Monetary Union, Entry Conditions and Economic Reform

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Abstract

This paper models the behaviour of a potential entrant into a monetary union where there is an inflation entry condition. In addition to making a monetary policy decision during a qualifying period, the potential entrant must make a decision about structural reform. The paper shows that the entry condition can have two undesirable effects. First, it can lead to multiple equilibria because inflationary expectations acquire a self-fulfilling property. Second, the entry condition can lead to a reduction in the amount of reform. This is because the entry condition reduces inflationary expectations and thus reduces the incentive to reform.

Key words: Monetary union, convergence, entry conditions.
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1. Introduction

This paper models the decision problem facing a potential entrant into a monetary union where admittance into the union is conditional on the entrant meeting a target inflation rate. The paper is thus an attempt to build a formal model of one of the criteria specified in the Maastricht Treaty.\(^1\) This is an obviously important issue in the lead up to the potential first phase of monetary union in Europe. It is a question of longer term importance as well. The expansion of European Monetary Union to include countries not admitted in the first phase is likely to be a recurring issue for years to come.

A primary aim of European monetary union is low and stable inflation within the new currency area. This concern for monetary discipline is present in the formulation of the Maastricht entry conditions.\(^2\) The Maastricht Treaty specifies conditions on variables having important implications for the disciplined operation of monetary policy. Entry conditions are set in terms of fiscal deficits, public debt, exchange rates, inflation rates and interest rates. The choice of this set of variables must be partially motivated by a desire to set conditions in terms of easily measured and observable outcomes. But this list of entry conditions does not cover all the potential factors which determine the inflation performance of a monetary union. The efficiency of the tax system and labour market flexibility in member countries can also have a major impact on inflation. Distortions produced by tax systems or labour markets can be removed by appropriate government policies. Thus an important issue is the extent that the entry conditions of the Maastricht Treaty encourage or discourage appropriate structural reforms by potential entrants. This is one of the main themes of this paper.

Our model focuses on an inflation entry condition and considers the impact of the condition on the behaviour of a potential entrant to a monetary union. Time is divided into two periods. Before the first

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1 While the Maastricht entry conditions have been the subject of much general debate in the literature (see for instance Artis (1996), Buiter et al (1993), Buiter (1995), De Grauwe (1995, 1996), Gros and Thygesen (1990) and Thygesen (1993)), there has been surprisingly little formal analysis of the specific implications of entry conditions. One exception is Winkler (1997) who considers a formal model of a potential entrant to a monetary union where a decision about “convergence effort” has to be made.

2 De Grauwe (1995, 1996) argues that the Maastricht entry conditions were designed specifically with a view to ensuring low inflation within the union. Similarly Artis (1996) suggests that the entry conditions are there to ensure that only countries which share Germany's low inflation objectives will be admitted to the union.
period starts, the potential entrant must make a decision about reforming its economy. Reform leads to a reduction in distortions in the tax system and labour market and therefore reduces the inflationary bias produced by discretionary monetary policy. Reform is costly and the amount of reform depends on the benefits to the entrant. In the first period the potential entrant sets monetary policy and this determines its inflation rate. If inflation is below the entry threshold then the entrant is allowed into monetary union in the second period. If period one inflation is above the threshold level, then the potential entrant is excluded from the union and sets its own monetary policy in period two.

The model shows that the entry condition can have two undesirable effects. Firstly, the entry condition can lead to multiple equilibria in period one because it generates a self-fulfilling property in inflationary expectations. If private-sector wage setters are confident that the potential entrant will choose to satisfy the entry condition, inflationary expectations will be low and this will make it easier for the potential entrant to satisfy the entry condition. If, on the other hand, wage setters expect the potential entrant to choose to break the entry condition, inflationary expectations will be high and it will be more difficult to satisfy the condition. Secondly, and paradoxically, a tighter entry condition can lead to a reduction in the amount of reform undertaken. This is because the entry condition can help reduce inflationary expectations in period one. Lower inflationary expectations bring down inflation in period one without the need for costly reform. The perceived benefits of reform are therefore lower and the potential entrant chooses to implement less reform.

The structure of the model is presented in section 2. The solution of the model has to be obtained recursively so section 3 derives the solution to period 2, section 4 derives the solution to period 1 and section 5 considers the reform decision. Section 6 concludes the paper.

2. The Model

The model is made up of an existing monetary union and a potential

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3 In modelling reform in this way we are building on a structure first introduced in Sibert (1997).

4 Winkler (1997) considers a reduced-form linkage between entry conditions and reform effort. We specify a much more detailed structure for the linkage between entry conditions, inflation and the incentives for reform.
entrant to the union. There are two periods, 1 and 2. Period 1 is a qualifying period during which the potential entrant remains outside the monetary union. The outside country is allowed into the union in period 2 if it satisfies an entry condition. The entry condition, which takes the form of a maximum inflation rate for period 1, denoted $\bar{\pi}$, is set by the monetary union before the start of period 1.\(^5\) After the entry condition is set, but before any other decisions are made, the outside country may carry out reform. We assume that reform is fully implemented before period 1 begins. The outside country then sets monetary policy for period 1.\(^6\) If inflation in period 1 is less than or equal to $\bar{\pi}$, the outside country joins the union in period 2. In this case the monetary authority for the union sets monetary policy in period 2 for the whole union. If the outside country fails to qualify, it stays outside the union and therefore sets its own monetary policy in period 2.

