A Portfolio Model of Capital Flows to Emerging Markets*

Michael B Devereux[†] and Alan Sutherland[‡]

June 2007, Revised June 2008

Abstract

Since the crises of the late 1990's, most emerging market economies have built up substantial positive holdings of US dollar treasury bills, while at the same time experiencing a boom in FDI capital inflows. This paper develops a DSGE model of the interaction between an emerging market economy and an advanced economy which incorporates two-way capital flows between the economies. The novel aspect of the paper is to make use of new methods for analyzing portfolio choice in DSGE models. We compare a range of alternative financial market structures, in each case computing equilibrium portfolios. We find that an asymmetric configuration where the emerging economy holds nominal bonds and issues claims on capital (FDI) can achieve a considerable degree of international risk-sharing. This risk-sharing can be enhanced by a more stable monetary policy in the advanced economy.

Keywords: Country portfolios, Emerging Markets.

JEL: E52, E58, F41

*Prepared for the IMF conference "New Perspectives on Financial Globalization", April 2007. We thank the editors, an anonymous referee, Martin Evans and participants at the conference for comments on an earlier draft. Supported by ESRC World Economy and Finance Programme, award 156-25-0027. Devereux thanks SSHRC, the Bank of Canada, and the Royal Bank of Canada for financial support. The views here are solely those of the authors, and do not reflect the position of the Bank of Canada.

[†]CEPR and Department of Economics, University of British Columbia, Canada. Email: devm@interchange.ubc.ca Web: www.econ.ubc.ca/devereux/mdevereux.htm

[‡]CEPR and School of Economics and Finance, University of St Andrews, UK. Email: ajs10@st-and.ac.uk Web: www.st-and.ac.uk/~ajs10/home.html

1 Introduction

In the decade since the crises of the 1990's, the international financial landscape has evolved and changed in ways that few observers would have predicted at that time. Emerging economies have generally experienced strong and uninterrupted economic growth with no major crises. Capital flows from industrial countries in the form of FDI, as well as portfolio and bond investment, have been strong. Sovereign spreads have been low by historic standards for a number of years. For most emerging countries, external accounts have swung sharply from positions of net deficits in the mid-1990's to generally strong surpluses at present. In addition, these countries have eliminated their financial vulnerabilities, displayed so clearly during the crisis years, by correcting the currency and maturity mismatches in their national balance sheets. Some countries have abandoned tight exchange rate pegs and moved towards flexible inflation targeting. More generally, the quality of policy-making in the fiscal and financial domain has improved greatly.

There is no single explanation for this surplus of good economic news from the emerging markets. High global saving has led to a prolonged period of low real interest rates, reducing the potential for crises. The build-up of strong positive net external positions as well as large stocks of foreign exchange rate reserves has had the same effect, and more generally has instilled a strong confidence in the investment potential of emerging economies. But in addition, real economic growth has been stimulated by high demand for exports from the industrial world (in particular the US), and commodity prices booms have generated huge net gains for many emerging countries.

One general feature of emerging economies' recent experience, that differs from previous episodes of high capital inflows and economic growth, is the degree to which they have been participants in the globalization of financial markets. Rather than simply being recipients of net capital inflows or generators of outflows, many emerging countries have displayed growth in gross external financial assets and liabilities that are much larger than net positions. In this sense, their experience mirrors that of many advanced economies, as documented in the seminal work of Lane and Milesi Ferretti (2001, 2005, 2006). While most recent discussion of global imbalances has focused on the size of net external surpluses of China and other emerging economies, indicating the apparently perverse situation of capital outflows from the developing world to developed economies (or more accurately, the US), in the background there is a large degree of two way capital flow. Emerging economies have been accumulating large stocks of US treasury bills going into official reserve assets, but they have also been receiving large inflows of FDI and portfolio equity investment, as well as private bond market inflows. Lane and Milesi Ferretti (2006) document this turnaround in the portfolio position of emerging market economies taken as a whole. From the situation in the mid 1990's, where many of these economies were substantial net debtors in non-contingent assets such as bank loans and short term US dollar bonds, now they have substantial net positive positions in fixed income assets, while being on the whole net debtors in FDI and portfolio equity investment. There is an argument that this is in fact a much more efficient form of financing development lending for emerging market economies, in terms of achieving the most desired degree of international sharing risk.

This paper analyses the impact of financial globalization in emerging market economies, paying particular attention to the determinants of country portfolio positions. We explore the factors underlying the determinants of an optimal risk-sharing portfolio for an emerging market economy and an advanced economy. The emerging economy is characterized by a high country-specific productivity risk. We explore how international risk-sharing can be achieved under a number of alternative financial market configurations, ranging from a situation of no financial markets (i.e. no portfolio diversification) to one of complete markets. Of particular interest is an intermediate financial structure, where there is international trade in equity-FDI claims of the emerging economy as well as nominal bonds denominated in the currency of the advanced economy. This is meant in a general sense to represent the present structure of two way capital flows between emerging market economies and the advanced economies as described in the previous paragraph.

For each financial market configuration, we compute the equilibrium optimal portfolio positions of each economy, given the financial instruments available, as well as the nature of country specific risks. Our analysis is built around a dynamic stochastic general equilibrium (DSGE) model of the interaction between an emerging market economy and the rest of the world. However, unlike the standard DSGE framework, we are able to incorporate two-way capital flows across countries. We do this by making use of new developments in the study of portfolio choice in general equilibrium environments (see Devereux and Sutherland 2006, 2007), in order to isolate the determinants of gross portfolio positions for an emerging market economy.[1]

Our results indicate that financial globalization, wherein an emerging market economy may simultaneously build up positive gross positions in non-contingent international bond assets, and negative positions in FDI and portfolio equity, may offer a considerable enhancement of international risk-sharing, relative to a case where only one-way intertemporal capital flows can take place. Nevertheless, we find that this financial structure is unable to attain a complete-markets full risk-sharing equilibrium. In our model, such an allocation can be attained if there is unrestricted trade in both country's equities and a non-contingent real bond.

The degree of risk-sharing attained with two way capital flows depends on some key features of the model. In particular, because agents engage in risk-sharing using nominal home currency bonds, the amount of price volatility will affect risk-sharing, even though all nominal prices are flexible. In particular, more volatility arising from price shocks will reduce the size of optimal portfolios, and reduce risk-sharing across countries.

We compare the response to productivity shocks under alternative asset market configurations. Optimal portfolio diversification tends to reduce the gap between the overall consumption responses to shocks, but at the same time tends to increase the movement in the net asset position. Because we can compute the optimal portfolio structure for each country, we can also decompose the change in net assets into the separate movement in gross liabilities (nominal bonds) and gross assets (FDI equity). In some cases, gross assets and liabilities tend to move in opposite directions, while in other cases they move in the same direction. Finally, we can break down the overall movement in gross portfolios (or capital flows), into price, (or valuation) effects, and quantity (or portfolio re-balancing) effects.

The rest of the paper is organized as follows. The next section discusses some recent developments in financial markets and country portfolios for emerging markets. Section 3 sets out the two country dynamic model, and explains the method of constructing optimal portfolios. Section 4 discusses the general method of portfolio computation. Section 5 presents the main results of the model for portfolio choice, risk-sharing, and international

[1] See also Engel and Matsumoto (2005), Evans and Hnatkovska (2005), Kollmann (2006) and Tille and Van Wincoop (2007) among other papers, for recent contributions to the literature on portfolio choice in general equilibrium.

capital flows. Some conclusions then follow.

2 Evidence

In this section we review some recent experience of the external balance and portfolio structure of developing and emerging market economies. The growing current account surpluses of emerging market economies, and in particular the Asian economies, is the counterpart in large degree to the increase in the US current account deficit. For many emerging countries, these current account surpluses date back to the crises of the late 1990's.

The model we analyze below highlights the importance of portfolio structure and the evolution of gross asset and liability positions. Nevertheless, we begin by focusing on the experience of net foreign assets. Figure 1 describes the evolution of the net foreign debt to GDP ratio for 76 (non oil-exporting) developing and emerging market economies over the period 1990-2004, where the data are obtained from the Lane and Milesi-Ferretti (2006) 'External Wealth of Nations' (EWN) data-set. The most striking aspect of Figure 1 is the sharp turnaround in the net foreign debt ratio immediately after the Asian crisis. The ratio grew sharply from the mid-1990's onwards, moving from about 35 percent to 48 percent between 1995 and 1999. But it fell quickly from this peak, and at the end of the sample (2004) had returned to its mid-1990's level. Although the EWN data set includes valuation effects of exchange rates and asset prices on gross assets and liabilities in order to construct the net debt measure, this rapid improvement reflects primarily the turnaround in the current accounts of emerging market economies from deficit to surplus following the Asian crisis.

A central reason to analyze the determinants of gross external positions is the phenomenon of 'financial globalization', or the simultaneous increase in stocks of gross external assets and liabilities. It is well-recognized that advanced economies have experienced rapid increases in gross external positions over the last decade. To what extent is this also true of developing economies? The top panel of Figure 2 illustrates the movement of gross external assets and liabilities, relative to GDP, for the same sample of countries as in Figure 1, over the 1990-2004 period. This confirms that on average developing countries are net debtors, as shown in Figure 1. But gross assets and liabilities both grew considerably over the period. In particular, average gross assets doubled from 34 to 68 percent of GDP over the period.

The lower panels of Figure 2 show gross asset and liability positions for China and India, the two emerging economy 'giants'. In both cases, we see a strong trend in the growth of both gross assets and liabilities. For China, the increase in gross assets was fast enough to turn it into a net creditor at the end of the sample. At the beginning of the sample, India is essentially in financial autarky, with very low ratios of gross assets and liabilities to GDP. But both series increase dramatically over the subsequent 15 years.

It is not just the presence of large gross external positions that is important, but also the composition of these assets and liabilities. A key implication of the model analyzed below is that emerging market countries participate in cross country risk-sharing through the accumulation of debt asset claims on advanced economies, while issuing claims in the form of equity and FDI liabilities that are held by residents of advanced economies. Lane and Milesi-Ferretti (2006) note that this dichotomy in portfolio composition is seen increasingly in the data over the last decade. The top panels of Figures 3 and 4 confirm this for the same group of developing and emerging market countries as before. Over the sample, the equity liabilities-GDP ratio increases four-fold. At the same time the ratio of debt assets to GDP increases from 32 to 52 percent. If we restrict our focus solely on foreign exchange rate reserves to GDP, the increase is more dramatic, going from 7 percent of GDP in 1990 to 21 percent in 2004.

The lower panels of Figures 3 and 4 confirm the same trends for China and India. In fact, the movements are much more pronounced for these two cases. In particular, China's equity liabilities to GDP ratio increased six-fold over the sample, and its debt assets to GDP ratio increased almost three fold. Notably, foreign exchange reserves increase quite sharply towards the end of the sample. Again, India's growth begins from a very low base, but we see a substantial increase in FDI liabilities, as well as debt assets and foreign exchange rate reserves.

