Inflation Targeting in a Small Open Economy

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December 2000

Abstract

A small open economy model is presented which allows explicit treatment of uncertainty and its effects on macroeconomic behaviour. Inflation targeting is compared to the welfare maximising monetary rule and to a fixed nominal exchange rate. It is found that flexible inflation targeting produces too little exchange rate volatility compared to the optimal rule but delivers higher welfare than a fixed nominal exchange rate. Strict inflation targeting also delivers higher welfare than a fixed rate. In addition it is found that the welfare maximising monetary rule can be replicated if the central bank's objective function includes the nominal exchange rate.

Keywords: inflation targeting, monetary policy, open economy
JEL Classification: E52, E58, F41

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This version: 19 December 2000

1. Introduction

In recent years "inflation targeting" has been adopted as a monetary policy strategy by many countries. This follows the apparent failure of other strategies, such as targeting of monetary aggregates or nominal exchange rates, which have proved to be insufficiently flexible to deal with major shocks. In parallel to this switch in policy making practice there has been a rapidly growing academic literature on inflation targeting.[1] An important feature of this literature (which is always a feature of new developments in monetary economics) is that it largely focuses on closed economies. Yet the experience of inflation targeting in a number of countries shows that open-economy issues are a major factor. In particular, for small open economies, the behaviour of the exchange rate has proved to be controversial. This paper aims to investigate the implications of inflation targeting for a small open economy. The stabilisation properties and the welfare implications of different forms of inflation targeting will be analysed and compared to a fixed nominal exchange rate. An important issue will be the role of the exchange rate in an inflation targeting regime.[2]

A proper welfare analysis of monetary policy requires a model based on consistent microeconomic foundations. The aspects of an economy that are crucial in this analysis are the links between monetary policy, the exchange rate, the response of the macroeconomy to stochastic shocks and welfare. The model developed by Obstfeld and Rogoff (1998, 2000) provides a consistent microfounded framework which addresses precisely these key issues.[3] The model contains many of the features now considered standard in the analysis of monetary policy in an open economy. The basic framework is one where goods and labour markets are imperfectly

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[1] See for instance Haldane (1995), Leiderman and Svensson (1995), Svensson (1999) and Svensson (2000).

[2] Clarida, Gali and Gertler (1999) conclude, in their survey of research on monetary policy, that examining open economy issues will be an important and fruitful line of future investigation. Recently a number of papers have started this process. See, for instance, Ball (1999), Batini and Haldane (1999), Benigno (2000), Devereux and Engel (1998, 2000), Gali and Monacelli (2000), McCallum and Nelson (1999), Smets and Wouters (2000) and Svensson (2000).

[3] Devereux and Engel (1998, 2000) have used a modified version of the Obstfeld and Rogoff framework to analyse the choice between fixed and flexible exchange rate regimes.

competitive and there is some degree of nominal stickiness. The model is particularly useful for the purposes of this paper because it allows the derivation of an explicit measure of welfare. The model developed in this paper is a small open economy version of Obstfeld and Rogoff (2000).

An important feature of the Obstfeld and Rogoff framework is that it allows an explicit and exact treatment of uncertainty. The model highlights an important link between the volatility of macroeconomic variables and the labour supply and consumption choices of agents. This link can have profound effects on the analysis of monetary regimes. Different monetary regimes produce different patterns of variances and covariances between macro-variables and can therefore induce different *ex ante* decisions about labour supply and consumption. In turn these differences can affect the relative welfare performance of regimes. Standard modelling approaches based on log-linearisation neglect these effects.[4]

The model presented below generalises Obstfeld and Rogoff (2000) in one crucial respect. Obstfeld and Rogoff assume that all wages are set one period in advance of the realisation of shocks. The degree of nominal stickiness is therefore completely fixed at the maximum possible level. In the model of this paper only a proportion of agents fix wages in advance. The remaining agents are allowed to set wages after shocks are realised. The degree of nominal stickiness is therefore determined by the proportion of agents who pre-fix wages. The implications of different degrees of nominal stickiness can therefore be investigated by varying the proportion of pre-fixed wages. This generalisation is crucial because, as will be apparent, the analysis of inflation targeting is not particularly interesting in a framework with complete nominal stickiness. It will also be apparent that the performance of any particular monetary regime can be strongly affected by the degree of nominal stickiness.

The model is simple enough to allow an explicit characterisation of optimal monetary policy. This is used as a benchmark of comparison

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[4] An important example of the alternative approach to analysing monetary policy is contained in Rotemberg and Woodford (1999). There a dynamic general equilibrium model is analysed using numerical simulations of the log-linearised equations of the model. A welfare measure is obtained using a second order approximation to the utility function of agents.

for other policy regimes. As is quite standard in these models, it turns out that the optimal policy replicates the flexible price equilibrium.

Following Svensson (1999, 2000) a regime of inflation targeting is represented by assuming that the monetary authority is required to minimise an objective function which consists of a weighted average of squared deviations of inflation from its target and squared deviations of output from its natural level. Svensson describes this as representing "flexible inflation targeting" because the monetary authority will choose to allow deviations from the inflation target in order to achieve some stabilisation of output. This can be contrasted with "strict inflation targeting" where the weight on output deviations in the objective function is zero and the monetary authority therefore chooses policy to minimise deviations from the inflation target at all times.

The solution of the model under both flexible and strict inflation targeting is obtained. It is found that in general both regimes perform less well, in terms of welfare, than the fully optimal policy. But in some special cases one or other regime is equivalent to the fully optimal policy. The welfare maximising weight to be placed on output in the central bank's objective function is derived. An interesting result is that this weight is negative for some parameter combinations (i.e. the central bank should induce more output volatility than implied by strict inflation targeting).

As has already been pointed out, in the context of an open economy an important issue is the role to be given to the exchange rate. One fear has been that inflation targeting gives rise to excessive exchange rate volatility and it has been argued that the monetary authority should be required to take account of exchange rate deviations in the setting of policy.[5] Surprisingly, however, the results from the model in this paper show that inflation targeting generally produces too little exchange rate volatility. And it is found that if the squared deviations of the exchange rate are included in the central bank's objective function the welfare maximising weight is in fact negative (i.e. the central bank should be induced to make exchange rates more volatile). It is found that including the exchange rate in the objective function in

[5] The debate about the role of the exchange rate is particularly relevant in the UK where widely differing opinions have been expressed by members of the Monetary Policy Committee. See for instance Vickers (2000) and Wadhwani (2000).

this way actually allows the fully optimal policy to be replicated for all parameter values.

As a further point of comparison the model is solved for the case of a completely fixed exchange rate. It is found that both flexible and strict inflation targeting dominate a fixed rate in welfare terms.

It must be emphasised that the model used in this paper is restricted in a number of respects. So the results just summarised need to be tested in more general frameworks before firm conclusions can be drawn. The concluding section of the paper discusses some of the restricted features of the present model and suggests lines for future research on this topic.

