

# Cost-Push Shocks and Monetary Policy in Open Economies\*

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## Abstract

This paper analyses the implications of cost-push shocks for the optimal choice of monetary policy target in a two-country sticky-price model. In addition to cost-push shocks, each country is subject to labour-supply and money-demand shocks. It is shown that the fully optimal coordinated policy can be supported by independent national monetary authorities following a policy of flexible inflation targeting. A number of simple (but non-optimal) targeting rules are compared. Strict producer-price targeting is found to be the best simple rule when the variance of cost-push shocks is small. Strict consumer-price targeting is best for intermediate levels of the variance of cost-push shocks. And nominal-income targeting is best when the variance of cost-push shocks is high. In general, money-supply targeting and fixed nominal exchange rates are found to yield less welfare than these other regimes.

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# 1 Introduction

What is the optimal choice of monetary target for an open economy? Recent analyses of closed-economy general equilibrium models tend to suggest that strict targeting of consumer prices will maximise aggregate utility.<sup>1</sup> Such a policy minimises relative price distortions when some prices are sticky and unable to respond to shocks in the short run. A similar result has been shown to hold in open economies, where welfare maximising monetary policy should focus on stabilising producer prices.<sup>2</sup>

A number of cases have, however, been identified where these basic results need to be modified. The presence of non-optimal ‘cost-push’ shocks is one such case. Cost-push shocks can be caused by factors such as fluctuations in monopoly mark-ups or changes in distortionary taxes which change prices but which do not imply any change in the socially optimal level of real output. Such shocks are particularly interesting and relevant because they create a potential policy dilemma for the monetary authority. On the one hand, stabilisation of the general price level is desirable (in order to avoid sub-optimal fluctuations in relative prices) but, on the other hand, price stabilisation in the face of cost-push shocks creates sub-optimal fluctuations in output. The presence of cost-push shocks therefore implies that optimal monetary policy should allow for some flexibility in general prices in order to allow some stabilisation of the output gap. This is often referred to as ‘flexible inflation targeting’ following the terminology suggested by Svensson (1999, 2000). In a closed economy context this implies that some flexibility should be allowed in the consumer price index. In an open economy context Clarida *et al* (2002) and Benigno and Benigno (2002, 2003b) show that a similar result holds with respect to producer prices.<sup>3</sup>

In a theoretical sense cost-push shocks are relatively easy to accommodate within the analysis of optimal policy. But from a practical point of view they raise an

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<sup>1</sup>See Aoki (2001), Goodfriend and King (2001), King and Wolman (1999) and Woodford (2003).

<sup>2</sup>See Aoki (2001), Benigno and Benigno (2003a) and Clarida *et al* (2001). In a closed economy without capital accumulation there is, by definition, no distinction between the consumer price index and the producer price index (or the GDP deflator). But in an open economy, international trade in consumer goods implies that the composition of the two price indices may differ. The important insight from the open economy literature is that it is the volatility of the producer price index which matters for welfare. In a closed economy with capital accumulation it would again be important to distinguish between the two price indices, and again it would be the volatility of producer prices which would matter for welfare.

<sup>3</sup>Further analysis of open economy models, where there is less than perfect pass-through from exchange rate changes to local currency prices, has shown that optimal monetary policy should involve some consideration of exchange rate volatility. (See Bacchetta and van Wincoop (2000), Corsetti and Pesenti (2001b), Devereux and Engel (1998, 2003), Engel (2002), Smets and Wouters (2002) and Sutherland (2002a).) In this case the monetary authority should allow some flexibility in producer prices in order to achieve some desired degree of stability in the nominal exchange rate. Furthermore, Sutherland (2002c) analyses the implications of the expenditure switching effect in a model where there is perfect pass-through. It is shown that, when the elasticity of substitution between home and foreign goods is greater than unity, exchange rate volatility can become an important factor in welfare even when there is full pass-through.

important issue, namely that the welfare maximising monetary strategy becomes more complex and more difficult to implement. It is clear that the optimality of a simple strategy of strict consumer-price or producer-price targeting does not carry over to more general cases. In addition, even when the optimal monetary strategy can be summarised by a relatively simple loss function (as is the case in Benigno and Benigno (2002) and in the model considered below), it becomes doubtful that the fully optimal monetary policy can in practice be made operational. The fully optimal policy may involve responding to unobservable or unmeasurable variables or require a complex balance between different targets where the optimal weights to be placed on different targets are unmeasurable or uncertain. In the face of such difficulties it may not be practical for a monetary authority to identify and follow the optimal monetary policy. The only practical policy strategy may be to adopt a non-optimal but simple targeting rule. It is therefore important to analyse the welfare performance of simple targeting rules. This is the central aim of the present paper and it is the main point of departure from the related papers of Clarida *et al* (2002) and Benigno and Benigno (2002, 2003b).

A two-country model is constructed where each country is subject to labour-supply, cost-push and money-demand shocks.<sup>4</sup> The paper focuses on the choice of a world monetary regime and the implications for world aggregate welfare.<sup>5</sup> It is shown that (as in Benigno and Benigno (2002)) the optimal coordinated policy can be supported by independent national monetary authorities following a policy of flexible inflation targeting. After deriving the theoretically optimal policy regime, a range of possible simple targeting rules are considered. The rules analysed include; money-supply targeting, strict targeting of producer prices, strict targeting of consumer prices, a fixed nominal exchange rate and nominal-income targeting. Nominal-income targeting is of particular interest when there are cost-push shocks. Nominal-income targeting implies that monetary policy stabilises both real output and prices to some extent. It is therefore a regime which shares some of the features of the fully optimal policy rule.<sup>6</sup>

It is found that money-supply targeting and fixed exchange rates are in general dominated by at least one of the other three regimes. Strict producer-price targeting is found to be the best simple rule when the variance of cost-push shocks is small. Strict consumer-price targeting is best for intermediate levels of the variance of cost-push shocks. And nominal-income targeting is best when the variance of cost-push

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<sup>4</sup>The model is in the ‘new open economy macro’ tradition (which originates with Obstfeld and Rogoff (1995)) in that it assumes monopolistic competition and sticky prices. The new open economy literature has been surveyed by Lane (2001).

<sup>5</sup>An obvious issue of interest (which is not tackled in this paper) is the welfare gain to coordinated monetary policy. In fact, in the basic version of the model presented here there are no welfare gains to coordination. There is therefore no distinction between the effects of regime choice on world welfare and the effects on individual country welfare. This contrasts with the models Benigno and Benigno (2002, 2003b) and Clarida *et al* (2002), where gains to coordination do arise.

<sup>6</sup>Nominal-income targeting has previously been analysed by McCallum and Nelson (1999) and Jensen (2002).

shocks is high.

The main advantage of the simple framework presented in this paper is that it yields explicit and exact analytical solutions for welfare. This is again a point of departure from Clarida *et al* (2002) and Benigno and Benigno (2002, 2003b). Yet, despite its simplicity, the model remains rich enough to allow a wide range of policy regimes to be compared. The model is however restricted in a number of respects. Two extensions to the basic model are considered in the final section of the paper but important restrictions remain. For instance, there are no asset accumulation or price dynamics, and there are no structural asymmetries between the countries. While it is certainly necessary to analyse more general models, consideration of more complex structures would inevitably require approximation techniques or numerical simulation. The model presented below yields important and explicit benchmark results which will make it easier to interpret the results generated by more complex models.

This paper proceeds as follows. Section 2 presents the model. Section 3 considers the general form of optimal monetary policy. Section 4 compares the welfare performance of a range of simple targeting rules. Section 5 discusses some extensions to the basic model. Section 6 concludes the paper.

## 2 The Model

### 2.1 Market Structure

The world exists for a single period<sup>7</sup> and consists of two countries, which will be referred to as the home country and the foreign country. Each country is populated by agents who consume a basket consisting of all home and foreign produced goods. Each agent is a monopoly producer of a single differentiated product. There is a continuum of agents of unit mass in each country. Home agents are indexed  $h \in [0, 1]$  and foreign agents are indexed  $f \in [0, 1]$ .

There are two categories of agent in each country. The first set of agents supply goods in a market where prices are set in advance of the realisation of shocks and the setting of monetary policy. Agents in this market are contracted to meet demand at the pre-fixed prices. Agents in this group will be referred to as ‘fixed-price agents’. The second set of agents supply goods in a market where prices are set after shocks

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<sup>7</sup>The model can easily be recast as a multi-period structure (where each period is simply a replication of the single period model) but this would be a purely cosmetic form of dynamics which would add no significant insights. A true dynamic model, with multi-period nominal contracts and asset stock dynamics would add interesting and potentially important aspects to the analysis but it would be considerably more complex and would require extensive use of numerical methods. Newly developed numerical techniques are available to solve such models and this is likely to be an interesting line of future research (see Kim and Kim (2000), Sims (2000), Schmitt-Grohé and Uribe (2004) and Sutherland (2002b)). However, the approach adopted in this paper yields useful insights which would not be available in a more complex model.

are realised and monetary policy is set. Agents in this group will be referred to as ‘flexible-price agents’.<sup>8</sup> The proportion of fixed-price agents in the total population is denoted  $\psi$ , so  $\psi$  is a measure of the degree of price stickiness in the economy. The total population of the home economy is indexed on the unit interval with fixed-price agents indexed  $[0, \psi]$  and flexible-price agents indexed  $(\psi, 1]$ . Prices and quantities relating to fixed-price agents will be indicated with the subscript ‘1’ while those relating to flexible-price agents will be indicated with the subscript ‘2’. The foreign economy has a similar structure.

