The Expenditure Switching Effect, Welfare and Monetary Policy in a Small Open Economy*

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Abstract

This paper analyses the implications of the 'expenditure switching effect' for the role of the exchange rate in monetary policy in a small open economy. It is shown that, when the elasticity of substitution between home and foreign goods is not equal to unity, welfare depends on the variances of producer prices and the terms of trade. Producer-price targeting is compared to consumer-price targeting and a fixed exchange rate. It is found that a fixed exchange rate yields higher welfare than the other regimes only when the elasticity of substitution between home and foreign goods is very high.
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1 Introduction

This paper analyses the implications of the expenditure switching effect for welfare maximising monetary policy in a small open economy. Many previous contributions to the literature have not addressed this issue because they are based on models where the elasticity of substitution between home and foreign goods is restricted to unity.[1] The model presented in this paper[2] allows for a non-unit elasticity of substitution between home and foreign goods and uses second-order approximation techniques to derive an explicit expression for welfare.[3] It is found that allowing for a non-unit elasticity of international substitution implies that terms-of-trade volatility becomes an important consideration for optimal monetary policy. Furthermore, welfare can be written as a weighted sum of two factors: the variance of producer prices and the variance of the terms of trade. The weight on terms-of-trade volatility is found to be increasing in the strength of the expenditure switching effect.

Previous literature on the welfare effects of monetary policy in closed economies has tended to suggest that strict targeting of consumer prices will maximise aggregate utility.[4] Such a policy minimises relative price distortions when some prices are sticky and unable to respond to shocks in the short run. Open economy contributions to the recent literature suggest that a welfare maximising monetary policy should focus on stabilising internal relative prices. This is achieved by strict targeting of producer prices.[5] Further analysis of open economy models, where there is less than perfect pass-through from exchange rate changes to local currency prices, has shown that optimal monetary policy should involve some consideration of exchange rate volatility.[6] In this case the monetary authority should allow some flexibility in producer prices in order to achieve some desired degree of volatility in the nominal exchange rate. The results of this paper show that terms-of-trade volatility (and thus exchange-rate volatility) can become an important factor in welfare maximising

[1] For example see Obstfeld and Rogoff (1998, 2000a, 2002), Devereux and Engel (2003), Devereux (2004), Corsetti and Pesenti (2001a, 2001b), Clarida, Gali and Gertler (2001, 2002). These papers focus on the unit elasticity case because of technical problems in deriving a full solution to a stochastic model when the elasticity is not equal to unity. Tille (2001), using a deterministic model, does analyse the role of international substitutability and shows that the international elasticity can have a significant effect on the transmission of welfare effects across countries. In a stochastic model it is possible to obtain an expression for world welfare when the international elasticity is not equal to unity (see Benigno and Benigno (2002, 2003a)). The specific technical problems relate to obtaining an expression for the welfare of an individual country. Some authors have been able to obtain some insights into optimal monetary policy for an individual country without obtaining a specific welfare function (see Benigno and Benigno (2003a)).

[2] The model is in the "new open economy macroeconomics" tradition (which originates with Obstfeld and Rogoff (1995)) in that it assumes monopolistic competition and sticky prices. The new open economy literature has been surveyed by Lane (2001).

[3] The technique used follows Kim and Kim (2003) and Sutherland (2002b).

[4] See Aoki (2001), Goodfriend and King (2001), King and Wolman (1999) and Woodford (2003).

[5] See Aoki (2001), Benigno and Benigno (2003a) and Clarida, Gali and Gertler (2001).

[6] See Bacchetta and van Wincoop (2000), Corsetti and Pesenti (2001a), Devereux and Engel (2003), Smets and Wouters (2002) and Sutherland (2002a).

monetary policy even when there is full pass-through.[7]

In the model described below the strength of the expenditure switching effect is determined by the elasticity of substitution between home and foreign goods. Before proceeding, it is worth considering the available empirical estimates for the value of this elasticity. Obstfeld and Rogoff (2000b) briefly survey some of the relevant literature. They quote estimates ranging between 1.2 and 21.4 for individual goods (see Trefler and Lai (1999)). Typical estimates for the average elasticity across all traded goods lie in the range 5 to 6 (see for instance Hummels (2001)). Anderson and van Wincoop (2003) also survey the empirical literature on trade elasticities and conclude that a value between 5 and 10 is reasonable.[8] There is thus considerable empirical evidence to suggest that the expenditure switching effect is potentially stronger than assumed in much of the recent open economy literature, where the elasticity between home and foreign goods is often restricted to unity.

One feature of more recent contributions to the literature on optimal monetary policy (which is shared by the model of this paper) is that the welfare maximising monetary strategy becomes more complex as more realistic aspects are added to the basic model. It quickly becomes apparent that the optimality of a simple strategy of strict consumer or producer-price targeting does not carry over to more general cases. In addition, even when the optimal monetary strategy can be summarised by a relatively simple loss function, it becomes doubtful that the fully optimal monetary policy can in practice be implemented. The fully optimal policy may involve responding to unobservable or unmeasurable variables or require a complex balance between different targets where the optimal weights to be placed on different targets are unmeasurable or uncertain. It is therefore useful to analyse the welfare performance of non-optimal but simple targeting rules. After deriving the theoretically optimal policy regime for the model economy, this paper considers three possible simple targeting rules, namely: strict targeting of producer prices, strict targeting of consumer prices and a fixed nominal exchange rate. It is found that, if the elasticity of substitution is low, producer-price targeting yields the highest welfare of the three simple rules. But for intermediate values of the elasticity of substitution, consumer-price targeting can be the best simple rule. And for (very) high degrees of substitutability, a fixed exchange rate can be the best simple rule.

The main focus of this paper is on the welfare effects of policy in a small open

[7] A further case where the basic price targeting result needs to be modified is where the economy is subject to non-optimal 'cost-push' shocks. In a closed economy context cost-push shocks imply that optimal policy allows for some flexibility in consumer prices in order to achieve some stabilisation of the output gap. This is often referred to as 'flexible inflation targeting' following the terminology suggested by Svensson (1999, 2000). Benigno and Benigno (2002) show that the same result holds in an open economy. Sutherland (2002c) also considers this issue and shows that nominal income targeting can be a good approximation for fully optimal policy when the variance of cost-push shocks is particularly high.

[8] These figures are all much higher than typically used in the real business cycle literature. For instance, Chari, Kehoe and McGrattan (2002) use a value of 1.5 in their investigation of real-exchange-rate volatility.

economy. Previous contributions to the literature have shown that the international spillover effects of monetary policy imply that the optimal policy from an individual country point of view may be suboptimal from a global perspective. Thus there are potential gains from policy coordination.[9] In a two country model, with a structure similar to the model presented here, Benigno and Benigno (2003a) have shown that the optimal coordinated policy involves strict targeting of producer prices in each country. The coordinated policy should not involve any attempt to stabilise the terms of trade. It is therefore important to note that the individual country concern for terms-of-trade stability identified in the model presented in this paper is inefficient from a global perspective.

This paper proceeds as follows. Section 2 presents the model. Section 3 discusses the welfare measure. Section 4 considers the general form of optimal monetary policy for the small open economy. Section 5 compares the welfare performance of the three simple targeting rules. Section 6 concludes the paper.