The paper proceeds as follows. Section 2 outlines the model. Section 3 presents the main results of the paper. Section 4 concludes.

2. The Model

In many respects the model follows Obstfeld and Rogoff (2000). The main differences are: the model in this paper is a small open economy; there is a variable degree of nominal stickiness; and the economy is subject to real demand shocks (as well as supply shocks).

Market Structure

The world consists of a small open economy (the home economy) and the rest of the world. The rest of the world is treated as exogenous. The home economy is populated by households which supply labour and consume a basket of goods consisting of all traded goods and home produced nontraded goods. There are three categories of goods in the home economy, home traded, foreign traded and nontraded. Home traded and nontraded products are produced in the home economy. Foreign traded goods are produced in the rest of the world (at exogenous foreign currency prices). Within each category of goods there is a continuum of differentiated products indexed on the unit interval. Each differentiated product is produced by a monopoly firm. Goods prices are perfectly flexible and are set by firms as a mark-up over marginal costs.

The only factor of production is labour, which is supplied by households. There are two categories of labour. Type 1 labour is supplied in a market where wages are set one period in advance.

Agents in this market are contracted to meet labour demand at the fixed wage. Type 2 labour is supplied in a market where agents are free to set wages in each period after shocks are realised and monetary policy is set. Within each category of labour there is a continuum of households which individually are monopoly suppliers of a particular variety of labour. Varieties are indexed on the unit interval. The proportion of type 1 households in the total population is denoted y, so y is a measure of the degree of wage stickiness in the economy.

Firms

Each firm is a monopoly producer of a particular variety of good. Production of each good requires all varieties of home labour. The production function for home traded good k is

$$Y_H(k) = \frac{L_{1H}^{y}(k) L_{2H}^{1-y}(k)}{y^{y}(1-y)^{1-y}} \tag{1}$$

where

$$L_{1H}(k) = \left[\int_0^1 L_{1H}(k,j)^{\frac{f-1}{f}} dj\right]^{\frac{f}{f-1}} \qquad L_{2H}(k) = \left[\int_0^1 L_{2H}(k,j)^{\frac{f-1}{f}} dj\right]^{\frac{f}{f-1}}$$

For the moment time subscripts are omitted. They will be introduced later as required. The production function for nontraded good k is

$$Y_N(k) = \frac{L_{1N}^{y}(k) L_{2N}^{1-y}(k)}{y^{y}(1-y)^{1-y}} \tag{2}$$

where

$$L_{1N}(k) = \left[\int_0^1 L_{1N}(k,j)^{\frac{f-1}{f}} dj\right]^{\frac{f}{f-1}} \qquad L_{2N}(k) = \left[\int_0^1 L_{2N}(k,j)^{\frac{f-1}{f}} dj\right]^{\frac{f}{f-1}}$$

These production functions imply a constant elasticity of substitution between different varieties of labour within a category and a unit elasticity of substitution between categories of labour. The latter assumption is important in allowing a simple solution to the model because it ensures that the share of labour income going to each category of labour is unaffected by shocks. This implies that all households have identical consumption behaviour regardless of their category.

The overall wage index corresponding to these production functions is

$$W = W_1^{y} W_2^{1-y} \tag{3}$$

where the sub-indices for each category of labour are

$$W_1 = \left[\int_0^1 W_1(j)^{1-f} \, dj \right]^{\frac{1}{1-f}} \qquad W_2 = \left[\int_0^1 W_2(j)^{1-f} \, dj \right]^{\frac{1}{1-f}}$$

It will be shown below that demand for each type of good has a constant price elasticity of q so firms set prices as a mark-up on costs as follows

$$P_N = P_H = \left(\frac{q}{q-1} \right) W \tag{4}$$

Households

There are two categories of household. Each type 1 household is the monopoly supplier of a particular variety of type 1 labour and each type 2 household is the monopoly supplier of a particular variety of type 2 labour. The proportion of type 1 households in the total population is y. All households have utility functions of the same form. The utility of household z is given by

$$U_t(z) = E_t \sum_{t=0}^{\infty} b^t \left[\ln C_{t+t} + \frac{c}{1-e} \left(\frac{M_{t+t}}{P_{t+t}} \right)^{1-e} - \frac{\mathrm{K}_t}{2} L_{t+t}^2(z) \right] \tag{5}$$

where C is a sub-utility function defined across all goods, M is holdings of nominal domestic money, P is the overall consumer price index, $L(z)$ is total labour supplied by household z, E is the expectations operator and K is a shock variable ($E[\ln \mathrm{K}] = 0$ and $Var[\ln \mathrm{K}] = s_K^2$). Aggregate labour demand for agent z is given by

$$L(z)=\int_0^1\left[L_N(z,j)+L_H(z,j)\right]dj \tag{6}$$

The sub-utility function for goods consumption is given by

$$C=\frac{C_T^{g}C_N^{1-g}}{g^{g}(1-g)^{1-g}} \tag{7}$$

where C_T and C_N are sub-utility function defined across all traded and nontraded goods respectively. In turn C_T is given by

$$C_T=\frac{C_H^{n}C_F^{1-n}}{n^{n}(1-n)^{1-n}} \tag{8}$$

where C_H and C_F are sub-utility function defined across all home traded and foreign traded goods respectively. The parameter *g* measures the share of traded goods in the consumption basket and the parameter *n* measures the share of home traded goods in the traded goods basket. In what follows the size of the home country is assumed to be very small so *n* is assumed to be very small. Thus in effect $C_T=C_F$. The parameter *g* can also be regarded as a measure of the degree of openness of the home economy. If *g*=0 then the economy is completely closed while *g*=1 implies a completely open economy.

The form of the utility function implies a unit elasticity of substitution between home and foreign tradables. This ensures that the current account is always in balance. There are therefore no changes in the net asset position of the home country following any shock.[6]

Utility from the three categories of goods is defined by the following sub-utility functions

$$C_H=\left[\int_0^1 C_H(j)^{\frac{q-1}{q}}dj\right]^{\frac{q}{q-1}}\quad C_F=\left[\int_0^1 C_F(j)^{\frac{q-1}{q}}dj\right]^{\frac{q}{q-1}}\quad C_N=\left[\int_0^1 C_N(j)^{\frac{q-1}{q}}dj\right]^{\frac{q}{q-1}}$$

[6] This assumption was first introduced into a deterministic open economy model by Corsetti and Pesenti (2000) and has proved to be a key assumption allowing a tractable solution to stochastic models of the type used in this paper.

