Foreign Money Shocks and the Welfare Performance of Alternative Monetary Policy Regimes

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Abstract

A two-country sticky-price general equilibrium model is used to examine the welfare properties of alternative monetary policy regimes for a country which is subject to foreign money shocks. The expenditure switching effect of exchange rate changes is found to be important for the comparison between money targeting and a fixed rate. Money targeting is welfare superior to a fixed rate when the expenditure switching effect is relatively weak, but a fixed rate is superior when the expenditure switching effect is strong. However, price targeting is found to be superior to both a fixed rate and money targeting for all values of the expenditure switching effect. Finally, a welfare maximising monetary feedback rule is found to yield lower output and exchange rate volatility than price targeting for a wide range of parameter values.

Keywords: monetary regimes, expenditure switching, welfare, foreign monetary shocks.

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1 Introduction

Friedman (1953) and other early proponents of floating exchange rates argued that floating exchange rates are desirable because they provide a degree of insulation against foreign monetary shocks. A floating rate regime allows a country to set monetary policy independently from monetary policy in other countries. This limits the transmission of foreign monetary policy shocks to the domestic economy. Thus, it was argued, floating exchange rates act as a 'shock absorber' which helps to stabilise the domestic economy in the face of foreign monetary shocks.

Recent literature on the choice of exchange rate regimes makes use of general equilibrium models with sticky-prices and explicit microeconomic foundations. This new modelling approach allows a re-examination of the shock-absorber role of the exchange rate. In particular, it allows a formal and systematic analysis of the impact of regime choice on the welfare of agents (which can be measured in terms of aggregate utility). In this paper we therefore use a two-country general equilibrium model to compare the welfare performance of fixed and a range of floating exchange rate regimes in the presence of foreign monetary shocks. In this way, we aim to assess whether Friedman's arguments in favour of the shock absorbing properties of floating rate regimes hold true in a stochastic general equilibrium model of the type used in the recent international macroeconomics literature.

Initially we define a floating rate to be a regime of money targeting. Our results show that the welfare performance of money targeting relative to a fixed rate depends on the responsiveness of demand to changes in the exchange rate (i.e. the strength of the expenditure switching effect). It is found that money targeting yields higher welfare than a fixed rate as long as the expenditure switching effect is relatively weak. But when the expenditure switching effect is strong it is found that a fixed rate can yield higher welfare than money targeting.

We show that there are two underlying factors which help explain this result. The first is that the welfare of the home population is strongly influenced by the volatility of home output (i.e. higher output volatility tends to reduce welfare). The second is that foreign monetary shocks create a volatility trade-off for home monetary policy. The volatility of home output depends on the volatility of world aggregate expenditure and the volatility of the nominal exchange rate. A monetary policy which stabilises the exchange rate tends to cause high volatility of world aggregate expenditure.[1] But a monetary policy which reduces the volatility of world aggregate expenditure tends to cause volatility of the nominal exchange rate. The relative impact of exchange rate volatility and world expenditure volatility on the volatility of output depends on the strength of the expenditure switching effect. The strength of the expenditure switching effect therefore has important implications for

[1] We focus on a two-country framework where the home country is large enough for its monetary policy to have an impact on world aggregate expenditure. In the case of a small open economy, with some degree of home bias in consumption expenditure, home monetary policy would have a comparable effect on the measure of aggregate expenditure relevant for the home economy. Thus a similar volatility trade-off would exist in a small open economy setting.

the relative welfare performance of different exchange rate regimes.[2] When the expenditure switching effect is relatively weak, exchange rate fluctuations have less impact on home output. Thus, in this case, money targeting yields higher welfare than a peg. But when the expenditure switching effect is strong, exchange rate fluctuations cause high volatility in home output so a peg yields higher welfare than money targeting.

Money targeting, while corresponding to the policy recommended by Friedman, is only one form of floating exchange rate. A particularly relevant alternative form of floating rate is a regime of price targeting. This (approximately) corresponds to inflation targeting - which is the regime adopted by many monetary authorities in recent years. Another important case to consider is the optimal policy regime, where the monetary instrument is adjusted in order to maximise welfare. We extend our analysis to consider the welfare performance of this wider set of policy regimes.

We find that price targeting yields higher welfare than both money targeting and a fixed rate for all values of the expenditure switching coefficient. This is because (for a wide range of parameter values) price targeting achieves a compromise between stabilisation of world aggregate demand and stabilisation of the exchange rate. It therefore lies at a superior point on the volatility trade-off compared to the money targeting and fixed rate regimes. The volatility of output thus tends to be lower, and welfare higher, than in the other two regimes.

But it is also found that price targeting is not (in general) fully optimal. It is shown that optimal policy (for a wide range of parameter values) implies some stabilisation of the nominal exchange rate relative to price targeting. So optimal monetary policy, even if it does not completely fix the exchange rate, should aim to stabilise it to some extent, i.e. optimal monetary policy is not entirely inward looking.

This paper is part of an extensive new literature which uses stochastic general equilibrium models to analyse the welfare effects of exchange rate and monetary regimes. Recent papers have focused on a range of different aspects of this issue. Devereux and Engel (2003), Corsetti and Pesenti (2001), Sutherland (2005a) and Bacchetta and van Wincoop (2000) analyse the role of imperfect pass-through. Benigno and Benigno (2003) and Sutherland (2005b) examine the impact of cost-push shocks on regime choice. And Devereux (2004) and Benigno (2001) consider the effects of different financial market structures. Much of this recent literature has focused on models where the elasticity of substitution between home and foreign goods

[2] Given the importance of the expenditure switching effect for the analysis which follows it is useful to note that the empirical literature does not provide any clear guidance on an appropriate value for the international elasticity of substitution between goods. Obstfeld and Rogoff (2000) briefly survey some of the relevant literature and quote estimates for the elasticity ranging between 1.2 and 21.4 for individual goods (see Trefler and Lai (1999)). Estimates for the average elasticity across all traded goods lie in the range 5 to 6 (see for instance Hummels (2001)). Anderson and van Wincoop (2003) also survey the empirical literature on trade elasticities and conclude that a value between 5 and 10 is reasonable. On the other hand, the real business cycle literature typically uses a much smaller value for this parameter. For instance Chari, Kehoe and McGrattan (2002) use a value of 1.5 in their analysis.

is restricted to unity. The model presented below, however, allows this parameter to differ from unity, and thus allows an analysis of the role of the expenditure switching effect. This is one important respect in which the current paper departs from the existing literature. Much of the existing literature focuses on the welfare effects at the world level of the symmetric choice of exchange rate regime. This paper, on the other hand, analyses the choice of regime from the point of view of a single (large) country. A third important departure from the existing literature is that this paper analyses the choice of monetary regime in the face of foreign monetary shocks. Other authors have focused on models where productivity or labour supply shocks predominate.[3]

The paper proceeds as follows: Section 2 presents the model; Section 3 describes the solution method and the derivation of the welfare measure; Section 4 discusses the links between monetary policy and welfare. Section 5 compares money targeting and a fixed nominal exchange rare. Sections 6 and 7 analyse price targeting and optimal policy respectively. Section 8 concludes the paper.

2 The Model

The model is a variation of the sticky-price general equilibrium structure which has been widely used in the recent open economy macroeconomics literature (following the approach developed by Obstfeld and Rogoff (1995, 1998)).[4] While the general structure is standard, an important difference between the model presented below and many others used in the recent literature is that the elasticity of substitution between home and foreign goods can differ from unity. This allows an analysis of the implications of the expenditure switching effect for the relative performance of different regimes. The only source of stochastic shocks in the model is the foreign money supply. Home monetary policy is modelled as a general feedback rule and the four possible regimes for the home monetary authority considered correspond to different settings for the coefficient in this feedback rule.

