These expressions can be used to derive the following expression for home welfare[20]

$$\begin{aligned}\Omega_D &= -\frac{\psi\mu(1-\nu)}{2(1-\psi)^2}\sigma_{P_H}^2-\frac{\psi\mu\nu}{2(1-\psi)^2}\sigma_{P_F^*}^2-\frac{\psi\mu\nu(1-\eta_2)}{(1-\psi)}\sigma_{S,P_F^*} \\ &\quad +\frac{\psi\nu}{2}\left[\left((\mu-1)(1-\nu^*)^2-\mu\right)(1-\eta_2)^2+(\mu-1)\nu(1-\nu)(1-\eta_1)^2\right]\sigma_S^2 \\ &\quad +O\left(\|\xi\|^3\right)\end{aligned} \tag{64}$$

The following expression for foreign welfare can also be derived

$$\begin{aligned}\Omega_D^* &= -\frac{\psi\mu(1-\nu^*)}{2(1-\psi)^2}\sigma_{P_F^*}^2-\frac{\psi\mu\nu^*}{2(1-\psi)^2}\sigma_{P_H}^2+\frac{\psi\mu\nu^*(1-\eta_1)}{(1-\psi)}\sigma_{S,P_H} \\ &\quad +\frac{\psi\nu^*}{2}\left[\left((\mu-1)(1-\nu)^2-\mu\right)(1-\eta_1)^2+(\mu-1)\nu^*(1-\nu^*)(1-\eta_2)^2\right]\sigma_S^2 \\ &\quad +O\left(\|\xi\|^3\right)\end{aligned} \tag{65}$$

The implications of these welfare functions for optimal monetary policy are discussed in the next section.

[20]Note that use has been made of the fact that $\sigma_{P_{H,2}}^2=\sigma_{p_H}^2/(1-\psi)^2$ and $\sigma_{P_{F,2}^*}^2=\sigma_{p_F^*}^2/(1-\psi)^2$.

4 Optimal Monetary Policy and Exchange Rate Volatility

It is useful to discuss the implications of the home welfare function, as derived in equation (64), by first considering the special case where the home economy is infinitesimally small relative to the foreign economy. This has the advantage that it is possible to treat the foreign economy as exogenous with respect to events in the home economy. In particular, it is possible to treat foreign monetary policy as independent from the choice of home monetary policy.

4.1 The Small Open Economy Case

The small open economy welfare functions can be derived by taking the limit of (64) and (65) as the population of the foreign country, ω, tends to infinity. This implies $n \to 0$, $\nu^* \to 0$ and $\nu \to \gamma$. The welfare functions therefore become

$$\begin{aligned}\Omega_D \;=\; & -\frac{\psi\mu(1-\gamma)}{2(1-\psi)^2}\sigma^2_{P_H} - \frac{\psi\mu\gamma}{2(1-\psi)^2}\sigma^2_{P_F^*} - \frac{\psi\mu\gamma(1-\eta_2)}{(1-\psi)}\sigma_{S,P_F^*} \\ & +\frac{\psi\gamma}{2}\left[-(1-\eta_2)^2 + (\mu-1)\gamma(1-\gamma)(1-\eta_1)^2\right]\sigma^2_S \\ & +O\left(\|\xi\|^3\right) \qquad (66)\end{aligned}$$

for the home economy and

$$\Omega_D^* = -\frac{\psi\mu}{2(1-\psi)^2}\sigma^2_{P_F^*} + O\left(\|\xi\|^3\right) \qquad (67)$$

for the foreign economy.

It is immediately apparent from (67) that the optimal monetary policy for the foreign country is completely to stabilise the price of foreign produced goods. The initial discussion of home welfare and monetary policy will be based on the assumption that the foreign monetary authority follows this monetary rule. Thus $\sigma^2_{P_F^*} = 0$ and $\sigma_{S,P_F^*} = 0$. The implications of relaxing this assumption will be considered later.

The first and most obvious result that emerges from (66) is that, when there is complete pass-through (i.e. $\eta_1 = \eta_2 = 1$), home welfare depends only on the variance of $\hat{P}_H$.[21] Thus the optimal policy rule for the home economy will stabilise $\hat{P}_H$. When there is complete pass-through $\hat{P}_H$ is equivalent to the producer price index. This result is in line with the result obtained by other authors for open economies.[22] Furthermore when $\gamma = 0$ (implying the home economy is completely closed) $\hat{P}_H$ is equivalent to the CPI so the optimal policy involves complete stabilisation of the

[21] When there is complete pass-through the equation for fixed-price output becomes log-linear so the solution becomes exact and the residual terms in all the above equations become zero.

[22] See Aoki (2001), Benigno and Benigno (2001a) and Clarida, Gali and Gertler (2001a).

CPI. This result corresponds to the result emphasised by other authors for closed economies.[23]

The reason why price stabilisation is optimal can be explained with reference to the risk premium in the prices of fixed-price agents. When $\eta_1 = \eta_2 = 1$ notice that $\lambda_1 = \lambda_2$, $\lambda_3 = \lambda_4$ and $\lambda_5 = \lambda_6 = 0$. The risk premium in the prices of fixed-price agents is generated by the volatility of work effort of fixed-price agents. This depends on the volatility of relative prices between fixed-price and flexible-price goods. *Ex post* the relative price depends entirely on the price of flexible-price goods. If the price of flexible-price goods is completely stabilised then all risk is removed from fixed-price agents so the risk premium is minimised.[24] This maximises welfare.