2 The Model

2.1 Market Structure

The world exists for a single period and consists of a small open economy (the home economy) and the rest of the world (the foreign economy).[10] The rest of the world is treated as exogenous. Each economy is populated by agents who consume a basket of goods consisting of all home and foreign produced goods. Each agent is a monopoly producer of a single differentiated product.[11]

There are two categories of agents in each country. The first set of agents supply

[9] Benigno and Benigno (2003a) show that gains from coordination exist when the elasticity of international substitution is not equal to unity. Sutherland (2002b), using a second-order solution technique, shows that these gains can be quantitatively quite large. This issue is considered in more detail in Sutherland (2004). Obstfeld and Rogoff (2002) and Clarida, Gali and Gertler (2002) show that there can be gains from coordination when the elasticity of intertemporal substitution is different from unity even when the elasticity of international substitution is equal to unity.

[10] The model can easily be recast as a multi-period structure but this adds no significant insights. A truly dynamic model, with multi-period nominal contracts and asset stock dynamics would be considerably more complex and would require much more extensive use of numerical methods. Newly developed numerical techniques are available to solve such models and this is likely to be an interesting line of future research (see Rotemberg and Woodford (1999), Woodford (2003), Kim and Kim (2003), Sims (2000), Schmitt-Grohé and Uribe (2004) and Sutherland (2002b) and Kollmann (2002)). However, the approach adopted in this paper yields useful insights which would not be available in a more complex model.

[11] By focusing on a small open economy it is possible to avoid the complications that would arise if the home economy was large enough to have an impact on the foreign economy. One way to think of the model presented here is that it is the limiting case of a two-country model where the foreign population is expanded to infinity while the home population is held constant. In this way the total home population remains large relative to an individual home agent (thus preserving the monopolistically competitive structure) while the home country becomes small in global terms.

goods in a market where prices are set in advance of the realisation of shocks and the setting of monetary policy. Agents in this market are contracted to meet demand at the pre-fixed prices. Agents in this group will be referred to as 'fixed-price agents'. The second set of agents supply goods in a market where prices are set after shocks are realised and monetary policy is set. Agents in this group will be referred to as 'flexible-price agents'. The proportion of fixed-price agents in the total population is denoted by ψ, so ψ is a measure of the degree of price stickiness in the economy. The total population of the home economy is indexed on the unit interval with fixed-price agents indexed on $[0,\psi]$ and flexible-price agents indexed on $(\psi, 1]$. Prices and quantities relating to fixed-price agents will be indicated with the subscript '1', while those relating to flexible-price agents will be indicated with the subscript '2'.[12]

This framework provides the minimal structure necessary to study the effects of price variability on welfare while allowing some degree of price stickiness. The fixed-price agents provide the nominal rigidity that is necessary to give monetary policy a role, while the flexible-price agents provide the partial aggregate price flexibility that allows an analysis of the connection between price volatility and welfare.

2.2 Preferences

All agents in the home economy have utility functions of the same form. The utility of agent z of type i is given by

$$U(z) = E\left[\log C(z) + \chi \log \frac{M(z)}{P} - K y_i(z)\right] \quad (1)$$

where $i = 1$ for a fixed-price agent and $i = 2$ for a flexible-price agent, C is a consumption index defined across all home and foreign goods, M denotes end-of-period nominal money holdings[13], P is the consumer price index, $y_i(z)$ is the output of good z, E is the expectations operator and K is a stochastic shock to labour supply preferences (where $E[\log K] = 0$ and $Var[\log K] = \sigma_K^2 > 0$ and $\log K \in [-\epsilon, \epsilon]$).[14]

[12] This structure can be thought of as a static version of the Calvo (1983) staggered price setting framework. A fixed/flexible-price structure similar to the one proposed here has previously been used in Aoki (2001) and Woodford (2003). The division of agents into fixed-price and flexible-price groups is taken to be a fixed institutional feature of the economy.

[13] The assumption that money enters the utility function is a proxy for modelling the transactions benefits of holding money. The assumption that it is end-of-period balances which enter utility has the somewhat unsatisfactory implication that agents finish the period holding unused money even when it has no future use (because there are no future periods). However, as previously stated, the model can easily be recast as an infinite horizon structure (where each period is identical in *ex ante* expected terms). In such a model, money held at the end of one period retains its value in future periods. An infinite horizon structure of this form would not alter any of the results presented below.

[14] The assumption of a finite support for the probability distribution of the shocks makes it possible to adopt a simple and precise notation when presenting the solution of the model, but it involves no loss of generality. Notice that, by definition, σ_K must be less than or equal to ϵ.

The consumption index C is defined as

$$C = \left[(1-\gamma)^{\frac{1}{\theta}} C_H^{\frac{\theta-1}{\theta}} + \gamma^{\frac{1}{\theta}} C_F^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}} \tag{2}$$

where C_H and C_F are indices of home and foreign produced goods and $0 \leq \gamma \leq 1$ and $\theta > 0$. The parameter γ measures the share of foreign goods in the consumption basket. The parameter γ can also be regarded as a measure of the degree of openness of the home economy. If $\gamma = 0$ then the economy is completely closed while $\gamma = 1$ implies a completely open economy.[15] The parameter θ is the elasticity of substitution between home and foreign goods. This is a key parameter which determines the strength of the expenditure switching effect.

Utility from consumption of home goods is defined as follows

$$C_H = \frac{C_{H,1}^{\psi} C_{H,2}^{(1-\psi)}}{\psi^{\psi}(1-\psi)^{(1-\psi)}} \tag{3}$$

where $C_{H,1}$and $C_{H,2}$ are indices of home fixed-price and flexible-price goods which are defined as follows

$$C_{H,1} = \left[\left(\frac{1}{\psi} \right)^{\frac{1}{\phi}} \int_0^{\psi} c_{H,1}(h)^{\frac{\phi-1}{\phi}} dh \right]^{\frac{\phi}{\phi-1}}, \ C_{H,2} = \left[\left(\frac{1}{1-\psi} \right)^{\frac{1}{\phi}} \int_{\psi}^{1} c_{H,2}(h)^{\frac{\phi-1}{\phi}} dh \right]^{\frac{\phi}{\phi-1}}$$

where $\phi > 1$, $c_{H,i}(h)$ is consumption of home good h produced by an agent of type i. C_F has a similar structure to C_H. The foreign economy is treated as exogenous from the point of view of the home economy so the detailed structure of C_F is irrelevant and is omitted.

The above functions imply a constant elasticity of substitution between different varieties of good of the same type and a unit elasticity of substitution between types of good.[16]

The budget constraint of agent z (where z is of type i) is given by

$$M(z) = M_0 + p_{H,i}(z)\, y_i(z) - PC(z) - T + PR(z) \tag{4}$$

[15] This structure is similar to the modelling of "home bias" in Gali and Monacelli (2000). It is also formally identical to the modelling of non-traded goods in Obstfeld and Rogoff (2000a) and Sutherland (2001). In the latter two papers the relative price of nontraded and home produced traded goods is fixed at unity so consumption of nontraded goods can be thought of as home bias in consumption.

[16] The assumption that the elasticity of substitution between fixed-price and flexible-price goods differs from the elasticity of substitution between goods within each type has the slightly odd implication that the degree of price stickiness is in effect embedded in the structure of preferences. It would be possible to relax this assumption (and, for instance, have a common elasticity of ϕ between all goods) but the present assumption allows some useful simplifications of the algebra (because it ensures that all home agents have identical income and consumption levels regardless of which type they are).

where M_0 and $M(z)$ are initial and final money holdings, T is a lump-sum government transfer, $p_{H,i}(z)$ is the price of good z, P is the aggregate consumer price index and $R(z)$ is the return on a portfolio of contingent assets (to be further described below).