2.1 Market Structure

The world exists for a single period and consists of two equal-sized countries, which will be referred to as the home country and the foreign country. There is a continuum of agents of unit mass in each country with home agents indexed $h \in [0, 1]$ and foreign agents indexed $f \in [0, 1]$. Agents consume a basket of goods containing all home and foreign produced goods. Each agent is a monopoly producer of a single differentiated product. All agents set prices in advance of the realisation of shocks

[3] Sutherland (2004), while analysing the impact of the expenditure switching effect on monetary policy in a small open economy, focuses exclusively on supply shocks and does not consider the role of foreign monetary shocks.

[4] See Lane (2001) for a survey of this literature.

and are contracted to meet demand at the pre-fixed prices. Prices are set in the currency of the producer.

The detailed structure of the home country is described below. The foreign country has an identical structure. Where appropriate, foreign real variables and foreign currency prices are indicated with an asterisk.

2.2 Preferences

All agents in the home economy have utility functions of the same form. The utility of agent h is given by

$$U(h) = E\left[\log C(h) + \chi \log \frac{M(h)}{P} - \frac{K}{2} y^2(h)\right] \tag{1}$$

where χ and K are positive constants, C is a consumption index defined across all home and foreign goods, M denotes end-of-period nominal money holdings, P is the consumer price index, $y(h)$ is the output of good h and E is the expectations operator.

The consumption index C for home agents is defined as

$$C = \left[\left(\frac{1}{2}\right)^{\frac{1}{\theta}} C_H^{\frac{\theta-1}{\theta}} + \left(\frac{1}{2}\right)^{\frac{1}{\theta}} C_F^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}} \tag{2}$$

where C_H and C_F are indices of home and foreign produced goods defined as follows

$$C_H = \left[\int_0^1 c_H(i)^{\frac{\phi-1}{\phi}} di\right]^{\frac{\phi}{\phi-1}}, \quad C_F = \left[\int_0^1 c_F(j)^{\frac{\phi-1}{\phi}} dj\right]^{\frac{\phi}{\phi-1}} \tag{3}$$

where $\phi > 1$, $c_H(i)$ is consumption of home good i and $c_F(j)$ is consumption of foreign good j. The parameter θ is the elasticity of substitution between home and foreign goods. This is the key parameter which determines the strength of the expenditure switching effect.

The budget constraint of agent h is given by

$$M(h) = M_0 + p_H(h)\, y(h) - PC(h) - T + PR(h) \tag{4}$$

where M_0 and $M(h)$ are initial and final money holdings, T is a lump-sum government transfer, $p_H(h)$ is the price of home good h, P is the aggregate consumer price index and $R(h)$ is the income from a portfolio of state contingent assets (to be described in more detail below).

The government's budget constraint is

$$M - M_0 + T = 0 \tag{5}$$

Changes in the money supply are assumed to enter and leave the economy via changes in lump-sum transfers.

2.3 Price Indices

The aggregate consumer price index for home agents is

$$P = \left[\frac{1}{2}P_H^{1-\theta} + \frac{1}{2}P_F^{1-\theta}\right]^{\frac{1}{1-\theta}} \tag{6}$$

where P_H and P_F are the price indices for home and foreign goods respectively defined as

$$P_H = \left[\int_0^1 p_H(i)^{1-\phi}\, di\right]^{\frac{1}{1-\phi}}, \quad P_F = \left[\int_0^1 p_F(j)^{1-\phi}\, dj\right]^{\frac{1}{1-\phi}} \tag{7}$$

The law of one price is assumed to hold. This implies $p_H(i) = p_H^*(i)\,S$ and $p_F(j) = p_F^*(j)\,S$ for all i and j where an asterisk indicates a price measured in foreign currency and S is the exchange rate (defined as the domestic price of foreign currency). Purchasing power parity holds in terms of aggregate consumer price indices, $P = SP^*$.

2.4 Consumption Choices

Individual home demand for representative home good, h, and foreign good, f, are given by

$$c_H(h) = C_H\left(\frac{p_H(h)}{P_H}\right)^{-\phi}, \quad c_F(f) = C_F\left(\frac{p_F(f)}{P_F}\right)^{-\phi} \tag{8}$$

where

$$C_H = \frac{1}{2}C\left(\frac{P_H}{P}\right)^{-\theta}, \quad C_F = \frac{1}{2}C\left(\frac{P_F}{P}\right)^{-\theta} \tag{9}$$

Foreign demands for home and foreign goods have an identical structure to the home demands. Individual foreign demand for representative home good, h, and foreign good, f, are given by

$$c_H^*(h) = C_H^*\left(\frac{p_H^*(h)}{P_H^*}\right)^{-\phi}, \quad c_F^*(f) = C_F^*\left(\frac{p_F^*(f)}{P_F^*}\right)^{-\phi} \tag{10}$$

where

$$C_H^* = \frac{1}{2}C^*\left(\frac{P_H^*}{P^*}\right)^{-\theta}, \quad C_F^* = \frac{1}{2}C^*\left(\frac{P_F^*}{P^*}\right)^{-\theta} \tag{11}$$

Each country has a population of unit mass so the total demands for goods are equivalent to individual demands. The total demand for home goods is therefore $Y = C_H + C_H^*$ and the total demand for foreign goods is $Y^* = C_F + C_F^*$.

2.5 Price Setting

Goods prices are chosen before shocks are realised and are fixed at these pre-chosen levels. Prices are set in the currency of the producer.[5] The first-order condition for price setting for home agents implies the following

$$P_H = \frac{\phi}{\phi - 1} \frac{KE\left[Y^2\right]}{E\left[Y/(PC)\right]} \tag{12}$$

A similar expression can be derived for foreign agents, as follows

$$P_F^* = \frac{\phi}{\phi - 1} \frac{KE\left[Y^{*2}\right]}{E\left[Y^*/(P^*C^*)\right]} \tag{13}$$

Notice that the expectations terms in these expressions imply that a form of risk premium is built into goods prices. This risk premium arises because agents, who are risk averse, face uncertainty over the level of work effort. The risk premium depends on the variances and covariances of work effort, the marginal utility of consumption and consumer prices.

In this framework, where all goods prices are fixed, a 'price-targeting' regime is defined in terms of targeting the price level that producers would set if they were able to respond to shocks. This price level is simply given by the expression in (12) after removing the expectations operators from the right hand side, i.e.

$$P_H^V = \frac{\phi}{\phi - 1} KYPC \tag{14}$$

where the superscript 'V' indicates that this is a 'virtual' or 'shadow' price level. This structure can be thought of as a limiting case of a more general model where the population is split into a set of flexible-price agents and a set of fixed-price agents. The shadow price P_H^V corresponds to the price of flexible-price producers as the proportion of flexible-price producers tends to zero.[6]

2.6 Financial Markets and Risk Sharing

The asymmetric structure of shocks and monetary policy, coupled with a non-unit elasticity of substitution between home and foreign goods, makes it necessary to adopt a more explicit structure for international asset markets than is usual in the recent literature.[7] It is assumed that sufficient contingent financial instruments exist

[5]Other contributions to the recent literature have shown that the currency of pricing is potentially important for the relative welfare performance of exchange rate regimes. This issue is not the main focus of the current paper, so, for simplicity, we concentrate on the case of producer currency pricing.

[6]See Sutherland (2004) for an example of a model where the proportion of flexible price producers is strictly greater than zero.

[7]When θ is equal to unity, the trade balance between the two countries automatically balances in all states of the world, in which case financial markets are irrelevant. When $\theta \neq 1$ it becomes necessary to consider the structure of financial markets. Additionally, when shocks are asymmetric and when the focus of interest is the policy choice and welfare of a single country, it becomes necessary explicitly to consider how policy choices affect asset prices and portfolio decisions.

to allow efficient sharing of consumption risks. All consumption is financed out of real income so the only source of consumption risk is variability in real income. Efficient sharing of consumption risk can therefore be achieved by allowing trade in two state-contingent assets, one which has a pay-off correlated with home aggregate real income and one with a pay-off correlated with foreign real income. For simplicity, it is assumed that each asset pays a return equal to the relevant country's real income, i.e. a unit of the home asset pays $y = YP_H/P$ and a unit of the foreign asset pays $y^* = Y^*P_F/P$.[8] The portfolio pay-offs for home and foreign agents are given by the following

$$R(h) = \zeta_H(h)(y - q_H) + \zeta_F(h)(y^* - q_F) \tag{15}$$

$$R^*(f) = \zeta_H^*(f)(y - q_H) + \zeta_F^*(f)(y^* - q_F) \tag{16}$$

where $\zeta_H(h)$ and $\zeta_F(h)$ are holdings of home agent h of the home and foreign assets, $\zeta_H^*(f)$ and $\zeta_F^*(f)$ are the holdings of foreign agent f of home and foreign assets and q_H and q_F are the unit prices of the home and foreign assets.