This framework provides the minimal structure necessary to study the effects of price variability on welfare while allowing some degree of price stickiness. The fixed-price agents provide the nominal rigidity that is necessary to give monetary policy a role while the flexible-price agents provide the partial aggregate price flexibility that allows an analysis of the connection between price volatility and welfare.

All prices are assumed to be set in the currency of the producer. There is thus full pass-through from changes in nominal exchange rates to the prices paid by consumers.<sup>9</sup>

The detailed structure of the home country is described below. The foreign country has an identical structure. Where appropriate, foreign real variables and foreign currency prices are indicated with an asterisk.

## 2.2 Preferences

All agents in the home economy have utility functions of the same form. The utility of agent  $z$  of type  $i$  is given by

$$U(z) = E \left[ \log C(z) + \chi \log \frac{M(z)}{P} - \frac{K}{\mu} y_i^\mu(z) \right] \quad (1)$$

where  $\mu \geq 1$ ,  $i = 1$  for a fixed-price agent and  $i = 2$  for a flexible-price agent,  $C$  is a consumption index defined across all home and foreign goods,  $M$  denotes end-of-period nominal money holdings,  $P$  is the consumer price index,  $y_i(z)$  is the output of good  $z$ ,  $E$  is the expectations operator,  $K$  is a log-normal stochastic labour-supply shock ( $E[\log K] = 0$  and  $Var[\log K] = \sigma_K^2$ ) and  $\chi$  is a log-normal stochastic money-demand shock ( $E[\log \chi] = 0$  and  $Var[\log \chi] = \sigma_\chi^2$ ).

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<sup>8</sup>The division of agents into fixed-price and flexible-price groups is taken to be a fixed institutional feature of the economy. A fixed/flexible-price structure similar to the one used here has previously been used in Aoki (2001) and Woodford (2003). Woodford (2003) focuses on the analysis of monetary policy in a closed economy while Aoki (2001) does consider some aspects of monetary policy in a small open economy. Neither of these papers analyse the implications of cost-push shocks - which is the main subject of the current paper.

<sup>9</sup>The implications of incomplete pass through have been extensively studied in the related literature cited in footnote 3.

The consumption index  $C$  for home agents is defined as

$$C = \frac{C_H^\nu C_F^{1-\nu}}{\nu^\nu (1-\nu)^{1-\nu}} \quad (2)$$

where  $C_H$  and  $C_F$  are indices of home and foreign produced goods and  $\nu = 1 - \gamma/2$  and  $0 \leq \gamma \leq 1$ . This formulation implies ‘home bias’ in consumption.<sup>10</sup> The parameter  $\gamma$  is effectively a measure of openness.  $\gamma = 0$  implies a completely closed economy while  $\gamma = 1$  implies a completely open economy.

Utility from consumption of home and foreign goods is defined as follows

$$C_H = \frac{C_{H,1}^\psi C_{H,2}^{(1-\psi)}}{\psi^\psi (1-\psi)^{(1-\psi)}}, \quad C_F = \frac{C_{F,1}^\psi C_{F,2}^{(1-\psi)}}{\psi^\psi (1-\psi)^{(1-\psi)}} \quad (3)$$

where  $C_{H,1}$  and  $C_{H,2}$  are indices of home fixed-price and flexible-price goods defined as follows

$$C_{H,1} = \left[ \left( \frac{1}{\psi} \right)^{\frac{1}{\phi}} \int_0^\psi c_{H,1}(h)^{\frac{\phi-1}{\phi}} dh \right]^{\frac{\phi}{\phi-1}}, \quad C_{H,2} = \left[ \left( \frac{1}{1-\psi} \right)^{\frac{1}{\phi}} \int_\psi^1 c_{H,2}(h)^{\frac{\phi-1}{\phi}} dh \right]^{\frac{\phi}{\phi-1}}$$

and  $C_{F,1}$  and  $C_{F,2}$  are indices of foreign fixed-price and flexible-price goods defined as follows

$$C_{F,1} = \left[ \left( \frac{1}{\psi} \right)^{\frac{1}{\phi}} \int_0^\psi c_{F,1}(f)^{\frac{\phi-1}{\phi}} df \right]^{\frac{\phi}{\phi-1}}, \quad C_{F,2} = \left[ \left( \frac{1}{1-\psi} \right)^{\frac{1}{\phi}} \int_\psi^1 c_{F,2}(f)^{\frac{\phi-1}{\phi}} df \right]^{\frac{\phi}{\phi-1}}$$

where  $c_{H,i}(h)$  is consumption of home good  $h$  produced by a home agent of type  $i$  and  $c_{F,i}(f)$  is consumption of foreign good  $f$  produced by a foreign agent of type  $i$ . The above functions imply a constant elasticity of substitution between different varieties of good of the same type and a unit elasticity of substitution between types of good.<sup>11</sup>

The structure of preferences implies a unit elasticity of substitution between home and foreign goods and a unit coefficient of relative risk aversion. These assumptions ensure that there is no idiosyncratic income risk between the home country and the rest of the world. The structure of financial markets is therefore irrelevant. This is a key simplification which makes it possible to obtain exact and

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<sup>10</sup>In this case ‘home bias’ implies home agents potentially give a higher weight to home goods than foreign goods and foreign agents give a higher weight to foreign goods than home goods.

<sup>11</sup>The assumption that the elasticity of substitution between fixed-price and flexible-price goods differs from the elasticity of substitution between goods within each type has the slightly odd implication that the degree of price stickiness is, in effect, embedded in the structure of preferences. However, the unit elasticity of substitution between fixed and flexible rice goods allows some useful simplifications of the algebra. The implications of allowing this elasticity to differ from unity are discussed in Section 5.

explicit solutions to the model and an exact and explicit expression for welfare.<sup>12</sup> Some of the implications of allowing a non-unit coefficient of relative risk aversion are discussed in Section 5.

The budget constraint of agent  $z$  (where  $z$  is of type  $i$ ) is given by

$$M(z) = M_0 + (1 + \alpha)p_{H,i}(z)y_i(z) - PC(z) - T \quad (4)$$

where  $M_0$  and  $M(z)$  are initial and final money holdings,  $T$  is lump-sum government transfers,  $p_{H,i}(z)$  is the price of home good  $z$ ,  $\alpha$  is a production subsidy and  $P$  is the aggregate consumer price index.

The government's budget constraint is

$$M - M_0 - \alpha P_H Y + T = 0 \quad (5)$$

where  $P_H$  is the aggregate price of home produced goods and  $Y$  is the aggregate output of the home economy.

## 2.3 Price indices

The aggregate consumer price index for home agents is

$$P = P_H^\nu P_F^{1-\nu} \quad (6)$$

where  $P_H$  and  $P_F$  are the price indices for home and foreign goods respectively.

The price indices of home and foreign goods are defined as

$$P_H = P_{H,1}^\psi P_{H,2}^{(1-\psi)}, \quad P_F = P_{F,1}^\psi P_{F,2}^{(1-\psi)} \quad (7)$$

where  $P_{H,1}$  and  $P_{H,2}$  are the price indices of home fixed-price and flexible-price goods defined as follows

$$P_{H,1} = \left[ \frac{1}{\psi} \int_0^\psi p_{H,1}(h)^{1-\phi} dh \right]^{\frac{1}{1-\phi}}, \quad P_{H,2} = \left[ \frac{1}{1-\psi} \int_\psi^1 p_{H,2}(h)^{1-\phi} dh \right]^{\frac{1}{1-\phi}}$$

and  $P_{F,1}$  and  $P_{F,2}$  are the price indices of foreign fixed-price and flexible-price goods defined as follows

$$P_{F,1} = \left[ \frac{1}{\psi} \int_0^\psi p_{F,1}(f)^{1-\phi} df \right]^{\frac{1}{1-\phi}}, \quad P_{F,2} = \left[ \frac{1}{1-\psi} \int_\psi^1 p_{F,2}(f)^{1-\phi} df \right]^{\frac{1}{1-\phi}}$$

The law of one price is assumed to hold. This implies  $p_{H,i}(j) = p_{H,i}^*(j)S$  and  $p_{F,i}(j) = p_{F,i}^*(j)S$  for all  $i$  and  $j$  where an asterisk indicates a price measured in foreign currency and  $S$  is the exchange rate (defined as the domestic price of foreign currency). Note that purchasing power parity does not hold because home and foreign agents have different preferences over consumption (because of home bias).