Now consider the effects of incomplete pass-through. It is useful to distinguish between the effects of η_1 and η_2. First consider the effects of $\eta_2 < 1$. It is clear from (66) that when $\eta_2 < 1$ home welfare depends negatively on the volatility of the exchange rate. The size of this effect depends on the openness of the economy and the degree of pass-through. Thus when $\eta_2 < 1$ it is no longer optimal to completely stabilise $\hat{P}_H$. Instead it is optimal to trade-off some volatility in $\hat{P}_H$ in order to reduce the volatility of the exchange rate.

The reason why exchange rate stabilisation becomes welfare improving with incomplete pass-through in import prices can be explained with reference to the risk premium in the prices of foreign fixed-price agents (i.e. the risk premium in import prices). Notice that the only effect of $\eta_2 < 1$ is to make $\lambda_3, \lambda_4 > 0$. The smaller is η_2 the more the exchange rate affects the home demand for foreign goods. Thus a reduction in exchange rate volatility reduces the risk premium in import prices and therefore increases home welfare.[25] This effect is larger when the share of foreign goods is large in the consumption basket of home agents (i.e. when γ is close to unity). Notice that in a completely open economy with $\eta_2 < 1$ a fixed nominal exchange rate is optimal.

Now consider the effects of $\eta_1 < 1$. Notice that if $\mu = 1$, η_1 has no effect on the home welfare function.[26] It is clear from (66) that when $\eta_2 = 1$, $\mu > 1$ and $\eta_1 < 1$ exchange rate volatility has a positive effect on welfare. Again it is no longer optimal simply to stabilise $\hat{P}_H$. But in this case it becomes optimal to increase (rather than reduce) exchange rate volatility.

[23] See Aoki (2001), Goodfriend and King (2001), King and Wolman (1999) and Woodford (2001).

[24] In terms of the Rotemberg and Woodford (1999) and Woodford (2001) interpretation of the welfare effects of monetary policy, minimising the variance of prices reduces the volatility of relative output levels between flexible-price and fixed-price producers. It is clear from the form in which the risk premia are expressed that the risk premia λ_1 and λ_2 are minimised exactly when relative output levels are stabilised.

[25] This is the result emphasised by Corsetti and Pesenti (2001b).

[26] Other authors who have considered incomplete pass-through in related models have assumed $\mu = 1$ (see for instance Devereux and Engel (1998, 2000), Devereux, Engel and Tille (1999) and Corsetti and Pesenti (2001b)). Notice that with $\mu = 1$ the equation for fixed-price output becomes irrelevant so the welfare function ceases to be an approximation (i.e. the residual term in the welfare function becomes zero).

The positive welfare effect of exchange rate volatility arises because of the risk premium in home prices for sales to foreign agents (i.e. the risk premium in export prices). When $\eta_1 < 1$ higher exchange rate volatility raises λ_2 and this makes a positive contribution to home welfare. λ_2 makes a positive contribution to welfare because higher prices for exports reduces foreign demand and this allows home agents to consume more home goods for a given level of work effort. Notice that the effect of $\eta_1 < 1$ on welfare is zero when the economy is completely closed ($\gamma = 0$) and when it is completely open ($\gamma = 1$). Obviously when the economy is completely closed the degree of pass-through is irrelevant. When the economy is completely open, home agents do not consume home goods so there is no welfare benefit from reducing foreign demand for home goods.

Some of the properties of the optimal volatilities of the exchange rate and home producer prices are illustrated in Figures 1 and 2. For the purposes of illustration the two pass-through parameters, η_1 and η_2, are assumed to be equal and are varied between 0 and 1. Baseline parameter values are $n = 0$ (i.e. the home country is a small open economy), $\gamma = 0.4$, $\mu = 2$, $\psi = 0.5$ and $\sigma_K^2 = \sigma_{K^*}^2 = 1$. The foreign economy is assumed to follow a monetary policy which fully stabilises foreign producer prices.[27]

Notice from Figure 1 that the optimal volatility of home prices is zero when there is full pass-through. The solid line in Figure 1 shows that the optimal volatility of home producer prices rises as the degree of pass-through is reduced below unity, while the solid line in Figure 2 shows that the optimal volatility of the exchange rate declines.

Figures 1 and 2 also illustrate the effects of varying the openness of the economy (as measured by the parameter γ). The dashed lines show the case where $\gamma = 0.8$, which represents a much more open economy than the baseline case (where $\gamma = 0.4$). The dashed line in Figure 1 shows that, as in the baseline case, the optimal volatility of home producer prices is zero when there is full pass-through and rises as the degree of pass-through falls below unity. But notice that in the $\gamma = 0.8$ case the optimal volatility of home producer prices rises much more than in the baseline case. Likewise, the dashed line in Figure 2 shows that, as in the baseline case, the optimal volatility of the exchange rate declines as the degree of pass-through falls below unity. But the decline in the optimal volatility of the exchange rate is much more than in the baseline case. In the $\gamma = 0.8$ case, the optimal volatility of the exchange rate falls by approximately half as the degree of pass-through is reduced from unity to zero. This illustrates the much greater welfare impact of exchange rate volatility in more open economies.