It is important to specify the timing of asset trade. It is assumed that asset trade takes place *after* the choice of exchange rate regime. This implies that agents can insure themselves against the risk implied by a particular exchange rate regime, but they can not insure themselves against the choice of regime.[9]

The Appendix shows that risk sharing implies the following relationship between consumption, asset prices and expected output levels in the two countries

$$\frac{C}{C^*} = \frac{q_H}{q_F} = \frac{E\left[\frac{y}{y+y^*}\right]}{E\left[\frac{y^*}{y+y^*}\right]} \tag{17}$$

2.7 Money Demand and Supply

The first-order condition for the choice of money holdings is

$$\frac{M}{P} = \chi C \tag{18}$$

The money supply in each country is assumed to be determined by the relevant national monetary authority. The foreign money supply is subject to stochastic

[8] Note that asset pay-offs are correlated with aggregate income. Individual agents therefore treat pay-offs as exogenous. This implies that the existence of contingent assets has no direct impact on optimal price setting.

[9] If, alternatively, asset trade takes place before the exchange rate regime is chosen, it would be possible for agents to insure themselves against the choice of regime. This could have very significant implications for the optimal choice of regime. The home monetary authority would be tempted to choose a regime which implies high volatility of demand for home goods. The high volatility of demand would discourage home labour supply and reduce home work effort but the level of home consumption would be protected by the risk-sharing arrangement. This alternative risk-sharing structure raises some interesting issues but it also involves some technical problems which go beyond the scope of this paper.

shocks such that $\log M^*$ is symmetrically distributed over the interval $[-\epsilon, \epsilon]$ with $E[\log M^*] = 0$ and $Var[\log M^*] = \sigma^2$.

Home monetary policy is defined in terms of a general feedback rule for the home money supply of the following form

$$M = \bar{M}\left(\frac{M^*}{\bar{M}^*}\right)^{\delta} \tag{19}$$

In what follows the value of δ will be determined by the monetary regime under consideration. We consider four different regimes: a fixed nominal exchange rate; money targeting; price targeting; and welfare maximising monetary rule. In the case of a fixed exchange rate, the home monetary authority chooses δ so that the exchange rate is maintained at a target level, $\bar{S}$. In the case of money targeting, the home monetary authority sets $\delta = 0$ so that the home money supply is constant at $\bar{M}$. In the case of price targeting, δ is chosen to maintain the virtual producer-price level, P_H^V, at a target level, $\bar{P}_H^V$. And finally, in the case of optimal policy, δ is chosen to maximise home welfare. The appropriate values of δ for each regime are derived below. Note that the values of $\bar{M}$, $\bar{S}$ and $\bar{P}_H^V$ only serve to provide an anchor for the equilibrium level of nominal variables. The equilibrium level of nominal variables has no effect on real variables or on welfare and is thus irrelevant to the analysis presented below.

3 Welfare and Model Solution

Our main objective is to analyse the welfare performance of different monetary policy regimes. The different regimes are represented by different choices for the policy feedback parameter δ, so the first task is to derive an expression for welfare in terms of this parameter.

Aggregate welfare of home agents is measured using the following[10]

$$\Omega = E\left[\log C - \frac{K}{2}Y^2\right] \tag{20}$$

It is not possible to derive an exact solution to the model described above.[11] A second-order approximation of the welfare expression is therefore derived.

Before proceeding it is necessary to define and explain some notation. The non-stochastic equilibrium of the model is defined as the solution which results when $M^* = 1$ with $\sigma^2 = 0$. For any variable X define $\hat{X} = \log\left(X/\bar{X}\right)$ where $\bar{X}$ is the value of variable X in the non-stochastic equilibrium. $\hat{X}$ is therefore the log-deviation of X from its value in the non-stochastic equilibrium.

[10] Following Obstfeld and Rogoff (1998, 2002) it is assumed that the utility of real balances is small enough to be neglected.

[11] The complication arising in this model is contained in equation (6). When θ is different from unity this equation is not linear in logs.

The only exogenous forcing variable in the model is the foreign money supply, M^*, so all log-deviations from the non-stochastic equilibrium are of the same order as the shocks to $\hat{M}^*$, which (by assumption) are of maximum size ϵ. When presenting an equation which is approximated up to order n it is therefore possible to gather all terms of order higher than n in a single term denoted $O\left(\epsilon^{n+1}\right)$.[12]

A second-order approximation of the welfare measure is given by

$$\tilde{\Omega} = E\left\{\hat{C} - K\bar{Y}^2\left[\hat{Y} + \hat{Y}^2\right]\right\} + O\left(\epsilon^3\right) \tag{21}$$

where $\tilde{\Omega}$ is the deviation of the level of welfare from the non-stochastic equilibrium.[13] Notice that this expression includes the first moments of consumption and output and the second moment of output. Welfare is increasing in the expected level of consumption and decreasing in the expected level and variance of output. Second-order accurate solutions for variances can be obtained from first-order accurate solutions for the relationships between endogenous variables and the shock variable. The analysis of volatility therefore involves working with a log-linearised (i.e. first-order approximated) version of the model. But a full second-order expression for welfare requires second-order accurate solutions for *both* the first and second moments of variables. So, a full analysis of welfare involves working with a second-order approximation of the model.

The detailed derivation of a second-order accurate solution of the model is described in the Appendix. The resulting expressions for the expected level of output and consumption and the variance of output are

$$E[\hat{C}] = \frac{1}{8}\left[-6 + 3\theta - \theta^2 + 2(-2 + \theta + \theta^2)\delta - (-2 + 5\theta + \theta^2)\delta^2\right]\sigma^2 + O\left(\epsilon^3\right) \tag{22}$$

$$E[\hat{Y}] = -\frac{1}{4}\left[1 + \delta - \theta + \theta\delta\right]^2\sigma^2 + O\left(\epsilon^3\right) \tag{23}$$

$$E[\hat{Y}^2] = \frac{1}{4}\left[1 + \delta - \theta + \theta\delta\right]^2\sigma^2 + O\left(\epsilon^3\right) \tag{24}$$

Thus welfare can be written as a function of δ, the feedback parameter in the policy rule, as follows

$$\tilde{\Omega} = \frac{1}{8}\left[-6 + 3\theta - \theta^2 + 2(-2 + \theta + \theta^2)\delta - (-2 + 5\theta + \theta^2)\delta^2\right]\sigma^2 + O\left(\epsilon^3\right) \tag{25}$$

[12] Thus, when the term $O\left(\epsilon^2\right)$ appears in an equation the variables in that equation should be understood to be accurate up to order one. While an equation which includes the term $O\left(\epsilon^3\right)$ should be understood to contain variables which are accurate up to order two. And an equation which does not include any term of the form $O\left(\epsilon^n\right)$ should be understood to hold exactly.

[13] In the non-stochastic equilibrium individual budget constraints imply that $\bar{P}\bar{C} = \bar{Y}\bar{P}_H$. Combining this expression with equation (12) shows that $\bar{Y} = \left[K\phi/\left(\phi - 1\right)\right]^{-1/2}$, thus $K\bar{Y}^2 = \left(\phi - 1\right)/\phi$. It is apparent from this expression that $\bar{Y}$ depends on the monopoly mark-up, $\phi/(\phi - 1)$. A common practice in the recent literature is to introduce a production subsidy to offset the monopoly distortion so that $\bar{Y}$ is at its welfare maximising level. It will become apparent below that the main welfare results in our paper are independent of the value of $\bar{Y}$ and are therefore independent of the monopoly distortion (and would therefore also be independent of any production subsidy).