While these figures scale the portfolio positions to overall GDP, a similar picture appears if we scale by the gross liability and asset stocks. In particular, for the overall country sample, as well as for China and India, there is a trend towards an increase in the share of liabilities in equity and FDI, and a trend towards an increase in the share of gross assets in debt instruments, and in particular in foreign exchange rate reserves. Hence, there is clear evidence not just of an increase in globalization for developing and emerging market economies, but also of a movement in portfolio composition. In the next section we examine the risk-sharing implications of these portfolio trends in a dynamic general equilibrium environment.

3 The Model

The model is based on a standard two-country version of the neo-classical growth model. There is a single world good which is produced by labor and capital in each country. Labor is immobile across countries, but there is no impediment to movement of goods. We think of the home country as being 'developed', and the foreign country as being 'developing'. In the context of the model, this distinction is mainly captured by two features. The first is technological. The home country has a lower volatility of GDP (or more precisely, a lower variance of productivity shocks). The second distinction between the countries relates to the configuration of assets markets. It is assumed that only home currency bonds are acceptable in international capital markets. This assumption is meant to capture the widely observed phenomenon that, at least up until recently, US dollar bonds dominated international capital flows, and more particularly that emerging market currency bonds are almost non-existent in international bond trade.

We will look at the implications for risk-sharing and capital flows of three different financial market arrangements. In the first, we assume no possibility of portfolio diversification at all. In this configuration all capital flows between the two countries can be financed only with a non state-contingent risk-free real bond. In this case, there is no distinction between gross and net foreign assets, or between gross and net capital flows. We denote this as the NP ('no portfolio') model. In the second configuration, we assume an asymmetric structure, based in a broad sense on the trend of emerging market portfolios discussed in the previous section. Under this arrangement, there are two assets traded between the countries; equity claims on the foreign country (if the home country takes a positive foreign equity position, we think of this as FDI investment), and home-currency denominated nominal bonds (which can be thought of as representing foreign exchange reserves, or more generally bond assets). This offers a more enhanced set of options for sharing risk, since equity returns will reflect productivity shocks in the foreign country, but also because real returns on home currency nominal bond will be affected by exchange rate movements (or equivalently, by movements in the home currency price level). Nevertheless, we find that in general, this does not allow for full risk-sharing. We denote this as the EB ('equity-bond'), model.

Finally, we look at a symmetric case, where the equities of both countries and a non-contingent real bond may be traded freely among home and foreign residents. In this model, we find that this financial structure is enough fully to exploit all gains from risk-sharing, and hence this represents complete markets. We denote this as the EQ ('equity') model.

This paper takes as given the financial market structure. The novel focus of the analysis is to explore the degree to which the intermediate financial arrangement (EB) falls between the two extremes of portfolio autarky (NP) and complete markets (EQ)[2].

3.1 Household choices

Households in each country receive wage income and asset income. In the NP model, the household can hold only a non-contingent real bond. Under this arrangement, the budget

[2] In principle, of course, it would be desirable to allow for the market structure to be determined endogenously. In the case of China, it might be argued that the presence of capital controls provides a direct explanation for the portfolio configuration whereby FDI is financed by positive holdings of rest-of-world bonds. But more generally, it is important to consider why the restricted range of asset holdings that are allowed in the EB configuration might arise as a result of differences across countries in transactions costs, financial market distortions, or informational asymmetries.

Two factors might be important in this consideration. First, as will be shown below, holding home country nominal bonds acts as a relatively efficient claim on home output, and so is a (imperfect) substitute for holding home equity. But if we allow for unrestricted trade in both countries equities, and nominal bonds, then markets would be complete (see section 5.4 below), and agents in the foreign country will take a large holding in domestic equity. However, if we were to introduce additional transactions costs to foreign agents holding of home equity, then agents in the foreign country would have a bias towards home bonds, and away from home equity. That is, it is possible to obtain a configuration which is close to the EB outcome in the presence of transactions costs of holding equity. A second possible explanation of the EB configuration may be developed along the lines of Mendoza et al. (2007). They show that, if different countries have different contract enforcement technologies, then residents of the country with better enforcement may take a long portfolio position in risky assets of the other country, financed partially by a negative position in risk-free bonds. This describes an outcome similar to our EB configuration.

constraint for home country households may be defined as,

$$C_t + q_{rt} B_t = W_t H_t + D_t + B_{t-1}, \tag{1}$$

where C is home consumption, B is the holding of the real bond, q_r is the price of the bond, and WH is wage income, with W being the real wage, and H labor supply. D represents dividend income received from the home firm. Since in this case, equity is non-traded, all dividend income from home firms is received directly by the home country household. Note that a unit of the bond purchased in period $t-1$ is assumed to pay a unit of the consumption good in period t.

In the EB model, the home country budget constraint is written to allow for holdings of both nominal bonds and foreign equity. Thus, we have:

$$C_t + q^*_{kt} S_{ft} + q_{bt} B_{ht} = W_t H_t + D_t + (D^*_t + q^*_{kt}) S_{ft-1} + \frac{B_{ht-1}}{P_t} \tag{2}$$

where q^*_k and q_b are respectively the price of foreign equity (or FDI) and the price of the home-currency nominal bond, S_f represents the home country holdings of foreign equity, and D^* represents the dividend payment on equity. We let S_f be of either sign, as the developed country residents may hold long or short positions on the emerging market firm. B_h is holdings of the nominal bond, which again may be of either sign, and P is the home country price level. A unit of the bond purchased in period $t-1$ is assumed to pay a unit of the home currency in period t. As in the NP case, in this financial arrangement, the home country still holds 100 percent of domestic equity[3].

[3]The model does not formally separate the emerging market economy's holdings of fixed income claims on the advanced economy from holdings of international reserves. As shown in section 2, much of the asset build-up of emerging economies can be attributed to the growth of official reserves. Most of the literature attempting to explain the large growth in reserves follows the approach we take; i.e. treating the reserve holding decision as done by a dynamic optimizing agent with intertemporal preferences. In particular, this is the case for Jeanne and Ranciere (2006) and Jeanne (2007), as well as Caballero and Panageas (2004). So long as governments are benevolent, this distinction should not make much difference. A separate issue concerns the widespread view that the growth in international reserves came from foreign exchange rate intervention, in an attempt to maintain undervalued nominal exchange rates. This mechanism would be substantially more difficult to model in our framework, since we would have to incorporate some type of failure of Ricardian equivalence. The reason is that with full Ricardian equivalence, the official holdings of foreign exchange rate reserves should not impact on the economy's net foreign bond holdings, since

In the EQ model, with symmetric trade in both equities and a real bond, the home country budget constraint may be written as

$$C_t + q^*_{kt} S_{ft} + q_{kt} S_{ht} + q_{rt} B_t = W_t H_t + (D_t + q_{kt}) S_{ht-1} + (D^*_t + q_{kt}) S_{ht-1} + B_{t-1} \quad (3)$$

where q_k is the price of home equity, and S_h is the home residents' share of the home firm.

We may re-write either (2) or (3) in terms of net foreign assets. In the case of (2), net foreign assets may be written as $NFA_t = q^*_{kt} S_{ft} + q_{bt} B_{ht}$, while for the (3) case, we have $NFA_t = q^*_{kt} S_{ft} + q_{kt}(S_{ht} - 1) + q_{rt} B_t$. Then we may re-write (2) as follows

$$C_t + NFA_t = W_t H_t + D_t + r_{xt}\alpha_{t-1} + r_{bt} NFA_{t-1} \quad (4)$$

Here $\alpha_{t-1} = q^*_{kt-1} S_{ft-1}$ is the real holding of equity, and $r_{xt} = r^*_{kt} - r_{bt}$ represents the excess return on equity relative to debt, where $r^*_{kt} = (D^*_t + q^*_{kt})/q^*_{kt-1}$ and r_{bt} is the real return on nominal debt, which is given by $r_{bt} = 1/(q_{bt-1} P_t)$.

In the case of (3), the equivalent of (4) is written as

$$C_t + NFA_t = W_t H_t + D_t + r'_{xt}\alpha_{t-1} + r_{kt} NFA_{t-1}, \quad (5)$$

where $r'_{xt} = [(r^*_{kt} - r_{kt}), (r_{rt} - r_{kt})]$, where $r_{rt} = 1/q_{rt-1}$ is the return on the non-contingent bond and $\alpha'_{t-1} = [q^*_{kt-1} S_{ft-1}, q_{rt-1} B_{t-1}]$.

This representation of the budget constraint in each case allows us to solve the portfolio problem for the choice of α, using the procedure of Devereux and Sutherland (2006, 2007).

Representative household z in the home country has a utility function of the form:

$$U_z = E_0 \sum_{t=0}^{\infty} \theta_t \left[\frac{C_{zt}^{1-\rho}}{1-\rho} - \frac{\chi}{1+\mu} H_{zt}^{1+\mu} \right] \quad (6)$$

where $\rho > 0$, $\mu > 0$, $\chi > 0$, C is consumption and E_t is the expectations operator conditional on time-t information. θ_t is the discount factor, which we assume to be endogenous

the private sector would internalize the present value of assets or liabilities implicit in the official reserve holdings. In addition, in order to explore this mechanism, we might need to introduce some type of price rigidity so that a fixed nominal exchange rate could successfully affect an economy's competitiveness. While it is likely that official intervention does play a large role in the build-up of nominal claims of emerging markets, we feel that these complications are enough that they are best dealt with in further research.

and determined as follows

$$\theta_{t+1} = \theta_t \beta(C_{zt}),\ \theta_0 = 1 \tag{7}$$

where $\beta(C_{zt}) = \omega C_{zt}^{-\eta}$.

The budget constraints in each case for the foreign country are written analogously, and foreign agents have similar preferences.

Following Schmitt Grohe and Uribe (2003), the role of θ_t is to ensure a stationary wealth distribution for the linear approximated dynamic model. As is well known, with constant identical time discount factors in each country, the distribution of world wealth is indeterminate in a steady state when financial markets are incomplete. Allowing for an endogenous time discount factor ensures a unique steady state distribution of wealth. In addition, for $\eta > 0$, the first-order dynamics in the neighborhood of a steady state display strict saddle path properties. The parameter ω allows for structural differences in savings propensities across the two countries. Below, we calibrate ω (and its foreign counterpart) so that the home country has a negative NFA in the steady state. We comment in more detail below on the computation and calibration of steady NFA positions.

3.2 Firms

In each country, competitive firms maximize the present value of dividends. In the home economy for instance, the firm's objective function is written as

$$E_t \sum_{i=0}^{\infty} \Theta_{t+i} D_{t+i}, \tag{8}$$

where $D_t = A_t F(K_t, H_t) - W_t H_t - I_t - \phi(I_t)$. Thus, dividends are comprised of output, less wage income, less investment, where we assume that investment is subject to adjustment costs. The adjustment cost function satisfies the conditions; $\phi' > 0$, $\phi'' > 0$, $\phi(\overline{I}) = \phi'(\overline{I}) = 0$, where $\overline{I}$ represents the steady state level of investment. Given a level of investment I, capital accumulation is described by:

$$K_{t+1} = I_t + (1-\delta)K_t. \tag{9}$$

In (8), the firm uses the stochastic discount factor Θ_{t+i} to evaluate its dividend stream. This will be related to the household's stochastic intertemporal marginal rate of substitution (SMRS). In the NP and the EB case, Θ_{t+i} is just equal to the home household's

SMRS, since the home firm is fully owned by the home household. In the EQ case, Θ_{t+i} is a convex combination of the home and foreign SMRS.[4]

We assume that the firm's production function is a standard Cobb-Douglas, so that $F(K_t, H_t) = K_t^{\gamma} H_t^{1-\gamma}$. In addition, A_t represents a stochastic productivity shock, which is characterized by

$$\log A_t = \zeta_A \log A_{t-1} + \varepsilon_{A,t}, \tag{10}$$

where $0 \leq \zeta_A \leq 1$, and ε_A is an i.i.d. shock symmetrically distributed over the interval $[-\epsilon, \epsilon]$ with $Var[\varepsilon_A] = \sigma_A^2$.