The government's budget constraint is

$$M - M_0 + T = 0 \tag{5}$$

2.3 Price Indices

The consumer price index for home agents is

$$P = \left[(1-\gamma) P_H^{1-\theta} + \gamma P_F^{1-\theta}\right]^{\frac{1}{1-\theta}} \tag{6}$$

and the price index of home goods is

$$P_H = P_{H,1}^{\psi} P_{H,2}^{(1-\psi)} \tag{7}$$

where $P_{H,1}$ and $P_{H,2}$ are the price indices of home fixed-price and flexible-price goods which are defined as follows

$$P_{H,1} = \left[\frac{1}{\psi}\int_0^{\psi} p_{H,1}(h)^{1-\phi}\, dh\right]^{\frac{1}{1-\phi}}, \quad P_{H,2} = \left[\frac{1}{1-\psi}\int_{\psi}^{1} p_{H,2}(h)^{1-\phi}\, dh\right]^{\frac{1}{1-\phi}}$$

The prices of all foreign goods are set in foreign currency and are assumed to be exogenous from the point of view of home agents. The law of one price is assumed to hold for each good so $P_F = SP_F^*$. Where P_F^* is the foreign currency price index of foreign goods - which is exogenous.

The terms of trade are defined to be relative producer prices, i.e. $\tau = P_H/P_F = P_H/(SP_F^*)$.

2.4 Consumption Choices

In a symmetric equilibrium the consumption decisions of all home agents are identical. Individual home demands for representative home fixed-price good h_1 and representative home flexible-price good h_2 are given by the following expressions

$$c_{H,1}(h_1) = \frac{1}{\psi} C_{H,1}\left(\frac{p_{H,1}(h_1)}{P_{H,1}}\right)^{-\phi}, \quad c_{H,2}(h_2) = \frac{1}{1-\psi} C_{H,2}\left(\frac{p_{H,2}(h_2)}{P_{H,2}}\right)^{-\phi}$$

where

$$C_{H,1} = \psi C_H \left(\frac{P_{H,1}}{P_H}\right)^{-1}, \quad C_{H,2} = (1-\psi) C_H \left(\frac{P_{H,2}}{P_H}\right)^{-1} \tag{8}$$

and

$$C_H = (1-\gamma) C \left(\frac{P_H}{P}\right)^{-\theta} \tag{9}$$

Notice that the elasticity of substitution between home and foreign goods, θ, emerges here as the price elasticity for home goods. The home demand for foreign goods is irrelevant to deriving equilibrium for the home economy so this expression is omitted.

The mass of home consumers is unity so the total home demand for each good is equal to the individual demands, i.e. $y_{H,1}(h_1) = c_{H,1}(h_1)$, $y_{H,2}(h_2) = c_{H,2}(h_2)$. It is also useful to define $Y_{H,1} = C_{H,1}$ and $Y_{H,2} = C_{H,2}$ to be the total home demands for home fixed-price and flexible-price goods and $Y_H = C_H$ to be the total home demand for all home goods.

2.5 The Foreign Economy

The foreign economy is assumed to have a structure similar to that of the home country and foreign agents are assumed to have utility functions similar to (1). Foreign demands for representative home fixed-price good h_1 and representative home flexible-price good h_2 are given by the following

$$y^*_{H,1}(h_1) = \frac{1}{\psi} Y^*_{H,1} \left(\frac{p^*_{H,1}(h_1)}{P^*_{H,1}} \right)^{-\phi}, \quad y^*_{H,2}(h_2) = \frac{1}{1-\psi} Y^*_{H,2} \left(\frac{p^*_{H,2}(h_2)}{P^*_{H,2}} \right)^{-\phi}$$

where

$$Y^*_{H,1} = \psi Y^*_H \left(\frac{P^*_{H,1}}{P^*_H} \right)^{-1}, \quad Y^*_{H,2} = (1-\psi) Y^*_H \left(\frac{P^*_{H,2}}{P^*_H} \right)^{-1} \tag{10}$$

and

$$Y^*_H = \gamma C^* \left(\frac{P^*_H}{P^*} \right)^{-\theta} \tag{11}$$

where P^* is the consumer price index in the rest of the world and C^* is *per capita* consumption in the rest of the world. $p^*_{H,1}(h_1)$ and $p^*_{H,2}(h_2)$ are the foreign currency prices of home goods h_1 and h_2. $P^*_{H,1}$ and $P^*_{H,2}$ are the foreign currency price indices of home fixed-price and flexible-price goods and P^*_H is the foreign currency price index of all home goods. The law of one price is assumed to hold so $p_{H,i}(h_i) = Sp^*_{H,i}(h_i)$ for all h_i, $i = 1, 2$ and $P_{H,i} = SP^*_{H,i}$ for $i = 1, 2$. But note that purchasing power parity across consumer price indices does not hold because home bias implies that home and foreign consumers have different consumption baskets. Note again that θ is the price elasticity of demand for home goods.

The rest of the world is assumed to be so large relative to the home country that all variables relating to foreign agents' behaviour are exogenous from the point of view of the home country. The foreign country behaves, in effect, as a closed economy. Foreign prices are assumed to be invariant. Foreign consumption and output are subject to shocks such that $E[\log C^*] = E[\log Y^*] = 0$ and $Var[\log C^*] = Var[\log Y^*] = \sigma^2_{C^*} > 0$ and $\log Y^* = \log C^* \in [-\epsilon, \epsilon]$. Such behaviour of foreign variables is consistent with an optimal monetary policy response to foreign productivity or labour supply shocks. It is simple to show (given the structure of price setting explained below) that the foreign monetary authority will maximise *per capita* utility of foreign agents by strictly targeting the aggregate price of foreign produced goods. Supply side shocks will therefore cause fluctuations in foreign output and consumption. For foreign agents there is no difference between aggregate producer prices and aggregate consumer prices so the consumer price index will also be fixed by such a monetary policy. Even though shocks to C^* are caused by supply side shocks in the foreign country, their effect on the home country is mainly through the impact on demand for home goods. These shocks can therefore be thought of as demand shocks from the point of view of home agents.

2.6 Risk Sharing

When the elasticity of substitution between home and foreign goods, θ, is equal to unity it is simple to show that the current account of the home economy is always in balance. In this special case the structure of financial markets is irrelevant. This is therefore a particularly useful special case which has frequently been analysed in recent literature. But in the more general case when $\theta \neq 1$ (which is the subject of this paper) the current account may be in deficit or surplus. It is therefore necessary to specify the structure of financial markets.

It is assumed that a sufficiently complete set of state-contingent financial instruments exists so that home agents are able to share consumption risk with foreign consumers. It is shown in the Appendix that equilibrium in the asset market implies

$$\frac{C}{C^*} = A\frac{P^*S}{P} \tag{12}$$

where A is given by

$$A = E\left[\frac{y}{y^*}\right] \tag{13}$$

where $y = YP_H/(SP^*)$ and $y^* = Y^*P_F^*/P^*$ (i.e. y and y^* are home and foreign real output levels expressed in terms of the foreign consumption basket).

It is important to specify the timing of asset trade. It is assumed that asset trade takes place after the choice of monetary policy rule. This implies that agents can insure themselves against the risk implied by a particular policy rule but they can not insure themselves against the choice of rule.[17]

[17] If, alternatively, asset trade takes place before the monetary policy rule is chosen, it would be possible for agents to insure themselves against the choice of rule. This could have very significant implications for the optimal choice of rule. The home monetary authority would be tempted to choose a rule which implies very high volatility of demand for home goods. The high volatility of demand would discourage home labour supply and reduce home work effort but the level of home consumption would be protected by the risk-sharing arrangement. Effects such as these certainly raise interesting questions about the interaction between policymaking and financial markets. But, in the context of the current exercise, they are a distraction from the main focus of interest. For this reason attention is confined to the case where asset trade takes place after the monetary rule is chosen.