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<sup>12</sup>In the case where there is no home bias (i.e.  $\gamma = 1$ ) financial markets would be irrelevant for any degree of relative risk aversion. The fact that a unit elasticity of substitution implies that financial markets are irrelevant was first noted by Cole and Obstfeld (1991) and was subsequently used as a key simplifying feature in a deterministic open economy model by Corsetti and Pesenti (2001a).

## 2.4 Consumption Choices

Individual home demands for representative home fixed-price good,  $h_1$ , and flexible-price good,  $h_2$ , are

$$c_1(h_1) = \frac{1}{\psi} C_{H,1} \left( \frac{p_{H,1}(h_1)}{P_{H,1}} \right)^{-\phi}, \quad c_2(h_2) = \frac{1}{1-\psi} C_{H,2} \left( \frac{p_{H,2}(h_2)}{P_{H,2}} \right)^{-\phi} \quad (8)$$

where

$$C_{H,1} = \psi C_H \left( \frac{P_{H,1}}{P_H} \right)^{-1}, \quad C_{H,2} = (1-\psi) C_H \left( \frac{P_{H,2}}{P_H} \right)^{-1} \quad (9)$$

and where

$$C_H = \nu C \left( \frac{P_H}{P} \right)^{-1} \quad (10)$$

Individual home demands for representative foreign fixed-price good,  $f_1$ , and flexible-price good,  $f_2$ , are

$$c_1(f_1) = \frac{1}{\psi} C_{F,1} \left( \frac{p_{F,1}(f_1)}{P_{F,1}} \right)^{-\phi}, \quad c_2(f_2) = \frac{1}{1-\psi} C_{F,2} \left( \frac{p_{F,2}(f_2)}{P_{F,2}} \right)^{-\phi} \quad (11)$$

where

$$C_{F,1} = \psi C_F \left( \frac{P_{F,1}}{P_F} \right)^{-1}, \quad C_{F,2} = (1-\psi) C_F \left( \frac{P_{F,2}}{P_F} \right)^{-1} \quad (12)$$

and where

$$C_F = (1-\nu) C \left( \frac{P_F}{P} \right)^{-1} \quad (13)$$

Each country has a population of unit mass so the total home demands for goods are equivalent to the individual demands.

Symmetry between the home and foreign countries implies that the foreign demands for home and foreign goods are given by

$$C_H^* = (1-\nu) C^* \left( \frac{P_H^*}{P^*} \right)^{-1}, \quad C_F^* = \nu C^* \left( \frac{P_F^*}{P^*} \right)^{-1} \quad (14)$$

Individual and total foreign demands for individual goods have an identical structure to the home demands given above.

Using the above relationships it is simple to verify that fixed-price and flexible-price agents have the same income levels (and therefore choose the same consumption levels). It is also possible to verify that financial markets are irrelevant. To see this latter point note that current account balance implies

$$SP_H^* C_H^* = P_F C_F \quad (15)$$

which, when combined with the above expressions for  $C_H^*$  and  $C_F$ , implies

$$\left(\frac{C^*}{C}\right)^{-1} = \frac{SP^*}{P} \quad (16)$$

This equation shows that, when the current account is in balance, the ratio of marginal utilities across the two countries is equal to the ratio of aggregate prices (i.e. the real exchange rate). This implies that there can be no Pareto improving reallocation of consumption across countries. Financial markets are therefore redundant.

## 2.5 Optimal Price Setting

Individual agents are each monopoly producers of a single differentiated good. They therefore set prices as a mark-up over marginal costs. The mark-up is given by  $\phi/(\phi - 1)$ . The mark-up is offset by a production subsidy,  $\alpha$ , which is paid to all producers (financed out of lump-sum taxes). It is useful to define the net mark-up as follows  $A \equiv \phi/[(\phi - 1)(1 + \alpha)]$ .

Cost-push shocks are assumed to enter the model through shocks to the net mark-up such that  $A$  is log-normally distributed with  $E[\log A] = 0$  and  $Var[\log A] = \sigma_A^2$ . The underlying source of these shocks may be assumed to be random changes in the production subsidy or the degree of monopoly power (i.e.  $\phi$ ). The important feature of these cost-push shocks is that they are non-optimal in the sense that they change private pricing and production incentives but they do not change the socially optimal level of production and work effort.<sup>13</sup>

Flexible-price producers are able to set prices after shocks have been realised and monetary policy has been set. In equilibrium all flexible-price producers set the same price so  $P_{H,2} = p_{H,2}(h_2)$  for all  $h_2$ . The first order condition for the choice of price is derived in the Appendix and implies the following

$$P_{H,2} = AKY_2^{\mu-1}PC \quad (17)$$

where  $Y_2$  is the output of home flexible-price producers (expressed *per capita* of the population of home flexible-price producers), which is given by

$$Y_2 = \frac{1}{1 - \psi} (C_{H,2} + C_{H,2}^*) = Y \left(\frac{P_{H,2}}{P_H}\right)^{-1} \quad (18)$$

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<sup>13</sup>The assumption that  $E[\log A] = 0$  implies that, on average, the production subsidy offsets the monopoly distortion. This is a convenient normalisation which has no implications for the welfare effects of monetary policy in the model used in the Sections 3 and 4. Thus, the average level of the production subsidy has no implications for fully optimal policy or for the welfare comparison between simple targeting rules. It also has no implications for the comparison between coordinated and non-coordinated policy. Section 5 discusses a modified version of the model with a more general utility structure. In that case the production subsidy does affect the welfare gain from policy coordination. But the production subsidy continues to be irrelevant for the comparison between simple targeting rules.

where  $Y$  is the total output of the home economy, which is given by

$$Y = C_H + C_H^* = C \left( \frac{P_H}{P} \right)^{-1} \quad (19)$$

Fixed-price agents must set prices before shocks have been realised and monetary policy is set. The first order condition for fixed-price producers is derived in the Appendix and is given by the following

$$P_{H,1} = \frac{E[KY_1^\mu]}{E[Y_1/(APC)]} \quad (20)$$

where  $Y_1$  is the output of home fixed-price producers (expressed *per capita* of the population of home fixed-price producers), which is given by

$$Y_1 = \frac{1}{\psi} (C_{H,1} + C_{H,1}^*) = Y \left( \frac{P_{H,1}}{P_H} \right)^{-1} \quad (21)$$

## 2.6 Money Demand and Supply

The first order condition for the choice of money holdings is

$$\frac{M}{P} = \chi C \quad (22)$$

It is assumed that the monetary authority adjusts the money stock so as to achieve whatever target is being considered. Furthermore, it is assumed that monetary policy is only used as a stabilisation instrument in reaction to shocks.

## 2.7 Home and Foreign Shocks

The foreign economy has a structure identical to the home economy. The foreign economy is subject to labour-supply, cost-push and money-demand shocks of the same form as the home economy. It is assumed that the variances of the shocks are identical across the two countries, i.e.

$$\sigma_K^2 = \sigma_{K^*}^2, \quad \sigma_A^2 = \sigma_{A^*}^2, \quad \sigma_\chi^2 = \sigma_{\chi^*}^2 \quad (23)$$

In addition the cross-country correlation of shocks is assumed to be identical for all three types of shocks, i.e.

$$\frac{\sigma_{KK^*}}{\sigma_K^2} = \frac{\sigma_{AA^*}}{\sigma_A^2} = \frac{\sigma_{\chi\chi^*}}{\sigma_\chi^2} = \zeta \quad (24)$$

where  $-1 \leq \zeta \leq 1$ .

## 2.8 Welfare

One of the main advantages of the model just described is that it provides a very natural and tractable measure of welfare which can be derived from the aggregate utility of agents. The focus of this paper is the coordinated choice of monetary policy and its implications for world aggregate welfare.<sup>14</sup> It is therefore necessary to consider an aggregation of utility across the world population. Following Obstfeld and Rogoff (1998, 2000) it is assumed that the utility of real balances is small enough to be neglected. It is therefore possible to measure world *ex ante* aggregate welfare using the following

$$\begin{aligned} \Omega = E \left[ \psi \left( \log C - \frac{K}{\mu} Y_1^\mu \right) + (1 - \psi) \left( \log C - \frac{K}{\mu} Y_2^\mu \right) \right. \\ \left. + \psi \left( \log C^* - \frac{K^*}{\mu} Y_1^{*\mu} \right) + (1 - \psi) \left( \log C^* - \frac{K^*}{\mu} Y_2^{*\mu} \right) \right] \end{aligned} \quad (25)$$

It proves useful to decompose welfare into two terms as follows

$$\Omega = \Omega_D + \Omega_X \quad (26)$$

where  $\Omega_D$  and  $\Omega_X$  contain all terms in the welfare expression which are, respectively, *dependent* and *independent* of the setting of monetary policy. The Appendix derives the following expression for  $\Omega_D$

$$\Omega_D = -\frac{\psi\mu}{2} \left\{ Var \left[ \hat{Y}_1 + \frac{\hat{K}}{\mu} \right] + Var \left[ \hat{Y}_1^* + \frac{\hat{K}^*}{\mu} \right] \right\} \quad (27)$$

where a hat indicates the log-deviation of a variable from a non-stochastic equilibrium (where  $\sigma_K^2 = \sigma_A^2 = \sigma_\chi^2 = 0$ ).<sup>15</sup> This expression shows that policy affects welfare only through the impact of policy on the variability of output levels of fixed-price agents.<sup>16</sup>

The link between the variability of fixed-price output and welfare can be explained in at least two ways. One explanation (which corresponds to the formal mathematical derivation of (27) given in the Appendix) is that output volatility generates risk premia in the level of prices set by fixed-price agents. Higher output volatility causes higher risk premia and therefore lowers the average level of

<sup>14</sup>In the basic version of the model there are no welfare gains to policy coordination so there is no distinction between the effects of regime choice on world welfare and the effects on individual country welfare. It is therefore sufficient to focus on world welfare.