The specification for the foreign economy is analogous. Foreign firms choose investment and employment to maximize the expected present value of dividends, evaluated at the relevant stochastic discount factor.

3.3 The Price Level

By assumption, only the home country's nominal bonds are tradable internationally. This means that the only relevant price level is that of the home country. In the home economy, the dollar price of the consumption good is assumed to determined by a simple quantity theory relationship of the form

$$M_t = \Omega_t P_t Y_t. \tag{11}$$

where M is the nominal money supply in the home country, Ω is a 'velocity' shock to money demand, and $Y_t = A_t K_t^{\gamma} H_t^{1-\gamma}$ is output in the home country. This is a short-cut approach to a more involved specification incorporating an explicit cash-in-advance constraint on expenditures.[5] For simplicity, we assume M_t grows at a constant rate m, and the velocity shock Ω is be determined by an autoregressive process of the form

$$\log \Omega_t = \log \Omega_{t-1} + \varepsilon_{M,t} \tag{12}$$

[4] For the foreign firm, the discount factor in the EB case is assumed to be a weighted average of the home and foreign consumers' SMRS.

[5] In the case of explicit cash-in-advance constraints, there would also appear distortionary wedges in the optimality equations for labor and capital. To make (11) exactly identical to a cash-in-advance economy, it would be necessary to assume that a separate lump-sum tax financed fiscal policy is used to eliminate these distortions.

where ε_Ω is an i.i.d. shock symmetrically distributed over the interval $[-\epsilon, \epsilon]$ with $Var[\varepsilon_\Omega] = \sigma_\Omega^2$. This is meant to be a general term which captures shocks to the transactions technology, such as financial innovation in the banking sector or shocks to the liquidity of the financial system. The key feature of this shock is that it generates uncertainty in the price level. In subsequent discussion, we refer to this as a velocity shock. In terms of the determination of the optimal portfolio structure, it has identical effects to a shock to the money supply.

3.4 Optimality conditions

Households in each economy choose consumption, hours worked, saving, and an optimal portfolio of assets under each different financial market structure. We state the optimality conditions for the home country. Labor supply is characterized by the first order condition:

$$\chi\theta_t H_t^\mu = \lambda_t W_t \tag{13}$$

In the NP economy, optimal consumption implies:

$$q_{rt}\lambda_t = \beta(C_t)E_t\lambda_{t+1}. \tag{14}$$

where λ_t is the Lagrange multiplier associated with the budget constraint.

In the EB economy (14) is replaced by the two conditions pertaining to the choice of equity holdings and nominal bonds:

$$q_{kt}^*\lambda_t = \beta(C_t)E_t\lambda_{t+1}(D_{t+1}^* + q_{kt+1}^*), \tag{15}$$

$$q_{bt}\lambda_t = \beta(C_t)E_t\lambda_{t+1}/P_{t+1}. \tag{16}$$

Finally, in the EQ economy, the relevant conditions are (14) and (15) for non-contingent bonds and foreign equity, while for home equity the optimality condition is:

$$q_{kt}\lambda_t = \beta(C_t)E_t\lambda_{t+1}(D_{t+1} + q_{kt+1}). \tag{17}$$

In all cases λ_t is given by

$$\lambda_t = C_t^{-\rho} + \varsigma_t\eta\omega C_t^{-(1+\eta)}$$

where ς_t is the Lagrange multiplier associated with (7). The first-order condition for the evolution of ς_t is given by

$$\varsigma_t = E_t[\varsigma_{t+1}\omega C_{t+1}^{-\eta} + C_{t+1}^{1-\rho}/(1-\rho) - \chi H_{t+1}^{1+\mu}/(1+\mu)]$$

Firms in each economy choose employment and investment in the standard fashion. For the home economy, the relevant conditions are:

$$(1-\gamma)A_t K_t^{\gamma} H_t^{-\gamma} = W_t, \tag{18}$$

$$\Theta_t[1+\phi'(I_t)] = E_t\Theta_{t+1}\left\{\gamma A_{t+1}K_{t+1}^{\gamma-1}H_{t+1}^{1-\gamma} + (1-\delta)[1+\phi'(I_{t+1})]\right\}. \tag{19}$$

3.5 Market Clearing Conditions

The model is closed by market clearing conditions for goods and asset markets.

$$Y_t + Y_t^* = C_t + C_t^* + I_t + \phi(I_t) + I_t^* + \phi(I_t^*), \tag{20}$$

$$B_t + B_t^* = 0 \tag{21}$$

$$S_{ft} + S_{ft}^* = 1, \tag{22}$$

$$S_{ht} + S_{ht}^* = 1, \tag{23}$$

$$B_{ht} + B_{ht}^* = 0. \tag{24}$$

Bonds are in zero net supply. We normalize so that the total supply of shares of the firm in each economy is unity. In the NP economy, only (20) and (21) apply, since there is no trade in nominal bonds or equity. In the EB economy, the conditions (20), (22), and (24) apply, while in the EQ economy, (20), (21), (22) and (23) apply.

3.6 The Current Account and ΔNFA

It is useful to define $\Delta NFA_t \equiv NFA_t - NFA_{t-1}$ to be the change in home net foreign assets. In the EB economy it is simple to show that

$$\Delta NFA_t = CA_t + r_{xt}'\alpha_{t-1}$$

where $CA_t \equiv W_t H_t - C_t + D_t + (r_{bt}-1)NFA_{t-1}$ corresponds (approximately) to the conventionally measured current account. Notice that ΔNFA_t differs from CA_t by the term $r'_{xt}\alpha_{t-1}$, which represents the unanticipated valuation effect arising from gross asset positions. A similar relationship holds in the EQ economy. But in the NP economy, where gross and net asset positions are identical, no unanticipated valuation effects exist, so $CA_t = \Delta NFA_t$ by definition.

4 Model Solution

In the NP case, equations (1), and (9), (13), (14), (18), (19), along with the analogous equations for the foreign country, as well as (20) and (21) may be solved to determine the path of B_t, B_t^*, C_t, C_t^*, I_t, I_t^*, K_t, K_t^*, H_t, H_t^*, W_t, W_t^*, and q_{rt}. In the EB economy, the conditions (2), and (9), (13), (15), (16), (18), (19), along with the analogous equations for the foreign country, as well as (11), (20) (22) and (24) may be solved to determine the path of B_{ht}, B_{ht}^*, S_{ft}^*, S_{ft}, C_t, C_t^*, I_t, I_t^*, K_t, K_t^*, H_t, H_t^*, W_t, W_t^*, P_t, q_{bt} and q_{kt}^*. Finally, for the EQ economy, the conditions (3), and (9), (13), (14), (15), (17) (18), (19), along with the analogous equations for the foreign country, as well as, (20), (21), (22) and (23) may be solved to determine the path of B_t, B_t^*, S_{ht}, S_{ht}^*, S_{ft}^*, S_{ft}, C_t, C_t^*, I_t, I_t^*, K_t, K_t^*, H_t, H_t^*, W_t, W_t^*, q_{kt} and q_{kt}^*.

The model determines a stochastic distribution of output, consumption, employment, and capital, as well as a time-varying path of debt and equity holdings. Of course, as in almost all applications of stochastic DGE models, it is not possible to obtain the exact solution of the model in any of these cases. The usual approach in general equilibrium modelling is to solve by linear approximation around a steady state. In the NP economy, this method may be applied without difficulty, because the only relevant endogenous asset variable is the net foreign asset holding. In the EQ case it is also relatively simple to solve a linear approximation of the model by exploiting the relationships implied by perfect risk sharing. But in the EB case, it is necessary to solve also for the portfolio composition of asset holdings between bonds and equity. To obtain this portfolio solution for the model, it is necessary to incorporate higher order aspects of the model approximation, involving the degree to which each asset is useful in hedging against income risk for each country.

We solve the model by the approximation methods developed in Devereux and Sutherland (2006, 2007). This involves a two part solution. First, using a second-order approximation of the portfolio equilibrium conditions, and the equivalent expression for the foreign economy, in combination with a first-order approximation of the rest of the model, we may solve for the zero-order, or steady-state portfolio division between equity and debt. This allows us to determine how the stochastic structure of the model determines portfolio allocation, and to characterize the economy's first-order response to stochastic shocks under an optimal portfolio. But we are also interested in how portfolio holdings themselves respond to stochastic shocks in the economy. To compute this, we follow Devereux and Sutherland (2007) in taking a third-order expansion of the portfolio conditions, in combination with a second-order expansion of the rest of the model.

4.1 Computing Optimal Portfolios

Before we analyse the solution of the model in detail, we briefly describe the approach to computing optimal portfolio behaviour. In any two-country DSGE model, there will be a set of portfolio optimality conditions for the two countries, such as:

$$E_t \lambda_{t+1} r_{xt+1} = 0, \quad E_t \lambda^*_{t+1} r_{xt+1} = 0, \tag{25}$$

where r_x represents the excess return on the asset portfolio, relative to a reference asset and λ and λ^* are the Lagrange multipliers associated with the budget constraints in the home and foreign countries. In addition, any DSGE model will have a set of equations which may be characterized as

$$E_t(X_{t+1}, X_t, Y_{t+1}, Y_t, Z_{t+1}, Z_t) = 0, \tag{26}$$

where X_t, Y_t,and Z_t represent respectively a vector of endogenous state variables, control variables, and exogenous shock processes.

The solution for (25) and (26) will give a vector of real portfolio holdings $\alpha(X_t, Z_t)$ for each traded asset. In general, it is difficult to obtain solutions for portfolios in DSGE models, or even to characterize the properties of the solutions. In Devereux and Sutherland, (2006, 2007), a simple method is developed for solving for the characteristics of $\alpha(X_t, Z_t)$, at the zero and first order.

