Monetary Policy and Portfolio Choice in an Open Economy Macro Model[*]

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Abstract

This paper explores the role of monetary policy in an open economy in an environment of endogenous portfolio choice. The model is simple enough to allow solutions for optimal portfolios to be derived analytically for a range of different asset market environments. We explore the impact of monetary policy on national bond and equity portfolios in environments where assets markets are either complete or incomplete.

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1 Introduction

Over the last decade, a large body of work in open economy macroeconomics has developed sticky price dynamic general equilibrium models that are useful for the analysis of monetary policy. Many papers have explored the determination of optimal monetary policy in different environments regarding price setting, asset market completeness, and the number of currencies assumed to exist. In some of these papers, it has been shown that a policy of strict domestic price stability represents an optimal monetary policy.[1]

This paper extends the literature on monetary policy in an open economy by introducing portfolio choice into the analysis of an open economy macro model. Making use of results developed in Devereux and Sutherland (2006), we illustrate how endogenous portfolio choice can be incorporated into a two-country 'work-horse' model of an open economy with Calvo price-setting, stochastic productivity and money shocks, and monetary policy governed by interest rate rules. We explore two asset market configurations. In one case there is trade in nominal bonds and equities, and given our stochastic environment, markets are complete. In the second case, only nominal bonds can be traded, so asset markets are incomplete.

The paper focuses on a simple model that admits an analytical solution for gross portfolio holdings under each asset market configuration. The aim of the paper is two-fold. First, we wish to compute equilibrium portfolios under complete and incomplete markets, and to explore how these are affected by the structural characteristics of the model. Secondly, we investigate the impact of monetary policy rules on portfolio choice.

The results for complete and incomplete asset market structures are very different. With complete markets, we find that the optimal portfolio is independent of the balance of shocks in the economy, and also independent of the monetary policy rule. With incomplete markets, on the other hand, the optimal portfolio represents a trade off between hedging against productivity shocks and money shocks. Moreover, we find that the portfolio is affected by the monetary policy rule. A policy rule that puts more weight on stabilizing producer price inflation leads to an increase in holdings of nominal bonds, and enhances the degree of international risk sharing. A novel feature of the results is that the monetary rule is important for portfolio composition even if nominal prices are fully flexible.

The next section describes the model, Section 3 presents the solution method, Section 4 discusses equilibrium portfolios and Section 5 concludes.

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[1] See Lane (2001), Bowman and Doyle, (2004) for surveys.

2 An open economy macro model

We develop a basic two-country 'work-horse' open economy model. There is a 'home' and 'foreign' country, with each country being specialized in a particular range of products. Households maximize utility over an infinite horizon. Consumers in each country can trade in a range of financial assets. Each country is subject to shocks to monetary policy and technology.

All agents in the home country have utility functions of the form

$$U = E_0 \sum_{t=0}^{\infty} \left[\frac{C_t^{1-\rho}}{1-\rho} - KL_t \right] \tag{1}$$

where $\rho > 0$, C is a consumption index defined across all home and foreign goods, L is labour supply and E is the expectations operator. The consumption index C for home agents is given by

$$C = \left[\left(\frac{1}{2} \right)^{\frac{1}{\theta}} C_H^{\frac{\theta-1}{\theta}} + \left(\frac{1}{2} \right)^{\frac{1}{\theta}} C_F^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}} \tag{2}$$

where C_H and C_F are indices of home and foreign produced goods with an elasticity of substitution between individual goods denoted ϕ, where $\phi > 1$. The parameter θ is the elasticity of substitution between home and foreign goods. The aggregate consumer price index for home agents is

$$P = \left[\frac{1}{2} P_H^{1-\theta} + \frac{1}{2} P_F^{1-\theta} \right]^{\frac{1}{1-\theta}} \tag{3}$$

where P_H and P_F are the price indices for home and foreign goods.

The budget constraint of the home country agent is

$$P_t C_t + W_{t+1} = w_t L_t + \Pi_t + P_t \sum_{k=1}^{N} \alpha_{kt-1} r_{kt} \tag{4}$$

where W_t is the real wealth for the home agent, w_t is the wage and Π_t is profit income from home-country firms. The final term represents the total return on the home country portfolio, which comprises N assets. The term α_{kt-1} represents the real holdings of asset k, brought into period t from the end of period $t-1$, and r_{kt} is the period t real return on this asset. The home consumer is assumed to be the default owner of home firms and receives all profits from home firms.[2] When an international equity market

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[2] Imperfect competition implies that firms earn monopoly profits.

exists, claims to home profits may be transferred to foreign consumers via trade in equity shares. From the definition of wealth, it must be the case that $W_t = \sum_{k=1}^{N} \alpha_{kt-1}$.