To understand some of the implications of asset market equilibrium it is useful to consider a second-order approximation of A. Using a hat to indicate the log deviation of a variable from a non-stochastic steady state, it is possible to write

$$\hat{A} = E\left[\hat{y} - \hat{y}^*\right] + \lambda_A + O\left(\epsilon^3\right) \tag{14}$$

where $\lambda_A = E\left[(\hat{y} - \hat{y}^*)^2\right]/2$ and the term $O\left(\epsilon^3\right)$ contains all terms of third order and higher in deviations from the non-stochastic steady state.[18] The expressions (12) and (14) show that (other things being equal) consumption in the home economy is increasing in the expected difference between home real output and foreign real output. In addition, it is apparent from the definition of λ_A, that home consumption is negatively related to the correlation between home and foreign real output. This last effect arises because, when output levels are less than perfectly correlated, home agents are able to provide insurance to agents in the foreign economy. The value of this insurance (and thus the value of shares in home output) declines as home and foreign output become more correlated.

2.7 Optimal Price Setting

Individual agents are each monopoly producers of a single differentiated product. They therefore set prices as a mark-up over marginal costs where the mark-up is given by $\phi/(\phi - 1)$.

Flexible-price producers are able to set prices after shocks have been realised and monetary policy has been set. In equilibrium all flexible-price producers set the same price so $P_{H,2} = p_{H,2}\left(h_2\right)$ for all h_2. The first order condition for the choice of price implies the following

$$P_{H,2} = \frac{\phi}{\phi - 1} KPC \tag{15}$$

Fixed-price agents must set prices before shocks have been realised and monetary policy is set. Prices are set in home currency (so there is full pass-through from exchange rate changes to foreign currency prices). The first order condition for fixed-price producers implies

$$P_{H,1} = \frac{\phi}{\phi - 1} \frac{E\left[KY_1\right]}{E\left[Y_1/(PC)\right]} \tag{16}$$

where Y_1 is the total output of fixed-price producers (expressed *per capita* of the population of fixed-price producers), which is given by

$$Y_1 = \frac{1}{\psi}\left(Y_{H,1} + Y_{H,1}^*\right) = Y\left(\frac{P_{H,1}}{P_H}\right)^{-1} \tag{17}$$

[18] The remainder term in a second-order expansion of any equation is at most of order $O(\epsilon^3)$ because the log deviations of all the endogenous variables of the model are proportional to the log deviations of the shocks and the shocks are of maximum absolute size ϵ.

where Y is total output of the home economy, which is given by

$$Y = Y_H + Y_H^* \tag{18}$$

In order to understand some of the implications of price setting, it is again useful to consider a second-order approximation. By defining $V_1 \equiv KY_1$ and $V_2 \equiv Y_1/(PC)$ so $P_{H,1} = E[V_1]/E[V_2]$ it is possible to approximate (16) with

$$\hat{P}_{H,1} = E\left[\hat{V}_1 - \hat{V}_2\right] + \lambda_{P_H} + O\left(\epsilon^3\right) \tag{19}$$

where $\lambda_{P_H} = E[(\hat{V}_1^2 - \hat{V}_2^2)/2]$, $\hat{V}_1 = \hat{K} + \hat{Y}_1$ and $\hat{V}_2 = \hat{Y}_1 - \hat{P} - \hat{C}$. In this equation the term λ_{P_H} can be interpreted as a form of risk premium which reflects the fact that prices are set in advance of shocks being realised.[19]

2.8 Money Demand

The first order condition for the choice of money holdings is

$$\frac{M}{P} = \chi C \tag{20}$$

It is assumed that the monetary authority adjusts the money stock so as to achieve whatever target is being considered. In general, monetary policy will take the form of a feedback rule, linking the money stock to the realisation of shocks, of the form

$$M = K^{\delta_K} C^{*\delta_{C^*}} \tag{21}$$

where the parameters δ_K and δ_{C^*} are chosen *ex ante* by the monetary authority (which is assumed to be able to commit to its choice of monetary rule).[20]

3 Welfare and Model Solution

One of the main advantages of the model just described is that it provides a very natural and tractable measure of welfare which can be derived from the aggregate utility of agents. Following Obstfeld and Rogoff (1998, 200a, 2002) it is assumed that the utility of real balances is small enough to be neglected. It is therefore possible to measure *ex ante* aggregate welfare using the following

$$\Omega = E\left[\psi\left(\log C - KY_1\right) + (1-\psi)\left(\log C - KY_2\right)\right] \tag{22}$$

It is shown in the Appendix that the price setting conditions imply the following relationships

$$E\left[KY_1\right] = E\left[KY_2\right] = \frac{\phi - 1}{\phi} \tag{23}$$

[19] This risk premium has previously been noted and analysed in Rankin (1998).

[20] It is necessary to assume commitment because the home monetary authority *ex post* faces a temptation to deviate from the announced monetary rule.

This allows welfare to be written more compactly as follows

$$\Omega = E[\log C] - \frac{\phi - 1}{\phi} \tag{24}$$

Thus welfare depends only on the expected log of consumption.

It is not possible to derive an exact expression for $E[\log C]$ (except in special cases). The complication arising in this model (which does not arise in other models used in recent literature) is contained in equations (6) and (18). When θ is not equal to unity neither of these equations is linear in logs. The model is therefore solved as a second-order approximation around a non-stochastic steady state. This allows a second-order accurate solution for $E[\log C]$ to be derived. A second-order approximation of the welfare measure is given by

$$\Omega_D = E[\hat{C}] + O\left(\epsilon^3\right) \tag{25}$$

where Ω_D is the deviation in the level of welfare from the non-stochastic steady state (and, as before, a hat indicates the log-deviation of a variable from the non-stochastic steady state).

Before deriving a solution for the welfare measure it is first useful to rewrite the main equations of the model in second-order log-deviation form. The relevant equations are summarised in Table 1. Most of the equations are linear in logs and thus do not require any approximation when converting to log-deviation form. There are just four equations which require approximation. These are: the definition of aggregate output (equation (6) in Table 1); the price setting condition for fixed-price producers (equation (7) in Table 1); the definition of aggregate consumer prices (equation (10) in Table 1); and the definition of A in the asset market equilibrium condition (equation (14) in Table 1). There are thus four equations where second-order terms enter the model, and four second-order terms, namely λ_{P_H}, λ_{CPI} , λ_Y and λ_A.

Welfare and λ_{P_H}, λ_{CPI} , λ_Y and λ_A

The determinants of λ_{P_H}, λ_{CPI} , λ_Y and λ_A will be discussed in detail below. First notice that the equations in Table 1 can be used to solve for $E[\hat{C}]$, and thus also for Ω_D, in terms of λ_{P_H}, λ_{CPI} , λ_Y and λ_A as follows

$$\Omega_D = E[\hat{C}] = \frac{-\psi[\theta(2-\gamma)-1]\lambda_{P_H} - \theta E[\lambda_{CPI}] + E[\lambda_Y] + \lambda_A}{\theta(2-\gamma)-1+\gamma} + O\left(\epsilon^3\right) \tag{26}$$

In what follows attention will be confined to parameter sets which ensure that the coefficient on λ_{P_H} in this expression is negative. Thus, it is assumed that $\theta > 1/(2-\gamma)$. The interpretation of this restriction is postponed until the full structure of the welfare measure has been described.

The impact on welfare of the four second-order terms, λ_{P_H}, λ_{CPI} , λ_Y and λ_A, is now discussed.