A brief description of the method is as follows. In a static, partial equilibrium environment with one investor, Samuelson (1970) shows that in order to obtain the properties of the portfolio at the order N, is necessary to approximate the investors utility function up to the order $N + 2$. Samuelson's method involves identifying the optimal portfolio for small shocks. We employ Samuelson's method in the case of a dynamic, general equilibrium environment. In our case, the optimal portfolio is approximated as

$$\alpha(X_t, Z_t) \approx \alpha(\overline{X}, \overline{Z}) + \alpha_x(\overline{X}, \overline{Z})\widehat{x}_t + \alpha_z(\overline{X}, \overline{Z})\widehat{z}_t,$$

where $\overline{X}$, $\overline{Z}$, represent the non-stochastic steady-state values of X and Z, and $\widehat{x}$, $\widehat{z}$ represent log deviations from the non-stochastic steady-state. The term $\alpha(\overline{X}, \overline{Z})$ represents the zero-order, or steady state portfolio, while the $\alpha_x(\overline{X}, \overline{Z})\widehat{x}$ and $\alpha_z(\overline{X}, \overline{Z})\widehat{z}$ terms represent the first-order components of the portfolio, capturing the way in which real portfolio holdings adjust to predictable changes in state variables. Devereux and Sutherland (2006) show that $\alpha(\overline{X}, \overline{Z})$ may be obtained by a combination of a second-order approximation of (25), and a first-order approximation of (26), where the approximation is taken at the non-stochastic steady-state point. Devereux and Sutherland (2007) show that $\alpha_x(\overline{X}, \overline{Z})$ and $\alpha_z(\overline{X}, \overline{Z})$ may be obtained by a third-order approximation of (25), in combination with a second-order approximation of (26).

4.2 Computing Steady State NFA

As previously noted, the endogeneity of the discount factor in (6) ensures stationarity in the NP and EQ models. While this ensures a well-defined steady state wealth distribution across the two countries, it provides no direct guidance regarding the appropriate calibration and computation of the steady state NFA position. It is therefore useful to describe our resolution of this problem before analysing our model in detail.

One way to tie down the steady state NFA position is to choose values for the parameter ω in (7), and its foreign counterpart ω^*, to deliver empirically plausible NFA holdings in the non-stochastic steady state of the model. The parameters ω and ω^* make it possible to capture structural differences in savings propensities across the two countries. Thus, for instance, if ω is less than ω^* the home country will have relatively lower steady state consumption, and the home NFA position will be negative in the non-stochastic steady state. In this way, a non-zero steady state NFA position is supported and explained by differences in time preferences between the two countries. In particular one could calibrate ω and ω^* in our model to yield a non-stochastic steady state NFA position for the home country approximately equal to the current US NFA position (i.e. approximately -20% of GDP).

This approach to determining steady state NFA is not fully satisfactory however, either from an empirical or theoretical perspective. Empirically, while there may be some grounds to believe that the currently observed negative US NFA position (and the corresponding positive NFA position of developing nations) is partly driven by differences in time preferences, it is likely that a substantial part of the NFA position is driven by precautionary savings on the part of developing countries (see Bernanke, 2005). In theoretical terms, it is unsatisfactory to base the approximation of our model around an exogenously determined steady state level of NFA. In the exact solution to the model, the distribution of NFA will be determined both by differences in time preferences, the menu of assets available for risk sharing, and the distribution of country specific risk faced by home and foreign households[6]. For given gross positions, differences in country specific risk give rise to differences in the degree of 'precautionary saving' among households in the two countries. But the menu of available assets also determines the size of the gross asset and liability positions that countries hold. With larger gross asset positions, differences between countries in time preference or country specific risk also give rise to larger net positions, or in other words larger NFA.

To capture these effects in the approximate solution to our model, we solve for the *stochastic* steady state of the second-order approximation of the model. This captures the impact of second moments of variables on their first moments, or equivalently, how risk affects the mean levels of endogenous economic variables. The difference between the stochastic steady state value for NFA and the value in a deterministic steady state in this second-order solution thus provides an approximate measure for the impact of uncertainty on NFA in the solution of the exact non-linear stochastic model. In computational terms, we follow an iterative procedure which starts with a second-order approximation of the model taken around an initial guess for the steady state NFA position. The stochastic steady state of this approximation is used to update the approximation point and the model is re-solved to yield a new estimate of the stochastic steady state. The procedure is repeated until the solution for the NFA position converges. This provides our approximate solution for the NFA position in the stochastic steady state of the exactly solved model.

[6] For an example of such an outcome, see Durdu et. al. (2007).

In the simulations reported below, we set $\eta = 0.01$ and we choose ω and ω^* so that the benchmark version of the EB model yields a home country NFA position of -20 percent of GDP in an approximate stochastic steady state (computed using the methodology just described). The values of ω and ω^* thus chosen are held constant across the NP, EB and EQ models, and across the other parameter variations reported below, and the approximate stochastic steady state is computed for each case. The steady state NFA position therefore varies endogenously in response to differences in risk across the different versions of the model and across the different parameter variations. The resulting values of steady state NFA are reported below.

5 Risk Sharing under Alternative Asset Market Configurations

5.1 Calibration

In order to solve the model, we first need to choose parameter values. In the absence of a more detailed study of emerging market economies, we choose a set of parameter values guided by previous literature. Table 1 describes the calibration

Table 1: Model Calibration

Parameter	Value	Parameter	Value
$\beta(\bar{C})$, $\beta(\bar{C}^*)$	0.96	ζ_A	0.9
ρ	1	ζ_{A^*}	0.9
μ	1	σ_A	0.02
γ	0.36	σ_{A^*}	0.05
γ^*	0.5	σ_M	0.05
δ	0.1	ϵ_ϕ	0.3
η	0.01	ω/ω^*	$\overline{NFA}$=-0.2$\bar{Y}$

We choose the parameters of $\beta(.)$ so that the steady-state real interest rate is 4 percent. We assume agents have log utility of consumption. The consumption constant elasticity of labor supply is set to unity. The share of capital in the production function in the home economy is set at 0.36, as is standard, but noting the lower share of labour in GDP in emerging market economies we set this parameter to 0.5 for the foreign country. We set the persistence parameter on the productivity shock equal to 0.9 for each country The capital depreciation rate is set at 0.1. We choose the volatility of productivity disturbances roughly to match the standard deviation of productivity shocks in the US, and the much higher volatility found in emerging markets (see, e.g. Garcia-Cicco et al, 2007). Hence the standard deviation of home productivity shocks are set at 2 percent, and the standard deviation of foreign productivity is set at 5 percent. The standard deviation of velocity shocks is set approximately to match the annual standard deviation of money velocity for the US economy, which is 0.05. We follow previous literature on investment adjustment costs in choosing the elasticity of the ϕ function, ϵ_{ϕ}, so that the implied elasticity of Tobin's q to investment is 0.3 (e.g. Bernanke et al. 1999). χ and χ^* are chosen so that $\bar{Y} = \bar{Y}^* = 1$.

5.2 The NP economy

In the absence of any portfolio diversification, the degree of risk-sharing is determined only by intertemporal borrowing and lending. In this sense, the NP model is essentially equivalent to that of Baxter and Crucini (1995). Figure 5 and Table 2 describe the basic risk-sharing properties of the model. Figure 5 illustrates the response of the trade balance, output, and consumption to shocks to home country productivity. Table 2 describes the volatility of consumption and output in the NP economy. Both output and consumption volatility are higher for the foreign economy, given that it experiences a higher variance of productivity shocks, and risk sharing is limited to a non-contingent bond. In Figure 5, a home productivity shock will initially generate a home country trade account deficit, as both consumption and investment in the home country rise relative to the foreign country. The rise in productivity also leads to a small rise in foreign country consumption, as investment falls in the foreign country, increasing foreign residents' dividend payments for any level of output. As is common in one-good neoclassical models, the home country productivity shock causes a rise in home output, but a slight fall in foreign output, since both investment and labor supply fall in the foreign economy.

Table 2: Volatility and correlations in the NP economy

$sd(C)$	$sd(C^*)$	$cor(C, C^*)$	$sd(Y)$	$sd(Y^*)$	$cor(Y, Y^*)$
1.19%	2.58%	0.35	2.32%	5.65%	-0.08

Note that, since there is no portfolio diversification in the NP economy, the behavior of gross and net foreign assets is equivalent. In addition, since the rate of return on the real bond is not state contingent, the response of the trade account and ΔNFA is equivalent. That is, there can be no valuation effects of shocks through movements in the ex-post return to external assets or liabilities.

The restricted risk sharing offered by the asset menu in the NP economy implies a strong incentive for precautionary savings for the developing country. We find that the developing country has a steady state NFA position of approximately 118% of GDP, which is considerably higher than the position implied by the EB model for the benchmark parameter set (which is calibrated to be 20% of GDP).

5.3 The EB economy

In the EB economy, we have to solve for the portfolio shares of equity and nominal bonds held by each country. These portfolio positions will in turn impact on the degree of risk-sharing that is achieved across countries. Following the procedure described in section 4 above, we first solve for the steady-state portfolio holdings.

Table 3 and Figure 6 describe the results for the EB economy. In equilibrium, the foreign economy takes a large positive position in home currency nominal bonds. That is, $B_h < 0$. At the same time, it sells equity-FDI to the home country. Since nominal bond returns are positively correlated with home GDP, and equity returns co-vary positively with foreign GDP, this represents an efficient risk-sharing portfolio allocation. The steady-state portfolio is measured relative to the steady-state capital stock of the foreign country. Table 3 indicates that $\overline{q_k S_f}/\overline{K^*} = 0.73$, so that the home country will hold about 73 percent of the foreign capital stock in FDI.[7]

[7] Since the capital output ratio in the foreign economy is about 3.5, this represents an equity holding of about 2.6 times steady-state output. Since $\overline{q_b B_h} + \overline{q_k S_f} = \overline{NFA}$ and $\overline{NFA} = -0.2\bar{Y}$ it follows that home country gross debt is about 2.8 times steady-state output. Note that the model does not incorporate any

Table 3: The equilibrium portfolio, NFA, volatility and correlations in the EB economy

	$\overline{q_k S_f}/\overline{K^*}$	$sd(C)$	$sd(C^*)$	$cor(C, C^*)$	$sd(Y)$	$sd(Y^*)$	$cor(Y, Y^*)$	$\overline{NFA}/\bar{Y}$
Benchmark	0.73	1.16%	2.50%	0.50	2.50%	5.75%	-0.16	-0.2
$\sigma_M = 0.025$	1.33	1.18%	2.28%	0.68	2.64%	5.89%	-0.22	-0.35
$\sigma_M = 0.0$	1.84	1.19%	2.03%	0.92	2.78%	6.02%	-0.28	-0.50
$\sigma_A = 0.01$	0.67	0.87%	2.46%	0.51	1.28%	5.73%	-0.24	-0.23

Why does the home country take a positive position in FDI? This is most easily explained by considering the case where $\eta = 0$ (i.e. where consumers' discount factors are exogenous). In this case the zero-order portfolio represents a solution to the orthogonality condition

$$E_t[(\widehat{C}_{t+1} - \widehat{C}^*_{t+1})\widehat{r}_{xt+1}] = 0 \tag{27}$$

where a hat indicates a log-deviation from the approximation point. The motive for portfolio diversification may be obtained by computing, from a first-order approximation of the model, the covariance between relative consumption and the ex-post excess return, when there is a *zero* portfolio, i.e. in the absence of any portfolio diversification. For a zero portfolio, a rise in A, the home country productivity, leads to a rise in relative home consumption, but a fall in the excess return on foreign equity. This is primarily due to fall in the home country price level, increasing the realised return on the holding of home currency bonds.[8] A rise in A^*, the foreign productivity shock, leads to a rise in the excess return on foreign equity, since it directly increases the dividend payment of the foreign firm, but a fall in relative home consumption. Thus, on both counts, holding a positive (negative) position in foreign equity (home bonds) represents an optimal diversification strategy for home-country residents.

trading costs or other frictions which could potentially reduce the degree of portfolio diversification. The aim is to explore the maximum degree of cross country risk-sharing that can be achieved in the EB economy relative to a complete markets environment, independent of other potential impediments to international financial investment.