Figures 3 and 4 show the effects of varying the elasticity of labour supply. The

[27] The optimal volatilities of prices and the exchange rate are derived by assuming the home monetary authority follows a rule of the form $\hat{M} = \delta_K \hat{K} + \delta_{K^*} \hat{K}^*$where δ_K and δ_{K^*} are feedback coefficients which are chosen *ex ante* by the monetary authority to maximise welfare. Attention is restricted to *ex ante* optimal monetary rules and it is assumed that the monetary authority is able to precommit to the optimal rule.

solid lines are the baseline case where $\mu = 2$ while the dashed lines are the case where $\mu = 10$ (i.e. where labour supply is less elastic). The dashed lines show that, when labour supply is inelastic, the optimal volatility of the exchange rate rises slightly as the degree of pass-through is reduced. The contrast between the $\mu = 2$ and $\mu = 10$ cases arises from the contrasting welfare effects of the risk premia in import and export prices. As explained above, when there is incomplete pass-through, exchange rate volatility increases both import and export prices. The increase in import prices reduces welfare but the increase in export prices increases welfare. The relative importance of these two effects depends on the elasticity of labour supply. When labour supply is very inelastic ($\mu = 10$) the export price effect dominates and it becomes optimal to increase the volatility of the exchange rate.[28]

4.2 Foreign Price Shocks

The analysis so far has been based on the assumption that the foreign monetary authority completely stabilises foreign producer prices. This would be the optimal policy for the foreign economy. It is interesting, however, to consider the case where the foreign monetary authority follows a non-optimal policy. This might, for instance, be a passive monetary rule which fixes the foreign money stock. Alternatively it may be an arbitrary rule which creates shocks in the foreign money supply. In either case the net result will be some volatility in foreign producer prices. The home welfare function (66) shows that any variance in foreign producer prices will have a negative effect on the welfare of home agents. Although there is nothing the home monetary authority can do to affect the volatility of foreign producer prices, it is apparent from (66) that, when there is imperfect pass-through in import prices, the home monetary authority can partly offset the negative welfare impact by creating a negative correlation between the nominal exchange rate and foreign producer prices. In other words the home monetary authority can use the exchange rate as a 'shock absorber' which partly insulates home welfare from foreign price shocks. Notice that this necessarily involves creating some volatility in the exchange rate. This will offset the incentive to stabilise the exchange rate discussed in the previous sub-section.

Figures 5 and 6 illustrate the implications of volatility of foreign producer prices. In the baseline case (illustrated with the solid lines) foreign producer prices are assumed to be fully stabilised by the foreign monetary authority. The dashed lines show the case where foreign monetary policy shocks generate shocks in foreign producer prices such that $\sigma_{P_F^*} = 0.5$. In this case, as the degree of pass-through falls, the optimal volatility of the exchange rate rises. The effect is quantitatively quite large. When the degree of pass-through is zero the optimal volatility is approximately 25%

[28] This incentive to create exchange rate volatility arises because of the incentive to reduce the volume of exports. As pointed out in footnote 17, this effect is mainly a consequence of the unit elasticity of demand for exports. In a more general model, where the elasticity of demand is greater than unity, it is likely that the incentive to reduce export volumes would be much less important.

higher than when there is full pass-through.

4.3 The Large Economy Case

Now consider the implications of moving away from the small open economy assumption. For the purposes of this analysis it is useful to revert to the assumption that the foreign monetary authority completely stabilises foreign producer prices. Strictly this would not be optimal for the foreign economy because the home economy would now be large enough to have spillover effects which would influence the foreign economy. Some of the implications of these spillover effects will be discussed in the next section.

The effects of increasing the size of the home country are very easy to determine from (64). First note that all the results so far discussed continue to hold for a large economy (i.e. when $n > 0$). Thus, exchange rate volatility has no impact on home welfare when there is complete pass-through. And it may have a negative or positive impact on welfare when there is incomplete pass-through depending on the openness of the economy, the elasticity of labour supply and the volatility of foreign producer prices.

But notice from (64) that, for given values of other parameters, increasing n increases the coefficient on the home price variance in the welfare function and reduces the coefficient on the exchange rate variance. Thus, other things being equal, a large economy should place less weight on exchange rate volatility and more weight on home price volatility in policy decisions. In other words a large economy should be more inward looking.

Figures 7 and 8 illustrate the effects of varying the size of the home economy. The solid lines are the baseline case where $n = 0$ while the dashed lines are the case where $n = 0.5$. As in previous cases, the optimal volatility of home prices rises as the degree of pass-through falls and the optimal volatility of the exchange rate falls. But these effects are less pronounced when the home economy is large.

5 Policy Coordination and Delegation

As pointed out above, other than in the small open economy case, there are spillover effects of home country monetary policy onto welfare in the foreign economy. It is therefore not plausible to assume that foreign monetary policy is exogenous to home country policy choices. In such circumstances there are potential gains to international policy coordination. In a model with perfect pass-through (and utility which is logarithmic in consumption) Obstfeld and Rogoff (2002) show that in fact such gains do not exist. They show that both non-cooperative policy making (represented by a Nash equilibrium in monetary policy rules) and optimal coordinated policymaking imply the same monetary policy rules.[29] Corsetti and Pesenti (2001b), on the other hand, show that, when there is less than perfect pass-through, there can indeed be welfare gains to monetary policy coordination.[30] A similar result holds in the model of this paper.

Rather than demonstrating this result, this section briefly considers how the coordinated policy outcome in this model can be supported by delegating monetary policy to independent monetary authorities in each county.[31] The coordinated policy is the choice of monetary rules which maximises aggregate world welfare, i.e. the choice of feedback parameters, δ_K, δ_{K^*}, δ_K^* and $\delta_{K^*}^*$, in monetary rules of the form

$$\hat{M} = \delta_K \hat{K} + \delta_{K^*} \hat{K}^* \tag{68}$$

$$\hat{M}^* = \delta_K^* \hat{K} + \delta_{K^*}^* \hat{K}^* \tag{69}$$

to maximise

$$\Omega_W = n\Omega_D + (1-n)\Omega_D^* \tag{70}$$

It is possible to show that the coordinated policy rules will be chosen if monetary policy is delegated to independent central banks in the home and foreign country where these central banks are required to minimise the following loss functions (respectively for the home and foreign central banks)

$$L = \sigma_{P_H}^2 + (1-\psi)^2 \nu\omega\sigma_S^2 \tag{71}$$

$$L^* = \sigma_{P_F^*}^2 + (1-\psi)^2 \nu^*\omega^*\sigma_S^2 \tag{72}$$

where

$$\omega = \frac{\left[(1-\nu+\mu\nu)(1-\eta_1) - \mu + \mu\nu^*(1-\eta_2)\right](1-\eta_1) + (1-\nu^*)(1-\eta_2)^2}{\mu\left[1-(1-\eta_1)\nu - (1-\eta_2)\nu^*\right]}$$

$$\omega^* = \frac{\left[(1-\nu^*+\mu\nu^*)(1-\eta_2) - \mu + \mu\nu(1-\eta_1)\right](1-\eta_2) + (1-\nu)(1-\eta_1)^2}{\mu\left[1-(1-\eta_1)\nu - (1-\eta_2)\nu^*\right]}$$