<sup>15</sup>Note that all the equations of the model are linear in logs. Thus re-expressing variables as log-deviations from a non-stochastic equilibrium is merely a convenient normalisation - it does not imply any loss of accuracy.

<sup>16</sup>The fact that welfare depends only on variables related to fixed-price producers is partly a consequence of the unit elasticity of substitution between fixed-price and flexible-price goods. If this elasticity differs from unity then welfare can not be written in this particularly simple form. Some of the implications of the more general case are discussed briefly in Section 5.

output and consumption.<sup>17</sup> The other explanation is that volatility in the relative price between fixed-price and flexible-price agents creates inefficient output dispersion across agents (which shows up in the welfare function as a cost of fixed-price output volatility).<sup>18</sup> These two different explanations are essentially equally valid ways to interpret the effects embodied in (27).

The next section explains how (27) can be used to analyse welfare maximising monetary policy.

### 3 Welfare Maximising Monetary Policy

In order to see what the welfare expression (27) implies for optimal policy it is necessary to consider the determinants of fixed-price output. Using (19), (21) and (22) it can be shown that home fixed-price output is given by

$$\hat{Y}_1 = \hat{M} - \hat{\chi} - \hat{P}_{H,1} \quad (28)$$

and foreign fixed-price output is given by

$$\hat{Y}_1^* = \hat{M}^* - \hat{\chi}^* - \hat{P}_{F,1}^* \quad (29)$$

(Where, as before, a hat indicates a log deviation from a non-stochastic equilibrium.) In these expressions the terms in  $\hat{P}_{H,1}$  and  $\hat{P}_{F,1}^*$  are fixed *ex ante* so the variances of fixed-price outputs depend only on money supplies and monetary shocks. It is also useful to note that equations (17) and (18) and their foreign counterparts imply that the output levels of home and foreign flexible-price producers are given by

$$\hat{Y}_2 = -\frac{\hat{K} + \hat{A}}{\mu}, \quad \hat{Y}_2^* = -\frac{\hat{K}^* + \hat{A}^*}{\mu} \quad (30)$$

It is immediately apparent from (27), (28) and (29) that welfare maximising monetary policy is given by

$$\hat{M} = \hat{\chi} - \frac{\hat{K}}{\mu}, \quad \hat{M}^* = \hat{\chi}^* - \frac{\hat{K}^*}{\mu} \quad (31)$$

These monetary rules yield  $\Omega_D = 0$ . The optimal monetary rules given in (31) imply full accommodation of money-demand shocks and a negative relationship between

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<sup>17</sup>The role of risk premia in sticky prices has previously been noted and analysed in Rankin (1998), Obstfeld and Rogoff (1998, 2000, 2002), Devereux and Engel (1998, 2003), Corsetti and Pesenti (2001b) and Sutherland (2000, 2002a).

<sup>18</sup>This alternative explanation is emphasised by Woodford (2003) who (in the context of more general dynamic models) uses a second-order approximation technique to derive an explicit welfare measure. A side-effect of the Woodford approximation technique is that it tends to highlight the role of price volatility in creating inefficient output dispersion across fixed-price and flexible-price agents. It is simple to show that the application of the Woodford solution technique to (25) results in the expression given in (27).

the money supply and labour-supply shocks. This is easily understood. Consider, for instance, a positive shock to  $K$ . This represents an increase in the marginal disutility of work effort which, in turn, must imply a reduction in the socially optimal level of work effort. Flexible-price agents can bring about this reduction in work effort by raising prices (as is implied by (17) and (30)). But fixed-price agents are locked into price contracts which cannot be adjusted in the light of shocks. The optimal monetary policy response in these circumstances is to reduce the money supply in a way which reproduces the socially optimal reduction in fixed-price output. The same logic applies to a negative shock to  $K$  (which implies an increase in the socially optimal level of work effort and an increase in the money supply).

Notice that optimal policy does not involve any response to cost-push shocks. Cost-push shocks (which in this model are assumed to arise from random fluctuations in the net mark-up) do not change the socially optimal level of work effort. There is therefore no need to use monetary policy to bring about changes in fixed-price output.

It is useful to consider the implications of optimal policy for producer prices. Using (7), (17) and (30) it is possible to show that home and foreign producer prices are given by

$$\hat{P}_H = \psi \hat{P}_{H,1} + (1 - \psi) \left( \frac{\hat{A} + \hat{K}}{\mu} + \hat{M} - \hat{\chi} \right) \quad (32)$$

$$\hat{P}_F^* = \psi \hat{P}_{F,1}^* + (1 - \psi) \left( \frac{\hat{A}^* + \hat{K}^*}{\mu} + \hat{M}^* - \hat{\chi}^* \right) \quad (33)$$

These expressions show that optimal policy completely eliminates any impact on producer prices of labour-supply and money-demand shocks but allows producer prices to vary in response to cost-push shocks.

Notice that optimal policy implies that fixed-price and flexible-price outputs respond differently to cost-push shocks. Fixed-price output is insulated against cost-push shocks while flexible-price output responds to cost-push shocks in exactly the same way as it responds to labour-supply shocks (see (30)). This highlights the fundamental differences between labour-supply shocks and cost-push shocks. Labour-supply shocks change both private and social incentives and therefore imply changes in the private and socially optimal levels of output. Cost-push shocks, on the other hand, change private incentives but have no impact on the socially optimal level of output. Welfare maximising monetary policy therefore ensures that fixed-price output responds only to socially optimal shocks. But flexible-price agents respond to the private incentives created by cost-push shocks and thus increase prices and reduce output in response to positive shocks to  $A$ .

Table 1 shows the variances of a number of variables in the optimal equilibrium. It is clear from Table 1 (and from (32) and (33)) that, in the presence of cost-push shocks, optimal policy does not imply completely stable producer prices. Thus strict targeting of producer price inflation in each country will not be consistent

Producer prices	$(1 - \psi)^2 \sigma_A^2 / \mu^2$
Consumer prices	$[(1 - \zeta)\nu^2 \sigma_K^2 + (1 - \psi)^2 (2 - (2 + \nu)(1 - \zeta)\nu) \sigma_A^2] / 2\mu^2$
Nominal income	$\sigma_K^2 / \mu^2$
Exchange rate	$2(1 - \zeta) \sigma_K^2 / \mu^2$
Output	$[\sigma_K^2 + (1 - \psi)^2 \sigma_A^2] / \mu^2$
Output gap $(\hat{Y} + \frac{\hat{K}}{\mu})$	$(1 - \psi)^2 \sigma_A^2 / \mu^2$

Table 1: Variances in the optimal equilibrium

with optimal policy.

It is simple to show, however, that a form of flexible inflation targeting in each country does reproduce the optimal policy rule. To see this, first define the natural levels of output for the home and foreign countries to be the following

$$\hat{Y}_N = -\frac{\hat{K}}{\mu}, \quad \hat{Y}_N^* = -\frac{\hat{K}^*}{\mu} \quad (34)$$

These are the socially optimal levels of total output. Notice that  $\hat{Y}_N$  and  $\hat{Y}_N^*$  depend only on labour-supply shocks and are unaffected by cost-push shocks. Define the ‘output gap’ for the home and foreign countries as follows

$$\hat{Y}_G = \hat{Y} - \hat{Y}_N = \hat{Y} + \frac{\hat{K}}{\mu}, \quad \hat{Y}_G^* = \hat{Y}^* - \hat{Y}_N^* = \hat{Y}^* + \frac{\hat{K}^*}{\mu} \quad (35)$$

It is now possible to define the following loss functions for the home and foreign monetary authorities

$$L = Var(\hat{P}_H) + \omega Var(\hat{Y}_G), \quad L^* = Var(\hat{P}_F^*) + \omega Var(\hat{Y}_G^*) \quad (36)$$

where  $\omega = (1 - \psi) / \psi$ . It is simple to show that, if the home monetary authority minimises  $L$  and the foreign monetary authority minimises  $L^*$ , the optimal policy rules will be reproduced.<sup>19</sup> A policy regime such as this could be described as a form of flexible inflation targeting because each monetary authority is stabilising a weighted sum of producer-price volatility and output-gap volatility.<sup>20</sup> This result corresponds to the results of Benigno and Benigno (2002), who show that, in the presence of

<sup>19</sup>The minimisation problem of the home central bank is to choose a state-contingent rule for  $\hat{M}$  to minimise  $L$  subject to equilibrium expressions relating home monetary policy to  $\hat{P}_H$  and  $\hat{Y}_G$ . An expression for  $\hat{P}_H$  is given in (32). Using (19), (22) and (35) it is possible to show that  $\hat{Y}_G = \hat{M} - \hat{\chi} - \hat{P}_H + \hat{K} / \mu$ . (Note that  $\hat{P}_{H,1}$  is non-stochastic and thus does not contribute to the variances of  $\hat{P}_H$  and  $\hat{Y}_G$ .) This minimisation problem yields the socially optimal home monetary policy rule given in (31). An equivalent minimisation problem for the foreign central bank yields the foreign monetary policy rule given in (31).