[8] A rise in home output will lead to a fall in P, as implied by (11). Since there is one world good and no barriers to trade, PPP will hold at all times and the rise in P is equivalent to an appreciation of the nominal exchange rate.

The equilibrium portfolio under the EB economy increases the degree of risk-sharing considerably relative to the NP economy. The cross correlation of consumption rises considerably, and consumption volatility falls in the foreign country, while consumption volatility in the home country is essentially unchanged.[9]

Figure 6 shows the impact of a shock to home country technology in the EB economy. The impact on home consumption is less, and that on foreign consumption is greater, than that of the NP economy. The impact of the shock on the trade account is now slightly positive, since home consumption increases by less as a result of the risk-sharing inherent in the optimal portfolio position.

Figure 6 shows that the degree of cross-country risk-sharing is intimately tied to 'valuation effects', coming from the response of the ex-post return on bonds, and the ex-post return on foreign equity. Following a shock to the home country technology, the home price level falls (or equivalently, the exchange rate appreciates). So the return on home currency bonds rises. Since the home productivity shock is persistent, the expected rate of return on all assets must rise. This reduces investment in the foreign country, leading to a fall in the return on the foreign equity. On both counts, the home country suffers a capital loss on its portfolio, since it has a negative position on its own currency bonds, and a positive position in foreign country equity. The sum of the slight positive movement in the trade account, and the large capital loss on the portfolio lead to a sharp deterioration in the home country ΔNFA. The movement in ΔNFA and the trade account differ due to this valuation effect on the pre-existing portfolio. Hence, net foreign assets fall abruptly, even though the trade account moves almost not at all. The fall in net foreign assets reduces home consumption (and increases foreign consumption) relative to the response in the NP economy. The critical point is that this valuation effect represents an optimal, ex-post, risk-sharing mechanism, given the distribution of productivity shocks in each country.

How does the degree of risk sharing depend upon the underlying distribution of shocks? Table 3 illustrates the effects of different levels of volatility of home-country velocity and technology. Lower volatility of home velocity leads to a much larger (more positive) home FDI position (and simultaneously a more negative bond position). A reduction in the volatility of velocity implies that the return on nominal bonds is more closely correlated

[9] Of course, in the case where one country has higher autarky consumption volatility, gains from asset trade do not necessarily imply that all countries reduce consumption volatility, relative to autarky.

to real shocks and thus bonds become a more useful risk-sharing instrument. This leads to more consumption risk sharing, i.e. the cross-country correlation of consumption increases.

Table 3 shows that lower volatility of home technology leads to a smaller home-country gross asset and liability position. This is because, for a given level of velocity volatility, lower volatility of technology implies that the home country nominal bond return is more dominated by velocity shocks. Hence, the nominal bond becomes a less efficient risk-sharing instrument. A lower volatility of home country technology therefore reduces the size of gross portfolio holdings.

The final column of Table 3 reports the steady state NFA positions implied by the different parameter variations. As previously explained, we choose values of ω and ω^* which imply that the steady state NFA position of the developed country is -20% of GDP in the benchmark case. The resulting steady state NFA position is therefore partly determined by differences in time preferences and partly determined by the endogenous responses to economic uncertainty in the model, including the role of precautionary savings. The absolute size of the net position is also influenced by the absolute size of gross portfolio positions. Ceteris paribus, for given differences in time preference, the larger is the gross asset and liability position, the larger will be the net position also. The relative importance of all these forces changes in the different cases illustrated in Table 3. There is thus no single simple explanation for the steady state NFA position in the different cases, except to note that, as the degree of risk sharing is increases, precautionary savings diminish and the NFA position is largely a result of differences in time preferences.

5.3.1 Capital Flows

The solution for steady-state optimal portfolio holdings is all that is required to derive the response of the real economy to shocks at the level of first-order approximation, and therefore characterizes the implications for cross country risk sharing in terms of the second moments of consumption. But, as discussed in section 3, the true portfolio holdings will in general, be time-varying, moving in response to changes in net foreign assets, as well as predictable changes in productivity and capital stocks in each country. The dynamics of home country holdings of foreign equity and domestic bonds will be related to movements in the three underlying shocks in the model; A, A^* and M, and the three predetermined endogenous state variables, NFA, K, and K^*. From section 3, we may describe the first-order movement in the home country holding of FDI, $\alpha_{FDIt} = q^*_{kt}S_{ft}$, in the EB case as

$$\widehat{\alpha}_{FDIt} = \alpha_{FDIt} - \overline{\alpha} \approx \gamma_1 \widehat{A}_t + \gamma_2 \widehat{A}^*_t + \gamma_3 \widehat{M}_t + \gamma_4 \widehat{NFA}_t + \gamma_5 \widehat{K}_{t+1} + \gamma_6 \widehat{K}^*_{t+1}. \tag{28}$$

Because portfolio holdings must add up to net wealth, it must also be the case that $\widehat{\alpha}_{FDIt} + \widehat{\alpha}_{Bt} = \widehat{NFA}_t$.[10] Hence, from (28), we may also determine the dynamics of real holdings of nominal bonds, given that $\widehat{NFA}_t$ is determined by the first-order approximation of the model described previously.

For the benchmark parameter set the γ coefficients are

$$\gamma_1 = 0.660,\ \gamma_2 = 1.267,\ \gamma_3 = 0.0,\ \gamma_4 = 0.865,\ \gamma_5 = 0.549,\ \gamma_6 = 0.763$$

The interpretation of the γ coefficients is as follows. A rise in either A or A^* will lead the home country to increase its positive position in foreign equity, as well as its negative position in home bonds. As described in Devereux and Sutherland (2007), this is due to the fact that persistent movements in productivity will lead to an increase in the covariance between relative consumption and excess returns, as described in (27), generating an increased desire for hedging relative consumption risk, which is satisfied by a rise in the size of gross portfolio positions. The movement in home and foreign capital stocks has a similar effect. Portfolio holdings are independent of the dynamics of velocity shocks, since anticipated velocity shocks are neutral in this model. In addition, we find that an increase in net foreign assets, $\widehat{NFA}$, tends to be disproportionally allocated towards foreign equity relative to home currency nominal bonds.

Figure 6 illustrates the response of gross assets to the productivity shock. From our previous discussion, we know that net foreign assets fall. Figure 6 shows that this fall in net foreign assets is broken down into a fall in the gross FDI assets, and a rise in gross nominal debt liabilities. Thus, an optimal response to a positive home productivity shock is to *reduce* the holdings of foreign equity, and increase gross nominal debt.

Recall that $\alpha_{Bt} = q_{Bt}B_t$ and $\alpha_{FDIt} = q^*_{kt}S_{ft}$. So real gross asset and liability movements, i.e. changes in α_{FDIt} and α_{Bt}, may be further broken down into price and volume

[10] α_{FDIt} is defined to be the holdings of foreign equity at the end of period t. Thus $\hat{\alpha}_{FDIt}$ is a function of the values of the state variables realised at the end of period t. Given the notational convention we have adopted for time subscripts, it follows that $\hat{\alpha}_{FDIt}$ is a function of $\widehat{A}_t$, $\widehat{A}^*_t$, $\widehat{M}_t$, $\widehat{F}_t$, $\widehat{K}_{t+1}$ and $\widehat{K}^*_{t+1}$.

movements. Changes in α_{Bt} can therefore be broken down into changes in price, q_{Bt}, and changes in volume, B_t. Likewise, changes in α_{FDIt} can be broken down into changes in q^*_{kt} and the changes in S_{ft}. To a first-order approximation the following relationships hold

$$\begin{aligned} \hat{\alpha}_{FDIt} &= \overline{q^*_k S_f} \times \hat{q}^*_{kt} + \hat{S}_{ft} \\ \hat{\alpha}_{Bt} &= \overline{q_B B} \times \hat{q}_{Bt} + \hat{B}_t \end{aligned} \tag{29}$$

where, for convenience, we define $\hat{S}_{ft} = \bar{q}^*_k(S_{ft} - \bar{S}_f)$ and $\hat{B}_t = \bar{q}_B(B_t - \bar{B})$. From the discussion above, it follows the rise in A leads to a rise in $\hat{q}_{Bt}$ (since the anticipated future price level falls) and a fall in $\hat{q}^*_{kt}$. We also know that both bond holdings, $\hat{\alpha}_{Bt}$, and equity holdings, $\hat{\alpha}_{FDIt}$, fall. It is simple to use (29) to calculate the implied movements in $\hat{B}_t$ and $\hat{S}_{ft}$. These are also plotted in Figure 6. As an example, consider period 1 in more detail. In period 1 the fall in bond holdings is 1.29% (of $\bar{Y}$) whereas bond prices, $\hat{q}_{Bt}$, rise by 1.23%. Using the fact that $\overline{q_B B} = -2.76$ equation (29) implies that $\hat{B}_t = 2.10$ in period 1, i.e. in volume terms $\hat{B}_t$ rises. The change in equity holdings in period 1 is -2.44% while the equity price, q^*_{kt}, falls by 0.16%. Combined with the fact that $\overline{q^*_k S_f} = 2.56$ it follows from (29) that $\hat{S}_{ft} = -2.05$. So the fall in equity holdings is accounted for by a combination of a fall in $\hat{q}_{kt}$ and a fall in $\hat{S}_{ft}$, i.e. both the price and volume of equity holdings fall.

5.3.2 Diversification with Real Bonds

An important part of the mechanism that determines portfolio diversification in the EB economy is movement in the home economy price level. This allows the home nominal bond to act partially as a claim on home output, but on the other hand, exposes the foreign holders of bonds to price level uncertainty coming from velocity shocks. How would the risk sharing potential change if, alternatively, all home nominal bond returns were fixed in real terms (rather than nominal)? This would still allow some potential risk-sharing, because the home country would still assume part of the foreign productivity risk by holding foreign equity, while the foreign country residents would hold a positive quantity of non-contingent claims on the home economy. It is easy to amend our model to the case of real (or indexed) bonds rather than nominal bonds. The results indicate the presence of a trade-off. On the one hand, if there were no velocity shocks to the price level, then nominal bonds would be more desirable in terms of risk-sharing potential, since they offer an indirect state contingent claim on home output, whereas the return on real bonds are fixed in output terms. On the other hand, if velocity shocks are high enough, then they substantially reduce the desirability of foreigners to hold nominal bonds, which in turn, in equilibrium, reduces the size of home equity holdings on the foreign economy. This can eliminate the overall ability of the EB economy to sustain risk sharing. As a result, the equilibrium with real bonds may offer more overall risk-sharing potential. Note also that the EB economy has implications for monetary policy. Take an equilibrium where monetary authorities targeted the domestic price level. If the domestic price level were fixed, the nominal bond would no longer act as a partial claim on domestic output. As discussed in Svensson (1989), when nominal bonds allow for risk sharing, price level targeting is not necessarily a desirable monetary rule.