The overall consumer price index corresponding to the utility function is

$$P = P_T^g P_N^{1-g} \tag{9}$$

where the index of traded goods prices is given by

$$P_T = P_H^n P_F^{1-n} \tag{10}$$

and the indices of home traded, foreign traded and nontraded prices are

$$P_H = \left[\int_0^1 P_H(j)^{1-q}\,dj\right]^{\frac{1}{1-q}} \quad P_F = \left[\int_0^1 P_F(j)^{1-q}\,dj\right]^{\frac{1}{1-q}} \quad P_N = \left[\int_0^1 P_N(j)^{1-q}\,dj\right]^{\frac{1}{1-q}}$$

The assumption that the home country is very small implies that effectively $P_T = P_F$.

The law of one price is assumed to hold for all traded goods so the domestic currency price of foreign traded goods is given by

$$P_F = SP_F* \tag{11}$$

where S is the nominal exchange rate of the domestic currency and P_F* is the foreign currency price of foreign traded goods. P_F* and all its component prices are assumed to be exogenous.

Agents can hold wealth in three forms: domestic currency, domestic currency bonds and foreign currency bonds.[7] The flow budget constraint of household *z* is given by

$$\begin{aligned} B_{t+1} + M_{t+1} = {} & (1+i_t)B_t + M_t + \mathrm{A}W_t(z)L_t(z) \\ & + \Pi_t - P_t C_t - T_t(z) - Q_{1,t} - Q_{2,t} \end{aligned} \qquad t = t\ldots\infty \tag{12}$$

[7] It is assumed that domestic bonds can only be held by domestic residents. Otherwise the presence of non-traded goods would give rise to conflicting arbitrage opportunities.

where B are bond holdings, Π is household z's share of the profits of domestic firms, T, Q_1, Q_2 and A are fiscal variables which are further explained below.

As already pointed out, the assumption of unit elasticity of substitution between type 1 and type 2 labour ensures that all households have identical income and consumption levels and the assumption of a unit elasticity of substitution between home traded and foreign traded goods ensures that net asset positions are unaffected by shocks.

Fiscal Policy

Government spending shocks are one of two sources of exogenous shocks in the model (the other being labour supply shocks operating through the utility function). Government spending is a basket of all consumer goods (with weights equal to the household basket) and is proportional to total household consumption.

$$G_t = (X_t - 1)C_t \tag{13}$$

where X is log normal and iid with $E[\ln X] = 0$ and $Var[\ln X] = s_X^2$. Identical shocks are assumed to hit foreign government spending

$$G_t{*} = (X_t - 1)C{*} \tag{14}$$

The assumption of multiplicative shocks is convenient for allowing a simple solution to the model but has the disadvantage of introducing a (time-varying) distortion into the economy. It turns out to be useful to offset this distortion by assuming that government spending is financed using time-varying proportional income taxation at rate $(X_t - 1)/X_t$. So for household z the following is true

$$T_t(z) = \frac{(X_t - 1)}{X_t}\left(\mathrm{A}W_t(z)L_t(z) + \Pi_t - Q_{2,t}\right) \tag{15}$$

It is also useful to offset the distortions induced by the presence of monopoly power in production and labour supply.[8] Output will be suboptimally low because of monopoly distortions. This will create a bias in policy towards expansion. This bias is partly offset by the desire of the home government to exploit the monopoly position of home produces *vis-à-vis* the rest of the world. The net bias is removed by assuming that a work subsidy of A is paid to households. A is given by the following expression

$$\mathrm{A} = \left(\frac{f}{f-1}\right)\left(\frac{q}{q-1}\right)(1-g) \tag{16}$$

The subsidy is financed through lump-sum taxes on households, denoted Q_2.

Finally it is assumed that any changes in the nominal money supply enter or leave the economy through lump-sum transfers to households, denoted Q_1. To summarise, there are three government budget constraints as follows

$$M_t - M_{t-1} = Q_{1,t} \tag{17}$$

$$Subsidy_t = Q_{2,t} \tag{18}$$

$$P_t G_t = T_t \tag{19}$$

where *Subsidy* is the total nominal cost of the production subsidy and *T* is the total revenue from proportional income taxation.

First order conditions

The first order condition for households' choice of consumption is

[8] The purpose of the present paper is to investigate optimal stabilisation policy. The presence of distortions which push the average level of output away from its welfare maximising level will tend to induce optimising policymakers to attempt to expand output. This bias is obviously a significant factor in the design of monetary policy regimes and institutions but it is not the primary focus of this paper. Hence it is useful to neutralise these biases for the purpose of the present analysis.

$$C_t^{-1} = b(1+i_t)P_t E_t\left[\frac{C_{t+1}^{-1}}{P_{t+1}}\right] \tag{20}$$

The allocation of wealth between domestic and foreign bonds implies an uncovered interest parity relationship as follows

$$(1+i_t) = (1+i^*)\frac{1}{S_t}\frac{E_t\left[\frac{S_{t+1}}{P_{t+1}}C_{t+1}^{-1}\right]}{E_t\left[\frac{1}{P_{t+1}}C_{t+1}^{-1}\right]} \tag{21}$$

where i^* is the nominal rate of interest on foreign currency bonds, which is assumed to be exogenous. The first order condition for the choice of money holdings is

$$\left(\frac{M_t}{P_t}\right)^e = c\left(\frac{1+i_t}{i_t}\right)C_t \tag{22}$$

The demand for nontradables is given by

$$C_{N,t} = (1-g)(C_t + G_t)\left(\frac{P_{N,t}}{P_t}\right)^{-1} \tag{23}$$

It is assumed that foreign households and governments behave in a symmetric way to domestic agents so the demand for home tradables is given by

$$C_{H,t} = g\,(C^* + G_t^*)\left(\frac{P_{H,t}}{S_t P^*}\right)^{-1} \tag{24}$$

where C* is total foreign consumption, which is assumed to be exogenous. This allows total output of domestic goods to be defined as

$$Y_t = C_{N,t} + C_{H,t} \tag{25}$$

This can also be regarded as the total demand for domestic labour. The levels of demand for the two individual types of labour are therefore given by

$$L_{1,t} = Y_t \left(\frac{W_{1,t}}{W_t} \right)^{-1} \quad L_{2,t} = Y_t \left(\frac{W_{2,t}}{W_t} \right)^{-1} \tag{26}$$

The first-order conditions for wages differ between the two types of labour. Type 1 households have to set their wages before shocks are realised. The first-order condition governing their choice of wages is therefore given by[9]

$$E_{t-1}\left[L_{1,t} C_t^{-1} \frac{W_{1,t}}{P_t} \frac{1}{X_t} \mathrm{A} \right] = \left(\frac{f}{f-1} \right) E_{t-1}\left[\mathrm{K}_t L_{1,t}^2 \right] \tag{27}$$

Type 2 households on the other hand can set their wages after shocks have been realised. The first order condition for type 2 wages is therefore

$$L_{2,t} C_t^{-1} \frac{W_{2,t}}{P_t} \frac{1}{X_t} \mathrm{A} = \left(\frac{f}{f-1} \right) \mathrm{K}_t L_{2,t}^2 \tag{28}$$

This holds *ex post* for all realisations of the shocks.[10]

The fact that type 1 wages are fixed before shocks are realised and monetary policy is set provides the nominal rigidity which is vital to giving a role to monetary policy. There is however, a further implication of the pre-setting of wages which is usually neglected in the traditional approach to analysing general equilibrium models,

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[9] All agents of a given type face similar wage setting problems and therefore, in a symmetric equilibrium, set identical wage levels. This fact has been used in the derivation of equations (27) and (28).