Thus the loss function for the home central bank is a weighted sum of the variance of home producer prices and the variance of the nominal exchange rate and the loss function for the foreign central bank is the weighted sum of the variance of foreign producer prices and the variance of the nominal exchange rate.

[29] Benigno and Benigno (2001a) show that the absence of gains from coordination in the Obstfeld and Rogoff (2002) model arises from the assumption of a unit elasticity of substitution between home and foreign goods. Sutherland (2001b) shows that the gains from coordination can be quite large when the elasticity of substitution differs from unity.

[30] More precisely, they show that there are gains from coordination when there is an intermediate degree of pass-through, but there are no gains from coordination when there is full pass-through or zero pass-through.

[31] That is, monetary authorities which are independent from the political authorities in each country and independent from each other.

Notice that the weight on the exchange rate is zero in each loss function when there is complete pass-through. Producer price targeting is therefore the optimal coordinated equilibrium when there is complete pass-through.[32] But when there is incomplete pass-through the weight on the exchange rate is non-zero. So, when there is incomplete pass-through, coordinated policy makers should deviate from producer price targeting in order to give some weight to the exchange rate variance.

Benigno and Benigno (2001b) have shown that, in the presence of 'cost-push' shocks, optimal coordinated policy can be supported by 'flexible inflation targeting'. In the examples studied by these authors the coordinated policy regime assigns each monetary authority a loss function which depends on the variance of producer price inflation and the variance of the 'output gap' (i.e. the deviation of real output from its flexible price level). The loss functions given by (71) and (72) can be thought of as an alternative form of flexible inflation targeting where the monetary authority allows deviations from strict inflation targeting in order to pursue some objective for the nominal exchange rate.[33]

Figure 9 plots the value of $(1-\psi)^2\nu\omega$ (i.e. the weight on the exchange rate variance in the home loss function) for two alternative parameter sets. The solid lines illustrate the case where $n = 0.5$, $\gamma = 0.4$, $\mu = 2$, $\psi = 0.5$ and $\sigma^2_K = \sigma^2_{K^*} = 1$. The dashed lines show the case where $\gamma = 0.8$. These plots show that the weight on the exchange rate can be large and positive when the degree of pass-through is low and the economy is very open. In these cases the exchange rate variance would be an important term in the loss function and coordinated policy would imply a significant degree of nominal exchange rate stabilisation and a significant deviation from strict producer price targeting.

6 Concluding Comments

The central feature of the model presented in this paper is that welfare can be written in terms of a weighted sum of the second moments of home and foreign producer prices and the nominal exchange rate. It is shown that the weight on the second moments of the exchange rate depends on the degree of pass-through and the size and openness of the economy and the elasticity of labour supply.

[32] Producer price targeting would also be the Nash equilibrium in this model when there is complete pass-through. Note, however, that this is only true because the elasticity of substitution between home and foreign goods is assumed to be unity. Sutherland (2002b) shows that, when there is a non-unit elasticity of substitution, a Nash equilibrium in delegated monetary regimes (where each country chooses a loss function for its central bank to maximise individual country welfare) would result in a non-zero weight on the exchange rate variance, even when there is full pass-through.

[33] A more general model, where there is both imperfect pass-through and cost-push shocks, would presumably imply a cooperative policy regime which is supported by loss functions which include the variance of producer prices, the variance of the nominal exchange rate and the variance of the output gap.

When there is complete pass-through the weight on the second moments of the exchange rate is zero. In this case the optimal monetary policy for the home country completely stabilises home producer prices. In a closed economy the producer price index is equivalent to the consumer price index, so consumer price targeting becomes optimal in this case.

When there is incomplete pass-through welfare depends on the variance of the exchange rate. There is a complicated (but fully characterised) relationship between the degree of pass-through, the degree of openness, the elasticity of labour supply and the volatility of foreign producer prices and the effect of exchange rate volatility on welfare. When labour supply is elastic and foreign prices are stable a reduction in the volatility of the exchange rate is unambiguously welfare improving. The size of the welfare effect depends on the degree of pass-through and the size and openness of the home economy. In contrast to this, when labour supply is relatively inelastic and/or foreign producer prices are not stable, it is found that increasing exchange rate variability can be welfare improving. Again, the size of this welfare effect depends on the degree of pass-through and the size and openness of the home economy.

These results are obtained by considering the welfare and the policy problem of the home economy while assuming the foreign economy follows a fixed monetary rule. The analysis is extended to consider some of the implications of incomplete pass-through for international policy coordination. It is shown that the coordinated outcome can be supported by requiring national central banks to minimise loss functions which are a weighted sum of the variances of producer prices and the nominal exchange rate.

It must be emphasised that the model analysed in this paper includes only a limited range of shocks, namely shocks to the disutility of labour in the home and foreign countries and shocks to foreign monetary policy. It is apparent from the results reported that the way in which the variance of the exchange rate enters the welfare function differs depending on the sources of shocks. An extension of the analysis of this paper to alternative sources of shocks will therefore be an interesting topic for further research.