(1)	$\hat{M} - \hat{P} = \hat{C}$
(2)	$\hat{M} = \delta_K \hat{K} + \delta_{C^*} \hat{C}^*$
(3)	$\hat{Y}_H = \hat{C} - \theta \left(\hat{P}_H - \hat{P} \right)$
(4)	$\hat{Y}_H^* = \hat{C}^* - \theta \left(\hat{P}_H - \hat{S} \right)$
(5)	$\hat{Y}_1 = \hat{Y} - \left(\hat{P}_{H,1} - \hat{P}_H \right)$
(6)	$\hat{Y} = (1-\gamma)\hat{Y}_H + \gamma \hat{Y}_H^* + \lambda_Y + O\left(\epsilon^3\right)$
(7)	$\hat{P}_{H,1} = E\left[\hat{V}_1 - \hat{V}_2 \right] + \lambda_{P_H} + O\left(\epsilon^3\right)$
(8)	$\hat{P}_{H,2} = \hat{K} + \hat{P} + \hat{C}$
(9)	$\hat{P}_H = \psi \hat{P}_{H,1} + (1-\psi)\hat{P}_{H,2}$
(10)	$\hat{P} = (1-\gamma)\,\hat{P}_H + \gamma \hat{S} + \lambda_{CPI} + O\left(\epsilon^3\right)$
(11)	$\hat{V}_1 = \hat{K} + \hat{Y}_1$
(12)	$\hat{V}_2 = \hat{Y}_1 - \hat{P} - \hat{C}$
(13)	$\hat{C} - \hat{C}^* = \hat{A} + \hat{S} - \hat{P}$
(14)	$\hat{A} = E\left[\hat{y} - \hat{y}^*\right] + \lambda_A + O\left(\epsilon^3\right)$
(15)	$\hat{y} = \hat{Y} + \hat{P}_H - \hat{S}$
(16)	$\hat{y}^* = \hat{Y}^* = \hat{C}^*$
(17)	$\lambda_{P_H} = \frac{1}{2} E\left[(\hat{K} + \hat{Y}_1)^2 - (\hat{Y}_1 - \hat{C} - \hat{P})^2 \right]$
(18)	$\lambda_{CPI} = \frac{1}{2}(1-\gamma)\gamma(1-\theta)\left[\hat{P}_H - \hat{S} \right]^2$
(19)	$\lambda_Y = \frac{1}{2}(1-\gamma)\gamma \left[\hat{Y}_H - \hat{Y}_H^* \right]^2$
(20)	$\lambda_A = \frac{1}{2} E\left[(\hat{y} - \hat{y}^*)^2 \right]$

Table 1: Second-order approximation of the model

As previously explained, λ_{P_H} represents a form of risk premium in the prices of fixed-price agents which reflects the fact that these prices are set before shocks are realised. The welfare expression (26) shows that, for values of θ greater than $1/(2-\gamma)$, λ_{P_H} has a negative effect on welfare. This can be understood as follows. An increase in λ_{P_H} raises the price of home goods. This reduces the demand for home goods and, when θ is large enough, this reduces home real income. This reduces the expected level of home consumption and thus reduces home welfare.[21]

The two terms λ_{CPI} and λ_Y arise from the process of approximating the definitions of aggregate consumer prices and aggregate output. The welfare expression (26) shows that λ_{CPI} has a negative effect on welfare, while λ_Y has a positive effect on welfare. A rise in λ_{CPI} raises the cost of the home consumption basket and reduces the expected level of consumption for home agents. On the other hand a rise in λ_Y increases real output and income of home agents and thus increases the expected level of home consumption.

As previously explained, λ_A captures the fact that home agents receive a benefit from being able to provide insurance to foreign agents when home and foreign output levels are not perfectly correlated. The welfare expression (26) shows that λ_A has a positive effect on welfare. An increase in λ_A represents an increase in the relative insurance value of home assets and this increases the expected home share of world aggregate consumption.

The determinants of λ_{P_H}, λ_{CPI} , λ_Y and λ_A

The determinants of λ_{P_H}, λ_{CPI} , λ_Y and λ_A are now discussed.

It is possible to derive the following expression for λ_{P_H} in terms of the variances[22] of home producer prices, P_H, and the terms of trade, τ:

$$\begin{aligned}\lambda_{P_H} &= \frac{1+\psi(\theta-1)(2-\gamma)\gamma}{2(1-\psi)^2}E[P_H^2]+\frac{(\theta-1)(2-\gamma)\gamma}{2\psi}E[\tau^2] \\ &\quad -\frac{(\theta-1)(2-\gamma)\gamma}{2\psi}(\sigma_K^2+\sigma_{C^*}^2)+O\left(\epsilon^3\right) \qquad (27)\end{aligned}$$

Given the restriction $\theta > 1/(2-\gamma)$ this expression shows that λ_{P_H} depends positively on $E[P_H^2]$ while the effect of $E[\tau^2]$ is positive for $\theta > 1$ and negative for $\theta < 1$. By definition volatility in aggregate producer prices represents volatility of the relative price between fixed-price and flexible-price producers. This increases the output

[21] When $\theta < 1/(2-\gamma)$ the effect on welfare is reversed. In this case an increase in the price of home goods causes an increase in home real income (because the demand for home goods is relatively inelastic) and thus an increase in λ_{P_H} raises home consumption and welfare. As previously stated, attention is focused on the case where $\theta > 1/(2-\gamma)$.

[22] Note that, up to a second-order approximation, $E[\hat{P}_H^2] = Var[\hat{P}_H]$ and $E[\hat{\tau}^2] = Var[\hat{\tau}]$. Second-order accurate expressions for variances can be obtained by solving a first-order approximation of the model (i.e. the set of equations in Table 1 omitting the second-order terms λ_{P_H}, λ_{CPI} , λ_Y and λ_A).

volatility of fixed-price producers and thus increases the risk premium in prices. The effect of terms-of-trade volatility is more complex. The terms of trade affect consumer prices, output and consumption (through the asset market equilibrium condition), so the impact of terms-of-trade volatility on λ_{P_H} is the outcome of a number of potentially offsetting factors. For high values of θ, the main factor appears to be the impact of terms-of-trade volatility on the volatility of fixed-price output. In this case terms-of-trade volatility has a positive effect on the risk premium in prices. But for low values of θ (specifically for $\theta < 1$) it is apparent that terms-of-trade volatility lowers the risk premium in prices.

It is possible to show that λ_{CPI} , λ_Y and λ_A can be written in terms of the volatility of the terms of trade, as follows

$$E[\lambda_{CPI}] = -\frac{1}{2}(1-\gamma)\gamma(\theta-1)E[\tau^2] + O\left(\epsilon^3\right) \tag{28}$$

$$E[\lambda_Y] = \frac{1}{2}(1-\gamma)^3\gamma(\theta-1)^2 E[\tau^2] + O\left(\epsilon^3\right) \tag{29}$$

$$\lambda_A = \frac{1}{2}(2-\gamma)^2\gamma^2(\theta-1)^2 E\left[\tau^2\right] + O\left(\epsilon^3\right) \tag{30}$$

$E[\lambda_{CPI}]$ depends negatively on $E[\tau^2]$ when $\theta > 1$ and positively on $E[\tau^2]$ when $\theta < 1$. When $\theta > 1$, aggregate consumer prices are concave in the log deviation of home and foreign prices. This concavity implies that, other things being equal, the expected level of aggregate consumer prices is reduced when home and foreign prices are less than perfectly correlated. The degree of correlation between home and foreign prices is inversely related to the volatility of the terms of trade. Another way to understand this is to note that, when home and foreign goods are good substitutes (i.e. $\theta > 1$), consumers can *ex post* switch consumption towards the lower priced set of goods, thus lowering the expected cost of the consumption basket. This effect is reversed when $\theta < 1$. In this case aggregate consumer prices are convex in the log deviation of home and foreign prices and the aggregate price level is increased when home and foreign prices are less than perfectly correlated. In this case home and foreign goods are relatively poor substitutes so agents are not willing to shift consumption to the lower priced set of goods.