[10] Equations (27) and (28) highlight an important difference in the way supply shocks affect the two groups of households. Type 1 households are committed to meet labour demand at the prefixed wage. Any unanticipated change in K affects their utility but cannot affect their behaviour. Type 2 households set wages after supply shocks are realised and are therefore able to set their wages and labour supply accordingly. If there were no type 2 households (so that all wages were prefixed) supply shocks would have no *ex post* effect on any variable except realised utility.

namely that there will be a risk premium in type 1 wages. An explicit expression for this risk premium can be obtained because the structure of the model and the assumption of log-normal shocks implies that all variables are log-normally distributed. It is therefore possible to take logs of equation (27) to yield the following (where lower case letters are used to denote the expected value of the log of variables)

$$l_{1,t} - c_t + w_{1,t} - p_t = \ln\left[f/(f-1)\right] - a + 2l_{1,t} + l \qquad (29)$$

where
$$l = \frac{1}{2}\left\{s_k^2 + 3s_{L_1}^2 - s_C^2 - s_P^2 - s_X^2\right\} + \left\{2s_{k,L_1} + s_{C,L_1} + s_{P,L_1} - s_{C,P} + s_{X,L_1} - s_{X,C} - s_{X,P}\right\}$$

is the risk premium in type 1 wages and where s_i^2 is the variance of the log of variable i and $s_{i,j}$ is the covariance between the logs of variable i and variable j.

The existence of this risk premium obviously affects the level of type 1 wages and therefore affects the *ex ante* choice of labour supply. In turn this affects the expected level of consumption and output. The expression for l just derived shows clearly that monetary policy can have an important channel of influence through the risk premium. The choice of monetary policy regime affects the variances and covariances which enter l and can therefore affect the level of type 1 wages and therefore the level of consumption and output. It will be shown below that this has a direct effect on the level of welfare.

Welfare

One of the main advantages of the model just described is that it provides a very natural and tractable measure of welfare in the shape of the utility function of agents. Following Obstfeld and Rogoff (2000) it is assumed that the utility of real balances is small enough to be neglected. It is therefore possible to measure *ex ante* aggregate welfare in period t using the following

$$\Omega = E_{t-1}\left[y\left(\ln C_t - \frac{K_t}{2}L_{1,t}^2\right) + (1-y)\left(\ln C_t - \frac{K_t}{2}L_{2,t}^2\right)\right] \qquad (30)$$

The structure of the model is such that, in expected terms, each period is identical, so it is sufficient to analyse welfare in terms of a single period.

The appendix shows that the following relationships hold

$$E_{t-1}\left[\mathrm{K}_t L_{1,t}^2\right] = \left(1-g\right) \qquad E_{t-1}\left[\mathrm{K}_t L_{2,t}^2\right] = \left(1-g\right) \tag{31}$$

so the welfare measure can be written as

$$\Omega = c_t - \frac{\left(1-g\right)}{2} \tag{32}$$

where $c_t = E_{t-1} \ln C_t$.[11] This is the measure of welfare used throughout the remainder of the paper.

Equation (32) at first appears to suggest that welfare does not depend at all on the variability of the economy. As explained above, however, variability is having an important indirect effect on welfare via the risk premium in wages. The risk premium pushes up type 1 wages and reduces the expected supply of labour and thus reduces the expected level of consumption. Any variances or covariances which affect the risk premium (defined under equation (29)) therefore have an effect on welfare. In fact, it will turn out that the expected value of the log of consumption will be proportional to the risk premium so welfare can, in effect, be measured by the risk premium itself.[12]

Outline of Solution Procedure

The model is closed by specifying the monetary policy regime. For each policy regime there are a number of alternative ways to characterise the policy instruments and actions of the central bank. For convenience it is assumed that the central bank uses the money stock as its instrument.[13] It is useful, however, to consider the

[11] It is interesting to note that both categories of households have the same level of welfare *ex ante*.

[12] The assumption that utility is logarithmic in consumption implies that welfare does not depend directly on risk. A more general utility function would introduce a direct link between the variability of macroeconomic variables and welfare.

[13] In reality central banks often use the short term nominal interest rate as a monetary instrument. Each monetary regime considered in this paper can equally well be

implications of each regime for the behaviour of the real interest rate, so for each regime an equilibrium relationship of the following form is derived[14]

$$\hat{i}_t - E_t(p_{t+1}) = a_K k_t + a_X x_t \tag{33}$$

where $\hat{i}_t$ is the deviation of the nominal interest rate from its *ex ante* expected level and p_{t+1} is the rate of inflation between period t and period $t+1$. This expression relates the real interest rate to the realisation of the shock variables. The values of coefficients a_K and a_X depend on the monetary policy regime.

The details of the solution of the model are outlined in the appendix. The general procedure is briefly outlined here. As already noted, the structure of the model implies that each period is identical in expected terms. It is therefore possible to derive all relevant results by solving for events in a single period. The solution of the model within a period can be divided into two stages. In the first stage the *ex post* solution[15] is derived conditional on a given value for the type 1 wage rate and for given realisations of the shock variables. The *ex post* solution obviously depends on the particular policy regime under consideration. The *ex post* solution is used to generate expressions for all the variances and covariances in the model. In the second stage of the solution procedure these variances and covariances are used to obtain an expression for the risk premium contained in type 1 wages and hence a solution for the *ex ante* expectation of all variables is obtained. This yields an expression for the level of welfare.

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supported by a rule for the nominal interest rate. If policy is specified in terms of the nominal interest rate then additional terms must be included in the rule in order to ensure a determinate rational expectations equilibrium. In this model an additional term of the form $dE_{t-1}[p_t - p_{t-1}]$ with d>1 would be sufficient to ensure determinacy.

[14] This is not a "Taylor rule" of the form suggested by Taylor (1993). Equation (33) is just the equilibrium relationship between the real interest rate and the underlying shocks hitting the economy.

[15] That is, the solution of the model after shocks are realised and policy is set for that period.