The conditions for utility maximization are standard. The home consumer's demand for home and foreign goods may be written as:

$$C_H = \frac{1}{2}\left(\frac{P_H}{P}\right)^{-\theta} C, \qquad C_F = \frac{1}{2}\left(\frac{P_F}{P}\right)^{-\theta} C.$$

The optimal consumption-leisure tradeoff implies:

$$w_t C_t^{-\rho} = P_t K. \tag{5}$$

And the optimal portfolio choice is characterised by the conditions:

$$C_t^{-\rho} = \beta E_t C_{t+1}^{-\rho} r_{Nt+1}, \tag{6}$$

$$E_t C_{t+1}^{-\rho} (r_{kt+1} - r_{Nt+1}) = 0, \qquad k = 1..N - 1. \tag{7}$$

Firms produce differentiated products. The production function for a good produced by firm i is $Y(i) = AL(i)$ where A is a stochastic productivity shock. We assume that $\log A_t = \zeta \log A_{t-1} + u_t$, where $0 \leq \zeta \leq 1$ and u_t is an i.i.d. shock with $E_{t-1}[u_t] = 0$ and $Var[u_t] = \sigma_u^2$.

Firms maximize profits. Sticky prices take the form of Calvo-style contracts with a probability of re-setting price given by $1 - \kappa$. To keep the model as close to the benchmark open economy formulation, we assume that all prices are set in terms of producer's currency. The dynamics of the newly-set price $\widetilde{P}_h$ and the home price index P_h are

$$\widetilde{P}_{ht} = \frac{\phi}{\phi - 1} \frac{E_t \sum_{i=0}^{\infty} \Omega_{t+i} \frac{w_{t+i}}{A_{t+i}} X_{ht+i}}{E_t \sum_{i=0}^{\infty} \Omega_{t+i} X_{ht+i}}, \qquad P_{ht}^{1-\phi} = (1-\kappa)\widetilde{P}_{ht}^{1-\phi} + \kappa P_{ht-1}^{1-\phi}, \tag{8}$$

where X_{ht+i} is the home firm's output and Ω_{t+i} is the discount factor.

Monetary authorities are assumed to follow interest rate rules which determine the nominal rates of return on nominal bonds. The home authority's monetary rule is:

$$R_{t+1} = \beta^{-1} (P_{ht}/P_{ht-1})^{\sigma} \exp(m_t) \tag{9}$$

where m_t is an i.i.d. shock such that, $E_{t-1}[m_t] = 0$, $Var[m_t] = \sigma_m^2$. The rule (9) determines the nominal interest rate as a function of historic domestic PPI inflation rates. We choose PPI rather than CPI inflation

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rates because it is well known that in a benchmark complete markets open economy (without 'cost-push' or government spending shocks), it is optimal (from a global welfare point of view) to stabilize PPI inflation rates. The main analysis of the paper will focus on the relationship between the stance of monetary policy, captured by the parameter σ, and the equilibrium portfolio holdings among countries.

Home equities represent a claim on home aggregate profits. The real payoff to a unit of the home equity purchased in period t is $\Pi_{t+1}/P_{t+1} + Z_{Et+1}$, where Z_{Et} is the real price of home equity. Thus the gross real rate of return on the home equity is $r_{Et+1} = (\Pi_{t+1}/P_{t+1} + Z_{Et+1})/Z_{Et}$.

Home nominal bonds represent a claim on a unit of home currency. The real payoff to a home nominal bond purchased at time t is therefore $1/P_{t+1}$. The real price of the bond is denoted Z_{Bt}, so the gross real rate of return on a home nominal bond is thus $r_{Bt+1} = 1/(P_{t+1}Z_{Bt})$. From the definition of the monetary policy rule, we note that it must be the case that $R_{t+1} = r_{Bt+1}(P_{t+1}/P_t) = 1/(P_t Z_{Bt})$.

Home GDP equals demand from home and foreign consumers:

$$Y_t = \frac{1}{2}\left(\frac{P_{ht}}{P_t}\right)^{-\theta} C_t + \frac{1}{2}\left(\frac{P_{ht}}{S_t P_t^*}\right)^{-\theta} C_t^* \tag{10}$$

The foreign economy has an analogous representation. Thus, foreign consumers choose labor supply and portfolio holdings in the same manner. Foreign firms adjust prices in the same way as (8), foreign equities and bonds are defined analogously, and the foreign monetary authority follows a rule of the form (9), except that it targets the foreign PPI inflation.