<sup>20</sup>Notice that the weight on the output gap depends only on the degree of price rigidity and not on the variance of cost-push shocks. If there were no cost-push shocks then producer prices and the output gap would be perfectly correlated so there would be no difference between producer-price

cost-push shocks, the world coordinated monetary policy can be supported by a policy of flexible inflation targeting in each country.<sup>21</sup>

Svensson (2002) has argued that assignment of objective functions of the form given in (36) is a practical way to induce independent central banks to follow socially optimal policy. Furthermore, he has argued that objective functions of this form can be used to derive ‘optimal targeting rules’ which provide a practical guide for the setting of policy.<sup>22</sup> The general procedure he describes for the derivation of such rules is to equate the marginal rate of substitution (MRS) between the target variables with the marginal rate of transformation (MRT) between the target variables (which is dictated by the underlying structure of the economy). Applying the procedure to the above model indicates that the ‘optimal targeting rules’ for the home and foreign central banks are of the form

$$\hat{P}_H + \hat{Y}_G = 0, \quad \hat{P}_F^* + \hat{Y}_G^* = 0 \quad (37)$$

That is, the home central banks should set monetary policy to achieve  $\hat{P}_H + \hat{Y}_G = 0$  and the foreign central bank should set monetary policy to achieve  $\hat{P}_F^* + \hat{Y}_G^* = 0$ . In the context of this model these rules are exactly equivalent to the socially optimal monetary policy.<sup>23</sup>

## 4 Simple Targeting Rules

It is apparent from the previous section that the optimal monetary strategy in this model economy is relatively easy to specify. A closed-form solution for the welfare function is derived and its implications for the optimal state-contingent monetary rule are clear. It is even possible to see that the optimal monetary rule can be implemented by independent national monetary authorities minimising loss functions which are weighted sums of producer-price volatility and output-gap volatility. Nevertheless, despite these clear results, there are reasons to suppose that the practical

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and output-gap targeting - both would produce the optimal policy. Notice also that these loss functions become inappropriate for extreme values of  $\psi$ . For  $\psi = 0$  the weight on the output gap is infinite. But in this case there are no sticky prices so monetary policy is powerless and the optimal monetary rule is undefined. In the opposite extreme, when  $\psi = 1$ , the weight on the output gap is zero, but producer prices are completely rigid so it is no longer meaningful to consider policy in terms of producer-price targeting.

<sup>21</sup>In contrast to Benigno and Benigno (2002, 2003b) and Clarida *et al* (2002), there are no welfare gains from international policy coordination in the basic form of the model used in this paper. A non-unit elasticity of substitution between home and foreign goods (as in Benigno and Benigno) and a utility function which is non-logarithmic in consumption (as in Benigno and Benigno and Clarida *et al*) imply cross-country spillover effects of monetary policy when there are cost-push shocks. It is these features which generate the gains from monetary policy coordination identified by these other authors.

<sup>22</sup>The general theoretical properties of such rules are also extensively analysed by Giannoni and Woodford (2002)

<sup>23</sup>Notice from Table 1 that optimal policy implies that the volatility of producer prices is identical to the volatility of the output gap. This is exactly what is required by the optimal targeting rules.

Regime	Targets
1. Money	$\hat{M} = \hat{M}^* = 0$
2. Producer prices	$\hat{P}_H = \hat{P}_F^* = 0$
3. Consumer prices	$\hat{P} = \hat{P}^* = 0$
4. Nominal income	$\hat{P}_H^* + \hat{Y}^* = \hat{P}_H + \hat{Y} = 0$
5. Symmetric fixed exchange rate	$(\hat{P} + \hat{P}^*)/2 = \hat{S} = 0$
6. Exchange rate/money	$\hat{M}^* = \hat{S} = 0$
7. Exchange rate/producer prices	$\hat{P}_F^* = \hat{S} = 0$
8. Exchange rate/consumer prices	$\hat{P}^* = \hat{S} = 0$

Table 2: Targeting regimes

implementation of such an optimal policy could be difficult. The optimal state-contingent rules (given in (31)) require observations of the underlying shocks and the ability to distinguish between labour-supply and cost-push shocks. If policy is implemented in the form of flexible producer-price targeting then it is necessary to measure the output gap. This is not a directly observable variable. Svensson (2002) argues that the optimal targeting rules (given in this model by (37)) yield a practical guide for policy which does not require observation of the underlying shocks. Nevertheless notice that these rules still require a measure of the output gap, and the derivation of the rules requires knowledge of the MRS and MRT between the target variables. The current model yields very simple optimal targeting rules where the relative weight on the two variables is unity. But this is the result of the very simple and somewhat restricted form of the model. In a more general model the relative weights would differ from unity and the derivation of the optimal weight would require detailed knowledge of the structure of the economy. Thus the optimal targeting rules are subject to many of the same practical difficulties which arise in the derivation and operation of the optimal state-contingent rules. For all these reasons it is useful to consider and compare a range of non-optimal but simple targeting rules. This is the objective of the present section.

In the context of the current model there are a number of possible target variables, namely: the money supply, producer prices, consumer prices, nominal income and the nominal exchange rate. These target variables can be combined in different ways to characterise a number of possible regimes. The full list of regimes considered is given in Table 2.<sup>24</sup>

The first four regimes to be considered are symmetric regimes where home mon-

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<sup>24</sup>For the purposes of this exercise, targeting variable  $X$  is taken to mean that the monetary authority adopts a rule which ensures that *ex post*  $\hat{X} = 0$ . Each of the targeting regimes in Table 2 can be achieved by adopting appropriate state-contingent money supply rules. Alternatively they can be supported by reaction functions of the form  $\hat{M} = -\delta\hat{X}$  where  $X$  is the target variable and  $\delta$  is a feedback coefficient. Rules of this form do not require policymakers to have information about the underlying shocks and the resulting equilibrium can be made arbitrarily close to the true targeting equilibrium by choosing a sufficiently high value of  $\delta$ .

etary policy is used to achieve a target for a home variable and foreign monetary policy is used to achieve a target for the corresponding foreign variable.

The last four regimes in Table 2 list a number of alternative fixed exchange rate regimes. Using (16) and (22) (and its foreign counterpart) it is simple to show that the nominal exchange rate is given by

$$\hat{S} = (\hat{M} - \hat{\chi}) - (\hat{M}^* - \hat{\chi}^*) \quad (38)$$

Thus any policy rule which implies  $\hat{M} - \hat{M}^* = \hat{\chi} - \hat{\chi}^*$  yields a fixed nominal exchange rate. A fixed exchange rate therefore only ties down the difference between home and foreign money supplies. To complete the characterisation of monetary policy it is necessary also to specify a rule which determines the absolute level of  $M$  or  $M^*$ . Regime 5 is a symmetric fixed exchange rate regime where the absolute levels of  $M$  and  $M^*$  are tied down by a target for aggregate consumer prices across the two countries.<sup>25</sup> Regimes 6, 7 and 8 are all asymmetric fixed exchange rate regimes where the absolute level of  $M^*$  is tied down by a target for a foreign nominal variable (either the money supply, producer prices or consumer prices) while home monetary policy is used to target the nominal exchange rate.

Note that nominal-income targeting (regime 4) implies  $\hat{P}^* + \hat{C}^* = \hat{P} + \hat{C} = 0$ . Nominal-income targeting is therefore supported by monetary rules of the form  $\hat{M} = \hat{\chi}$  and  $\hat{M}^* = \hat{\chi}^*$ , which (from (38)) imply a fixed nominal exchange rate. Thus, within this model, nominal-income targeting is also a form of fixed exchange rate regime. Nominal-income targeting is interesting in the present context because it implies stabilising a combination of real output and producer prices. It therefore shares some of the characteristics of the fully optimal policy rule.