5.4 The EQ Economy

How does the degree of risk-sharing in the asymmetric case of the last section compare with a world asset market where there is unrestricted trade in equities? We now focus on the EQ economy, where there is trade in both home and foreign equities and non-contingent bonds. In this case, there is effectively complete markets[11].

For the benchmark parameter set, we find the optimal steady-state portfolio holdings imply that the home country holds foreign equity equivalent to 234% of the foreign capital stock while the foreign country holds home equity equivalent to 350% of the home capital stock.[12] Thus each country takes a large positive position in the equity of the other country. In fact, as in Baxter and Jermann (1997), each country holds a larger position in foreign equity than in home equity, since home equity returns and (non-diversifiable) home labor income are positively correlated. The implied volatility and correlation of consumption and output is shown in Table 4. There is perfect risk sharing across countries. But note that

[11] It is necessary to allow for holdings of real bonds because the two economies are asymmetric - having different shares of capital in production, and thus different shares of equity returns in total income, in an autarky situation. We could also allow for free equity trade and trade in the home country nominal bond. In this case, the portfolio outcome is almost identical to that reported below. That is, agents take large equity positions, and very small positions in nominal bonds - and the market is complete. Hence, the assumption of no foreign holdings of foreign equity in the EB model is important for the results. The rationalization for this assumption has been discussed in footnote 2.

[12] Thus the home country holds a short position in home equity and the foreign country holds a short position in foreign equity.

the cross-country correlation of consumption is not equal to unity because of the effects of endogenous discounting.

Table 4: Volatility and correlations in the EQ economy

$sd(C)$	$sd(C^*)$	$cor(C, C^*)$	$sd(Y)$	$sd(Y^*)$	$cor(Y, Y^*)$
1.64%	1.40%	0.98	2.64%	6.16%	-0.32

We find that the steady state NFA position of the home economy in the EQ model is -45% of GDP. With complete markets and perfect risk sharing there is clearly no role for precautionary savings, so this NFA position largely reflects the large size of absolute gross positions, in combination with differences in time preferences.

The mechanics of risk-sharing in the EQ case are very similar to those of the EB case. Figure 7 illustrates that a rise in home productivity leads to an immediate capital loss on the home portfolio, since the home country is short (long) in the home (foreign) equity. With unrestricted trade in equity, it is possible to hold gross equity assets and liabilities to the extent that these equity valuation effects perfectly share consumption risk.

We may also describe capital flows in the EQ model. Figure 7 illustrates that the fall in net foreign assets following a rise in A is accounted for by a reduction in gross foreign equity assets and a rise in home equity liabilities. But again, the decomposition into price and volume effects differs between assets and liabilities. The rise in the price of home equity automatically increases the liability position of the home country, for a given equity holding. Since the rise in the equity price increases gross liabilities more than the equilibrium response of home equity liabilities, in the adjustment to the shock, the home country actually increases its holdings of home equity.

5.5 Discussion

In comparing the three financial structures we may conclude that the EB configuration, which can be thought of in a very crude sense as reflecting the portfolio inter-relationship between emerging economies and advanced economies, can sustain a considerable degree of cross-country risk-sharing. But the risk-sharing achieved is always less than that under full unrestricted trade in equity, as in the EQ economy. Moreover, the maximum feasible risk-sharing is limited by velocity volatility in the home economy. The higher is the volatility of velocity, the less successful are home-currency nominal bonds in sustaining risk-sharing, and the closer is the EB economy to the NP economy. Moreover, paradoxically, we find that, for a given degree of velocity volatility, a fall in output volatility in the home economy will reduce gross portfolio holdings, since it reduces the efficiency of nominal home currency bonds in sharing risk.

We have also shown that the implications of optimal portfolio structure of the gross flows of international capital, and the breakdown of portfolio adjustment into price and volume effects may be quite complex, depending on the types of shocks as well as other details of the model.

6 Conclusions

The recent engagement of emerging market economies in the international financial system has led to greater optimism with respect to the ability of these countries to reduce the consumption and income implications of their high level of country risk. But the consequences of financial market integration for international risk-sharing, at least when financial markets remain incomplete, are complicated, and depend on the available menu of assets and the size of optimal financial portfolios. Previous literature in this area has been limited by the difficulty in incorporating portfolio choice in policy-relevant dynamic general equilibrium environments. This paper represents a first step in addressing these limitations, using some recent theoretical advances in dynamic portfolio choice modelling. Future developments in modelling are required more accurately to account for the size and evolution of observed country portfolios. This will likely require a more general model, potentially incorporating multiple goods, home bias in consumption and trading costs.

References

[1] Baxter, M. and M. Crucini (1995),"Business Cycles and the Asset Structure of Foreign Trade", *International Economic Review,* 36, 821-54.

[2] Baxter, M. and U. Jermann (1997) "The International Diversification Puzzle is Worse than you Think", *American Economic Review,* 87, 170-180.

[3] Bernanke, B. (2005) "The Global Saving Glut and the U.S. Current Account Deficit", Federal Reserve Board, Speeches Archive.

[4] Bernanke, B., M. Gertler, and S. Gilchrist (1999), "The Financial Accelerator in a Quantitative Business Cycle Model", in J. Taylor and M. Woodford, ed. Handbook of Macroeconomics, Elsevier.

[5] Caballero, R. and S. Panageas, (2004), "Contingent Reserves Management: An Applied Framework", NBER Working Paper No 10786.

[6] Garcia-Cicco, J., R. Pancrazzi, and M. Uribe, (2007) "Real Business Cycles in Emerging Economies?", unpublished manuscript, Duke University.

[7] Devereux, M. and A. Sutherland (2006) "Solving for Country Portfolios in Open Economy Macro Models" CEPR Discussion Paper No 5966.

[8] Devereux, M. and A. Sutherland (2007) " Country Portfolios Dynamics" CEPR Discussion Paper No 6208.

[9] Durdu, C. B., E. Mendoza, and M. Terrones (2007) "Precautionary Demand for Foreign Assets in Sudden Stop Economies: An Assessment of the New Mercantilism", FRB International Finance Discussion Paper No. 911.

[10] Engel, C. and A. Matsumoto (2005) "Portfolio Choice in a Monetary Open-Economy DSGE Model" unpublished manuscript, University of Wisconsin and IMF.

[11] Evans, M. and V. Hnatkovska (2005) "International Capital Flows, Returns and World Financial Integration" NBER Working Paper No 11701.

[12] Jeanne, O. (2007), "International Reserves in Emerging Markets: Too Much of a Good Thing?", Brookings Papers on Economic Activity, 1.

[13] Jeanne, O. and R. Ranciere, (2006) "The Optimal Level of International Reserves for Emerging Markets: Formulas and Applications", IMF Working Paper No 06/229.

[14] Kollmann, R. (2006) "International Portfolio Equilibrium and the Current Account" unpublished manuscript, University of Paris XII.

[15] Lane, P., and G-M. Milesi-Ferretti (2001) "The External Wealth of Nations: Measures of Foreign Assets and Liabilities for Industrial and Developing Countries" *Journal of International Economics* 55, 263-94.

[16] Lane, P., and G-M. Milesi-Ferretti (2005) "A Global Perspective on External Positions", IIIS Discussion Paper No 79, Trinity College Dublin.

[17] Lane, P., and G-M. Milesi-Ferretti (2006) "The External Wealth of Nations Mark II" IMF Working Paper No 06-69.

[18] Mendoza, E. V. Quadrini, and V. Rios Rull (2007),"Financial Integration, Financial Deepness, and Global Imbalances", NBER Working Paper No 12909.

[19] Schmitt-Grohe, S. and M. Uribe (2003) "Closing Small Open Economy Models", Journal of International Economics, 61, 163-85.

[20] Samuelson, P. A. (1970) "The Fundamental Approximation Theorem of Portfolio Analysis in terms of Means, Variances and Higher Moments" *Review of Economic Studies*, 37, 537-542.

[21] Svensson, L. (1989) "Trade in Nominal Assets", *Journal of International Economics,* 26, 1-28.

[22] Tille, C and E van Wincoop (2007) "International Capital Flows" unpublished manuscript, University of Virginia.

Figure 1: Developing and Emerging Market Countries
Net Foreign Debt

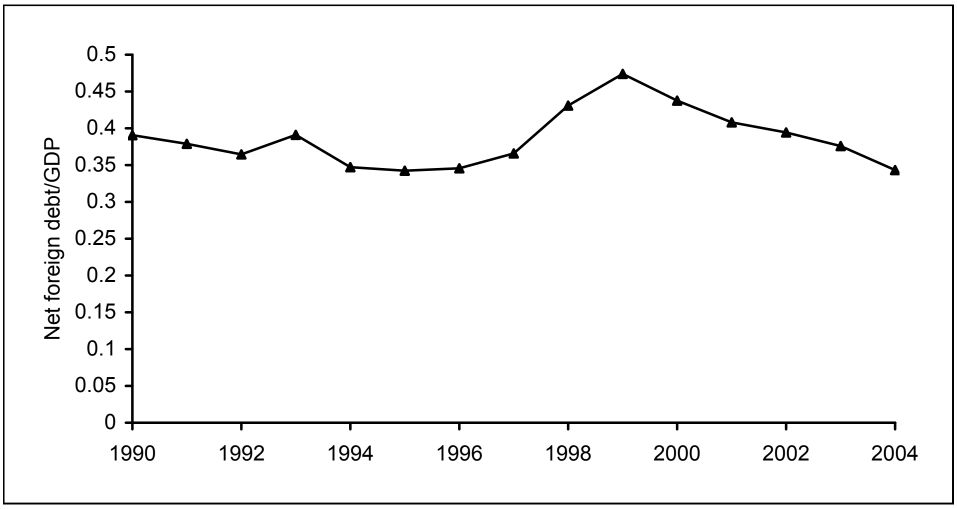
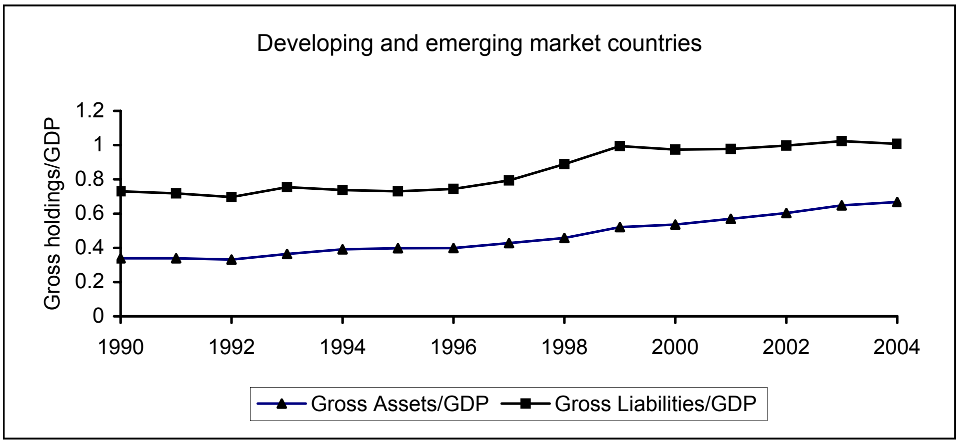
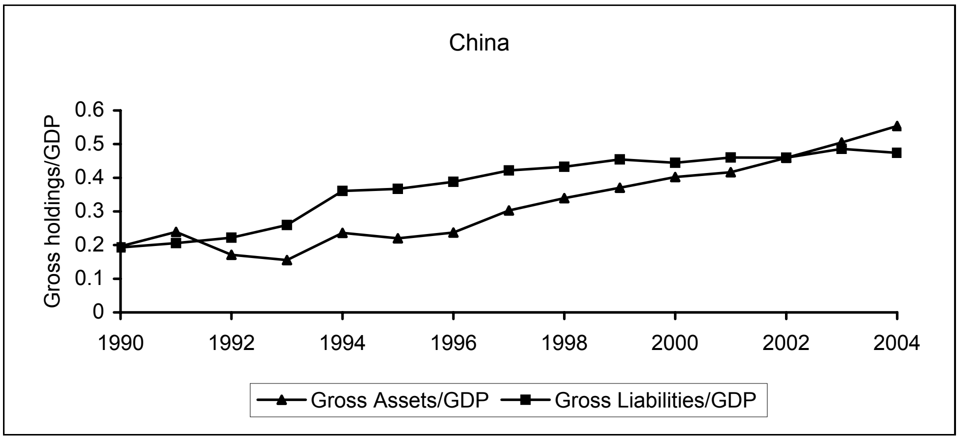