Note from (23) and (24) that $E[\hat{Y}] + E[\hat{Y}^2] = 0$, so welfare is effectively determined by $E[\hat{C}]$ alone.[14]

4 Monetary Policy and Welfare

Before analysing the individual monetary policy rules it is useful to trace and explain the main linkages between monetary policy and welfare.

As shown above, because $E[\hat{Y}] + E[\hat{Y}^2] = 0$, welfare depends directly (and only) on the expected level of consumption, $E[\hat{C}]$. The expected level of consumption depends, via individual country and world resource constraints, on the expected level of output. And the expected level of output depends (amongst other things) on monetary policy via the impact of monetary policy on the volatility of output. As a general rule, higher output volatility tends to discourage work effort (since agents are risk averse). Thus, a monetary rule which leads to high output volatility tends to lead to a low expected level of output. And, conversely, a monetary rule which leads to low output volatility tends to lead to high expected output.[15] The impact of the expected level of output on the expected level of consumption and welfare can be positive or negative depending on the value of θ (the elasticity of substitution between home and foreign goods).

There are thus two important links between monetary policy and welfare. First, the link between monetary policy and the volatility of output. And second, the link between the expected level of output and the expected level of consumption. It is useful to set out some of the important equations which explain these two links before considering each of the individual monetary policy rules.

4.1 Monetary Policy and Output

When considering the impact of monetary policy on the volatility of output it is sufficient to look at first-order accurate solutions to the model.[16]

A first-order expansion of equation (17) shows that risk sharing implies the following relationship between realised consumption levels in the two countries

$$\hat{C} - \hat{C}^* = 0 + O\left(\epsilon^2\right)$$

where, as explained above, the term $O\left(\epsilon^2\right)$ indicates that the variables in this relationship should be understood to be accurate up to a first-order approximation. When combined with the purchasing power parity relationship (which implies

[14] Thus, as previously stated, the value of $\bar{Y}$ is irrelevant.

[15] It is important to note that the level of expected output does not depend exclusively on the volatility of output. Other factors are also important, such as the volatility of consumption and prices and the covariances between output, consumption and prices.

[16] Terms of order two and higher in expressions for realised values become terms of order three and higher in expressions for variances. Higher order terms in expressions for realised values are therefore irrelevant for the second-order accurate analysis of welfare.

$\hat{S} = \hat{P} - \hat{P}^*$) and the expressions for home and foreign money demand (which imply $\hat{M} = \hat{P} + \hat{C}$ and $\hat{M}^* = \hat{P}^* + \hat{C}^*$), the following expression for the exchange rate is obtained

$$\hat{S} = \hat{M} - \hat{M}^* + O\left(\epsilon^2\right) \tag{26}$$

Thus the nominal exchange rate depends on *relative monetary policy*, i.e. the difference between the home money supply and the foreign money supply.

The assumption of fixed goods prices implies $\hat{P}_H = \hat{P}_F^* = 0 + O\left(\epsilon^2\right)$, so aggregate consumer prices are given by

$$\hat{P} = \frac{1}{2}\hat{S} + O\left(\epsilon^2\right), \quad \hat{P}^* = -\frac{1}{2}\hat{S} + O\left(\epsilon^2\right) \tag{27}$$

These expressions, combined with the money demand relationships, imply that consumption levels are

$$\hat{C} = \hat{C}^* = \frac{1}{2}\left(\hat{M} + \hat{M}^*\right) + O\left(\epsilon^2\right) \tag{28}$$

Thus consumption in the two countries responds equally (because of the risk-sharing structure) to *aggregate world monetary policy*, i.e. the sum of home and foreign money supplies.

First-order approximations for home and foreign aggregate output levels yield

$$\hat{Y} = \frac{1}{2}\left(\hat{C} + \hat{C}^*\right) + \frac{\theta}{2}\hat{S} + O\left(\epsilon^2\right) \tag{29}$$

$$\hat{Y}^* = \frac{1}{2}\left(\hat{C} + \hat{C}^*\right) - \frac{\theta}{2}\hat{S} + O\left(\epsilon^2\right) \tag{30}$$

These expressions show that the realised value of output depends on aggregate monetary policy (via $\hat{C} + \hat{C}^*$) and relative monetary policy (via $\hat{S}$). Thus home-country monetary policy affects the volatility of home output via its impact on the volatility of aggregate monetary policy ($\hat{M} + \hat{M}^*$) and its impact on relative monetary policy ($\hat{M} - \hat{M}^*$). Notice that, given the exogenous stochastic shocks affecting M^*, the home monetary authority faces a trade-off: a monetary rule which stabilises ($\hat{M} + \hat{M}^*$) will necessarily destabilise ($\hat{M} - \hat{M}^*$) and vice versa. The relative importance of these two factors for the volatility of output will depend on the value of θ. The larger the value of θ (i.e. the stronger the expenditure switching effect) the more important is volatility in ($\hat{M} - \hat{M}^*$).

The link between monetary policy and output volatility therefore depends on the value of θ. This is one respect in which the welfare performance of different monetary policy regimes is affected by the strength of the expenditure switching effect.

4.2 Expected Output and Consumption

Expected levels of variables depend on second moments so, when considering the link between the expected level of output and the expected level of consumption, it is necessary to consider a second-order accurate solution to the model.

Second-order expansions of the home and foreign output relationships yield[17]

$$E\left[\hat{Y}\right] = E\left[\frac{1}{2}\left(\hat{C} + \hat{C}^{*}\right) - \frac{\theta}{2}\hat{\tau}\right] + O\left(\epsilon^{3}\right) \tag{31}$$

$$E\left[\hat{Y}^{*}\right] = E\left[\frac{1}{2}\left(\hat{C} + \hat{C}^{*}\right) + \frac{\theta}{2}\hat{\tau}\right] + O\left(\epsilon^{3}\right) \tag{32}$$

From which it follows that

$$E\left[\hat{C} + \hat{C}^{*}\right] = E\left[\hat{Y} + \hat{Y}^{*}\right] + O\left(\epsilon^{3}\right) \tag{33}$$

Thus the expected level of *world aggregate consumption* $E[\hat{C} + \hat{C}^{*}]$ must equal the expected level of *world aggregate output* $E[\hat{Y} + \hat{Y}^{*}]$. This shows one part of the link between home output and home consumption. An increase in home output raises the total of output available for consumption in the world.

The second part of the link between home output and home consumption is determined by the consumption risk sharing relationship, (17). This relationship determines how expected consumption is shared between the two countries. A second-order expansion of equation (17) shows that risk sharing implies that the first moments of consumption and output in the two countries are related as follows

$$E\left[\hat{C} - \hat{C}^{*}\right] = E\left[\hat{Y} - \hat{Y}^{*} + \hat{\tau}\right] + O\left(\epsilon^{3}\right) \tag{34}$$

where τ is the terms-of-trade. Thus *relative consumption*, $(\hat{C} - \hat{C}^{*})$, depends on *relative output*, $(\hat{Y} - \hat{Y}^{*})$, and the terms of trade.