When $\theta \neq 1$, $E[\lambda_Y]$ depends positively on $E[\tau^2]$. As already explained, λ_Y captures the fact that aggregate output is convex in the log deviations of home and foreign demand. This convexity has the implication that, other things being equal, the expected level of aggregate output is higher when home and foreign demands for home goods are less than perfectly correlated. The size of this effect depends on the volatility of the terms of trade and θ. When $\theta = 1$, home and foreign demands for home goods are perfectly correlated, in which case $\lambda_A = 0$.

When $\theta \neq 1$, λ_A also depends positively on $E[\tau^2]$. λ_A captures the benefit received by home agents from being able to provide insurance to foreign agents. The size of this benefit is increasing in the volatility of home real income. In turn, when $\theta \neq 1$, the volatility of home real income is proportional to the volatility of the terms of trade. When $\theta = 1$, the terms of trade have no effect on home real income, in which case $\lambda_A = 0$.

A closed-form solution for welfare

Using equations (26) to (30) it is possible to write the aggregate welfare measure in the following form

$$\Omega_D = \omega_0 - \omega_L L + O\left(\epsilon^3\right) \tag{31}$$

where

$$L = E[\hat{P}_H^2] + \omega_\tau E[\hat{\tau}^2] \tag{32}$$

and where

$$\omega_0 = \frac{\left(\sigma_K^2 + \sigma_{C^*}^2\right)(\theta - 1)[\theta(2-\gamma) - 1](2-\gamma)\gamma}{2[\theta(2-\gamma) - 1 + \gamma]} \tag{33}$$

$$\omega_L = \frac{\psi[\theta(2-\gamma) - 1][1 + \psi(\theta - 1)(2-\gamma)\gamma]}{2(1-\psi)^2[\theta(2-\gamma) - 1 + \gamma]} \tag{34}$$

$$\omega_\tau = \frac{(1-\psi)^2 \gamma(1-\gamma)^2(\theta - 1)(2\theta - 1)}{\psi[\theta(2-\gamma) - 1][1 + \psi(\theta - 1)(2-\gamma)\gamma]} \tag{35}$$

Notice that, while ω_0, ω_L and ω_τ are functions of the parameters of the model, they are independent of the parameters of the monetary policy rule. In addition ω_L and ω_τ are independent of the variances of the shocks. These expressions show that monetary policy affects welfare only through the effects of policy on the variances of producer prices and the terms of trade. Writing the welfare function in this form makes it particularly simple to describe the form of optimal monetary policy and to interpret the effects of varying parameter values.

4 Welfare Maximising Monetary Policy

The following proposition can now be stated and proved:

Proposition 1 *When* $0 < \gamma < 1$, $0 < \psi < 1$ *and* $\theta > 1/(2-\gamma)$*:*
(a) ω_L *is strictly positive;*
(b) ω_τ *is increasing in* θ, *strictly negative for* $1/(2-\gamma) < \theta < 1$, *equal to zero for* $\theta = 1$ *and strictly positive for* $\theta > 1$.

Proof. The proof follows from expressions (34) and (35). ■

Proposition 1 has two implications. First, part (a) ensures that the weight ω_τ is the only coefficient in the welfare function that is relevant for determining optimal policy.[23] And second, part (b) shows that the size and sign of ω_τ depends on the strength of the expenditure switching effect, i.e. θ.

[23] Notice that fully optimal monetary policy (from the point of view of the home country) can be achieved by delegating monetary policy to an independent monetary authority and assigning a loss function of the form (32).

First consider the implications of $\theta = 1$. In this case ω_τ is zero and it is clear that optimal monetary policy completely stabilises the producer price index. This corresponds to the case which has frequently been analysed in recent literature. The underlying explanation for the optimality of price stabilisation in this case is contained in expressions (26) to (30). Expressions (27) to (30) show that, when $\theta = 1$, terms-of-trade volatility disappears from all four of the second-order terms. Indeed, in this case $E[\lambda_{CPI}] = E[\lambda_Y] = \lambda_A = 0$. Thus welfare depends only on the risk premium in prices, λ_{P_H}, and λ_{P_H} depends only on the volatility of producer prices. Thus the risk premium is minimised and welfare is maximised by completely stabilising prices.[24]

Now consider the implications of $\theta > 1$. In this case optimal policy will require some balance between stabilising producer prices and stabilising the terms of trade. Furthermore, the more powerful is the expenditure switching effect (i.e. the larger is θ) the more weight the monetary authority should place on stabilising the terms of trade. Again the underlying explanation for these effects is contained in expressions (26) to (30). Expressions (28) to (30) show that $E[\lambda_{CPI}]$, $E[\lambda_Y]$ and λ_A all depend on the volatility of the terms of trade when $\theta > 1$. Notice, however, that all three of the terms contribute a positive effect of terms-of-trade volatility to the overall welfare expression given in (26). These terms thus create an incentive to increase, rather than decrease, the volatility of the terms of trade. The fundamental cause of the *overall* incentive to stabilise the terms of trade therefore must come from the effect of terms-of-trade volatility on the risk premium in prices, λ_{P_H}. Expression (27) shows that, when $\theta > 1$, terms-of-trade volatility increases the risk premium in prices and (26) shows that this has a negative effect on welfare. This effect outweighs the positive effect of terms-of-trade volatility coming from the other second-order terms.

Notice that, even when $\theta > 1$, ω_τ is zero when $\gamma = 0$. Obviously in a closed economy the international terms of trade are irrelevant. In this case only the volatility of producer prices matters for welfare. Thus the optimal monetary policy is completely to stabilise the producer price index.[25]

Notice that ω_τ is also zero when $\gamma = 1$. Thus terms-of-trade volatility has no impact on welfare in a fully open economy. This is a somewhat surprising and counterintuitive result. It would, perhaps, be natural to expect the impact of terms-of-trade volatility to rise monotonically with the degree of openness and to be at a maximum for a fully open economy. This intuition, however, neglects the fact that terms-of-trade volatility has several (potentially offsetting) effects on welfare.

[24] It is important to emphasise that the optimality of price stabilisation in the $\theta = 1$ case is not a general result. It would break down, for instance, if there were stochastic shocks to the monopoly mark-up, or if prices were set in the currency of the consumer (rather than in terms of the currency of the producer). These mechanisms are omitted from the current model in order to isolate and highlight the role of the expenditure switching effect.

[25] Recall that this is the monetary policy assumed for the foreign economy. Again it is important to emphasise that the optimality of price targeting for a closed economy is not a general result.

As previously noted, (28) to (30) show that $E[\lambda_{CPI}]$, $E[\lambda_Y]$ and λ_A all generate a positive welfare effect from terms-or-trade volatility when $\theta > 1$. When γ is less than unity this positive welfare effect is offset by the negative welfare effect coming from the risk premium in prices, λ_{P_H}. But the relative balance between these effects changes as the degree of openness is increased. Most importantly, λ_A rises as γ increases. This is because, the more open is the home economy, the more similar the consumption baskets of home and foreign agents. This raises the welfare gains from risk sharing and thus raises the value of shares in home output. In the extreme case of a fully open economy, the welfare benefit from terms-of-trade volatility coming from λ_A perfectly offsets the welfare cost of terms-of-trade volatility coming through λ_{P_H}. The net result is that the welfare effect of term-of-trade volatility is zero when $\gamma = 1$.