Figure 2: Gross Portfolios





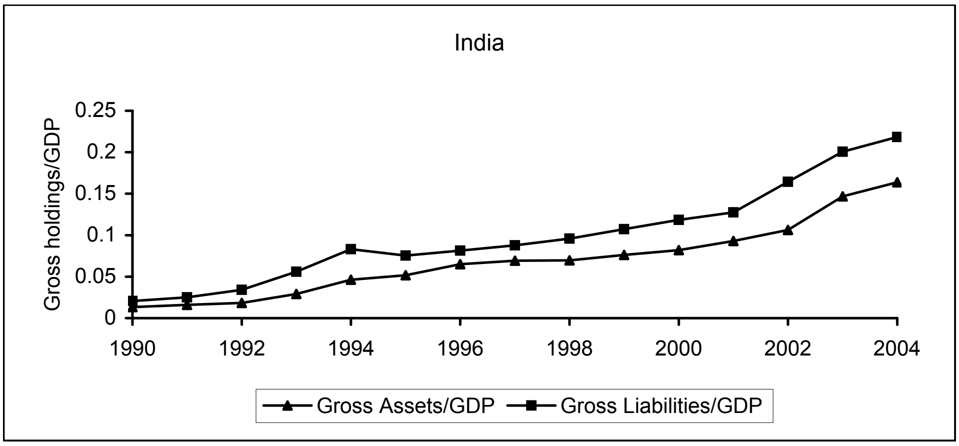
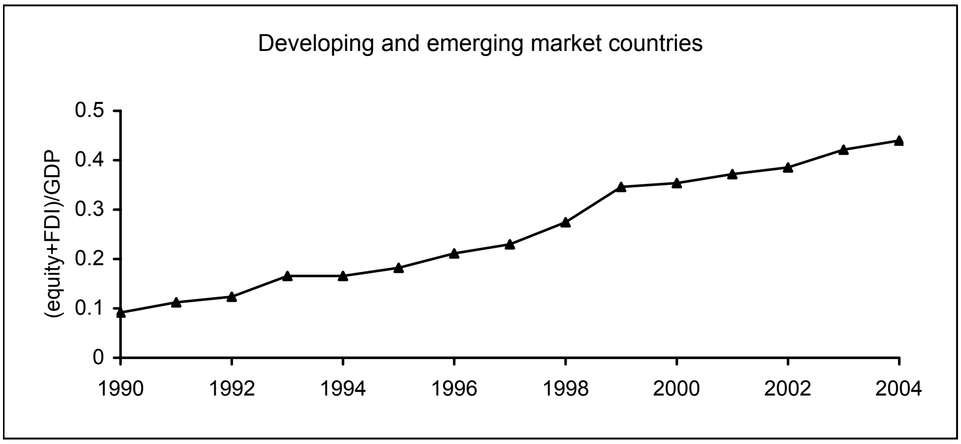
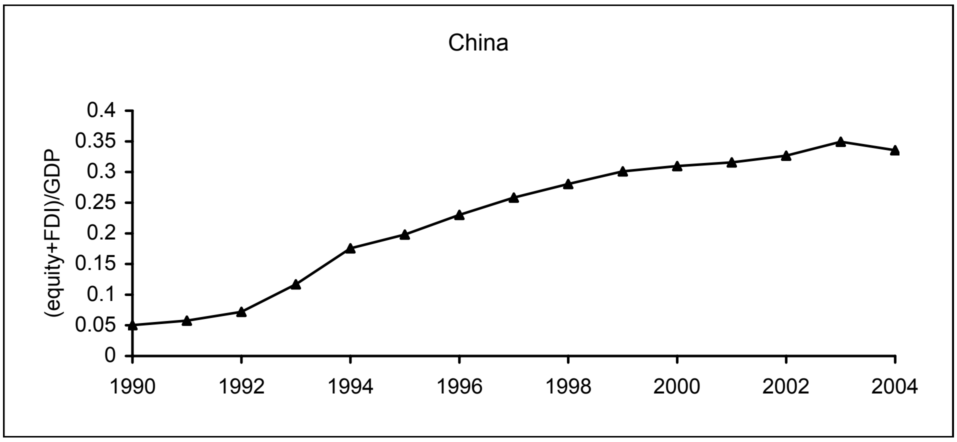


Figure 3: Equity and FDI Liabilities





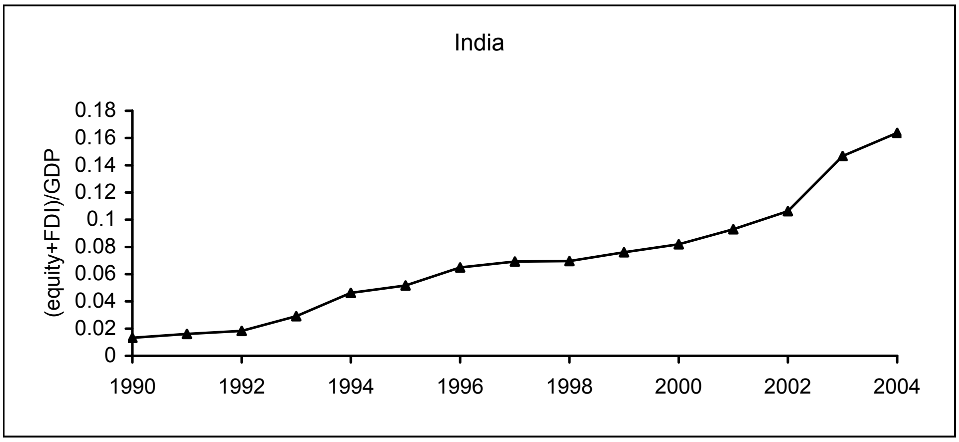
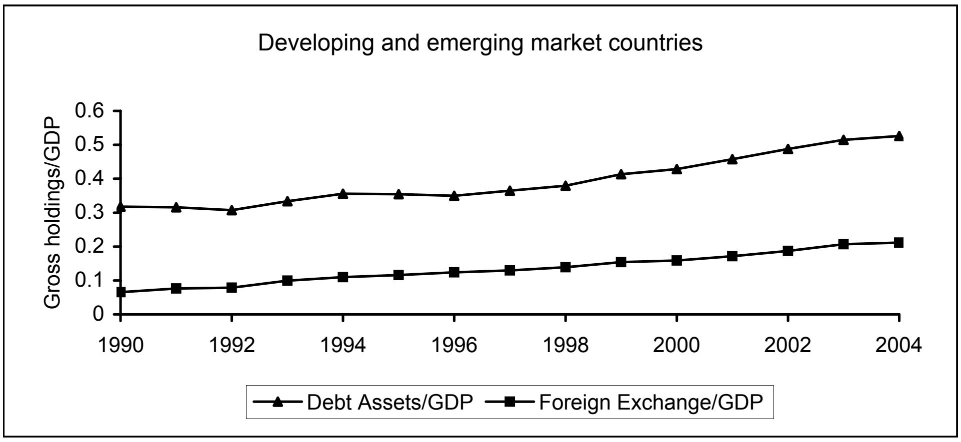
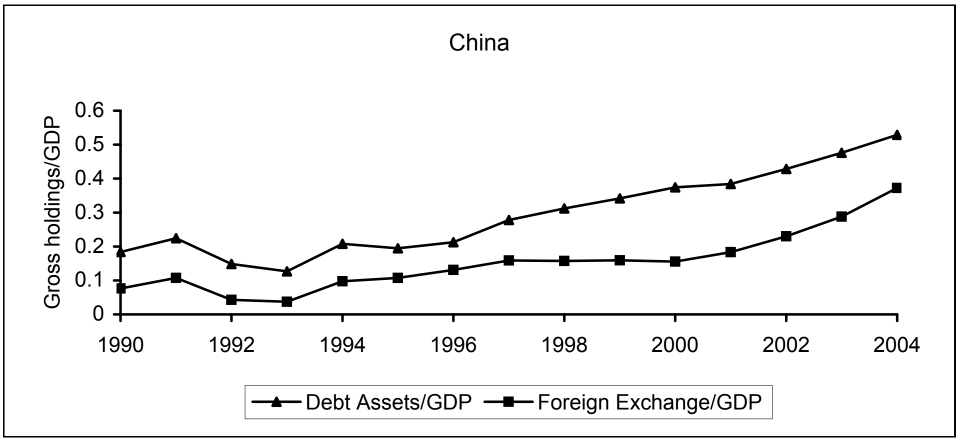
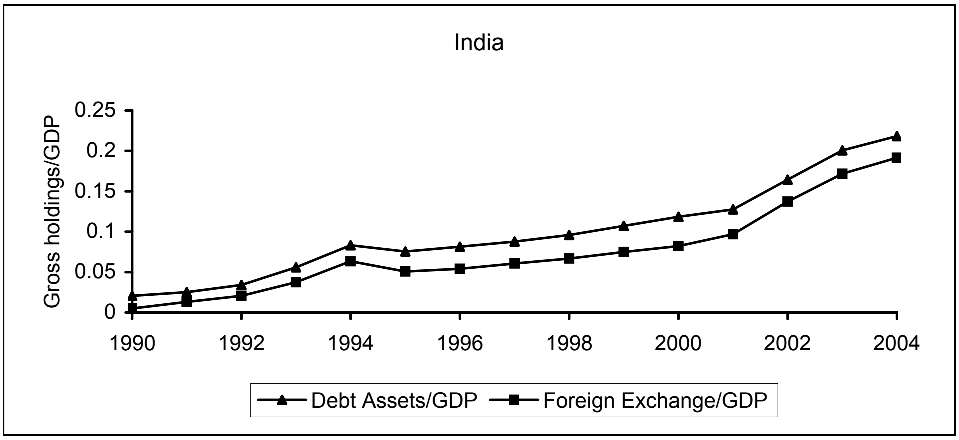