In period $t$, output in the outside country is given by

$$y_t = \pi_t - \pi^e_t - \tau,$$

(1)

where $y$ is output measured as the log deviation from the socially optimal level, $\pi$ is inflation, the superscript "e" denotes a subjective expectation by private sector agents and $\tau$ represents the output loss due to distortions.

The distortions represented by $\tau$ are factors such as market failures in the labour market or inefficiencies in the tax system which tend to push output below its socially optimal level. Notice that equation (1) implies an equilibrium output deviation of $-\tau$. The value of $\tau$ is determined by the amount of reform undertaken by the outside country before the start of period 1 and is fixed at that level throughout periods 1 and 2.

Total output of the initial members of the monetary union is determined by a similar supply function

$$y^* = \pi^*_t - \pi^*_t^e.$$  

(2)

\(^5\) We take it as given that the entry condition is set in terms of inflation. It will become apparent that it would be more effective if the entry condition were set in terms of an amount of reform. However, we assume that the reform effort of the entrant is not verifiable by the monetary union, hence contracts written in terms of reform are not practical.

\(^6\) The monetary authority in the monetary union sets monetary policy for period 1 for the existing members of the union. In the model we present below, this decision has no impact on the behaviour of the outside country so we do not consider it explicitly.
The main difference between the supply functions is that the monetary union faces no distortions.

It is assumed that policymakers can choose inflation. Thus, monetary policy amounts to selecting an inflation rate in each period. Within each period the sequence of events is as follows. First, private sector agents form expectations of inflation and enter into nominal contracts based on those expectations. Second, the monetary authority sets inflation. It is assumed that neither monetary authority can make binding announcements about monetary policy at the start of the period. Private sector agents therefore predict inflation on the basis of their knowledge of policymakers' optimising behaviour.

Policymakers set monetary policy to minimise losses arising from inflation and output deviations. Period-\(t\) losses for the outside country are

\[ L_t = \left( \pi_t^2 + y_t^2 \right)/2. \] (3)

To reduce the number of parameters, we assume that equal weight is placed on output deviations relative to inflation. For the initial members of the monetary union losses are

\[ L_t^* = \left( \pi_t^{*2} + y_t^{*2} \right)/2. \] (4)

Notice that policymakers are assumed to be targeting output at the socially optimal level. In the case of the outside country, where there is an output distortion, the policymaker is targeting an output level which is above that desired by wage setters. As is well known in models of this type (see, for example, Barro and Gordon (1983) and Persson and Tabellini (1990)), this gives rise to an inflationary bias in the outside country. The existing members of the union have no such bias.

In addition to losses arising from output and inflation deviations, the outside policymaker faces reform costs which depend on the amount of reform undertaken. It is assumed that if there is no reform, the distortion is equal to \(\bar{\tau}\). Attaining a distortion of \(\tau\) entails a cost of

\[ C = \left( \bar{\tau} - \tau \right)^2/2. \] (5)
The total loss faced by the policymaker when the reform decision is made is
\[
\Gamma = \eta L_1 + (1-\eta)L_2 + C, \tag{6}
\]
where \(\eta\) is a parameter which measures the weight placed on period 1 relative to period 2 and \(0 \leq \eta \leq 1.\)

Given the sequence of events, the model is solved recursively, i.e. starting with period 2 and working backwards. This process is described in the next sections.

3. Equilibrium in Period 2

There are two possible outcomes that can arise in period 2. Either the outside country has qualified for entry into the union or it has not. In the case where the outside country has not qualified the solution is very simple. The monetary authority in the outside country sets inflation so as to minimise \(L_2\) while taking \(\pi^*_2\) as given. Imposing rational expectations implies the following equilibrium inflation rate
\[
\pi_2 = \pi^*_2 = \tau. \tag{7}
\]

Thus, as stated above, the outside country suffers from an inflation bias which depends on the level of output distortions in the economy.

By (1), (3) and (7)
\[
L_2^N = \tau^2, \tag{8}
\]
where the superscript \(N\) indicates that this is the case where the outside

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\(^7\) The value of \(\eta\) measures the importance of the qualifying period relative to the monetary union period and hence captures a number of factors. The monetary union is presumably expected to last for a very long time so it might be argued that period 2 should have a much greater weight than period 1. Discounting will, however, tend to offset this. It could also be the case that further chances to qualify for monetary union will arise in the future so period 2 could be thought of as the time period between the end of the current qualifying period and the next chance to qualify. This again will tend to reduce the importance of period 2 relative to period 1.

We assume that reform costs are incurred at the start of period 1 but we assume that the size of these costs is independent of the importance of the qualifying period. Hence \(C\) is not weighted by \(\eta\) in the loss function.
country has not joined the union.