3. A Comparison of Monetary Policy Regimes

Optimal Monetary Policy

As a benchmark consider the case of welfare maximising (or first best) monetary policy. In this case it is assumed that the money stock is set period by period to maximise (32). The resulting behaviour of the real interest rate is

$$\hat{i}_t - E_t[p_{t+1}] = \frac{(1-g)}{2}k_t + (1-g)x_t \tag{34}$$

Thus the optimal response to both supply shocks and demand shocks is to generate a rise in the real interest rate.[16] It is apparent that this rule is independent of the degree of price stickiness (i.e. it is independent of y). Thus the optimal rule simply replicates the flexible price equilibrium regardless of the actual degree of price stickiness.

It is worth noting that the optimal policy rule produces completely stable wage rates and completely stable prices of home produced goods. It therefore follows that the optimal rule is equivalent to a policy of strict targeting of domestic prices (or domestic price inflation). It is worth considering the explanation for this result in a little more detail. It was noted above that welfare is effectively determined by the level of the risk premium in type 1 wages. The underlying cause of the risk premium is the variability in labour effort generated by changes in the relative wage of type 1 workers. If policy completely stabilises the overall level of wages it obviously prevents all changes in relative wages and therefore removes the underlying risk faced by type 1 workers. The risk premium is therefore minimised and welfare is maximised when wages are completely stabilised. Domestic prices are a fixed mark-up over domestic wages

[16] The coefficient on supply shocks is smaller than that on demand shocks because supply shocks (which take the form of changes in the disutility of labour) lead to changes in the welfare maximising level of output. The optimal rule allows these changes to take place. The coefficients on both shock variables are declining in the degree of openness and reach zero when the economy is completely open. In this model, when the economy is completely open, there is effectively full risk sharing between domestic and foreign residents. There is therefore no need for an active stabilisation policy.

so a policy of stabilising domestic prices will, by definition, yield maximum welfare.[17]

The level of welfare yielded by the optimal rule is shown in Table 1. The implied variances of (the logs of) consumption, output and the nominal exchange rate are shown in Table 2.

Table 1: Welfare levels

	Welfare
Optimal policy	$\frac{(1-g)}{2}\ln(1-g)-\frac{(1-g)}{2}$
Flexible inflation targeting	$\frac{(1-g)}{2}\ln(1-g)-\frac{(1-g)}{2}-\frac{(1-g)yg^{2}s_{K}^{2}s_{X}^{2}}{(1-y)^{2}s_{K}^{2}+4y^{2}g^{2}s_{X}^{2}}$
Strict inflation targeting	$\frac{(1-g)}{2}\ln(1-g)-\frac{(1-g)}{2}-\frac{(1-g)yg^{2}\left(s_{K}^{2}+4s_{X}^{2}\right)}{4[1-y(1-g)]^{2}}$
Fixed nominal exchange rate	$\frac{(1-g)}{2}\ln(1-g)-\frac{(1-g)}{2}-\frac{(1-g)}{4}y\left(s_{K}^{2}+4s_{X}^{2}\right)$

Inflation Targeting

In the case of inflation targeting it is assumed that the monetary authority has full independence and discretion over the setting of its policy instrument but its objectives are set by some higher governmental authority in the form of the following loss function

$$L_t=\left(p_t-p_{t-1}-\bar{p}\right)^2+m_Y\left(y_t-\bar{y}\right)^2+m_S\left(s_t-\bar{s}\right)^2 \qquad (35)$$

[17] This result also arises in more general models of nominal stickiness such as models based on the Calvo (1983) structure. See for instance Benigno (2000).

Table 2: Variances

	$Var(c)$	$Var(y)$	$Var(s)$
Optimal policy	$\frac{(1-g)^2}{4}\left(s_K^2+4s_X^2\right)$	$\frac{1}{4}s_K^2$	$\frac{1}{4}\left(s_K^2+4s_X^2\right)$
Flexible inflation targeting	$\frac{(1-g)^2\left[\left[(1-y)s_K^2-4ygs_X^2\right]^2+4(1-y)^2 s_K^2 s_X^2\right]}{4\left[(1-y)^2 s_K^2+4y^2g^2s_X^2\right]}$	$\frac{(1-g)^2 s_K^4}{4\left[(1-y)^2 s_K^2+4y^2g^2s_X^2\right]}$	$\frac{\left[(1-y)s_K^2-4ygs_X^2\right]^2+4(1-g)^2(1-y)^2 s_K^2 s_X^2}{4\left[(1-y)^2 s_K^2+4y^2g^2s_X^2\right]}$
Strict inflation targeting	$\frac{(1-g)^2(1-y)^2\left(s_K^2+4s_X^2\right)}{4\left[1-y(1-g)\right]^2}$	$\frac{(1-y)^2 s_K^2+4y^2g^2s_X^2}{4\left[1-y(1-g)\right]^2}$	$\frac{(1-g)^2(1-y)^2\left(s_K^2+4s_X^2\right)}{4\left[1-y(1-g)\right]^2}$
Fixed nominal exchange rate	$\frac{(1-g)^2}{4}(1-y)^2\left(s_K^2+4s_X^2\right)$	$\frac{1}{4}\left[(1-y)^2 s_K^2+4y^2s_X^2\right]$	0

where $\overline{p}$, $\overline{y}$ and $\overline{s}$ are target levels of inflation, output and the nominal exchange rate respectively. In what follows the target level of inflation is zero[18] and the target levels of output and the exchange rate are set at the *ex ante* expected level of these variables. The monetary authority chooses its policy instrument to minimise this loss function period by period. The weights m_Y and m_S reflect the importance that is placed on output and nominal exchange rate deviations respectively. In the analysis which follows initially m_S is fixed at zero. Following Svensson (1999, 2000) the term "strict inflation targeting" is used to denote the case where m_Y is also zero and the term "flexible inflation targeting" is used to denote the case where $m_Y \neq 0$.[19]

Notice that the inflation target is assumed to be in terms of the consumer price index. As has already been noted, in this model a policy of targeting the inflation rate of domestic prices would replicate the first best policy rule. An immediate conclusion could be that setting central bank objectives in the form of (35) contains a very basic error and that the welfare performance of policy could be improved simply by changing the targeted price variable rather than by modifying other aspects of the objective function. It is the case however that governments seem to prefer to set inflation targets in terms of consumer price inflation. So it is interesting to consider how changes to other aspects of the objective function can compensate for this inefficient choice of targeting variable.[20]

Flexible inflation targeting is considered first (where m_S=0 and $m_Y \neq 0$). For an arbitrary value of m_Y the policy choices of the central bank will generate the following relationship between the real interest rate and the underlying shock variables

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[18] Note that the loss function depends on current inflation rather than expected future inflation. In this model expected inflation is always zero so no meaningful analysis would be possible if the inflation target was in terms of expected inflation.

[19] There is a long tradition in macroeconomics of using functions similar to (35) as measures of social welfare. It is clear that the utility based welfare function used in this paper is very different from *ad hoc* formulations of this sort. A loss function of the form of (35) is however still a useful way to model the objectives of an inflation targeting central bank. In this case the loss function is a representation of the incentive structure imposed on the central bank, and is therefore not intended to be a measure of social welfare.