It is simple to show, using equations (31) and (32), that relative output and the terms of trade are related as follows

$$E\left[\hat{Y} - \hat{Y}^{*}\right] = -\theta E\left[\hat{\tau}\right] + O\left(\epsilon^{3}\right) \tag{35}$$

This shows that an increase in home output relative to foreign output requires a fall in the price of home goods relative to the price of foreign goods (i.e. a fall in the terms of trade). The size of the required adjustment in $\hat{\tau}$ clearly depends on θ, the elasticity of substitution between home and foreign goods. Combining (34) and (35) implies

$$E\left[\hat{C} - \hat{C}^{*}\right] = (\theta - 1)E\left[\hat{Y} - \hat{Y}^{*}\right] + O\left(\epsilon^{3}\right) \tag{36}$$

Equations (33) and (36) show that the impact of the expected level of home output on the expected level of home consumption depends on the impact of home output on world aggregate consumption (equation (33)) and the impact on relative consumption (equation (36)). When $\theta > 1$ an increase in home output raises both

[17] In general, equations (31) and (32) should include terms which depend on the second moments of home and foreign consumption. However, the perfect cross-country correlation of consumption levels implies that these terms are equal to zero. They are therefore omitted from (31) and (32).

aggregate consumption, $E[\hat{C}+\hat{C}^*]$, and relative consumption, $E[\hat{C}-\hat{C}^*]$, so there is an unambiguous increase in $E[\hat{C}]$. But when $\theta < 1$ an increase in home output leads to an increase in aggregate consumption but a reduction in relative consumption. In this case, there is an ambiguous effect on $E[\hat{C}]$. If θ is sufficiently small an increase in home output can cause a decrease in $E[\hat{C}]$.

The link between expected output and expected consumption thus depends on the value of θ. This is another respect in which the relative welfare performance of different monetary policy regimes will depend on the strength of the expenditure switching effect.

5 Money Targeting versus a Fixed Rate

The essential feature of each of the targeting rules considered in this paper is that each rule ensures that the variance of the targeted variable is zero. Thus the money targeting rule ensures that the variance of M is zero. And the fixed exchange rate rule ensures that the variance of S is zero. The relevant value of δ, the policy feedback parameter, in each case can be derived by setting the variance of the relevant targeted variable to zero. Second-order accurate expressions for variances can be obtained by considering first-order accurate solutions for realised values. So, the value of δ for each rule can be derived by considering first-order accurate expressions for the targeted variables.

The money supply rule (19) implies $\hat{M} = \delta \hat{M}^*$. Thus, in the case of money targeting, the variance of $\hat{M}$ is zero when $\delta = 0$.

It was shown above (equation (26)) that the realised value of the exchange rate is given by $\hat{S} = \hat{M} - \hat{M}^* + O(\epsilon^2)$. This immediately shows that the variance of the exchange rate is zero when the home money supply is set equal to the foreign money supply, i.e. $\hat{M} = \hat{M}^*$. So the fixed rate regime implies $\delta = 1$.

The welfare level yielded by the money targeting and fixed rate regimes can now be derived by substituting the relevant value of δ into the welfare expression (25). The resulting expressions are shown in Table 1. This table also shows the implied variances of output, the exchange rate and consumption in the fixed rate and money targeting regimes.

It follows from the welfare expressions shown in Table 1 that the money targeting regime yields higher welfare than the fixed rate regime when $\theta \lesssim 3.56$. Thus money targeting yields higher welfare when the expenditure switching effect is relatively weak, but a fixed exchange rate regime is superior when the expenditure switching effect is strong.

These results can be understood by considering the impact of exchange rate volatility and the expenditure switching effect on output volatility. It was shown above in equation (29) that monetary policy affects the volatility of home output via its impact on aggregate consumption and via its impact on the exchange rate. Money targeting (i.e. $\delta = 0$) implies a low variance of aggregate consumption but high variance of the exchange rate. This contrasts with a fixed exchange rate regime

	Fix	MT	PT	OPT
δ	1	0	$\frac{\theta-1}{\theta+3}$	$\frac{\theta^2+\theta-2}{\theta^2+5\theta-2}$
$\tilde{\Omega}$	$-\sigma^2$	$\frac{1}{8}\left[-6+3\theta-\theta^2\right]\sigma^2$	$\left(\frac{-5-4\theta+\theta^2}{(3+\theta)^2}\right)\sigma^2$	$\left(\frac{2-5\theta+\theta^2}{-2+5\theta+\theta^2}\right)\sigma^2$
$E\left[\hat{Y}^2\right]$	σ^2	$\left(\frac{1-\theta}{2}\right)^2\sigma^2$	$\left(\frac{1-\theta}{3+\theta}\right)^2\sigma^2$	$\left(\frac{2-3\theta+\theta^2}{-2+5\theta+\theta^2}\right)^2\sigma^2$
$E\left[\hat{S}^2\right]$	0	σ^2	$\left(\frac{4}{3+\theta}\right)^2\sigma^2$	$\left(\frac{4\theta}{-2+5\theta+\theta^2}\right)^2\sigma^2$
$E\left[\hat{C}^2\right]$	σ^2	$\frac{1}{4}\sigma^2$	$\left(\frac{1+\theta}{3+\theta}\right)^2\sigma^2$	$\left(\frac{-2+3\theta+\theta^2}{-2+5\theta+\theta^2}\right)^2\sigma^2$

Table 1: Key Expressions for the Four Monetary Policy Regimes

(i.e. $\delta = 1$) which implies a high variance of consumption and a completely stable exchange rate.

The relative impact of consumption variability and exchange rate variability on the volatility of output depends on the strength of the expenditure switching effect. For low values of θ, the variability of the exchange rate is relatively unimportant so money targeting implies relatively low output volatility.[18] But for high values of θ, exchange rate volatility becomes relatively more important so a fixed exchange rate tends to deliver much lower output volatility than money targeting. The net result is that welfare in the money targeting regime declines as the expenditure switching effect becomes stronger. This is because of the negative impact of output volatility on the expected level of output and consumption. For large values of θ, this effect can become so strong that it implies that a fixed rate regime is welfare superior to a money targeting regime.

6 Price Targeting

The previous section compared money targeting with a fixed rate. However, money targeting is only one form of floating rate regime. A particularly relevant alternative form of floating rate regime is one where the monetary authority uses policy to stabilise prices, i.e. a price targeting regime. This section analyses a regime of this type and compares it to money targeting and a fixed rate.

In the price targeting case the feedback term in the rule (equation (19)) is chosen to stabilise the producer 'price level'. Because all prices are assumed to be pre-fixed,

[18]In fact, because consumption and the exchange rate are negatively correlated in a money targeting regime, the variance of output will be very low for θ close to unity.

price targeting is defined in terms of stabilising 'virtual' prices (i.e. the prices that producers would choose if prices were fully flexible).[19] In order to derive the value of δ it is necessary to derive a first-order accurate expression for P_H^V as a function of the money supply. A first-order expansion of (14), combined with (26), (27), (28) and (29) implies that

$$\hat{P}_H^V = \frac{3+\theta}{2}\hat{M} + \frac{1-\theta}{2}\hat{M}^* + O\left(\epsilon^2\right)$$

This expression shows that the variance of $\hat{P}_H^V$ is zero when

$$\delta = \frac{\theta-1}{\theta+3} \tag{37}$$

When this value for the δ is substituted into (25), an expression for welfare in the price targeting regime is obtained. This is shown in Table 1.

A comparison of the welfare expressions in Table 1 shows that price targeting, for all $\theta \neq 1$, yields higher welfare than both money targeting and the fixed rate. (In the case where $\theta = 1$, money targeting and price targeting are equivalent.)[20]

In order to understand these results, first consider the case where $\theta > 1$. Notice that, in this case, price targeting implies that $0 < \delta < 1$. Thus monetary policy responds positively to a foreign monetary shock, but less than one-for-one. In this sense, price targeting is a compromise between money targeting (where $\delta = 0$) and a fixed exchange rate (where $\delta = 1$). This in turn implies that price targeting yields lower exchange rate volatility than money targeting and lower aggregate consumption volatility than a fixed exchange rate. The net effect is that price targeting yields lower output volatility than both money targeting and a fixed rate. So, price targeting leads to lower risk for producers and therefore a higher expected level of output and consumption. And higher expected consumption implies a higher level of welfare than both the money targeting and a fixed exchange rate regimes.

In the case when $\theta < 1$, the explanation is more complex. In this case $\delta < 0$. This implies that price targeting yields higher exchange rate and consumption volatility, and thus higher output volatility, than the other two regimes. Yet, price targeting still yields higher welfare than the other two regimes.