In the case where the outside country does qualify for monetary union the outcome is more complicated. In this case monetary policy is set by the monetary authority in the union. The new entrant in the union therefore shares the same actual and expected inflation rate as the rest of the union. Output for the new entrant is therefore given by

\[ y_2 = \pi_2^* - \pi_2^{e*} - \tau. \]  

(9)

It is assumed that the monetary authority in the union sets monetary policy to minimise a weighted-average of the losses of the new entrant and the existing members

\[ L_y = \frac{\theta}{2} \left( \pi_1^{*2} + y_1^* \right) + \frac{1-\theta}{2} \left( \pi_2^{*2} + y_2^* \right), \]

(10)

where \( \theta \) is the weight placed on the new entrant's losses and \( 0 \leq \theta < 1. \)

The monetary authority minimises (10) while taking \( \pi_2^{e*} \) as given. Imposing rational expectations implies

\[ \pi_2^* = \pi_2^{e*} = \theta \tau. \]

(11)

By (1), (3) and (11),

\[ L'_U = \left( 1 + \theta^2 \right) \tau^2 / 2, \]

(12)

where the superscript \( U \) indicates that this is the case where the outside country joins the union.

It is clear from equations (8) and (12) that for any \( \tau \), the outside country benefits from joining the union. A comparison of equations (7) and (11) shows that the monetary union delivers lower inflation than independent monetary policy. By (1), (7) and (11), equilibrium output is the same in both cases. The explanation is as follows. The size of the inflation bias is determined by the level of distortions. The existing monetary union has no output distortion. When the new member enters,

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8 Alternatively it could be assumed that the monetary union's losses depend on the weighted average of output deviations (rather than being a weighted average of \( L \) and \( L^* \)). This makes no significant difference to the results presented in this paper.
the average distortion in the new union is lower than the level of the distortion in the outside country on its own. The new monetary union, therefore, has a lower inflation bias than the outside country. The smaller the weight the monetary union puts on the outside country the more the outside country benefits from membership.

Existing members of the monetary union are made worse off by the entry of the new member. Without the new member, inflation would be zero in the monetary union. The new member raises inflation to $\theta \tau$, which is positive for $\theta > 0$. Therefore, this model does not explain why the monetary union would ever allow the new member to join. We suppose that there are other benefits to monetary union, not modelled here, which outweigh the costs to existing members of allowing the new member to enter.\(^9\) Since the main focus of this paper is on the behaviour of the outside country it is not necessary to consider these other factors explicitly. In any case, it turns out that all our results are valid for the case where $\theta = 0$ so henceforth we set $\theta = 0$ (in order to minimise the number of parameters in the model).

Before moving on we note one further implication of equations (8) and (12). The coefficient on $\tau^2$ is larger in equation (8), the case where the outside country does not enter, than in equation (12), the case where it does enter. If entry is to take place with certainty the outside country has a lower incentive to reform than if it knew it would be excluded.\(^{10}\)

4. Equilibrium in Period 1

This section will show that there are three possible types of rational expectations equilibria that can arise in period 1 and that, in some situations, more than one of these equilibria can exist. The relevant equilibrium depends on the combination of the level of the entry condition, $\bar{\pi}$, and the level of distortions, $\tau$, that are set prior to the start of period 1. The objective of this section is to divide up $(\bar{\pi}, \tau)$ space into regions where each of the different equilibria is relevant.

\(^{9}\) It is clear that, given the structure of our model, the optimal choice of $\bar{\pi}$ is $-\infty$. A more meaningful analysis of the optimal choice of $\bar{\pi}$ would require a model which explicitly included other motives for allowing the outside country to enter the union.

\(^{10}\) The explanation is that future membership of the monetary union allows the outside country to shift some of the bad effects of its distortions onto the other members. It therefore has less incentive to reform when it knows entry is certain. Sibert and Sutherland (1998) analyse this issue more fully.
When the policymaker sets policy for period 1 the relevant loss function to consider is \( \eta L_1 + (1-\eta)L_2 \). The choice of period 1 inflation has an impact on period 2 losses because period 1 inflation determines whether or not the outside country will be allowed into the monetary union. As shown above \( L_2 \) takes on one of two values \( L_2 = L_2^N \) if \( \pi_1 > \bar{\pi} \) and \( L_2 = L_2^U \) if \( \pi_1 \leq \bar{\pi} \).

In solving the policymaker's problem it is convenient initially to ignore the entry condition and to consider the policymaker's optimal "unconstrained" choice of period 1 inflation. In this case the policymaker simply minimises \( L_1 \) taking \( \pi_1 \) as given. This yields

\[
\pi^* = \frac{\pi + \pi^*_1}{2},
\]

where the superscript "o" indicates that this is the optimal "unconstrained" inflation rate. Losses arising from choosing the unconstrained inflation rate are

\[
L^o = \left( \pi + \pi^*_1 \right)^2 / 4,
\]

where again the superscript "o" indicates the unconstrained optimum.

It is also useful to consider the case where the outside country decides to satisfy the entry condition exactly, i.e. to set \( \pi = \bar{\pi} \). In this case losses for period 1 are

\[
L_1^c = \left[ \bar{\pi}^2 + (\bar{\pi} - \pi^*_1 - \tau)^2 \right] / 2,
\]

where the superscript "c" indicates that this is the "constrained" case.

It is now possible to outline three possible cases. In the first case the optimal unconstrained inflation rate is no higher than the entry condition, i.e. \( \pi^*_1 \leq \bar{\pi} \). In this case there is no conflict between optimal inflation policy in period 1 and the desire to enter monetary union in period 2. The outside country can simply set the optimal unconstrained inflation rate.