Figure 4: Debt Assets and Foreign Echange Reserves







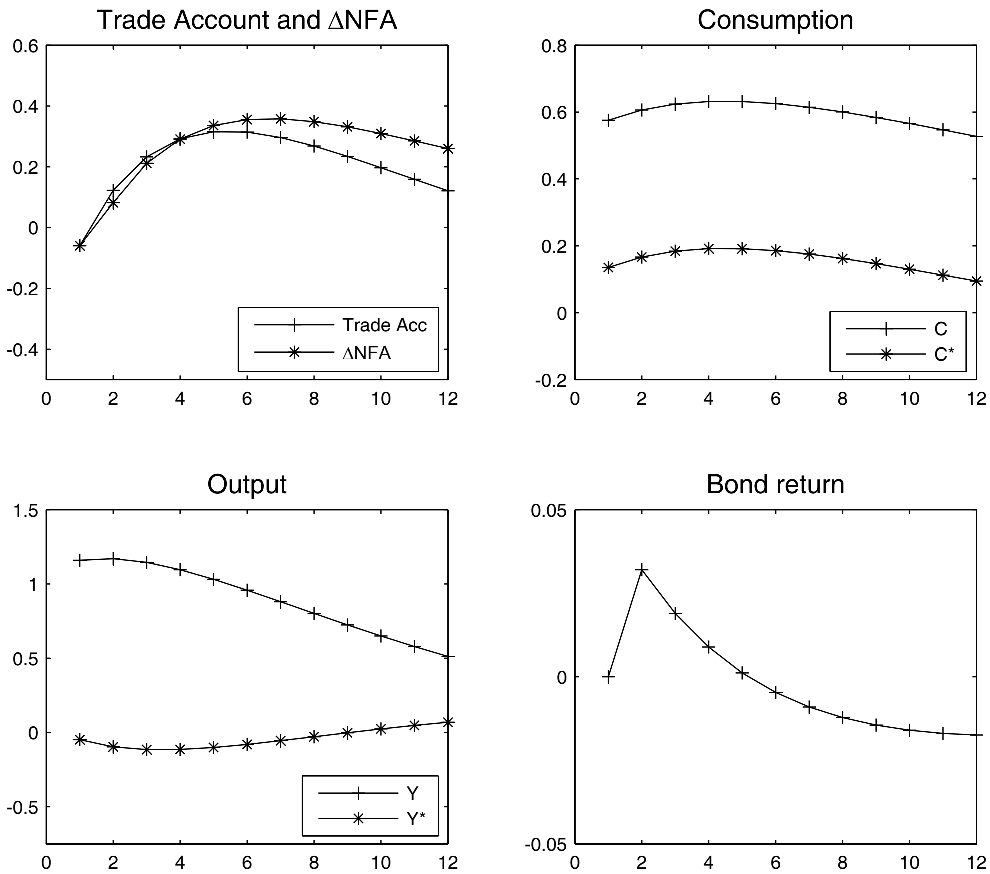


Figure 5: Trade in non-contingent bonds (the NP economy)

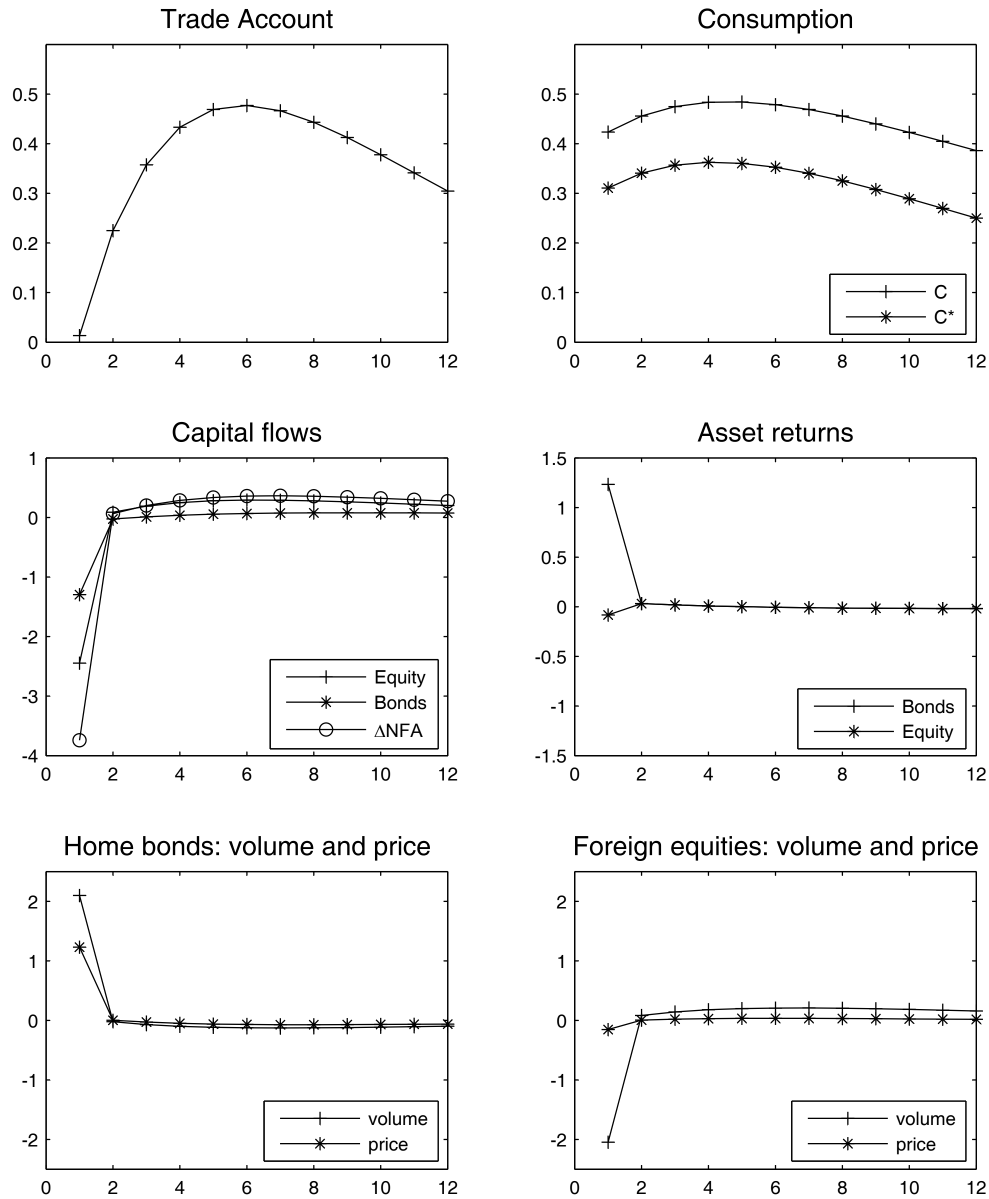


Figure 6: Trade in home nominal bonds and foreign equity (the EB economy)

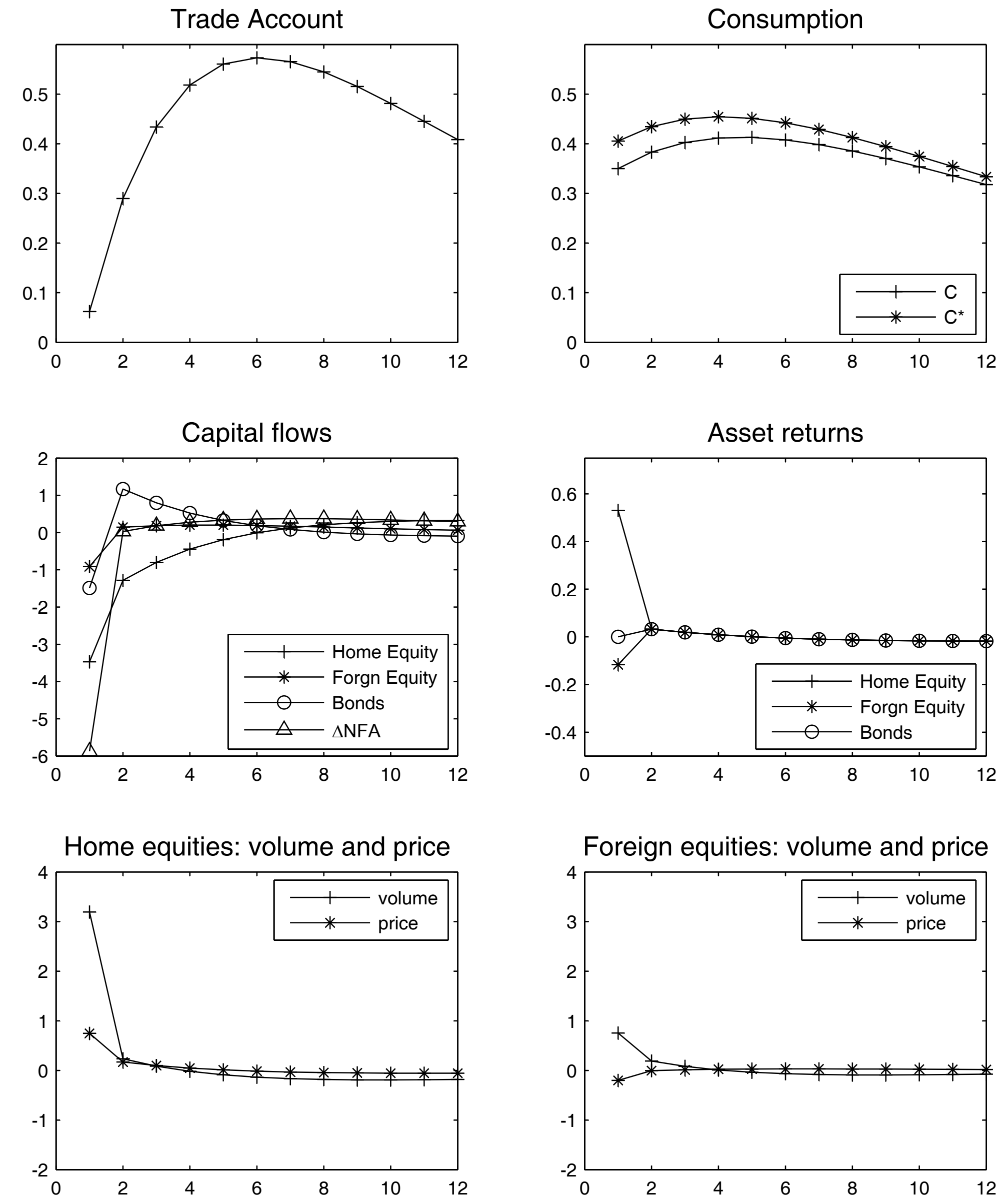


Figure 7: Trade in home and foreign equities and non-contingent bonds (the EQ economy)