The key to understanding the contrast between the $\theta > 1$ case and the $\theta < 1$ case lies in the link between the expected level of output and the expected level of home consumption. When $\theta > 1$, an increase in home output leads to an increase in both aggregate expected consumption and relative expected consumption (see equations (36) and (33)). This means that a monetary policy which leads to an increase in

[19] An alternative price targeting regime would be one that stabilises consumer prices. However, we focus on producer price targeting because it is not straightforward, in the context of our fixed-price model, to define a 'virtual' consumer price index.

[20] Sutherland (2004) shows that, in a similar model, for very high values of θ a fixed rate can yield higher welfare than price targeting. Sutherland, however, focuses on a small open economy case where the main sources of disturbances are home and foreign labour supply shocks.

the expected level of home output will also lead to an increase in the expected consumption possibilities for home consumers and thus an increase in home welfare.

On the other hand, equation (33) shows that when $\theta < 1$ an increase in home output causes a fall in relative consumption for home agents which can outweigh the rise in aggregate consumption. Thus, in contrast to the $\theta > 1$ case, a monetary policy which leads to a decrease in the expected level of home output can, in the $\theta < 1$ case, nevertheless lead to an increase in expected home consumption. This explains why price targeting, which generates high exchange rate and output volatility when $\theta < 1$, nevertheless yields higher welfare than the other two regimes. The high output volatility reduces expected home output which in turn, increases expected home consumption and welfare.

7 Optimal Policy

The previous sections have compared three simple monetary policy regimes. Each regime implies a different value for the policy feedback parameter δ. A natural benchmark against which to compare these simple policy rules is the optimal regime where the value of δ is chosen to maximise welfare.

The optimal value of δ is obtained by maximising welfare, as given in (19), with respect to δ. The first order condition implies

$$\delta^{OPT} = \frac{\theta^2 + \theta - 2}{\theta^2 + 5\theta - 2} \tag{38}$$

It is useful to note that the second-order condition for this maximisation problem implies that $2 - 5\theta - \theta^2 < 0$, which in turn implies that a valid maximum only exists for $\theta > (-5 + \sqrt{33})/2 \approx 0.372$.

The welfare level yielded by the optimal feedback rule is shown in Table 1. A comparison of the welfare results in Table 1 shows that, in general, price targeting is not fully optimal. Only in the case where $\theta = 1$ does price targeting correspond to the welfare maximising policy. Notice that, in this case money targeting also corresponds to the welfare maximising policy. A fixed exchange rate, on the other hand, does not correspond to the optimal policy rule for any value of θ.[21]

Other authors (e.g. Goodfriend and King (2001), King and Wolman (1999) and Clarida, Gali and Gertler (2001)) have argued that price targeting is a welfare maximising policy because it reproduces the flexible price equilibrium and thus perfectly

[21] Another way of representing optimal monetary policy is to derive the 'optimal targeting rule'. The optimal targeting rule expresses policy in terms of a rule for stabilising a weighted average of variables. In the current model it can be shown that the optimal targeting rule is the rule which stabilises T^{OPT}, where $T^{OPT} \equiv \theta \hat{P}_H^V + (\theta - 1)\hat{S}$. It is simple to show that the monetary feedback rule with $\delta = \delta^{OPT}$ can equivalently be represented in terms of the optimal feedback rule or in terms of the optimal targeting rule. Notice that the optimal targeting rule shows that price targeting is not fully optimal in general but it is optimal when $\theta = 1$. The optimal targeting rule also shows that a fixed rate will not be fully optimal for any value of θ.

offsets the distortions created by sticky prices. Why then does price targeting not correspond to the welfare maximising policy for all values of θ? There are two reasons for this. Firstly, price targeting in the home economy does not reproduce the flexible price equilibrium because there are distortions that arise from sticky prices in the foreign country. Secondly, the flexible price equilibrium is not welfare maximising from the home country's point of view because the home country has monopoly power over the supply of home goods. In a sticky-price world, the home country can use monetary policy to exploit this monopoly power to raise welfare above the level yielded by the flexible price equilibrium.

The size of the welfare difference between price targeting and optimal policy is relatively small for all values of θ. This is illustrated in Figure 1 which plots the welfare levels of the four policy regimes against values of θ.[22] The welfare difference with the other regimes diverges for large values of θ. The level of welfare yielded by money targeting declines particularly rapidly as θ is increased.

It is simple to check using the expressions in Table 1 that, when $\theta > 1$, optimal policy yields lower exchange rate and output volatility than price targeting. This is illustrated in Figures 2 and 3, which plot exchange rate and output volatility for the four regimes.[23] Thus, while a fixed exchange rate is not optimal, some degree of exchange rate stabilisation is optimal relative to pure price targeting (in the $\theta > 1$ case). In other words, optimal policy is not purely inward-looking. The degree of exchange rate stabilisation compared to the price targeting rule is however relatively small.

As explained in the previous section, for $\theta < 1$ the welfare effects of exchange rate and output volatility are reversed - i.e. output volatility can be welfare increasing. So the optimal policy rule tends to generate high output and exchange rate volatility when $\theta < 1$. Figure 2 shows that optimal policy tends to lead to more exchange rate volatility than price targeting. And, for particularly low values of θ, the optimal policy tends to generate substantially more exchange rate and output volatility than all three of the other regimes.

8 Conclusions

This paper has analysed the welfare properties of a range of monetary regimes in the presence of foreign monetary shocks. Our objective was to investigate whether Friedman's (1953) arguments in support of floating exchange rate regimes continue

[22] For the purposes of this figure σ is set equal to 0.1. The values of welfare plotted on the vertical axis can be interpreted in terms of steady-state consumption units. For instance, $\tilde{\Omega} = -1$ represents a welfare deviation equivalent to a one percent deviation of consumption from its non-stochastic equilibrium value.

[23] The explanation for the link between monetary policy and welfare given above emphasises the importance of the volatility of output. This may suggest that a monetary policy rule which stabilises output will be optimal. The results reported in Table 1 and illustrated in Figure 3 shows, however, that optimal policy does not, in general, fully stabilise output. This is because, as noted in footnote 15, the volatility of output is not the only determinant of the expected level of output.

to hold true in a stochastic general equilibrium model with a microfounded welfare criterion. In our analysis the degree of expenditure switching was found to be an important factor in the comparison between regimes. An analysis of the fixed rate and money targeting regimes shows that money targeting yields higher welfare when the expenditure switching effect is relatively weak, but a fixed exchange rate regime is superior when the expenditure switching effect is strong. A price targeting regime, however, yields higher welfare than both a fixed rate and money targeting for all values of the expenditure switching parameter. Nevertheless, price targeting is not fully optimal. An analysis of a welfare maximising monetary rule showed that optimal policy should involve some measure of exchange rate stabilisation relative to the price targeting regime when the elasticity of substitution between home and foreign goods is greater than unity (i.e. the expenditure switching effect is relatively strong). On the other hand, when the elasticity of substitution is less than unity, optimal policy leads to higher exchange rate volatility than the other regimes.

Our results suggest, that while Friedman's intuition is supported in the sense that a fixed rate regime is always dominated by a floating rate regime of one form or another, money targeting (Friedman's preferred monetary regime) is not always better than a fixed rate. This is mainly because money targeting can generate high exchange rate volatility which has destabilising effects on output. Other floating rate regimes, such as price targeting, welfare dominate both money targeting and a fixed rate regime.