In the second and third cases the optimal unconstrained inflation rate is above the entry condition and the policymaker must decide whether or not to satisfy the entry condition. The policymaker will choose to satisfy the condition if doing so results in lower costs over periods 1 and 2, i.e. if
\[ \eta L_i^e + (1 - \eta) L_j^e \leq \eta L_i^L + (1 - \eta) L_j^N. \]  

(16)

The policymaker will choose to break the entry condition if inequality (16) does not hold.

To summarise, the three cases are (1) the outside country sets \( \pi_i = \pi_i^o \) and joins monetary union, (2) the outside country sets \( \pi_i = \pi_i^o \) and joins monetary union and (3) the outside country sets \( \pi_i = \pi_i^o \) and does not join monetary union.

It is now necessary to derive rational expectations equilibria and conditions under which these equilibria exist. Figure 1 shows the regions in \((\pi, \tau)\) space where the different equilibria exist. In order to focus on the important mechanisms at work, in constructing Figure 1 we set \( \eta = 1/2 \). The effect of varying \( \eta \) will be analysed later. Consider each of the three cases in turn.

Case 1: \( \pi_i = \pi_i^o \) so it follows that \( \pi_i^e = \pi_i^o \) and thus \( \pi_i^e = \tau \). This is an equilibrium provided \( \pi_i^e \leq \pi_i^o \), i.e. when \( \tau \leq \pi_i^o \). The line OA on Figure 1 marks values of \( \tau \) and \( \pi_i \) for which \( \tau = \pi_i^o \). The region where the case 1 equilibrium exists is therefore the region below OA.

Case 2: \( \pi_i = \pi_i^o \) so it follows that \( \pi_i^e = \pi_i^o \). This is an equilibrium if \( \pi_i^o > \pi_i \) (i.e. the entry condition is binding) and condition (16) holds. After substitution the following condition is derived

\[ (1 - 2\gamma) \tau \leq \pi < \tau, \]  

(17)

where \( \gamma = \sqrt{(1 - \eta)/2\eta} \) (see Appendix 1 for details of the derivation). The relevant region is therefore the area above schedules OA and OB in Figure 1 (where OB plots \( \pi = (1 - 2\gamma) \tau \)).

Case 3: As in case 1 \( \pi_i = \pi_i^o \) so it again follows that \( \pi_i^e = \pi_i^o \) and thus \( \pi_i^e = \tau \). This is an equilibrium if \( \pi_i^o > \pi_i \) (i.e. the entry condition is binding) and condition (16) does not hold. After substitution and rearrangement the following condition is obtained

\[ \pi \leq (1 - \gamma) \tau, \]  

(18)

(See Appendix 1 for details of the derivation.) The relevant region in Figure 1 is therefore the area above schedule OC where OC plots \( \pi = (1 - \gamma) \tau \).

Figure 1 shows that there are four different regions in which different period 1 equilibria (or combinations of equilibria) can arise. In the area
to the right of OA the entry condition is not binding and the outside country joins monetary union. In the region between OA and OC the entry condition is binding and the outside country chooses to satisfy it exactly and therefore enters monetary union. In the region to the left of OB the entry condition is binding but the outside country chooses not to satisfy it and therefore stays outside monetary union. In each of these regions there is a unique rational expectations equilibrium. In contrast to this, in the fourth region, the area between OC and OB, there are two possible equilibria. In this area the entry condition is binding but the outside country will choose whether or not to satisfy the condition depending on the level of private sector expectations. If private agents choose to set inflationary expectations at $\bar{\pi}$, it will be optimal for the outside country to satisfy the entry condition exactly and thus join the union. If, on the other hand, private agents choose to set inflationary expectations at $\tau$, it will be optimal for the outside country to break the entry condition and to stay outside the union. Expectations therefore have a self-fulfilling property, "optimistic" expectations lead to the outside country joining monetary union, while "pessimistic" expectations lead to it failing to join.

Figure 1 demonstrates the first result emphasised in the introduction to this paper, namely that the entry condition can generate multiple equilibria.\textsuperscript{11}

From a theoretical point of view, multiple equilibria raise questions about the selection of appropriate equilibria to analyse. There are two theoretical complications which arise here. Firstly, when there are many private sector agents who form inflationary expectations independently, it is not clear that all agents will choose the same equilibrium. Secondly, in this model there is a decision stage prior to period 1, where the outside policymaker chooses a level of reform, and the behaviour of the policymaker at this stage will be influenced by the equilibrium expected to hold in period 1. This complication will become apparent in the next section. These are interesting issues which will not be pursued in detail in this paper.

From a policy point of view multiple equilibria may be regarded as an undesirable side-effect of using an inflation entry condition as a device for determining membership of a monetary union. Multiple equilibria

\textsuperscript{11} There is a close parallel between the multiple equilibria arising here and the multiple equilibria that arise in models of currency crises such as those of Obstfeld (1994) and Ozkan and Sutherland (1998).
create uncertainty amongst private agents and can therefore aggravate volatility as private agents switch from one level of expectations to another. If this is true then policymakers should take actions which help private agents focus on one equilibrium rather than another.\textsuperscript{12}

5. The reform decision

The final step in solving the model is to analyse the outside country's choice of reform (represented by the choice of \( \tau \)). In making this choice the policymaker aims to minimise the sum of losses over the two periods and the costs of reform, as specified in (6).