It is also apparent from the analysis of this paper that the welfare maximising monetary strategy becomes more complex as more realistic aspects are added to the basic model. It quickly becomes obvious that the optimality of a simple strategy of strict consumer or producer price targeting does not carry over to more general cases.[34] In addition, even when the optimal monetary strategy can be summarised by a relatively simple loss function, it becomes doubtful that the fully optimal monetary policy can in practice be implemented. The fully optimal policy may involve responding to unobservable or unmeasurable variables or require a complex balance between different targets where the optimal weights to be placed on different targets are unmeasurable or uncertain. It would therefore be interesting to use the

[34] This becomes more obvious if such issues as cost-push shocks and the expenditure switching effect are considered (see Sutherland (2002a, 2002b).

model developed in this paper to analyse the welfare performance of non-optimal but simple targeting rules. This may also be a productive topic for further research.

Appendix

Optimal Price Setting: The price setting problem facing flexible-price producer z is the following:

$$MaxU(z) = \log C(z) + \log\left(\frac{M}{P}\right) - \frac{K}{\mu} y_2^{\mu}(z) \tag{73}$$

subject to

$$PC(z) = p_{H,2}(z)\, y_2(z) + M_0 - M - T \tag{74}$$

$$p_{H,2}(z) = Sp^*_{H,2}(z) \tag{75}$$

$$P_{H,2} = SP^*_{H,2} \tag{76}$$

$$y_2(z) = y_{H,2}(z) + y^*_{H,2}(z) \tag{77}$$

$$y_{H,2}(z) = c_{H,2}(z) = \frac{1}{1-\psi} C_{H,2}\left(\frac{p_{H,2}(z)}{P_{H,2}}\right)^{-\phi} \tag{78}$$

$$y^*_{H,2}(z) = \omega c^*_{H,2}(z) = \frac{\omega}{1-\psi} C^*_{H,2}\left(\frac{p^*_{H,2}(z)}{P^*_{H,2}}\right)^{-\phi} \tag{79}$$

The first order condition with respect to $p_{H,2}(z)$ is

$$\frac{y_2(z)}{PC(z)} - \phi\left[\frac{p_{H,2}(z)}{PC(z)} - Ky_2^{\mu-1}(z)\right]\frac{y_2(z)}{p_{H,2}(z)} = 0 \tag{80}$$

In equilibrium all flexible-price agents choose the same price and consumption level so

$$\frac{Y_2}{PC} - \phi\left[\frac{P_{H,2}}{PC} - KY_2^{\mu-1}\right]\frac{Y_2}{P_{H,2}} = 0 \tag{81}$$

where

$$Y_2 = \frac{1}{1-\psi}\left(C_{H,2} + \omega C^*_{H,2}\right) \tag{82}$$

Rearranging yields the expression in the main text.

The price setting problem facing fixed-price producer z is the following:

$$MaxU(z) = E\left\{\log C(z) + \log\left(\frac{M}{P}\right) - \frac{K}{\mu} y_1^{\mu}(z)\right\} \tag{83}$$

subject to

$$PC(z) = p_{H,1}(z)\, y_{H,1}(z) + Sp^*_{H,1}(z)\, y^*_{H,1}(z) + M_0 - M - T \tag{84}$$

$$y_1(z) = y_{H,1}(z) + y^*_{H,1}(z) \tag{85}$$

$$y_{H,1}(z) = c_{H,1}(z) = \frac{1}{\psi} C_{H,1} \left(\frac{p_{H,1}(z)}{P_{H,1}} \right)^{-\phi} \tag{86}$$

$$y^*_{H,1}(z) = \omega c^*_{H,1}(z) = \frac{\omega}{\psi} C^*_{H,1} \left(\frac{p^*_{H,1}(z)}{P^*_{H,1}} \right)^{-\phi} \tag{87}$$

$$p^*_{H,1}(z) = \frac{\breve{p}_{H,1}(z)}{S} \left(\frac{S}{S_E} \right)^{1-\eta_1} \tag{88}$$

The first order condition with respect to $p_{H,1}(z)$ is

$$E \left\{ \frac{y_{H,1}(z)}{PC(z)} - \phi \left[\frac{p_{H,1}(z)}{PC(z)} - K y_1^{\mu-1}(z) \right] \frac{y_{H,1}(z)}{p_{H,1}(z)} \right\} = 0 \tag{89}$$

In equilibrium all flexible-price agents choose the same price and consumption level so

$$E \left\{ \frac{C_{H,1}}{PC} - \phi \left[\frac{P_{H,1}}{PC} - K Y_1^{\mu-1} \right] \frac{C_{H,1}}{P_{H,1}} \right\} = 0 \tag{90}$$

where

$$Y_1 = \frac{1}{\psi} \left(C_{H,1} + \omega C^*_{H,1} \right) \tag{91}$$

Rearranging yields the expression given in the main text.

The first order condition with respect to $\breve{p}_{H,1}(z)$ implies

$$E \left\{ \frac{y^*_{H,1}(z) S^{1-\eta_1}}{PC(z) S_E^{1-\eta_1}} - \phi \left[\frac{p^*_{H,1}(z) S}{PC(z)} - K y_1^{\mu-1}(z) \right] \frac{y^*_{H,1}(z)}{p^*_{H,1}(z) S^{\eta_1} S_E^{1-\eta_1}} \right\} = 0 \tag{92}$$

In equilibrium all flexible-price agents choose the same price and consumption level so

$$E \left\{ \frac{C^*_{H,1} S^{1-\eta_1}}{PC S_E^{1-\eta_1}} - \phi \left[\frac{P^*_{H,1} S}{PC} - K Y_1^{\mu-1} \right] \frac{C^*_{H,1}}{P^*_{H,1} S^{\eta_1} S_E^{1-\eta_1}} \right\} = 0 \tag{93}$$

Rearranging yields the expression given in the main text.