[20] There may be political pressures or arguments related to accountability or transparency which constrain governments to set targets in terms of consumer prices rather than domestic goods prices.

$$\hat{i}_t - E_t\left[p_{t+1}\right] = \frac{(1-g)(1-y)\left[1-y(1-g)\right]}{2\left[1+y^2\left(m_Y+(1-g)^2\right)-2y(1-g)\right]}k_t + \frac{(1-g)\left[1+y^2\left(m_Y+1-g\right)-y(2-g)\right]}{1+y^2\left(m_Y+(1-g)^2\right)-2y(1-g)}x_t \tag{36}$$

It can be seen from this expression that the coefficient on the supply shock is negatively related to m_Y. This is easily explained. A supply shock tends to raise prices. A rise in interest rates tends to depress demand and therefore offsets the rise in prices, but also causes a recession. The more weight the central bank places on output the less willing it will be to accept the recession, so its response to the supply shock will be less aggressive. The effect of a change in m_Y on the coefficient on demand shocks is ambiguous in sign but will in any case be very small. A demand shock tends to raise both prices and output so the central bank will be willing to raise the interest rate in response to a demand shock regardless of the weight placed on output in the loss function.[21]

It is useful to consider the welfare maximising value of m_Y. This is easily obtained and is given by the following expression

$$m_Y^* = \frac{g\left[1-y(1-g)\right]\left[4ygs_X^2-(1-y)s_K^2\right]}{(1-y)ys_K^2} \tag{37}$$

When this value of m_Y is substituted into the expression for the real interest rate the following is obtained

[21] Equation (36) illustrates the importance of allowing for less than full price stickiness in this analysis. If there were no type 2 households (i.e. y=1) the coefficient on supply shocks would be zero in the interest rate expression. This is because, as already explained, when all wages are prefixed, supply shocks have no effect on the *ex post* value of any macroeconomic variable because households are committed to meeting labour demand at the prefixed wage. In such a situation the central bank will have no need to respond to supply shocks. This result is apparent in all the non-first-best regimes considered in this paper. Supply shocks do, however, affect welfare (because they affect utility directly) so in the first best case the monetary rule does include a response to supply shocks even when wages are completely preset.

$$\hat{i}_t - E_t[p_{t+1}] = \frac{(1-g)(1-y)^2 s_K^2}{2\left[(1-y)^2 s_K^2 + 4y^2 g^2 s_X^2\right]} k_t + \\ \frac{(1-g)\left[(1-y)(1-y-yg) s_K^2 + 4y^2 g^2 s_X^2\right]}{(1-y)^2 s_K^2 + 4y^2 g^2 s_X^2} x_t \tag{38}$$

The welfare level achieved by this regime is shown in Table 1 (labelled “flexible inflation targeting”) while the variances of (the logs of) consumption, output and the nominal exchange rate are shown in Table 2.

Before discussing the implications of these results it is useful to derive the corresponding results for the case of strict inflation targeting. These are obtained simply by setting m_Y=0. The implied expression for the real interest rate is the following

$$\hat{i}_t - E_t[p_{t+1}] = \frac{(1-g)(1-y)}{2[1-y(1-g)]} k_t + \frac{(1-g)(1-y)}{1-y(1-g)} x_t \tag{39}$$

while the levels of welfare and variances are shown in Tables 1 and 2 (labelled “strict inflation targeting”).

The results just presented have a number of clear-cut and important implications. These are presented and discussed as a series of propositions.

Proposition 1: If *0<g<1* and y, s_K^2, s_X^2>0 then flexible inflation targeting yields lower welfare than the first best even when m_Y is chosen optimally. If *g*=0 or *g*=1 or y=0 or s_K^2=0 or s_X^2=0 then flexible inflation targeting with m_Y chosen optimally achieves the first best welfare.

Proof: The proof follows directly from the expressions for welfare given in Table 1.

Proposition 1 states that there are some special cases where flexible inflation targeting is equivalent to the first best monetary policy rule. But in general it is worse. The special cases where it does deliver first best policy are: if the economy is completely closed (i.e. *g*=0); or completely open (g=1); if prices are fully flexible (y=0); or if shocks come from only one source (s_K^2=0 or s_X^2=0). Other aspects of these special cases will be further discussed below.

Proposition 2: If $0<g<1, y>0$ and m_Y is chosen optimally then flexible inflation targeting yields higher welfare than strict inflation targeting. If g=0 or g=1 or y=0 then strict inflation targeting achieves the first best welfare.

Proof: The proof follows directly from the expressions for welfare given in Table 1.

In general strict inflation targeting is worse than flexible inflation targeting except in some special cases. These special cases are: if the economy is completely closed (i.e. g=0); or completely open (g=1); or if prices are fully flexible (y=0).

Propositions 1 and 2 establish a clear welfare ranking of regimes. Flexible inflation targeting is generally worse than first best and strict inflation targeting is generally worse than flexible inflation targeting. These results are easily understood. As has been noted already, inflation targeting in terms of domestic prices would be first best so it is not surprising that targeting consumer price inflation is worse than first best. It is also not surprising that strict inflation targeting should be worse than flexible inflation targeting. In crude terms, strict inflation targeting is a special case of flexible inflation targeting with m_Y set to zero. If m_Y is chosen optimally it is clear that higher welfare can in general be achieved.

The next two propositions deal with the effects of inflation targeting on the volatility of key macro variables.

Proposition 3: Flexible inflation targeting (with m_Y chosen optimally) produces lower volatility of consumption, output and the exchange rate than the first best policy.

Proof: The proof follows directly from the expressions for variances given in Table 2.

Proposition 4: Strict inflation targeting produces lower volatility of consumption and the exchange rate than flexible inflation targeting, but may produce higher or lower volatility of output.

Proof: The proof follows directly from the expressions for variances given in Table 2.

Propositions 3 and 4 are somewhat surprising in the light of the debate about the appropriate role of the exchange rate in the setting of monetary policy. The conventional wisdom is that, if anything, inflation targeting leads to too much exchange rate volatility. The effects of inflation targeting on the volatility of the exchange rate can be explained with reference the definition of the consumer price index (equation (10)) which can more conveniently be expressed in terms of log deviations as follows

$$\hat{p}_t = g\hat{s}_t + (1-g)\,\hat{p}_{N,t}$$

which has been simplified by making use of the fact that traded goods prices are entirely determined by the nominal exchange rate. It is immediately apparent from this equation that a central bank which is attempting to stabilise the consumer price index will also tend to stabilise the nominal exchange rate.[22] And, in addition, the more strictly the central bank is required to stabilise consumer price inflation the more it will tend to stabilise the nominal exchange rate.