Before analysing the optimal choice of \( \tau \) some preliminary comments are necessary. The previous section showed that there were three possible outcomes for periods 1 and 2, labelled cases 1, 2 and 3. Each of the three cases implies a level of total losses (i.e. \( \Gamma = \eta L_1 + (1-\eta)L_2 + C \)) as follows

\[
\Gamma_1 = \left[ 2\eta \tau^2 + (1-\eta)\tau^2 + (\tau - \tau)^2 \right]/2, \tag{19}
\]

\[
\Gamma_2 = \left[ \eta(\pi^2 + \tau^2) + (1-\eta)\tau^2 + (\tau - \tau)^2 \right]/2, \tag{20}
\]

\[
\Gamma_3 = \left[ 2\tau^2 + (\pi - \tau)^2 \right]/2, \tag{21}
\]

where the subscripts indicate cases 1, 2 and 3 (see Appendix 1 for details of the derivation).

It was also shown in the previous section that \((\pi, \tau)\) space can be divided into four regions. Three regions where cases 1, 2 and 3 respectively exist as a unique equilibria and one region where either case 2 or case 3 can be an equilibrium depending on expected inflation. These four regions are shown in Figure 1. It is apparent from that figure that, for any given level of \( \bar{\pi} \), the choice of \( \tau \) determines which of the four regions is relevant.

\textsuperscript{12} In the region of multiple equilibria the policymaker in the outside country clearly prefers the equilibrium which results in entry to the union. This equilibrium not only delivers the benefits of monetary union in period 2, it also delivers lower inflation in period 1 when compared to the case 3 equilibrium. It is therefore in the outside country's interests to focus expectations on the case 2 equilibrium.
Within each region there is a cost minimising level of $\tau$. One of these local minima will also be a global minimum and this will be the policymaker’s optimal choice. However, note that the loss function faced by the outside policymaker has discontinuities at the boundaries between the regions. This implies that, for some levels of $\bar{\pi}$, corner solutions can occur where the outside policymaker chooses a level of $\tau$ exactly on the boundary between two regions.

As already noted at the end of the previous section, the existence of multiple equilibria creates additional complications for analysing the optimal choice of $\tau$. In the region of multiple equilibria two possible equilibria can arise, an optimistic equilibrium (where entry takes place) and a pessimistic equilibrium (where entry does not take place). At the time when the reform decision is being made the policymaker in the outside country cannot predict which of these two equilibria will occur, so it is not clear how the reform decision should be made. Here we sidestep this problem and simply assume that one or other of the equilibria is expected to occur with certainty.\textsuperscript{13}

Having noted these complications it is now possible to consider the optimal choice of $\tau$. It proves convenient to start the analysis by minimising the cost functions (19), (20) and (21) with respect to $\tau$ to obtain the following expressions

$$
\tau_1 = \bar{\pi}/(2 + \eta), \quad (22)
$$

$$
\tau_2 = \bar{\pi}/2, \quad (23)
$$

$$
\tau_3 = \bar{\pi}/3, \quad (24)
$$

where again the subscripts indicate the relevant case.

It is now possible to analyse how the level of reform is related to $\bar{\pi}$. The results differ depending on whether an optimistic or a pessimistic equilibrium is expected in period 1. The results for the optimistic equilibrium are illustrated in Figure 2 and the results for the pessimistic equilibrium are illustrated in Figure 3. In both figures $\bar{\pi} = 1$ and $\eta = 1/2$. In each case the relationship between the optimal $\tau$ and $\bar{\pi}$ is piece-wise

\textsuperscript{13} In addition we assume that private sector agents all choose the same equilibrium. A slightly more general approach would be to assume that the policymaker expects the two equilibria to occur with probabilities $p$ and $(1-p)$ respectively. Our approach amounts to looking at the two extremes, $p=1$ and $p=0$. 
linear and discontinuous. There are five critical values of \( \bar{\pi} \) marked on each figure which mark the breaks in the piece-wise linear relationships.\(^{14}\)

If an optimistic equilibrium is expected in period 1 then the optimal choices of \( \tau \) are

\[
\begin{align*}
\tau &= \tau_1 & \text{for } \bar{\pi} > \bar{\pi}_5, \\
\tau &= \tau_2 & \text{for } \bar{\pi}_2 < \bar{\pi} < \bar{\pi}_5, \\
\tau &= \bar{\pi}/(1-2\gamma) & \text{for } \bar{\pi}_1 < \bar{\pi} < \bar{\pi}_2, \\
\tau &= \tau_3 & \text{for } \bar{\pi} < \bar{\pi}_1.
\end{align*}
\] (25)

This relationship is marked with the solid grey line in Figure 2. It can be seen that, for values of \( \bar{\pi} \) higher than \( \bar{\pi}_5 \), the value of \( \tau \) chosen leads to a case 1 equilibrium where the outside country is unconstrained by the entry condition and entry takes place. For values of \( \bar{\pi} \) between \( \bar{\pi}_1 \) and \( \bar{\pi}_5 \) the value of \( \tau \) chosen leads to a case 2 equilibrium where the outside country is constrained by the entry condition and chooses to satisfy it.\(^{15}\) While for values of \( \bar{\pi} \) below \( \bar{\pi}_1 \) the value of \( \tau \) chosen leads to a case 3 equilibrium where the outside country chooses to break the entry condition and stays outside monetary union.