The solution to the model is a sequence $\{C_t, C_t^*, \widetilde{P}_{ht}, \widetilde{P}_{ft}, P_{ht}, P_{ft}, S_t, Y_t, Y_t^*, R_t, R_t^*, r_{kt}, r_{kt}^*, \alpha_t\}$ which solves equations (5)-(7), (8)-(10) and the equivalent equations for the foreign economy.[3] The literature typically analyzes a model such as this by first-order approximation around a non-stochastic steady state. It is well known that, up to a first-order approximation, all assets are perfect substitutes so the value of α_t is indeterminate. The existing literature therefore confines attention to asset market structures where the portfolio allocation problem is not relevant. In the following section, we describe our method for obtaining optimal portfolio shares by means of a second-order approximation approach. This solution method makes it possible to analyse the above model with any asset market structure.

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[3] α_t is the vector of portfolio holdings.

3   Solving the model

Devereux and Sutherland (2006) provide a full description of the solution method. Here we present just a brief account of the approach.

The model is approximated around a symmetric zero-net-wealth non-stochastic steady state, where the values of portfolio holdings at the steady state, denoted $\overline{\alpha}_k$, are treated as unknowns. The solution procedure yields solutions for the $\overline{\alpha}_k$'s. Notice that, in a zero-net-wealth steady state, asset holdings sum to zero, so only $N - 1$ of the $\overline{\alpha}_k$'s are free variables.

By taking second-order approximations of (7) and its foreign counterpart around this approximation point, it is possible to show that:

$$E_t[(\widehat{C}_{t+1} - \widehat{C}^*_{t+1} + \widehat{Q}_{t+1}/\rho)(\widehat{r}_{k,t+1} - \widehat{r}_{N,t+1})] = 0, \quad k = 1...N-1 \quad (11)$$

where a hat indicates a log-deviation from the approximation point.

Notice that (11) contains only second-order terms in the log-deviations of variables. Thus, in order to evaluate the left-hand side of (11) up to second-order accuracy, one only requires first-order accurate expressions for the realised values of consumption, the real exchange rate and asset returns. First-order accurate solutions for these variables can be obtained from a linearised version of the model.

A linearised version of the model is entirely standard except that gross asset holdings, $\overline{\alpha}_k$, enter approximated budget constraints. For instance, a linear approximation of the home budget constraint (4) yields

$$\widehat{W}_{t+1} = \frac{1}{\beta}\widehat{W}_t + \widehat{Y}_t - \widehat{C}_t + \sum_{k=1}^{N-1} \widetilde{\alpha}_k(\widehat{r}_{k,t} - \widehat{r}_{N,t}) \quad (12)$$

where $\widetilde{\alpha}_k = \overline{\alpha}_k/\beta\overline{Y}$.[4] Equation (12) can be combined with the other linearised equations of the model and solved to yield first-order accurate expressions for the all the variables of the model, conditional on the unknown portfolio holdings, $\overline{\alpha}_k$. Expressions for the first-order accurate behaviour of consumption, the real exchange rate and asset returns can thus be derived and substituted into the left-hand side of (11). The resulting $N - 1$ equations can then be solved to yield expressions for the $N - 1$ free $\overline{\alpha}_k$'s.[5] The resulting solutions represent steady-state asset holdings. The next section analyses these solutions for different asset market structures.

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[4]$\widehat{W}_t$ is defined as a difference relative to steady state GDP.

[5]At first sight, it might appear that (11) provides a different set of $N - 1$ equations for each time period. It is simple to show, however, that (11) depends only on one-period ahead second-moments, which will be non-time varying provided the variances of the exogenous shocks are non-time-varying. Thus, the solution for the $\overline{\alpha}_k$'s will be non-time-varying.

4 Equilibrium Portfolios

We examine two different asset market structures. First, we allow only nominal bonds in either currency to be traded across countries. This allows some international risk sharing so long as the returns on nominal bonds differ across currencies. But markets are incomplete, since there are four independent shocks (money and productivity shocks in each country) but only two assets. Secondly, we allow for trade in both nominal bonds and equity. In this case markets are complete, since there are four assets with independent returns. We refer to these structures as the 'nominal bonds economy' (NB), and the 'nominal bonds and equity' economy (NBE), respectively.