Simplifying the Welfare Function: The price setting conditions for fixed-price producers can be rewritten as follows

$$E \left[\frac{P_{H,1} C_{H,1}}{PC} \right] = \frac{\phi}{\phi - 1} E \left[K Y_1^{\mu-1} C_{H,1} \right] \tag{94a}$$

$$E \left[\frac{S P^*_{H,1} C^*_{H,1}}{PC} \right] = \frac{\phi}{\phi - 1} E \left[K Y_1^{\mu-1} C^*_{H,1} \right] \tag{95}$$

so

$$E\left[\frac{P_{H,1}C_{H,1}+\omega SP^*_{H,1}C^*_{H,1}}{PC}\right]=\frac{\phi}{\phi-1}\psi E\left[KY_1^{\mu}\right] \tag{96}$$

The price setting condition for flexible-price producers is as follows

$$\frac{P_{H,2}}{PC}=\frac{\phi}{\phi-1}KY_2^{\mu-1} \tag{97}$$

and it follows that

$$E\left[\frac{P_{H,2}C_{H,2}}{PC}\right]=\frac{\phi}{\phi-1}E\left[KY_2^{\mu-1}C_{H,2}\right] \tag{98}$$

$$E\left[\frac{SP^*_{H,2}C^*_{H,2}}{PC}\right]=\frac{\phi}{\phi-1}E\left[KY_2^{\mu-1}C^*_{H,2}\right] \tag{99}$$

and

$$E\left[\frac{P_{H,2}C_{H,2}+\omega SP^*_{H,2}C^*_{H,2}}{PC}\right]=\frac{\phi}{\phi-1}\left(1-\psi\right)E\left[KY_2^{\mu}\right] \tag{100}$$

The budget constraint for individual agents combined with the government budget constraint implies

$$P_{H,1}C_{H,1}+\omega SP^*_{H,1}C^*_{H,1}=\psi PC \tag{101}$$

for fixed-price producers and

$$P_{H,2}C_{H,2}+\omega SP^*_{H,2}C^*_{H,2}=(1-\psi)PC \tag{102}$$

for flexible-price producers. Using the above equations the following are obtained

$$E\left[KY_1^{\mu}\right]=\frac{\phi-1}{\phi},\quad E\left[KY_2^{\mu}\right]=\frac{\phi-1}{\phi} \tag{103}$$

These relationships are used in the main text to simply the welfare measure.

Approximating fixed-price Output: The output of fixed-price agents is given by

$$Y_1=\frac{1}{\psi}\left(C_{H,1}+\omega C^*_{H,1}\right) \tag{104}$$

In a symmetric deterministic equilibrium

$$\frac{1}{\psi}\frac{\bar{C}_{H,1}}{\bar{Y}_1}=1-\nu \text{ and } \frac{1}{\psi}\frac{\omega\bar{C}^*_{H,1}}{\bar{Y}_1}=\nu \tag{105}$$

Taking a second-order approximation of the left and right hand sides of (104) yields

$$Y_1-\bar{Y}=\frac{1}{\psi}\left(C_{H,1}-\bar{C}_{H,1}\right)+\frac{1}{\psi}\left(C^*_{H,1}-\bar{C}^*_{H,1}\right)+O\left(\|\xi\|^3\right) \tag{106}$$

where $O\left(\|\xi\|^3\right)$ indicates a residual which includes terms of order three and above in deviations from the non-stochastic steady state. To a second-order approximation $X-\bar{X}=\bar{X}(\hat{X}+\hat{X}^2/2)+O\left(\|\xi\|^3\right)$ so

$$\begin{aligned}\hat{Y}_1 &= (1-\nu)\left(\hat{C}_{H,1}+\frac{1}{2}\hat{C}_{H,1}^2\right) \\ &\quad +\nu\left(\hat{C}_{H,1}^*+\frac{1}{2}\hat{C}_{H,1}^{*2}\right)-\frac{1}{2}\hat{Y}_1^2+O\left(\|\xi\|^3\right) \qquad (107)\end{aligned}$$

Squaring the right hand side of this expression and deleting terms of order higher than two yields

$$\hat{Y}_1^2=(1-\nu)^2\hat{C}_{H,1}^2+\nu^2\hat{C}_{H,1}^{*2}+2(1-\nu)\nu\hat{C}_{H,1}\hat{C}_{H,1}^*+O\left(\|\xi\|^3\right) \qquad (108)$$

substituting this back into (107) yields

$$\begin{aligned}\hat{Y}_1 &= (1-\nu)\hat{C}_{H,1}+\nu\hat{C}_{H,1}^* \\ &\quad +\frac{1}{2}(1-\nu)\nu\left[\hat{C}_{H,1}^2+\hat{C}_{H,1}^{*2}-2\hat{C}_{H,1}\hat{C}_{H,1}^*\right]+O\left(\|\xi\|^3\right) \qquad (109)\end{aligned}$$

Note that for any variable X it is possible to write

$$\hat{X}=(\log X-E\left[\log X\right])+\left(E\left[\log X\right]-\log\bar{X}\right)$$

or

$$\hat{X}=(\log X-E\left[\log X\right])+E\left[\hat{X}\right]$$

so

$$(\log X-E\left[\log X\right])=\hat{X}-E\left[\hat{X}\right]$$

Furthermore note that $E\left[\hat{X}\right]$ is of second-order magnitude (because it depends only on second-moment terms). So, to a second-order approximation

$$\sigma_X^2=E\left[(\log X-E\left[\log X\right])^2\right]=E\left[\hat{X}^2\right]+O\left(\|\xi\|^3\right)$$

Taking expectations of (109) therefore yields

$$\begin{aligned}E\left[\hat{Y}_1\right] &= (1-\nu)E\left[\hat{C}_{H,1}\right]+\nu E\left[\hat{C}_{H,1}^*\right] \\ &\quad +\frac{1}{2}(1-\nu)\nu\left[\sigma_{C_{H,1}}^2+\sigma_{C_{H,1}^*}^2-2\sigma_{C_{H,1}C_{H,1}^*}\right]+O\left(\|\xi\|^3\right) \qquad (110)\end{aligned}$$

which is the approximation used in the text.