If a pessimistic equilibrium is expected in period 1 then the optimal choices of \( \tau \) are

\[
\begin{align*}
\tau &= \tau_1 & \text{for } \bar{\pi} > \bar{\pi}_5, \\
\tau &= \tau_2 & \text{for } \bar{\pi}_4 < \bar{\pi} < \bar{\pi}_5.
\end{align*}
\] (26)

\(^{14}\) See Appendix 2 for details of the derivation of these critical values. The results reported relate to a numerical example and are easily confirmed by calculation. More general results can be obtained but they are rather cumbersome and add few insights beyond those contained in the example discussed here.

\(^{15}\) The interval between \( \bar{\pi}_1 \) and \( \bar{\pi}_5 \) is an example of a corner solution. When an optimistic outcome is expected in period 1, setting \( \tau \) at a level on or above OB leads to a case 2 equilibrium (where entry takes place), while setting \( \tau \) below OB leads to a case 3 equilibrium (where entry does not take place). In the interval between \( \bar{\pi}_1 \) and \( \bar{\pi}_5 \) the benefits of entry provide an incentive which is strong enough to induce the policymaker to set \( \tau \) exactly on OB. For values of \( \bar{\pi} \) below \( \bar{\pi}_1 \) the costs of setting \( \tau \) on OB outweigh the benefits of entry, so \( \tau \) is set at \( \tau_3 \) and entry does not take place.
\[ \tau = \frac{\pi}{1 - \gamma} \quad \text{for} \quad \pi_3 < \pi < \pi_4, \]
\[ \tau = \tau_3 \quad \text{for} \quad \pi < \pi_3. \]

This relationship is marked with the solid grey line in Figure 3. The difference between the pessimistic case and the optimistic case is that, in the pessimistic case, the interval over which a case 2 equilibrium occurs is reduced to values of \( \pi \) between \( \pi_3 \) and \( \pi_4 \). While the interval where a case 3 equilibrium occurs is expanded to those values of \( \pi \) below \( \pi_3 \).\(^{16}\)

Figures 2 and 3 demonstrate the second result emphasised in the introduction to this paper, namely that the imposition of an entry condition does not necessarily lead to more reform in the outside country. For high levels of \( \pi \) the outside country chooses reform level \( \tau_1 \). But tightening the entry condition so that \( \pi \) lies between \( \pi_4 \) and \( \pi_3 \) leads to the level of reform being set at \( \tau_2 - \tau_3 \). Thus a tighter entry condition leads to less reform. If an optimistic equilibrium is expected in period 1, tightening the entry condition further does not result in more reform until \( \pi \) is less than \( \pi_1 \). But for values of \( \pi \) less than \( \pi_1 \) entry does not take place. The only case where tightening the entry condition results in more reform, while allowing entry to take place, is when a pessimistic equilibrium is expected in period 1. In this case, tightening the entry condition below \( \pi_4 \) results in more reform. But reducing \( \pi \) still further so that it is below \( \pi_3 \) results in the outside country failing to join monetary union.

The reason why the entry condition potentially results in less reform can be understood by considering (22), (23) and (24). It is easy to see that \( \tau_2 \geq \tau_1 > \tau_3 \). This ranking is explained as follows. In case 3 the outside country does not join monetary union. When a country does not join monetary union it faces the full consequences of its own distortions and therefore has a strong incentive to carry out reform. In case 1 the outside country is unconstrained by the entry condition and joins monetary union. Here the outside country knows that some of the drawbacks associated with distortions will be shifted onto other monetary

\[^{16}\] The interval between \( \pi_3 \) and \( \pi_4 \) is another example of a corner solution. When a pessimistic outcome is expected in period 1, setting \( \tau \) on or below OC leads to a case 2 equilibrium (where entry takes place), while setting \( \tau \) above OC leads to a case 3 equilibrium (where entry does not take place). In the interval between \( \pi_3 \) and \( \pi_4 \), the benefits of entry provide an incentive just strong enough to induce the policymaker to set \( \tau \) exactly on OC. For values of \( \pi \) below \( \pi_3 \), the costs of setting \( \tau \) on OC outweigh the benefits of entry, so \( \tau \) is set at \( \tau_3 \) and entry does not take place.
union members. There is therefore a lower incentive to reform. This mechanism also operates in case 2. But in this case there is an additional effect which further reduces the amount of reform. Here the outside country is constrained by the entry condition and sets lower inflation in period 1 than it would do otherwise. The inflation rate in period 1 is therefore determined by $\pi$ and is unrelated to the level of reform. This implies that the incentive to reform is further reduced. This is the crucial mechanism which causes the entry condition to lead to less reform.