Before analysing the welfare performance of these simple rules, it is useful to consider the behaviour of some of the potential target variables in the optimal equilibrium. This will suggest the circumstances in which different target variables will be good or bad approximations for the fully optimal policy. The optimal equilibrium volatilities of a producer prices, consumer prices, the exchange rate and nominal income are reported in Table 1. It is immediately obvious that, in general, none of the simple targeting rules will be exactly identical to optimal policy because fully optimal policy generates variations in all the target variables. There are, however, a number of special cases where one or other of the targeting rule will coincide with optimal policy. Two very obvious examples are the extreme cases where there are no cost-push shocks (i.e.  $\sigma_A^2 = 0$ ) and where there are no labour supply shocks (i.e.  $\sigma_K^2 = 0$ ). In former case producer price targeting coincides with optimal policy and in the latter case nominal income targeting coincides with optimal policy.

Table 1 also shows that there are two special cases where optimal policy implies a stable nominal exchange rates. One is where the variance of labour supply shocks

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<sup>25</sup>If the nominal exchange rate is fixed then the fact that  $P$  and  $P^*$  are in different currencies is irrelevant for forming a common consumer-price target. It can be shown that targeting world aggregate consumer prices is equivalent to targeting world aggregate producer prices.

Regime	World Welfare ( $\Omega_D$ )
1. Money	$-\frac{\psi(\sigma_K^2 + \mu^2 \sigma_X^2)}{\mu}$
2. Producer prices	$-\frac{\psi \sigma_A^2}{\mu}$
3. Consumer prices	$-\frac{\psi[(1-\zeta)\gamma^2 \sigma_K^2 + [2B^2 - (2B-\gamma)(1-\zeta)\gamma] \sigma_A^2]}{2\mu B^2}$
4. Nominal income	$-\frac{\psi \sigma_K^2}{\mu}$
5. Symmetric exchange rate	$-\frac{\psi[(1-\zeta)\sigma_K^2 + (1+\zeta)\sigma_A^2]}{2\mu}$
6. Exchange rate/money	$-\frac{\psi(\sigma_K^2 + \mu^2 \sigma_X^2)}{\mu}$
7. Exchange rate/producer prices	$-\frac{\psi[(1-\zeta)\sigma_K^2 + \sigma_A^2]}{\mu}$
8. Exchange rate/consumer prices	$-\frac{\psi[(1-\zeta)[2-\gamma(2-\gamma)]\sigma_K^2 + [2-(1-\zeta)\gamma(2-\gamma)]\sigma_A^2]}{2\mu}$
9. Fully optimal policy	0

Table 3: Welfare Levels

is zero (i.e.  $\sigma_K^2 = 0$ ). The other is where the labour supply shocks are perfectly correlated across the two countries (i.e.  $\zeta = 0$ ). The first of these cases is consistent with results reported in Benigno and Benigno (2002, 2003b) (who show that, in their model, a fixed rate may be optimal when cost-push shocks are the only disturbances affecting the economy). Note, however, that, even in these special cases, imposing a fixed nominal exchange rate is not sufficient to ensure fully optimal policy (because there are many pairs of monetary rules which are consistent with a fixed nominal exchange rate).

The derivation of the world welfare level implied by each targeting regime is given in the Appendix. The welfare levels are given in Table 3 (where  $B \equiv 1 - \psi(1 - \gamma)$ ). As a point of comparison, the level of welfare yielded by fully optimal policy (as represented by the monetary rules (31)) is also given in Table 3.

Using the welfare levels given in Table 3 it is possible to state and prove a series of propositions which establish the welfare ranking of the different regimes. Proposition 1 compares the welfare yielded by each of the asymmetric fixed exchange rate regimes with the corresponding symmetric regime.

**Proposition 1** (a) *The level of welfare yielded by symmetric targeting of money supplies (regime 1) is greater than or equal to the level of welfare yielded by asymmetric targeting of the exchange rate and the foreign money supply (regime 6); (b) The level of welfare yielded by symmetric targeting of producer prices (regime 2) is greater than or equal to the level of welfare yielded by asymmetric targeting of the exchange rate and foreign producer prices (regime 7); (c) The level of welfare yielded by symmetric targeting of consumer prices (regime 3) is greater than or equal to the level of welfare yielded by asymmetric targeting of the exchange rate and foreign consumer prices (regime 8).*

**Proof.** Proofs follow from comparison of the expressions given in Table 3 ■

This proposition demonstrates that each symmetric regime yields equal or higher welfare than the corresponding asymmetric regime for all parameter values.<sup>26</sup> It is therefore possible to disregard the asymmetric regimes in further comparison of the other regimes.

Proposition 2 compares the welfare yielded by the money targeting and nominal-income targeting regimes.

**Proposition 2** *The level of welfare yielded by nominal-income targeting (regime 4) is greater than or equal to the level of welfare yielded by symmetric money targeting (regime 1).*

**Proof.** The proof follows from comparison of the expressions given in Table 3

■

Nominal-income targeting yields equal or higher welfare than money-supply targeting because of money-demand shocks. Money-supply targeting is identical to nominal-income targeting except that money-supply targeting does not accommodate money-demand shocks. Money-supply targeting therefore allows money-demand shocks to create volatility in real output while nominal-income targeting completely offsets such shocks. It is therefore possible to disregard money-supply targeting in the further comparison of the other regimes.

Proposition 3 compares the symmetric fixed rate regime with producer-price targeting and nominal-income targeting.

**Proposition 3** *(a) The level of welfare yielded by nominal-income targeting is greater than or equal to the level of welfare yielded by a symmetric fixed exchange rate when  $\sigma_A^2 \geq \sigma_K^2$ . (b) The level of welfare yielded by producer-price targeting is greater than or equal to the level of welfare yielded by a symmetric fixed exchange rate when  $\sigma_A^2 \leq \sigma_K^2$ .*

**Proof.** Proofs follow from comparison of the expressions given in Table 3 ■

This proposition establishes that the symmetric fixed exchange rate regime (with world consumer-price targeting) is equivalent to or dominated by at least one of the other targeting regimes for all parameter values. It is therefore possible to disregard this regime in the further comparison of the other regimes.

There are now just three remaining regimes to consider: symmetric producer-price targeting (regime 2), symmetric consumer-price targeting (regime 3) and symmetric nominal-income targeting (regime 4). The relative ranking of these three regimes depends on the relative volatility of cost-push and labour-supply shocks. It is therefore useful to analyse the relative welfare performance in terms of  $\sigma_A^2/\sigma_K^2$ .

In what follows  $X_1$  is used to denote the value of  $\sigma_A^2/\sigma_K^2$  at which producer-price targeting yields the same level of welfare as consumer-price targeting and  $X_2$  is used

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<sup>26</sup>This result is perhaps not surprising in a model where the two countries have identical structures and parameter values and are subject to the same sources of shocks.

to denote the value of  $\sigma_A^2/\sigma_K^2$  at which consumer-price targeting yields the same level of welfare as nominal-income targeting. Using the welfare expressions in Table 3 it follows that

$$X_1 = \frac{\gamma}{2B - \gamma} \quad (39)$$

$$X_2 = \frac{2B^2 - (1 - \zeta)\gamma^2}{2B^2 - (2B - \gamma)(1 - \zeta)\gamma} \quad (40)$$

The following proposition can now be proved.

**Proposition 4** *When  $-1 \leq \zeta < 1$  and  $\gamma > 0$ : (a)  $X_1 < 1 < X_2$ . (b) When  $\sigma_A^2/\sigma_K^2 < X_1$  producer-price targeting yields higher welfare than consumer-price targeting and nominal-income targeting. (c) When  $X_1 < \sigma_A^2/\sigma_K^2 < X_2$  consumer-price targeting yields higher welfare than producer-price targeting and nominal-income targeting. (d) When  $\sigma_A^2/\sigma_K^2 > X_2$  nominal-income targeting yields higher welfare than producer-price targeting and consumer-price targeting.*

**Proof.** Proofs follow from comparison of the expressions given in Table 3 ■

This proposition shows that, when cost-push shocks are relatively unimportant, producer-price targeting is a reasonably good approximation for fully optimal policy. But when cost-push shocks are very important, nominal-income targeting becomes a good approximation for fully optimal policy. And for an intermediate range of values for  $\sigma_A^2/\sigma_K^2$ , consumer-price targeting is the best approximation for fully optimal policy.

It is clear from (31), (32) and (33) that fully optimal policy involves completely stabilising producer prices when there are no cost-push shocks. It therefore follows that producer-price targeting is a good approximation for fully optimal policy when the variance of cost-push shocks is low. It is also clear from the discussion in the previous section that, in the presence of cost-push shocks, fully optimal policy involves stabilising a weighted sum of output-gap and producer-price volatility. Nominal-income targeting implies stabilising both output and prices so it follows that nominal-income targeting is a good approximation for fully optimal policy when the variance of cost-push shocks is very high. In the intermediate region, where the variance of cost-push shocks is relatively high, but not yet high enough to make nominal-income targeting best, consumer-price targeting offers a nominal anchor which allows some flexibility in producer prices. This flexibility is sufficient to allow consumer-price targeting to yield higher welfare than producer-price targeting for intermediate values of  $\sigma_A^2/\sigma_K^2$ .