Figure 1 illustrates the quantitative effects on ω_τ of varying θ and γ (for $\psi = 1/2$). It can be seen that terms-of-trade volatility can become a significant term in the welfare function for empirically relevant values of θ and γ. For many small or medium sized countries $\gamma = 0.5$ is a reasonable approximation. This suggests a value of ω_τ in the range 0.1 to 0.15 for values of θ in the range suggested by the empirical evidence quoted in the Introduction (i.e. θ between 5 and 10).

Finally consider the case where $1/(2-\gamma) < \theta < 1$. In this case, terms-of-trade volatility has a positive effect on welfare and thus optimal monetary policy should allow some volatility in producer prices in order to generate additional volatility in the terms of trade. Again the underlying explanation for these effects is contained in expressions (26) to (30). Expressions (26), (27), (29) and (30) show that λ_{P_H}, $E[\lambda_Y]$ and λ_A all contribute a positive welfare effect of terms of trade volatility when $1/(2-\gamma) < \theta < 1$. Taken together these terms outweigh the negative welfare effect of terms-of-trade volatility coming from $E[\lambda_{CPI}]$.

Notice that, in the extreme case when $\theta = 1/(2-\gamma)$, the coefficient on λ_{P_H} in (26) becomes zero. In this case welfare depends only on the volatility of the terms of trade, and, furthermore, welfare is increasing in terms-of-trade volatility. For values of θ very close to this extreme case it is optimal for the monetary authority to create an infinite amount of terms-of-trade volatility. Thus, as can be seen in Figure 1, ω_τ tends to $-\infty$ as θ tends to $1/(2-\gamma)$. The underlying explanation for this effect is that, for a sufficiently low value of θ, the home country's monopoly power in the world market becomes so strong that it is optimal to drive the supply of home goods down to zero. It is for these reasons that attention is restricted to values of θ greater than $1/(2-\gamma)$.[26]

[26] Notice that this lower bound on θ is positively related to γ (the degree of openness). For a very closed economy (i.e. low γ) the desire to exploit the monopoly power of the home economy in the world market is offset by the fact that the share of home goods in the home consumption basket is very high. For very open economies (i.e. high γ) this offsetting effect is weaker.

5 Simple Targeting Rules

It is apparent from the previous section that the optimal monetary strategy in this model economy is relatively easy to specify. A closed-form solution for the welfare function is derived and its implications for the optimal monetary rule are clear. It is even possible to see that the optimal monetary rule can be implemented by an independent monetary authority minimising a loss function which includes producer-price volatility and terms-of-trade volatility. Nevertheless, despite these clear results, there are reasons to suppose that the practical implementation of such an optimal policy may be difficult. Even in this very simple model the coefficients in the optimal loss function are quite complicated combinations of the parameters of the model. The structure of the optimal loss function is sensitive to uncertainty about the structure of the model and to uncertainty about the true values of the underlying model parameters. Even if these problems can be overcome it is not clear how minimising a loss function can be translated into the practical day to day business of setting monetary policy. For these reasons it is useful to consider and compare a range of non-optimal but simple targeting rules.

In the context of the current model there are three possible simple rules.[27] These are: strict targeting of the producer price index, strict targeting of the consumer price index and strict targeting of the nominal exchange rate.[28] In each case these rules provide a nominal anchor which is based on an easily observable variable. Given the welfare measure derived above, it is interesting to investigate the relative welfare performance of these rules for different values of the model parameters.

Numerical solutions are used to illustrate the welfare comparison. It is found that the welfare ranking of the three regimes depends on parameter values. To illustrate the results, in what follows X_1 is used to denote the value of θ at which producer-price targeting yields the same welfare level as consumer-price targeting, and X_2 is used to denote the value of θ at which consumer-price targeting yields the same level of welfare as a fixed exchange rate. The welfare ranking can be summarised as follows. For values of θ less than X_1 producer-price targeting is found to yield the highest welfare. For values of θ between X_1 and X_2 consumer-price targeting yields the highest welfare. And for values of θ higher than X_2 a fixed exchange rate yields the highest welfare.

The reason for this ranking can be explained with reference to Figure 1. For small values of θ, the volatility of the terms of trade is relatively unimportant for welfare, i.e. ω_τ is quite small. In this case producer-price targeting is a reasonable approximation to fully optimal policy. For higher values of θ terms-of-trade volatility

[27] For the purposes of this exercise, targeting variable Z is taken to mean that the monetary authority adopts a rule which ensures that *ex post* $\hat{Z} = 0$.

[28] It would also be possible to consider nominal income targeting, as advocated in McCallum and Nelson (1999). If the model were recast as a dynamic framework with an explicit interest rate, it would be possible to analyse interest rate feedback rules of the form suggested by Taylor (1993). Nominal income targeting is considered in a related model in Sutherland (2002c).

becomes more important in welfare, i.e. ω_τ rises. Since the consumer price index is effectively a weighted sum of producer prices and the nominal exchange rate, stabilising the consumer price index implicitly involves a certain degree of exchange rate stabilisation. This helps to stabilise the terms of trade. So, when θ takes on an intermediate value, consumer-price targeting is a better approximation to the optimal policy than producer-price targeting. When θ is large the weight on the terms of trade becomes large (see Figure 1) so stabilisation of the terms of trade becomes an important consideration for policy. In this case a fixed exchange rate is the best approximation of fully optimal policy because it helps to stabilise the terms of trade most effectively.

To illustrate the quantitative implications of these results Figure 2 plots X_1 and X_2 against γ. The solid lines show the case where $\psi = 1/2$ while the dashed lines show the case where $\psi = 1/4$. It is apparent from this figure that X_2 is relatively high. In neither of the cases illustrated is X_2 less than 10. It therefore appears that a fixed exchange rate is the best simple rule only for implausibly high values of θ.[29] On the other hand, for empirically relevant values of γ, X_1 can be well within the empirically relevant range of values for θ. Thus it is plausible that consumer price targeting is the best simple rule for empirically relevant parameter combinations.[30]

6 Concluding Comments

This paper has presented a simple model which allows for a non-unit elasticity of substitution between home and foreign goods. It is shown that aggregate welfare for the home economy is a weighted sum of the variances of producer prices and the terms of trade. The fully optimal monetary policy can be achieved by assigning a loss function to the monetary authority which includes these two terms.

Three simple non-optimal targeting rules are compared and the implications of the degree of international substitutability are analysed. For low values of substitutability, producer-price targeting is best. At intermediate values of substitutability, consumer-price targeting is best. And for very high values of substitutability, a fixed nominal exchange rate is best. The underlying reason for these results is that terms-of-trade shocks tend to cause inefficiently large fluctuations in the demand for home produced goods, and therefore home work effort, when the expenditure switching effect is strong. This creates an incentive for the home policy maker to stabilise the terms of trade. However, terms-of-trade fluctuations are only inefficient from the point of view of home agents. From the perspective of world welfare,

[29] Notice that the plot of X_1 is U-shaped. Thus a fixed rate is a very poor substitute for optimal policy for a very closed economy and also for a relatively open economy. Recall that ω_τ is very small for both γ close to zero and γ close to unity i.e. terms-of-trade variability is relatively unimportant for both a very closed economy and a very open economy.