Before concluding it is necessary to consider the effects of varying $h$. It has just been explained that the case 2 equilibrium, where the outside country is constrained by the entry condition, yields less reform than the other cases because the constraint on inflation in period 1 removes one of the benefits of reform. It should be clear that the importance of this effect must be related to the weight placed on period 1 in the policymaker's welfare function, i.e. the parameter $\eta$. A comparison of equations (22) and (23) shows that, as $\eta$ is reduced to zero, the levels of reform in these two cases converge towards equality. In the limit, when $\eta=0$, the two equilibria yield the same levels of reform. In this extreme case it is no longer true that the entry condition in itself causes less reform. This extreme case is, however, unlikely to be relevant since there is no point in having a qualifying period for monetary union if events within that period have no weight in policymakers' objective functions.17

6. Conclusion

This paper presents a formal model of an entry condition into a monetary union. Two results are emphasised. First, the entry condition creates the possibility of multiple equilibria in the qualifying period. Second, the entry condition can lead to a reduction in structural reform implemented by the potential entrant. Both these effects are undesirable. Multiple equilibria lead to instability and uncertainty while a reduction in the amount of structural reform implies a failure to achieve convergence.

There are a number of policy implications of this analysis. The model

17 Changing the value of $\eta$ also affects the slopes of the OB and OC schedules and therefore affects the critical values of $\pi$. Specifically, reducing $\eta$ pivots OB and OC in an anti-clockwise direction and therefore tends to increase the ranges of $\pi$ for which a case 2 equilibrium is relevant. The intuitive explanation for this is simple. Reducing the importance of the qualifying period relative to period 2 increases the incentive to satisfy the entry condition.
shows that if an inflation entry condition is used then the level at which it is set is crucial for determining its effects. If a very high level is set then the entry condition is irrelevant. If a very low level is set it will tend to exclude countries. If an intermediate level is set it may cause less structural reform. There is a range of values where an entry condition will be successful in achieving more reform while allowing the outside country to join the union, but this may be a relatively small range. And even within this range the desirable outcome can only be achieved if private sector wage bargainers are pessimistic about the prospects of the outside country satisfying the condition.

There are a number of directions in which this analysis could be extended. This paper has concentrated on just one of the entry criteria specified in the Maastricht Treaty. It is likely that combinations of entry conditions will produce rather different effects from those emphasised in this paper. For instance, the combination of an inflation condition and a fiscal deficit condition will tend to reduce a potential entrant's ability to avoid reform of the tax system. Thus one of the undesirable effects identified in this paper may be avoided. A more detailed formal analysis of this issue would be useful.

Another way in which the model could be developed is to consider alternative ways of representing structural reform. In this paper structural reform has been viewed as a process of removing distortions that cause inflationary biases. Another way of viewing reforms is that they make the economy more flexible so that it can more easily absorb stochastic shocks. Monetary union reduces the ability of policymakers in individual countries to deal with country specific shocks, so there are likely to be important interactions between monetary union and entry conditions and reform which is directed at improving market flexibility. Again, a formal analysis of this issue would be very useful.
Appendix 1

This appendix provides details of the derivation of equations (17) – (21).

Derivation of equation (17)

The second inequality follows from the assumption that the constraint is binding. The first inequality follows from (16). Substitution of (8), (12), (14) and (15) into (16) yields (note that $\theta$ is set equal to 0)

$$
\eta \left[ \pi^2 + (\pi - \pi^*_t - \tau)^2 \right] / 2 + (1 - \eta) \tau^2 / 2 \leq
\eta \left( \tau + \pi^*_t \right)^2 / 4 + (1 - \eta) \tau^2
$$

(A1)

In case 2 $\pi^*_t = \bar{\pi}$. After substituting for $\pi^*_t$ and simplifying the following is found

$$
\bar{\pi}^2 - 2\tau \bar{\pi} + \left[ 1 - 2 \left( \frac{1 - \eta}{\eta} \right) \right] \tau^2 \leq 0
$$

(A2)

which is true for

$$(1 - 2\gamma) \tau \leq \bar{\pi} \leq (1 + 2\gamma) \tau$$

(A3)

where $\gamma = \sqrt{(1 - \eta)/2\eta}$. The second inequality in (A3) is irrelevant because $\bar{\pi} \leq \tau$ in case 2. Thus a case 2 equilibrium holds when

$$(1 - 2\gamma) \tau \leq \bar{\pi} < \tau.$$  

(A4)

Derivation of equation (18)

This inequality follows from assuming that (16) does not hold i.e. that

$$
\eta \left[ \pi^2 + (\pi - \pi^*_t - \tau)^2 \right] / 2 + (1 - \eta) \tau^2 / 2 \geq
\eta \left( \tau + \pi^*_t \right)^2 / 4 + (1 - \eta) \tau^2
$$

(A5)
In case 3 $p_1 = \tau$. Substitution and rearrangement yields

$$2\pi^2 - 4\pi\pi + \left[ 2 - \left( \frac{1 - \eta}{\eta} \right) \right] \pi^2 \geq 0$$  \hspace{1cm} (A6)

which implies

$$\pi \leq (1 - \gamma)\tau \text{ and } \pi \geq (1 + \gamma)\tau$$  \hspace{1cm} (A7)

The second of these inequalities is irrelevant because $\pi \leq \tau$ in case 3.