Figures 1, 2 and 3 illustrate the effects of some of the model's parameters on  $X_1$  and  $X_2$  (the threshold values of  $\sigma_A^2/\sigma_K^2$  at which the ranking of regimes changes). The benchmark parameter values are  $\mu = 2$ ,  $\gamma = \psi = 1/2$  and  $\zeta = 0$ . Figure 1 shows that the degree of price stickiness,  $\psi$ , has relatively little impact on the values of  $X_1$  and  $X_2$ . The main effect is that the range of values of  $\sigma_A^2/\sigma_K^2$  for which consumer-price targeting is best shrinks to zero for very high values of  $\psi$ .

Figure 2 shows the effects of varying the degree of openness as measured by  $\gamma$ . The degree of openness has its main effect on  $X_1$ . For very closed economies  $X_1$  is very low so consumer-price targeting is better than producer-price targeting for quite low values of  $\sigma_A^2/\sigma_K^2$ . But for very open economies  $X_1$  is high so  $\sigma_A^2/\sigma_K^2$  needs to be quite large before consumer-price targeting dominates producer-price targeting.

Figure 3 shows the effects of varying the degree of cross-country correlation of shocks (as measured by  $\zeta$ ). Varying  $\zeta$  only affects  $X_2$  but the effect is quite substantial. For strong negative cross-country correlation  $X_2$  is very high. When  $\zeta = -1$ ,  $X_2$  is approximately equal to five, so cost-push shocks need to be more than five times more volatile than labour-supply shocks before nominal-income targeting dominates consumer-price targeting. But for  $\zeta = 1$ ,  $X_2$  is approximately equal to unity, so nominal-income targeting is the best simple rule for much lower values of  $\sigma_A^2/\sigma_K^2$ .

## 5 Extensions to the Basic Model

The above model is restricted in a number of ways. This section briefly discusses the implications of two extensions.<sup>27</sup> The first extension allows for a non-unit coefficient of relative risk aversion in consumption. In this case the preference function is given by

$$U(z) = E \left[ \frac{C^{1-\rho}(z)}{1-\rho} + \chi \log \frac{M(z)}{P} - \frac{K}{\mu} y_i^\mu(z) \right] \quad (41)$$

where  $\rho > 0$ . The logarithmic structure adopted in the earlier sections of this paper corresponds to the case where  $\rho = 1$ .

The second extension allows for a non-unit elasticity of substitution between fixed-price and flexible-price goods. In this case the sub-utility functions over fixed-price and flexible-price goods are given as follows

$$C_H = \left[ \psi^{\frac{1}{\varphi}} C_{H,1}^{\frac{\varphi-1}{\varphi}} + (1-\psi)^{\frac{1}{\varphi}} C_{H,2}^{\frac{\varphi-1}{\varphi}} \right]^{\frac{\varphi}{\varphi-1}}, \quad C_F = \left[ \psi^{\frac{1}{\varphi}} C_{F,1}^{\frac{\varphi-1}{\varphi}} + (1-\psi)^{\frac{1}{\varphi}} C_{F,2}^{\frac{\varphi-1}{\varphi}} \right]^{\frac{\varphi}{\varphi-1}} \quad (42)$$

where  $\varphi > 0$  is the elasticity of substitution between fixed-price and flexible-price goods.<sup>28</sup> The Codd-Douglas structure adopted in the earlier sections of the paper corresponds to the case where  $\varphi = 1$ .

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<sup>27</sup>The technical details of the solutions to the extended model are available in an appendix which is available separately.

<sup>28</sup>Note that, in this new structure, it is no longer automatically true that fixed and flexible price producers have the same level of income in all states of the world. In this case it is useful to assume the existence of an insurance market which allows agents within a country to insure their incomes. This makes it possible to maintain the (analytically convenient) assumption that fixed and flexible price producers have the same level of consumption.

In order to achieve a sufficiently tractable solution, the two extensions are considered separately. Furthermore, it is necessary to restrict some of the other parameters of the model. In the discussion that follows it is assumed that both economies are fully open (i.e.  $\gamma = 1$ ), the disutility of work effort is linear (i.e.  $\mu = 1$ ) and the cross-country correlation of shocks is zero (i.e.  $\zeta = 0$ ).<sup>29</sup>

First consider the impact of the value of  $\rho$  on the comparison between simple targeting rules.<sup>30</sup> It is necessary to distinguish between the case where  $\rho > 1$  from the case where  $\rho < 1$ . When  $\rho$  is greater than unity, all the relevant results discussed in Section 4 continue to hold (in qualitative terms). Thus producer price targeting is the best simple rule for low values of  $\sigma_A^2/\sigma_K^2$  and nominal income targeting is the best simple rule for high values of  $\sigma_A^2/\sigma_K^2$ .<sup>31</sup> When  $\rho$  is less than unity, however, it is only possible to prove a weaker set of results. In this case, producer price targeting is the best simple rule when  $\sigma_A^2 = 0$  and nominal income targeting is the best simple rule when  $\sigma_K^2 = 0$ , but it is not possible to prove any conclusive statement about the best simple rule for intermediate values of  $\sigma_A^2/\sigma_K^2$ .

Now consider the implications of allowing  $\varphi$  to differ from unity.  $\varphi$  plays a potentially important role because it determines the impact of changes in relative prices on the distribution of demand across fixed and flexible price producers. Thus, the larger is  $\varphi$ , the larger is the degree of output dispersion across fixed and flexible price producers caused by any given degree of price dispersion. This increases the welfare cost of price dispersion and therefore increases the welfare cost of price volatility. The impact of  $\varphi$  can be most easily illustrated by considering the weight to be put on output-gap volatility in the optimal loss functions (36). It is possible to show that weight is now as follows

$$\omega = \frac{(1 - \psi)}{\psi\varphi} \quad (43)$$

This expression shows clearly that  $\omega$  is declining in  $\varphi$ , i.e. a high value of  $\varphi$  implies that the monetary authority should put more weight on price stabilisation and less on output-gap stabilisation. The implied optimal targeting rules are as follows

$$\varphi\hat{P}_H + \hat{Y}_G = 0, \quad \varphi\hat{P}_F^* + \hat{Y}_G^* = 0 \quad (44)$$

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<sup>29</sup>When  $\rho$  differs from unity an exact solution to the model can only be obtained when  $\gamma = 1$ . When  $\varphi$  differs from unity it is necessary to use a second-order approximation technique (of the form described in Sutherland (2002b)) to solve the model.

<sup>30</sup>The value of  $\rho$  also has an impact on the scope for welfare gains from international policy coordination. As shown by Clarida *et al* (2002), when utility is not logarithmic in consumption, there are international spillover effects of monetary policy and there can be welfare gains from coordination. (But note that Clarida *et al* consider policymaking under discretion while here it is assumed that commitment is possible.) If  $\gamma = 0$  gains exist as long as  $\sigma_A^2$  is greater than zero. If  $\gamma < 0$  then gains exist even if  $\sigma_A^2 = 0$ . Note that the size of the gains from coordination depends on the average level of the production subsidy. But the production subsidy continues to be irrelevant for the comparison between simple targeting rules.

<sup>31</sup>Recall that the discussion in this section is focusing on the case where  $\gamma = 1$ . This implies that the intermediate range of values of  $\sigma_A^2/\sigma_K^2$ , where consumer price targeting is best, shrinks to zero.

Again, these rules show clearly that the optimal weight on price stabilisation is increasing in the value of  $\varphi$ .

It is now simple to describe the implications of the value of  $\varphi$  for the welfare comparison between simple targeting rules. For small values of  $\varphi$  the results derived in Section 4 continue to apply (in qualitative terms), i.e. producer price targeting is the best simple rule for low values of  $\sigma_A^2/\sigma_K^2$ , and nominal income targeting is the best simple rule for high values of  $\sigma_A^2/\sigma_K^2$ . But if  $\varphi$  is large enough, the welfare impact of price volatility can become so large that producer price targeting is the best simple rule for all values of  $\sigma_A^2/\sigma_K^2$ .

## 6 Conclusion

This paper has shown that, in a simple two-country model where each country is subject to labour-supply, cost-push and money-demand shocks, the optimal coordinated policy can be supported by independent national monetary authorities following a policy of flexible inflation targeting. Such a policy minimises a loss function which depends on a weighted sum of producer-price volatility and output-gap volatility. After deriving the theoretically optimal policy regime, the paper compares a number of possible simple targeting rules. It is found that money-supply targeting and fixed exchange rates are in general dominated by either consumer-price targeting, producer-price targeting or nominal-income targeting. When the variance of cost-push shocks is small, strict producer-price targeting is found to be best of all the regimes considered. For intermediate levels of the variance of cost-push shocks, strict consumer-price targeting is best. For high levels of the variance of cost-push shocks nominal-income targeting is best.