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Appendix

Portfolio Allocation, Asset Prices and Risk Sharing

There are four first-order conditions for the choice of asset holdings. After some rearrangement they imply the following four equations

$$E\left[C^{-1}y\right] = E\left[C^{-1}\right] q_H, \quad E\left[C^{-1}y^*\right] = E\left[C^{-1}\right] q_F \tag{39}$$

$$E\left[C^{*-1}y\right] = E\left[C^{*-1}\right] q_H, \quad E\left[C^{*-1}y^*\right] = E\left[C^{*-1}\right] q_F \tag{40}$$

The combination of the private and government budget constraints and the portfolio pay-off functions for each country imply that aggregate home and foreign consumption levels are given by

$$C = y + \zeta_H (y - q_H) + \zeta_F (y^* - q_F) \tag{41}$$

$$C^* = y^* + \zeta_H^* (y - q_H) + \zeta_F^* (y^* - q_F) \tag{42}$$

where in a symmetric equilibrium $\zeta_H(h) = \zeta_H$ and $\zeta_F(h) = \zeta_F$ for all h and $\zeta_H^*(f) = \zeta_H^*$ and $\zeta_F^*(f) = \zeta_F^*$ for all f. Equilibrium in asset markets implies $\zeta_H + \zeta_H^* = 0$ and $\zeta_F + \zeta_F^* = 0$. These equations can be used to solve for q_H, q_F, ζ_H, ζ_F, ζ_H^*, ζ_F^*, C and C^* in terms of y and y^*.

Using the solution procedure outlined in Obstfeld and Rogoff (1996, pp. 302-3) it is possible to show that the two asset prices are given by

$$q_H = \frac{E\left[\frac{y}{y+y^*}\right]}{E\left[\frac{1}{y+y^*}\right]}, \quad q_F = \frac{E\left[\frac{y^*}{y+y^*}\right]}{E\left[\frac{1}{y+y^*}\right]} \tag{43}$$

and consumption levels in the two countries are given by

$$C = \frac{q_H (y + y^*)}{q_H + q_F}, \quad C^* = \frac{q_F (y + y^*)}{q_H + q_F} \tag{44}$$

Thus

$$\frac{C}{C^*} = \frac{q_H}{q_F} = \frac{E\left[\frac{y}{y+y^*}\right]}{E\left[\frac{y^*}{y+y^*}\right]} \tag{45}$$

which is equation (17) in the main text.

Model Solution

A second-order accurate solution to the model is derived using a single-period version of the solution method described in Sutherland (2002). Second-order expansions of (12) and (13) yield

$$\hat{P}_H = E\left[\hat{Y} + \hat{P} + \hat{C}\right] + \lambda_{P_H} + O\left(\epsilon^3\right) \tag{46}$$

$$\hat{P}_F^* = E\left[\hat{Y}^* + \hat{P}^* + \hat{C}^*\right] + \lambda_{P_F^*} + O\left(\epsilon^3\right) \tag{47}$$

where

$$\lambda_{P_H} = \frac{1}{2}E\left[4\hat{Y}^2 - \left(\hat{Y} - \hat{C} - \hat{P}\right)^2\right] \tag{48}$$

$$\lambda_{P_F^*} = \frac{1}{2}E\left[4\hat{Y}^{*2} - \left(\hat{Y}^* - \hat{C}^* - \hat{P}^*\right)^2\right] \tag{49}$$

Notice that these expressions both include terms (denoted λ_{P_H} and $\lambda_{P_F^*}$) which depend on the second moments of output, consumption and consumer prices. These terms represent a form of risk premium which is built into goods prices by risk-averse agents who have to set prices before shocks are realised. The risk premium depends on the variances and covariances of work effort, the marginal utility of consumption and the consumer prices.

The money demand and supply relationships imply that

$$E\left[\hat{P} + \hat{C}\right] = 0, \quad E\left[\hat{P}^* + \hat{C}^*\right] = 0 \tag{50}$$

Note that the money demand relationships are linear in logs so they do not require any approximation. The expressions for home and foreign goods prices therefore simplify to

$$\hat{P}_H = E\left[\hat{Y}\right] + \lambda_{P_H} + O\left(\epsilon^3\right), \quad \hat{P}_F^* = E\left[\hat{Y}^*\right] + \lambda_{P_F^*} + O\left(\epsilon^3\right) \tag{51}$$

These expressions can be combined with second-order expansions of the definitions of consumer prices to yield

$$E\left[\hat{P}\right] = \frac{1}{2}\lambda_{P_H} + \frac{1}{2}\lambda_{P_F^*} + \frac{1}{2}E\left[\hat{Y} + \hat{Y}^* + \hat{S}\right] + E\left[\lambda_{CPI}\right] + O\left(\epsilon^3\right) \tag{52}$$

$$E\left[\hat{P}^*\right] = \frac{1}{2}\lambda_{P_H} + \frac{1}{2}\lambda_{P_F^*} + \frac{1}{2}E\left[\hat{Y} + \hat{Y}^* - \hat{S}\right] + E\left[\lambda_{CPI}\right] + O\left(\epsilon^3\right) \tag{53}$$

where

$$\lambda_{CPI} = \frac{1}{8}(1-\theta)\hat{S}^2 \tag{54}$$

Notice that the non-log-linearity of consumer prices gives rise to another second-order term (denoted λ_{CPI}).

The expressions for consumer prices can be combined with the money market equations to yield the following expressions for consumption

$$E\left[\hat{C}\right] = -E\left[\hat{P}\right] = -\frac{1}{2}\lambda_{P_H} - \frac{1}{2}\lambda_{P_F^*} - \frac{1}{2}E\left[\hat{Y} + \hat{Y}^* + \hat{S}\right] - E\left[\lambda_{CPI}\right] + O\left(\epsilon^3\right) \tag{55}$$

$$E\left[\hat{C}^*\right] = -E\left[\hat{P}^*\right] = -\frac{1}{2}\lambda_{P_H} - \frac{1}{2}\lambda_{P_F^*} - \frac{1}{2}E\left[\hat{Y} + \hat{Y}^* - \hat{S}\right] - E\left[\lambda_{CPI}\right] + O\left(\epsilon^3\right) \tag{56}$$

A second-order expansion of equation (17) shows that risk sharing implies that the first moments of consumption and output in the two countries are related as follows

$$E\left[\hat{C}-\hat{C}^{*}\right]=E\left[\left(\hat{Y}-\hat{Y}^{*}\right)+\left(\hat{P}_{H}-\hat{P}_{F}^{*}\right)-\left(\hat{P}-\hat{P}^{*}\right)\right]+O\left(\epsilon^{3}\right) \tag{57}$$

while second-order expansions of the home and foreign output relationships yield

$$E\left[\hat{Y}\right]=E\left[\frac{1}{2}\left(\hat{C}+\hat{C}^{*}\right)-\theta\left(\hat{P}_{H}-\hat{P}\right)\right]+O\left(\epsilon^{3}\right) \tag{58}$$

$$E\left[\hat{Y}^{*}\right]=E\left[\frac{1}{2}\left(\hat{C}+\hat{C}^{*}\right)-\theta\left(\hat{P}_{F}^{*}-\hat{P}^{*}\right)\right]+O\left(\epsilon^{3}\right) \tag{59}$$

Combining (57), (58) and (59) with the purchasing power parity condition yields the following expression for the expected level of the exchange rate

$$E\left[\hat{S}\right]=\frac{\theta-1}{2\theta}\left(\lambda_{P_{H}}-\lambda_{P_{F}^{*}}\right) \tag{60}$$

Using the above equations it is possible to write consumption and output levels entirely in terms of λ_{P_H}, $\lambda_{P_F^*}$ and λ_{CPI} as follows

$$E\left[\hat{C}\right]=-\left(\frac{2\theta-1}{4\theta}\right)\lambda_{P_{H}}-\frac{1}{4\theta}\lambda_{P_{F}^{*}}-\left(\frac{1+\theta}{2}\right)E\left[\lambda_{CPI}\right]+O\left(\epsilon^{3}\right) \tag{61}$$

$$E\left[\hat{Y}\right]=-\frac{1}{2}\lambda_{P_{H}}-\frac{(1-\theta)}{2}E\left[\lambda_{CPI}\right]+O\left(\epsilon^{3}\right) \tag{62}$$