A similar procedure can be used to derive a second-order approximation for foreign fixed-price output.

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Figure 1: Producerprices

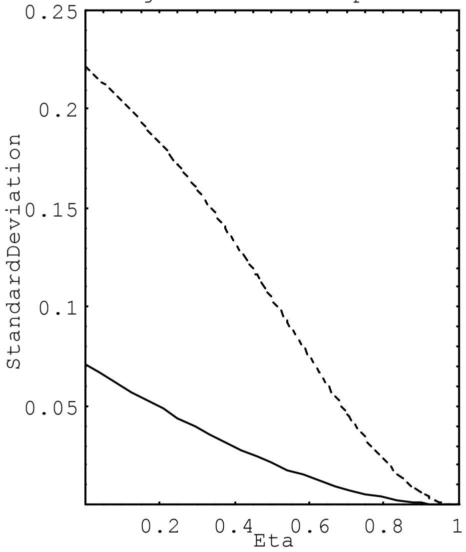
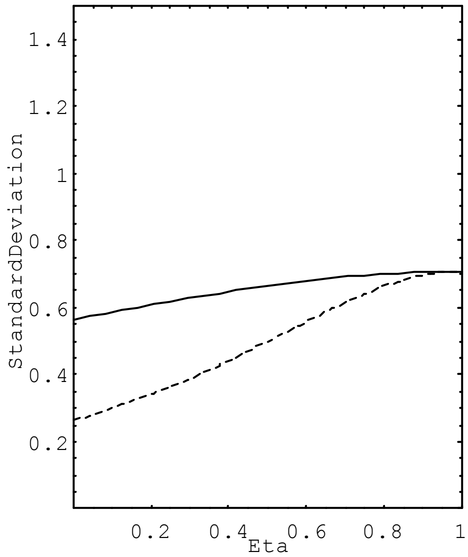


Figure 2: Exchangerate



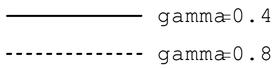


Figure 3: Producerprices

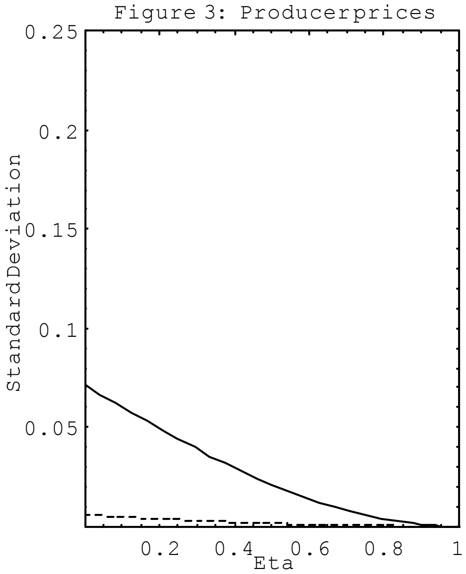
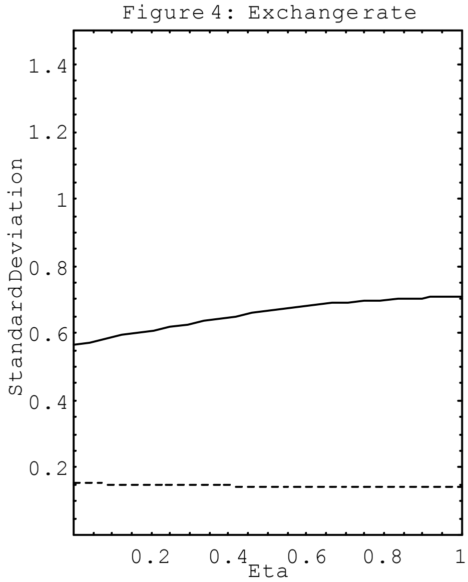