In general the solutions for asset holdings are highly complicated expressions that can only be described numerically. Because of this, we focus on a special case of the model which admits interpretable expressions. This special case assumes a) $\rho = 1$, or log utility, and b) $\zeta_A = 1$, so that productivity shocks are random walks. Table 1 reports equilibrium portfolio holdings in the NB and NBE economies for this special case.[6]

Table 1: Optimal Portfolio Holdings		
NB	$\widetilde{\alpha}_{B,NB}$	$0.5\frac{(\theta-1)}{(1-\beta)}\frac{(\sigma+\lambda)^2\sigma_A^2+[\lambda+\theta-\beta\lambda(1-\theta)]\lambda(1-\beta)\sigma_m^2}{(\sigma+\lambda)^2\sigma_A^2+[\lambda+\theta-\beta\lambda(1-\theta)](1+\lambda)\sigma_m^2}$
NBE (Bonds)	$\widetilde{\alpha}_{B,NE}$	$0.5\frac{\lambda(\theta-1)(\phi-1)}{(\theta-1)(1+\lambda\beta)+\lambda(\phi-1)(1-\beta)}$
NBE (Equity)	$\widetilde{\alpha}_{E,NE}$	$0.5\frac{1}{1-\beta}\frac{(\theta-1)(1+\lambda\beta)}{(\theta-1)(1+\lambda\beta)+\lambda(\phi-1)(1-\beta)}$

4.1 The NB Economy

Take the NB case first. Table 1 shows that the optimal holding of the home currency bond is positive, but the value of $\widetilde{\alpha}_{B,NB}$ depends on the importance of productivity shocks relative to monetary shocks. When productivity shocks are predominant, so that $\sigma_A^2/\sigma_m^2 \longrightarrow \infty$, bond holdings tend to $0.5(\theta-1)/(1-\beta)$, while as $\sigma_A^2/\sigma_m^2 \longrightarrow 0$, bond holdings tend to $0.5\lambda(\theta-1)/(1+\lambda)$. The intuitive explanation behind this portfolio position can be seen by focusing on the response of relative consumption to money and productivity shocks if each country held a zero portfolio, and comparing this to relative bond returns. If countries had a zero portfolio,

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[6]Symmetry implies that, for the home country, $\widetilde{\alpha}_{B,NB} + \widetilde{\alpha}_{B,NB}^* = 0$ in the NB economy, and $\widetilde{\alpha}_{B,NE} + \widetilde{\alpha}_{B,NE}^* = 0$ and $\widetilde{\alpha}_{E,NE} + \widetilde{\alpha}_{E,NE}^* = 0$ in the NBE economy, where an asterisk denotes holdings of foreign assets, and other notation is self-explanatory.

then relative consumption and relative bond returns are:

$$\widehat{C}_t - \widehat{C}_t^* = \frac{(\theta - 1)}{\theta} \left[(u_t - u_t^*) - \frac{\lambda(1 - \beta)}{(\sigma + \lambda)}(m_t - m_t^*) \right] \tag{13}$$

$$\widehat{r}_{B^*t} - \widehat{r}_{Bt} = \frac{1}{\theta} \left[(u_t - u_t^*) - \frac{(\theta + \lambda(1 + \beta(\theta - 1)))}{(\sigma + \lambda)}(m_t - m_t^*) \right] \tag{14}$$

From (13), a home country productivity shock will increase home relative consumption. Home consumers would like to hedge this consumption risk. From (14), we see that a home productivity shock will cause an increase in the return on foreign bonds, relative to home bonds. This is because the productivity shock generates a nominal exchange rate depreciation. Thus, the foreign bond is a relatively bad hedge against consumption risk due to productivity shocks, while reciprocally, the home bond is a relatively good hedge. Thus, in the NB economy, home consumers hold positive quantities of home currency bonds as a hedge against productivity shocks. The scale of bond holdings must be proportional to $1/(1 - \beta)$ since the payoff on a one period bond represents a one-time, transitory return, while the productivity shock is permanent. Thus, in order to hedge the consumption risk from productivity shocks, bond holdings must be large.