It is useful to discuss the welfare maximising value of m_Y in the light of Propositions 1-4. The first point to note is that m_Y* is not always positive. Indeed when the variance of supply shocks is high relative to the variance of demand shocks it is found that m_Y*<0. In other words it is optimal to induce the central bank to create more output volatility. On the other hand, in the case where the variance of demand shocks is relatively high it is optimal to induce the central bank to stabilise output.[23] This explains why flexible inflation

[22] Of course to be strictly correct this explanation relies on the assumption that the nominal exchange rate and domestic prices are not negatively correlated. This is in general true in this model.

[23] Supply shocks in this model are changes to the marginal disutility of labour. Supply shocks therefore change the welfare maximising level of labour supply and output. If supply shocks are relatively more important than demand shocks then a welfare maximising government should allow output to fluctuate in the face of supply shocks. Hence the optimal value of m_Y will be negative.

targeting may produce either higher or lower output volatility than strict inflation targeting.

It is also useful to consider the value of m_Y* in the special case where the economy is completely closed (i.e. g=0). In this case the optimal value of m_Y is zero. In other words strict inflation targeting is optimal. It was noted above that the first best rule is equivalent to a policy of targeting the price of domestically produced goods. In a closed economy there is no distinction between the price index of domestically produced goods and the consumer price index so it is clear that a policy of strictly targeting the consumer price index will deliver the first best policy.[24]

Propositions 1-4 and the expressions in Tables 1 and 2 have established a number of clear cut results. But they also demonstrate the general importance of allowing for an intermediate degree of price stickiness. It is apparent that the parameter y has an important effect on the optimal value of m_Y and on the absolute welfare performances of flexible and strict inflation targeting regimes. It is also apparent that the degree of openness and the source of shocks can have important effects on the optimal value of m_Y and welfare performance.

Including the Exchange Rate in the Central Bank's Loss Function

Consider now the implications of including the exchange rate in the loss function of the central bank, i.e. allow $m_S \neq 0$. In this case, if m_Y and m_S are chosen in order to maximise welfare, it is found that the optimal value of m_Y is zero and the optimal value of m_S is given by

$$m_S^* = g\left[y\left(1-g\right)-1\right] \tag{40}$$

Furthermore this regime replicates the first best policy rule for all parameter sets and combinations of shocks. Thus an inflation targeting regime which explicitly takes account of exchange rate

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[24] In the special case where wages are fully flexible (y=0) the optimal value of m_Y is $-\infty$. In other words first best policy will be achieved by maximising the variance of output. In the cases where there are no supply shocks or where wages are fully fixed (y=1) the optimal value of m_Y is $+\infty$. In these cases the weight on inflation in the objective function goes to zero. In effect these are no longer cases of inflation targeting but are cases of strict output targeting.

volatility in the central bank's loss function achieves the first best level of welfare. However, note that the optimal value of m_S is unambiguously negative. The central bank is therefore induced to create more exchange rate volatility than would be generated by a strict inflation targeting regime. This result is not surprising in the light of Propositions 3 and 4 which showed that inflation targeting generates too little exchange rate volatility.

It is also not surprising that a central bank loss function which includes inflation and the nominal exchange rate is able to achieve the first best outcome. It was noted above that the first best outcome is achieved by strict targeting of the price of domestic goods. The two components of the consumer price index are the price of domestic goods and the price of foreign goods. The latter is determined by the nominal exchange rate. By including the nominal exchange rate in the loss function (with an optimal weight) the foreign price element of the consumer price index is effectively being offset. The net result is a loss function which depends only on the price of domestic goods.

A Fixed Nominal Exchange Rate

Finally the various inflation targeting regimes are compared to a fixed exchange rate regime. In this case the money stock is set period by period to ensure $s_t = \bar{s}$. The implied behaviour of the real interest rate is as follows

$$\hat{i}_t - E_t[p_{t+1}] = \frac{(1-g)(1-y)}{2} k_t + (1-g)(1-y)x_t \tag{41}$$

The level of welfare achieved by a fixed rate is shown in Table 1 and the variances of (the logs of) consumption and output are shown in Table 2. The following propositions can now be stated and discussed.

Proposition 5: Both flexible inflation targeting (when m_Y is chosen optimally) and strict inflation targeting produce higher welfare than a fixed rate.

Proof: The proof follows directly from the expressions for welfare given in Table 1.

Proposition 6: A fixed rate produces less volatile consumption than both strict and flexible inflation targeting (and therefore less volatile consumption than optimal). A fixed rate may produce more or less volatile output than either flexible inflation targeting or the optimum but less volatile output than strict inflation targeting.

Proof: The proof follows directly from the expressions for variances given in Table 2.

A fixed exchange rate is a useful point of comparison for inflation targeting for two reasons. First, inflation targeting was adopted by a number of countries in response to the failure of fixed exchange rate regimes during the 1990s. Second, a number countries in Europe are currently operating inflation targeting regimes but are considering joining the Euro zone.

The fact that a fixed nominal exchange rate yields lower welfare than both flexible and strict inflation targeting can again be explained with reference to the definition of the consumer price index. The first best policy involves stabilising the domestic price level. Targeting consumer price inflation involves some stabilisation of the nominal exchange rate but does at least partly result in some stabilisation of domestic prices. A fixed nominal exchange rate directs all the central bank's efforts towards stabilising the nominal exchange rate and therefore completely removes any attempt to stabilise domestic prices. It is therefore clear that inflation targeting (of consumer prices) is a better approximation of the first best policy.

4. Conclusions

This paper presents a model of a small open economy which takes explicit account of the effects of risk on economic behaviour. The model is used to analyse the welfare and stabilisation effects of inflation targeting. It is found that flexible inflation targeting yields lower welfare than first best policy for most parameter combinations. It is also found that inflation targeting implies lower than optimal volatility of the nominal exchange rate and other key macro variables. When compared to a fixed exchange rate, both flexible and strict inflation targeting yield higher welfare.

Some of these results contrast sharply with assumptions evident in the policy debate and they are remarkably clear and consistent across parameter values in this model. However, it is important to emphasise that the model used in this paper is restricted and simplified in a number of potentially significant ways. So it is necessary to conclude by discussing some of these restrictions.

First, the form of utility function used here is very special. The assumption that utility is logarithmic in consumption and quadratic in labour supply implies a particular form of behaviour towards risk. It is possible and necessary to generalise the analysis of this paper to deal with a more general utility function.

Second, the model is heavily restricted in order to eliminate changes in net asset positions. At present there is no tractable method for analysing more general models. But it is likely that relaxation of the parameter restrictions necessary to eliminate current account imbalances (if such analysis were possible) would modify the results of this paper. One possible way to gain some insight into the more general case would be through numerical simulation of a log-linearised model.[25] But such an approach neglects some of the risk induced effects which are important in the model and analysis of this paper.