It is useful at this stage to consider what these expressions reveal about the determination of the expected levels of consumption and output. Equations (61) and (62) show that the risk premia, λ_{P_H} and $\lambda_{P_F^*}$, have a negative impact on expected output and consumption. Any factor which increases the risk faced by producers (such as an increase in the volatility of output) will discourage the supply of work effort and therefore depress output. By definition this also reduces the quantity of goods available for consumption and therefore reduces the expected level of consumption.[24]

The only remaining task is to derive expressions for the second-moment terms λ_{P_H}, $\lambda_{P_F^*}$ and λ_{CPI}. This requires (first-order accurate) expressions for realised output, consumption, prices and the exchange rate. First-order accurate expressions for the money supply and demand relations are as follows

$$\hat{M}=\delta\hat{M}^{*} \tag{63}$$

$$\delta\hat{M}^{*}=\hat{P}+\hat{C},\ \ \hat{M}^{*}=\hat{P}^{*}+\hat{C}^{*} \tag{64}$$

[24]Equations (61) and (62) also show that the λ_{CPI} term implies that, when $\theta>1$, exchange rate volatility has a positive impact on the expected level of consumption and a negative impact on the expected level of output. Exchange rate volatility tends to reduce the average cost of the consumption basket when $\theta>1$. This allows agents to reduce work effort and consume more goods.

and PPP implies

$$\hat{S} = \hat{P} - \hat{P}^* \tag{65}$$

so the exchange rate is given by

$$\hat{S} = (\delta - 1)\hat{M}^* - (\hat{C} - \hat{C}^*) \tag{66}$$

A first-order expansion of equation (17) shows that risk sharing implies that $\hat{C} - \hat{C}^* = 0 + O(\epsilon^2)$. Thus

$$\hat{S} = (\delta - 1)\hat{M}^* + O\left(\epsilon^2\right) \tag{67}$$

By assumption goods prices are fixed in advance so $\hat{P}_H = \hat{P}_F^* = 0 + O(\epsilon^2)$. Home and foreign consumer prices are therefore given by

$$\hat{P} = -\hat{P}^* = \frac{1}{2}\hat{S} + O\left(\epsilon^2\right) = \frac{1}{2}(\delta - 1)\hat{M}^* + O\left(\epsilon^2\right) \tag{68}$$

The risk sharing relationship combined with the money market relationships and the solution for the exchange rate imply that consumption is given by

$$\hat{C} = \hat{C}^* = \frac{1}{2}(1 + \delta)\hat{M}^* + O\left(\epsilon^2\right) \tag{69}$$

A first-order approximation for the home output level yields

$$\hat{Y} = \frac{1}{2}\left(\hat{C} + \hat{C}^*\right) - \theta\left(\hat{P}_H - \hat{P}\right) + O\left(\epsilon^2\right) \tag{70}$$

Combining these expressions with the solutions for consumption and price levels implies

$$\hat{Y} = \frac{[(1+\theta)\delta + (1-\theta)]\hat{M}^*}{2} + O\left(\epsilon^2\right) \tag{71}$$

and

$$\hat{Y}^* = \frac{[(1+\theta) + (1-\theta)\delta]\hat{M}^*}{2} + O\left(\epsilon^2\right) \tag{72}$$

The expressions for $\hat{S}$, $\hat{P}$, $\hat{P}^*$, $\hat{C}$, $\hat{C}^*$, $\hat{Y}$ and $\hat{Y}^*$ given (67), (68), (69), (71) and (72) can be substituted into (48), (49) and (54) to yield the following expressions for λ_{P_H}, $\lambda_{P_F^*}$ and λ_{CPI}

$$\lambda_{P_H} = \frac{1}{8}[3(\theta - 1)^2 - 2\delta(-5 + 2\theta + 3\theta^2) + \delta^2(3 + 10\theta + 3\theta^2)]\sigma^2 \tag{73}$$

$$\lambda_{P_F^*} = \frac{1}{8}[3 + 10\theta + 3\theta^2 - 2\delta(-5 + 2\theta + 3\theta^2) + \delta^2 3(\theta - 1)^2]\sigma^2 \tag{74}$$

$$\lambda_{CPI} = \frac{1}{8}(1 - \theta)(\delta - 1)\hat{M}^{*2} \tag{75}$$

These expressions can be substituted into (61) and (62) to yield the expressions for $E\left[\hat{C}\right]$ and $E\left[\hat{Y}\right]$ given in (22) and (23) in the main text.

Figure 1: Welfare

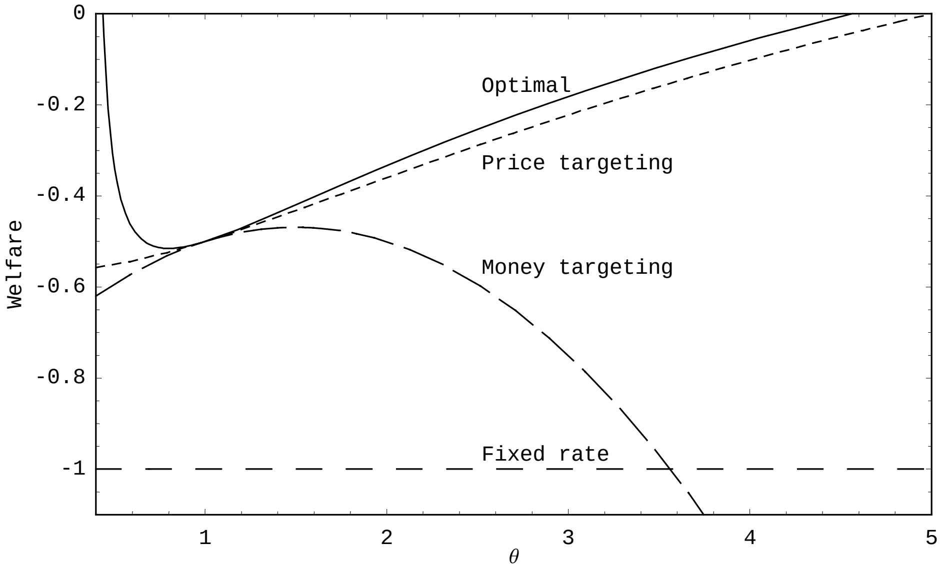

Figure 2: Exchange Rate Variability

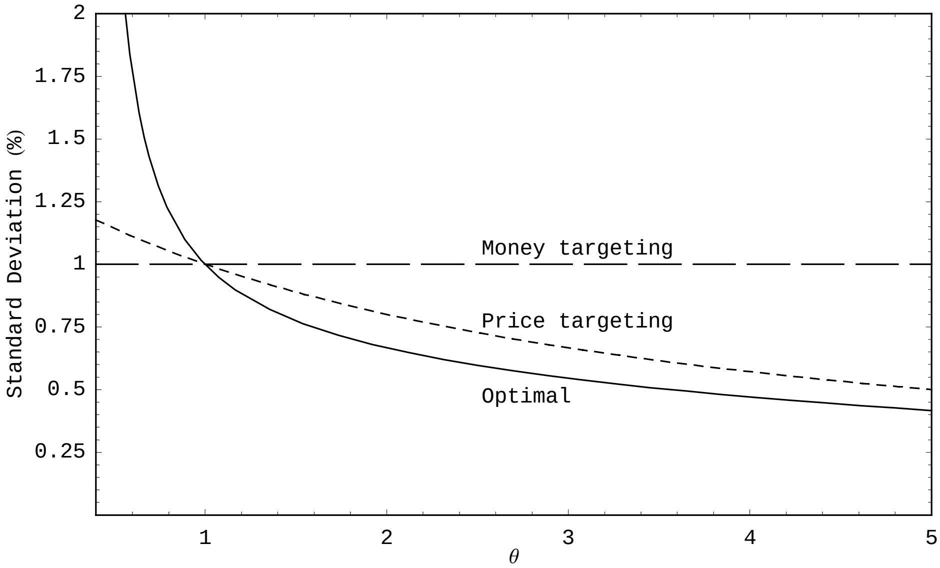

Figure 3: Output Variability