Figure 4: Exchangerate



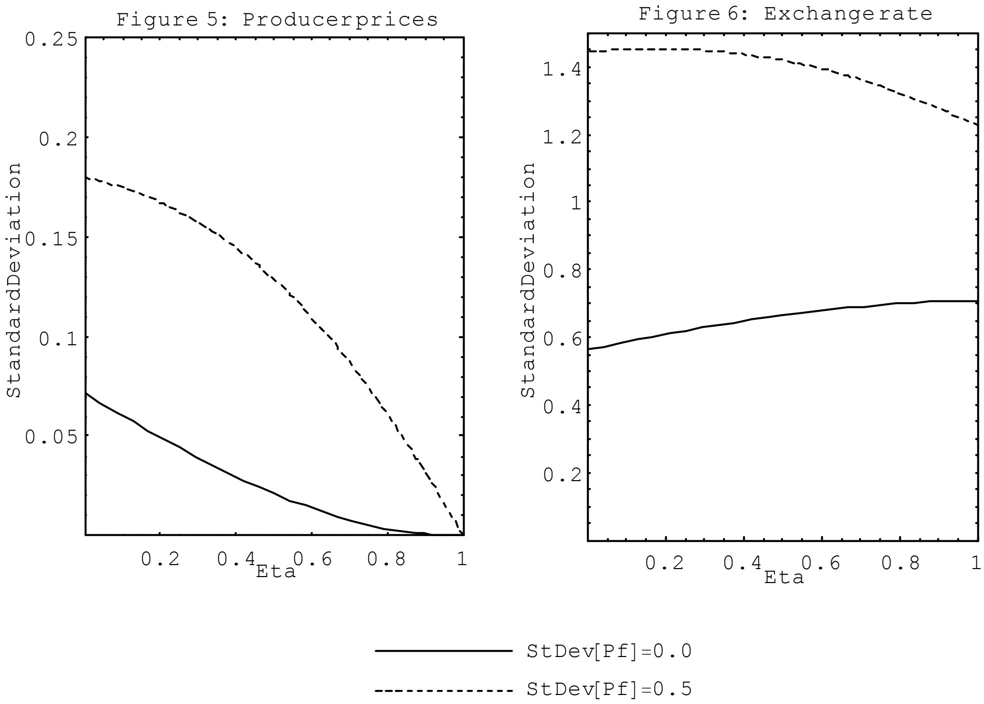


Figure 5: Producerprices
StandardDeviation
Eta
0.25
0.2
0.15
0.1
0.05
0.2
0.4
0.6
0.8
1
Figure 6: Exchangerate
StandardDeviation
Eta
1.4
1.2
1
0.8
0.6
0.4
0.2
StDev[Pf]=0.0
StDev[Pf]=0.5

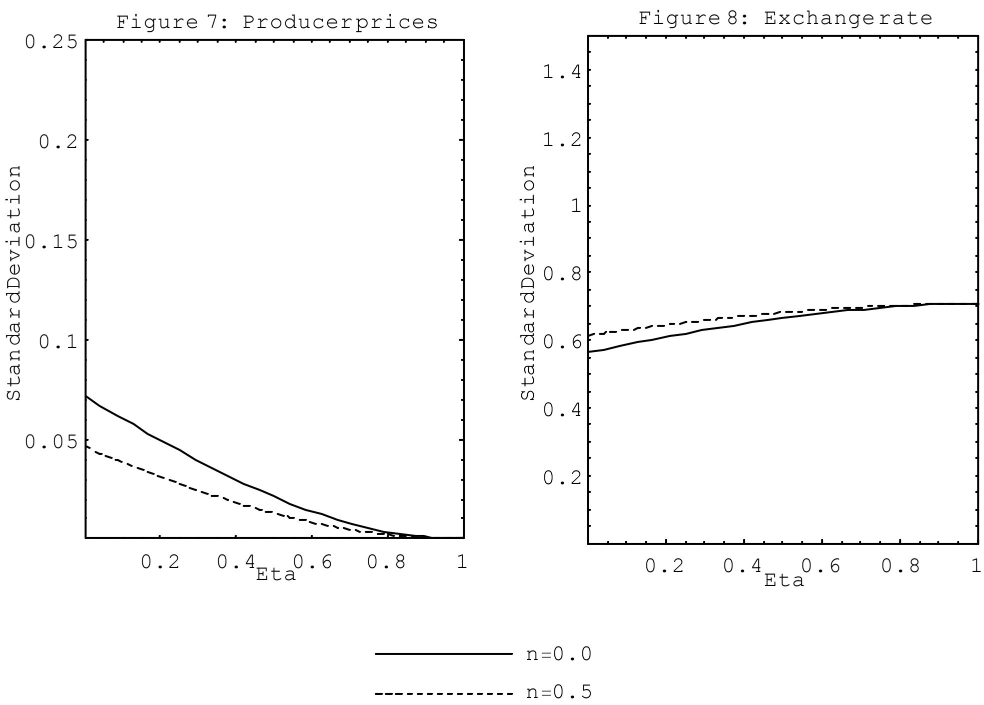


Figure 7: Producerprices
StandardDeviation
Eta
0.25
0.2
0.15
0.1
0.05
0.2
0.4
0.6
0.8
1
Figure 8: Exchangerate
StandardDeviation
Eta
1.4
1.2
1
0.8
0.6
0.4
0.2
n=0.0
n=0.5

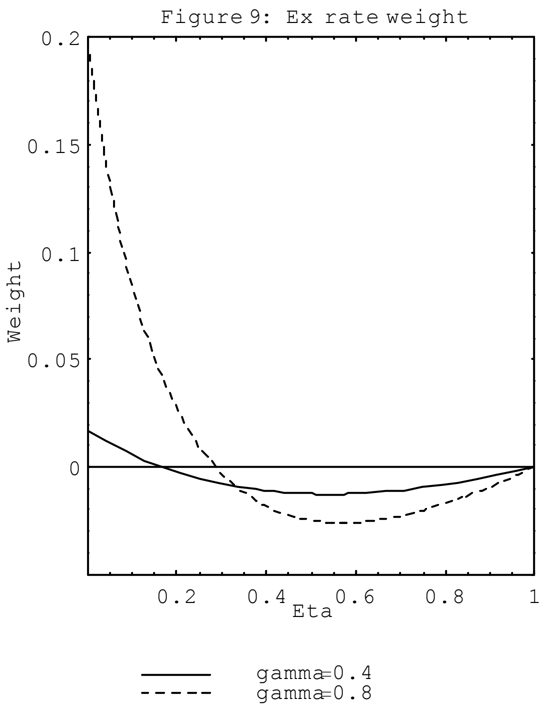


Figure 9: Ex rate weight
0.2
0.15
0.1
0.05
0
Weight
0.2
0.4
0.6
0.8
1
Eta
gamma=0.4
gamma=0.8