Third, the analysis of this paper does not allow for exogenous shocks that originate in financial markets. In the model presented here the exchange rate responds to shocks originating in goods and labour markets and acts as a transmission channel of monetary policy but it is not itself a source of shocks. Again it is likely that the results in this paper will be altered if foreign exchange market shocks are included. It is, however, technically difficult to include such shocks while preserving a tractable and consistent structure.

Finally, the model assumes perfect factor mobility between the traded and non-traded sectors of the economy. One of the reasons exchange rate volatility is thought to be important is that large swings in exchange rates cause large swings in output and employment in the traded goods sector. If unemployed factors could easily switch to the non-traded sector (as is true in this model) exchange rate movements would obviously be relatively unimportant. It is likely therefore that the results of this paper would be modified if there were barriers or costs to intersectoral factor mobility.

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[25] Using the techniques employed in Rotemberg and Woodford (1999) for instance.

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Appendix

The Ex Post Solution of the Model

It is useful to take logs of all equations.[26] Lower case letters with hats are used to denote the log deviation of variables from their *ex ante* expected levels. Given that W_1 is unaffected by shocks by definition $\hat{w}_1 = 0$. The logs equivalents of all the equations necessary for the *ex post* solution to the model are listed below.

The first order condition for consumption becomes

$$-\hat{c}_t = -E_t\hat{c}_{t+1} + \hat{i}_t - \left(E_t\hat{p}_{t+1} - \hat{p}_t\right) \tag{A1}$$

The UIP relationship yields

$$\hat{i}_t = E_t\hat{s}_{t+1} - \hat{s}_t \tag{A2}$$

The current account is always in balance so the following relationship always holds

$$\hat{y}_t + \hat{p}_{N,t} = \hat{c}_t + \hat{p}_t + \hat{x}_t \tag{A3}$$

The CPI definition

$$\hat{p}_t = g\hat{p}_{T,t} + (1-g)\hat{p}_{N,t} \tag{A4}$$

Aggregate wages

$$\hat{w}_t = (1-y)\hat{w}_{2,t} \tag{A5}$$

Prices of home produced goods

$$\hat{p}_{N,t} = \hat{p}_{H,t} = \hat{w}_t \tag{A6}$$

[26] Note that the model (apart from the money demand equation) is log-linear by construction. The money demand equation plays no part in deriving the equilibria relevant to this paper so taking logs of the equations of the model does not involve any approximation.

Prices of foreign produced goods

$$\hat{p}_{T,t} = \hat{p}_{F,t} = \hat{s}_t \tag{A7}$$

Wages of type 2 workers

$$\hat{l}_{2,t} - \hat{c}_t + \hat{w}_{2,t} - \hat{p}_t = \hat{k}_t + \hat{x}_t + 2\hat{l}_{2,t} \tag{A8}$$

Demand for type 2 labour

$$\hat{l}_{2,t} = \hat{y}_t - \left(\hat{w}_{2,t} - \hat{w}_t\right) \tag{A9}$$

The interest rate relationship is

$$\hat{i}_t - E_t\left(\hat{p}_{t+1}\right) = a_K\hat{k}_t + a_X\hat{x}_t \tag{A10}$$

Equations (A1) to (A10) can be solved to yield *ex post* solutions for all the main equations of the model. The *ex post* solution can be used to obtain expression for the variances and covariances of all variables and hence an expression for the risk premium in type 1 wages can be generated.

The Ex Ante Solution of the Model

Lower case letters without hats are used to denote the log of the *ex post* expected value of a variable. For equations which hold *ex post* in all states of the world it is possible to take logs and then expectations to yield equations in the expected value of the logs of variables. The equations necessary for the *ex ante* solution of the model are listed below.

Current account balance implies

$$c_t + p_t = c_t* + s_t \tag{A11}$$

Current account balance also implies the following relationship between output and consumption

$$y_t + p_{N,t} = c_t + p_t \tag{A12}$$

The CPI is given by

$$p_t = gp_{T,t} + (1 - g)p_{N,t} \tag{A13}$$

Aggregate wages

$$w_t = y w_{1,t} + (1 - y)w_{2,t} \tag{A14}$$

Prices of home produced goods

$$p_{N,t} = p_{H,t} = w_t + \ln[q/(q-1)] \tag{A15}$$

Prices of foreign produced goods

$$p_{T,t} = p_{F,t} = s_t \tag{A16}$$

Type 1 wages

$$l_{1,t} - c_t + w_{1,t} - p_t = \ln[f/(f-1)] - a + 2l_{1,t} + l \tag{A17}$$

Type 2 wages

$$l_{2,t} - c_t + w_{2,t} - p_t = \ln[f/(f-1)] - a + 2l_{2,t} \tag{A18}$$

Labour demand

$$l_{1,t} = y_t - (w_{1,t} - w_t) \tag{A19}$$

$$l_{2,t} = y_t - (w_{2,t} - w_t) \tag{A20}$$

Equations (A11) to (A20) can be solved to yield solutions for the *ex ante* expectations of all the main variables of the model.

Simplifying the welfare measure

The wage setting equations imply the following

$$E_{t-1}\left[L_{1,t}C_t^{-1}\frac{W_{1,t}}{P_t}\frac{1}{X_t}\mathrm{A}\right]=\left(\frac{f}{f-1}\right)E_{t-1}\left[\mathrm{K}_t L_{1,t}^2\right] \quad \text{(A21)}$$

$$E_{t-1}\left[L_{2,t}C_t^{-1}\frac{W_{2,t}}{P_t}\frac{1}{X_t}\mathrm{A}\right]=\left(\frac{f}{f-1}\right)E_{t-1}\left[\mathrm{K}_t L_{2,t}^2\right] \quad \text{(A22)}$$

But from the following relationships

$$L_{1,t}=Y_t\left(\frac{W_{1,t}}{W_t}\right)^{-1} \qquad L_{2,t}=Y_t\left(\frac{W_{2,t}}{W_t}\right)^{-1} \quad \text{(A23)}$$

$$P_{H,t}=\left(\frac{q}{q-1}\right)W_t \quad \text{(A24)}$$

$$Y_t P_{H,t}=C_t P_t X_t \quad \text{(A25)}$$

$$\mathrm{A}=\left(\frac{f}{f-1}\right)\left(\frac{q}{q-1}\right)(1-g) \quad \text{(A26)}$$

the following can be derived

$$L_{1,t}\frac{W_{1,t}}{P_t}\frac{1}{X}=C_t\frac{q-1}{q} \qquad L_{2,t}\frac{W_{2,t}}{P_t}\frac{1}{X}=C_t\frac{q-1}{q} \quad \text{(A27)}$$

so

$$E_{t-1}\left[\mathrm{K}_t L_{1,t}^2\right]=(1-g) \qquad E_{t-1}\left[\mathrm{K}_t L_{2,t}^2\right]=(1-g) \quad \text{(A28)}$$

These relationships are used in the main text to simply the welfare measure.