In response to a home money shock, relative home consumption falls, since the monetary shock causes a rise in the home country's nominal interest rate. At the same time, domestic inflation falls relative to foreign inflation, and the exchange rate appreciates. This increases the return on home currency bonds relative to foreign currency bonds. Again, this implies that home currency bonds are a better hedge against consumption risk due to monetary shocks than are foreign currency bonds. But the scale of nominal bond holdings will depend on the degree of price stickiness. As λ falls, there is less price stickiness. Then consumers tend to ignore the direct consumption fluctuations due to monetary policy shocks, and bond holdings used to hedge the risk from money shocks will be lower. Note also that $\lambda(\theta - 1)/(1 + \lambda) < (\theta - 1)/(1 - \beta)$. Since money shocks are transitory, households need to hold a smaller bond position to hedge money shocks than productivity shocks. Thus, as σ_A^2/σ_m^2 rises, gross bond portfolios will rise in both countries.

The results reported in Table 1 also have some interesting implications for the case of fully flexible prices. Setting $\lambda = 0$ implies $\widetilde{\alpha}_{B,NB} = 0.5 \left[(\theta - 1)\sigma^2\sigma_A^2 \right] / \left[(1 - \beta)\sigma^2\sigma_A^2 + \theta\sigma_m^2 \right]$. This depends negatively on the volatility of money shocks. As $\sigma_A^2/\sigma_m^2 \longrightarrow 0$, $\widetilde{\alpha}_{B,NB}$ goes to zero (for $\lambda = 0$), and bonds become useless as a hedge against productivity risk. Hence

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monetary policy volatility has a significant impact on the portfolio, even though when $\lambda = 0$ money is neutral in the sense that, for a zero-portfolio, all real variables would be independent of monetary policy disturbances. But (14) shows that even if all prices were flexible, the relative return on nominal bonds is affected by money shocks. In the NB economy, agents can only use nominal bonds to hedge against productivity risk. The more volatile are money shocks, the less useful are nominal bonds in achieving this risk-sharing.

4.2 The NBE Economy

In the NBE economy, households will also hold a positive nominal bond position in home currency bonds (negative in foreign currency bonds), but will also now hold a positive share of foreign equity. Because this represents a complete market configuration, this efficient hedging also achieves full cross-country risk sharing. Now we find that portfolio shares are independent of the relative size of productivity shocks to money shocks. The reason is that the portfolio is structured so that full risk-sharing is achieved for all possible realizations of shocks. Hence there is no trade-off between hedging one shock relative to another as in the NB economy. Because the return on equity is affected by both productivity shocks and money shocks, the weight of equity and bonds in the optimal portfolio is affected by the degree of price stickiness. In the limit, when $\lambda = 0$, money shocks have no affect on consumption, and equity alone can be used to hedge consumption risk from productivity shocks; the weight of nominal bond holdings goes to zero. Thus, unlike the NB economy, with flexible prices money is *fully neutral* in the NBE economy.

More generally, we find that as λ rises, portfolio shares held in equity fall, while the portfolio share in bonds rises. Even with extreme levels of price stickiness however, equity holdings still remain positive, since there is still consumption risk due to productivity shocks[7].

4.3 Monetary Policy and Portfolio Structure

How does the stance of monetary policy affect portfolio holdings? We use the parameter σ as a measure of the tightness of monetary policy. As σ rises, the variance of PPI inflation falls. Thus, a higher σ can be interpreted as a policy placing more emphasis on price stability. From Table

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[7]This contrasts with the results of Engel and Matsumoto (2005), who employ both a different form of price setting, as well as a different monetary policy rule.

1 we note a key distinction between the economies with incomplete and complete markets. In the NB economy, a rise in σ increases the gross holdings of home and foreign currency bonds. In the NBE economy however, the portfolio shares in bonds and equities are independent of σ.

In the NB economy, markets are incomplete, and gross bond holdings have to act as a hedge against both productivity shocks and money shocks. The higher is σ, the less impact will money shocks have on the variance of consumption in each country. As a result, bond holdings are dedicated more and more to the hedging of productivity shocks, which require higher gross holdings. On the other hand, with complete markets, both money and productivity shocks are fully hedged for all realizations of shocks. Changes in σ only scale up or down the importance of money shocks in macro volatility. Since the NBE portfolio fully hedges all possible money shocks independent of their volatility, then it is independent of σ.

5   Conclusion

This paper shows how a simple work-horse model of a two country sticky price open economy macro model can be amended so as to incorporate endogenous portfolio choice. We solve for the optimal portfolio holdings of equities and nominal bonds, and show how these depend on the magnitude of stochastic shocks, the degree of price stickiness, and the stance of monetary policy. We show that the asset market structure has a substantial impact on the types of portfolios that agents hold, and the effect of the stance of monetary policy on these portfolios.

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