**Derivation of equations (19), (20) and (21)**

If there is a case 1 equilibrium in period 1 then $y_1 = -\tau$ and $\pi_1 = \tau$ so $L_1 = \tau^2$. The outside country joins the union in period 2 so $L_2 = L_2' = \tau^2/2$ thus

$$\Gamma_1 = L_1 + L_2 + C = \left[ 2\eta\tau^2 + (1 - \eta)\tau^2 + (\pi - \tau)^2 \right]/2.$$  \hspace{1cm} (A8)

If there is a case 2 equilibrium in period 1 then $y_1 = -\tau$ and $\pi_1 = \pi$ so $L_1 = \left( \pi^2 + \tau^2 \right)/2$. The outside country joins the union in period 2 so $L_2 = L_2' = \tau^2/2$ thus

$$\Gamma_2 = L_1 + L_2 + C = \left[ \eta(\pi^2 + \tau^2) + (1 - \eta)\tau^2 + (\pi - \tau)^2 \right]/2.$$  \hspace{1cm} (A9)

If there is a case 3 equilibrium in period 1 then $y_1 = -\tau$ and $\pi_1 = \tau$ so $L_1 = \tau^2$. The outside country does not join the union in period 2 so $L_2 = L_2'' = \tau^2$ thus

$$\Gamma_3 = L_1 + L_2 + C = \left[ 2\tau^2 + (\pi - \tau)^2 \right]/2.$$  \hspace{1cm} (A10)
Appendix 2

This appendix describes the calculations necessary to derive the critical values of $\bar{\pi}$ shown in Figures 2 and 3. In these figures $\eta=1/2$ so the loss functions given in equation (19), (20) and (21) can be simplified as follows

\[ \Gamma_1 = \frac{5}{4} \tau^2 - \tau + \frac{1}{2}, \quad (A11) \]
\[ \Gamma_2 = \frac{1}{4} \bar{\pi}^2 + \tau^2 - \tau + \frac{1}{2}, \quad (A12) \]
\[ \Gamma_3 = \frac{3}{2} \tau^2 - \tau + \frac{1}{2}, \quad (A13) \]

and the optimal levels of $\tau$ in the three cases are: $\tau_1 = 2/5$, $\tau_2 = 1/2$ and $\tau_3 = 1/3$.

The critical value $\bar{\pi}_s$ is where the outside country is indifferent between setting $\tau=\tau_1$ and $\tau=\tau_2$. $\bar{\pi}_s$ is therefore the value of $\bar{\pi}$ which solves

\[ \frac{5}{4} \tau_1^2 - \tau_1 + \frac{1}{2} = \frac{1}{4} \bar{\pi}^2 + \tau_2^2 - \tau_2 + \frac{1}{2}, \quad (A14) \]

which yields $\bar{\pi}_s = 1/\sqrt{5} \approx 0.447$. It is simple to check numerically that $\tau=\tau_1$ is optimal for values of $\bar{\pi}$ greater than $\bar{\pi}_s$ and $\tau=\tau_2$ is optimal for $\bar{\pi}$ less than $\bar{\pi}_s$.

The critical value $\bar{\pi}_s$ is the point where the outside country is indifferent between setting $\tau$ on OC and receiving $\Gamma_1$ as a payoff and setting $\tau=\tau_3$ and receiving $\Gamma_3$ as a payoff. $\bar{\pi}_s$ is therefore the value of $\bar{\pi}$ which solves

\[ \frac{1}{4} \bar{\pi}^2 + (2 + \sqrt{2}) \bar{\pi}^2 - (2 + \sqrt{2}) \bar{\pi} + \frac{1}{2} = \frac{3}{2} \tau_3^2 - \tau_3 + \frac{1}{2}, \quad (A15) \]

which yields $\bar{\pi}_s \approx 0.062$. It is simple to check numerically that when a pessimistic equilibrium is expected in period 1 then $\tau = \pi/(1-\gamma)$ is optimal for $\bar{\pi}$ greater than $\bar{\pi}_s$ and $\tau=\tau_1$ is optimal for $\bar{\pi}$ less than $\bar{\pi}_s$.

The critical value $\bar{\pi}_s$ is the point where the outside country is
indifferent between setting $\tau$ on OB and receiving $\Gamma_2$ as a payoff and setting $\tau=\tau_3$ and receiving $\Gamma_3$ as a payoff. $\bar{\pi}_1$ is therefore the value of $\bar{\pi}$ which solves

$$\frac{1}{4}\bar{\pi}^2 + \left(1 + \sqrt{2}\right)^2 \bar{\pi}^2 + \left(1 + \sqrt{2}\right)\bar{\pi} + \frac{1}{2} = \frac{3}{2} \tau_3^2 - \tau_3 + \frac{1}{2}, \tag{A16}$$

which yields $\bar{\pi}_1 \approx -0.308$. It is simple to check numerically that when an optimistic equilibrium is expected in period 1 then $\tau = \bar{\pi} / (1 - 2\gamma)$ is optimal for $\bar{\pi}$ greater than $\pi_1$ and $\tau = \tau_3$ is optimal for $\bar{\pi}$ less than $\pi_1$. 
References

Sibert, Anne and Alan Sutherland (1998) "Monetary Union and Labour Market Reform" mimeo, University of York.
Figure 1: Period 1 equilibria.

\[
\tau = \frac{\pi}{1 - 2\gamma}
\]

\[
\tau = \frac{\pi}{1 - \gamma}
\]
Figure 2: Reform levels - optimistic case.
Figure 3: Reform levels - pessimistic case.