[30] Notice that X_2 is close to zero for small γ. This is because, for very closed economies, there is very little difference between producer-price targeting and consumer-price targeting.

terms-of-trade fluctuations are optimal. So a world policy maker would not attempt to stabilise the terms of trade.[31]

The model presented in this paper is restricted in a number of respects, and relaxing some of these restrictions is likely to have an impact on the results. For instance, the degree of risk aversion in consumption is fixed by the assumption that utility is logarithmic in consumption. Previous authors have noted that risk aversion is an important parameter in determining the welfare effects of exchange rate volatility.[32] The source of shocks can also be important in determining the welfare impact of exchange rate policy. Non-optimal variations in the degree of monopoly power or shocks to foreign monetary policy can have an important impact on the relative performance of different nominal target variables. These topics are likely to form interesting lines of future research.[33]

Appendix

Portfolio allocation and asset prices

In order to derive equilibrium asset prices it is initially convenient to assume that the share of the home population in the world population is n (where $n > 0$). The small-open-economy assumption employed in the main body of the paper corresponds to the limit as n tends to zero.

There are two independent sources of shocks in the model, so efficient sharing of consumption risk can be achieved by allowing trade in two (independent) state-contingent assets. Assume that one asset has a payoff correlated with home output and the other has a payoff correlated with foreign output, where both home and foreign output are expressed in terms of the foreign consumption basket. Thus a unit of the home asset pays y and a unit of the foreign asset pays y^* where $y = YP_H/(SP^*)$ and $y^* = Y^*P_F^*/P^*$. The portfolio payoffs for home and foreign agents are given by the following

$$R(h) = \zeta_H(h)[y - q_H]SP^*/P + \zeta_F(h)[y^* - q_F]SP^*/P \tag{36}$$

$$R^*(f) = \zeta_H^*(f)[y - q_H] + \zeta_F^*(f)[y^* - q_F] \tag{37}$$

where $\zeta_H(h)$ and $\zeta_F(h)$ are holdings of home agent h of the home and foreign assets, $\zeta_H^*(f)$ and $\zeta_F^*(f)$ are the holdings of foreign agent f of home and foreign assets and

[31] The weight given to terms-of-trade stabilisation in the home country's welfare function represents the home country's temptation to deviate from the coordinated policy. Notice that this temptation is zero when $\theta = 1$. In this case there are no gains from policy coordination (as previously demonstrated by Obstfeld and Rogoff (2002)).

[32] See Devereux and Engel (2003).

[33] Clarida, Gali and Gertler (2002), Benigno and Benigno (2003b) and Sutherland (2002c) consider the implications of 'cost-push' shocks for the choice of international monetary regime, and Senay and Sutherland (2004) consider foreign monetary shocks.

q_H and q_F are the unit prices of the home and foreign assets. There are four first-order conditions for the choice of asset holdings. After some rearrangement they imply the following four equations

$$E\left[C^{-1}y\right] = E\left[C^{-1}\right] q_H, \quad E\left[C^{-1}y^*\right] = E\left[C^{-1}\right] q_F \tag{38}$$

$$E\left[C^{*-1}y\right] = E\left[C^{*-1}\right] q_H, \quad E\left[C^{*-1}y^*\right] = E\left[C^{*-1}\right] q_F \tag{39}$$

Using the solution procedure outlined in Obstfeld and Rogoff (1996, pp 302-3) it is possible to show that consumption levels in the two countries are given by

$$C = \frac{q_H\left[ny + (1-n)y^*\right]}{nq_H + (1-n)q_F}\frac{P^*S}{P}, \quad C^* = \frac{q_F\left[ny + (1-n)y^*\right]}{nq_H + (1-n)q_F} \tag{40}$$

Notice that (40) implies

$$\frac{C}{C^*} = \frac{q_H}{q_F}\frac{P^*S}{P} \tag{41}$$

The two asset prices are given by

$$q_H = \frac{E\left[\frac{y}{ny+(1-n)y^*}\right]}{E\left[\frac{1}{ny+(1-n)y^*}\right]}, \quad q_F = \frac{E\left[\frac{y^*}{ny+(1-n)y^*}\right]}{E\left[\frac{1}{ny+(1-n)y^*}\right]} \tag{42}$$

Taking the limit of (42) as n tends to zero implies

$$\frac{q_H}{q_F} = E\left[\frac{y}{y^*}\right] \tag{43}$$

Simplifying the welfare expression

Using (15), (16) and (17) (and the expression for flexible-price output) it is simple to show that

$$E\left[KY_1\right] = E\left[KY_2\right] = \frac{\phi-1}{\phi}E\left[\frac{P_HY}{PC}\right] \tag{44}$$

Using the definition of y (given above equation (36)) together with (40) (with $n=0$) and (43) it follows that

$$E\left[\frac{P_HY}{PC}\right] = E\left[y\frac{SP^*}{PC}\right] = \frac{q_F}{q_H}E\left[\frac{y}{y^*}\right] = 1 \tag{45}$$

which confirms the relationships in (23) which are used to simplify the welfare expression.

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Figure 1: Terms-of-trade weight in welfare (omega-tau)

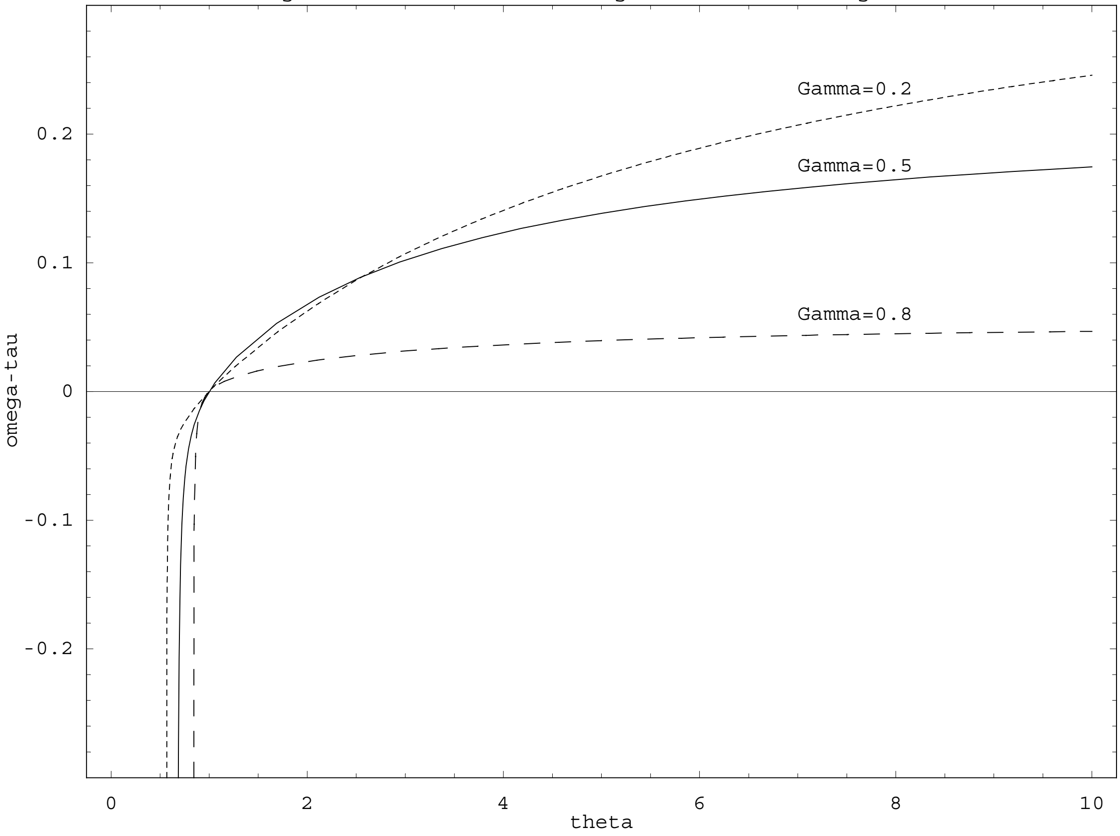

Figure 2: X1 and X2