The basic model presented in this paper is restricted in a number of respects. Two extensions to the model have been discussed but there are further extensions which are likely to prove interesting. Firstly, in the model presented here there are no real demand shocks. The international macroeconomics literature has previously emphasised the importance of real demand shocks for the choice of exchange rate regime. The introduction of real demand shocks is therefore an obvious way to develop the model and analysis presented above. Secondly, the utility function used in this paper (even after the extensions discussed in the previous section) is restricted in the sense that the elasticity of substitution between home and foreign goods is fixed at unity. This implies that the strength of the expenditure switching effect is constrained. The strength of the expenditure switching effect obviously has important implications for the welfare effects of monetary policy and the choice of monetary policy regime.<sup>32</sup> Again this issue suggests an interesting route to develop the model and analysis presented in this paper.

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<sup>32</sup>Sutherland (2002c) considers the implications of the expenditure switching effect in a small open economy model where there are no cost-push shocks.

# Appendix

## Optimal Price Setting

The price setting problem facing flexible-price producer  $z$  is the following:

$$MaxU(z) = \log C(z) + \log \left( \frac{M}{P} \right) - \frac{K}{\mu} y_2^\mu(z) \quad (45)$$

subject to

$$PC(z) = (1 + \alpha) p_{H,2}(z) y_2(z) + M_0 - M - T \quad (46)$$

$$y_2(z) = c_{H,2}(z) + c_{H,2}^*(z) = \frac{1}{1 - \psi} (C_{H,2} + C_{H,2}^*) \left( \frac{p_{H,2}(z)}{P_{H,2}} \right)^{-\phi} \quad (47)$$

The first order condition with respect to  $p_{H,2}(z)$  is

$$(1 + \alpha) \frac{y_2(z)}{PC(z)} - \phi \left[ (1 + \alpha) \frac{p_{H,2}(z)}{PC(z)} - K y_2^{\mu-1}(z) \right] \frac{y_2(z)}{p_{H,2}(z)} = 0 \quad (48)$$

In equilibrium all flexible-price agents choose the same price and consumption level so

$$(1 + \alpha) \frac{Y_2}{PC} - \phi \left[ (1 + \alpha) \frac{P_{H,2}}{PC} - K Y_2^{\mu-1} \right] \frac{Y_2}{P_{H,2}} = 0 \quad (49)$$

where

$$Y_2 = \frac{1}{1 - \psi} (C_{H,2} + C_{H,2}^*) = Y \left( \frac{P_{H,2}}{P_H} \right)^{-1} \quad (50)$$

where

$$Y = C_H + C_H^* = C \left( \frac{P_H}{P} \right)^{-1} \quad (51)$$

Rearranging yields the expression in the main text.

The price setting problem facing fixed-price producer  $z$  is the following:

$$MaxU(z) = E \left\{ \log C(z) + \log \left( \frac{M}{P} \right) - \frac{K}{\mu} y_1^\mu(z) \right\} \quad (52)$$

subject to

$$PC(z) = (1 + \alpha) p_{H,1}(z) y_1(z) + M_0 - M - T \quad (53)$$

$$y_1(z) = c_{H,1}(z) + c_{H,1}^*(z) = \frac{1}{\psi} (C_{H,1} + C_{H,1}^*) \left( \frac{p_{H,1}(z)}{P_{H,1}} \right)^{-\phi} \quad (54)$$

The first order condition with respect to  $p_{H,1}(z)$  is

$$E \left\{ (1 + \alpha) \frac{y_1(z)}{PC(z)} - \phi \left[ (1 + \alpha) \frac{p_{H,1}(z)}{PC(z)} - K y_1^{\mu-1}(z) \right] \frac{y_1(z)}{p_{H,1}(z)} \right\} = 0 \quad (55)$$

In equilibrium all fixed-price agents choose the same price and consumption level so

$$E \left\{ (1 + \alpha) \frac{Y_1}{PC} - \phi \left[ (1 + \alpha) \frac{P_{H,1}}{PC} - KY_1^{\mu-1} \right] \frac{Y_1}{P_{H,1}} \right\} = 0 \quad (56)$$

where

$$Y_1 = \frac{1}{\psi} (C_{H,1} + C_{H,1}^*) = Y \left( \frac{P_{H,1}}{P_H} \right)^{-1} \quad (57)$$

Rearranging yields the expression given in the main text.

## The Welfare Function

The welfare expression can be considerably simplified by noting that equations (17) to (21) (and the equivalent equations for the foreign economy) imply the following

$$E [KY_1^\mu] = E [KY_2^\mu] = E \left[ \frac{1}{A} \right] \quad (58)$$

$$E [K^*Y_1^{*\mu}] = E [K^*Y_2^{*\mu}] = E \left[ \frac{1}{A^*} \right] \quad (59)$$

Thus welfare can be rewritten as

$$\Omega = E \left[ \hat{C} + \hat{C}^* \right] + 2 \ln \bar{C} + E \left[ \frac{1}{\mu A} + \frac{1}{\mu A^*} \right] \quad (60)$$

where  $\bar{C}$  is the level of home and foreign consumption in a non-stochastic equilibrium. Expressions for  $E[\hat{C}]$  and  $E[\hat{C}^*]$  are now derived. First note that all the equations of the model are linear in logs and all the exogenous shocks are log-normally distributed. This implies that all the endogenous variables are also log-normally distributed. Equations (17), (18) and (19) and their foreign counterparts imply that

$$E[\hat{P}_{H,2}] = E \left[ \hat{P}_{F,2} \right] = 0 \quad (61)$$

and, using (20), (21) and (19), together with their foreign counterparts, it is possible to show that

$$\hat{P}_{H,1} = \frac{\lambda_{P_H}}{\mu}, \quad \hat{P}_{F,1}^* = \frac{\lambda_{P_F^*}}{\mu} \quad (62)$$

where

$$\lambda_{P_H} = \left\{ Var \left[ \hat{K} + \mu \hat{Y}_1 \right] - Var \left[ \hat{Y}_1 - \hat{P} - \hat{C} - \hat{A} \right] \right\} / 2 \quad (63)$$

$$\lambda_{P_F^*} = \left\{ Var \left[ \hat{K}^* + \mu \hat{Y}_1^* \right] - Var \left[ \hat{Y}_1^* - \hat{P}^* - \hat{C}^* - \hat{A}^* \right] \right\} / 2 \quad (64)$$

Comparing (61) and (62) shows that the prices of fixed-price agents contain a form of risk premium which reflects the fact that prices are set in advance of shocks being

realised. Using (19) and (21), and their foreign counterparts, it is further possible to simplify (63) and (64) as follows

$$\lambda_{P_H} = \left\{ Var \left[ \hat{K} + \mu \hat{Y}_1 \right] - \sigma_A^2 \right\} / 2 \quad (65)$$

$$\lambda_{P_F^*} = \left\{ Var \left[ \hat{K}^* + \mu \hat{Y}_1^* \right] - \sigma_{A^*}^2 \right\} / 2 \quad (66)$$

(where use has been made of the fact that, by definition, the variances of the prices of fixed-price agents are zero, i.e.  $Var \left[ \hat{P}_{H,1} \right] = Var \left[ \hat{P}_{F,1}^* \right] = 0$ ).

The money market equilibrium condition (22), together with the assumption that monetary policy only reacts to shocks, implies that

$$E \left[ \hat{C} \right] = -E \left[ \hat{P} \right], \quad E \left[ \hat{C}^* \right] = -E \left[ \hat{P}^* \right] \quad (67)$$

These expressions, combined with (16), imply that  $E \left[ \hat{S} \right] = 0$ .

Using the definitions of the various price indices, together with (61) and (62), it follows that

$$E \left[ \hat{C} \right] = -\frac{\psi}{\mu} \left[ \nu \lambda_{P_H} + (1 - \nu) \lambda_{P_F^*} \right] \quad (68)$$

An equivalent expression for foreign consumption is

$$E \left[ \hat{C}^* \right] = -\frac{\psi}{\mu} \left[ \nu \lambda_{P_F^*} + (1 - \nu) \lambda_{P_H} \right] \quad (69)$$

thus

$$E \left[ \hat{C} + \hat{C}^* \right] = -\frac{\psi}{\mu} \left[ \lambda_{P_H} + \lambda_{P_F^*} \right] \quad (70)$$

After substituting for  $\lambda_{P_H}$  and  $\lambda_{P_F^*}$  using (65) and (66) it is possible to write welfare in the form  $\Omega = \Omega_D + \Omega_X$  where

$$\Omega_D = -\frac{\psi \mu}{2} \left\{ Var \left[ \hat{Y}_1 + \frac{\hat{K}}{\mu} \right] + Var \left[ \hat{Y}_1^* + \frac{\hat{K}^*}{\mu} \right] \right\} \quad (71)$$

$$\Omega_X = \frac{\psi}{\mu} \sigma_A^2 + 2 \ln \bar{C} + E \left[ \frac{1}{\mu A} + \frac{1}{\mu A^*} \right] \quad (72)$$

All the terms in  $\Omega_X$  are independent of the setting of monetary policy and are thus irrelevant for welfare comparisons between targeting regimes.

## Welfare Levels

**1. Money targeting:** Money targeting implies  $\hat{M} = \hat{M}^* = 0$  so  $\hat{Y}_1 = -\hat{\chi} - \hat{P}_{H,1}$  and  $\hat{Y}_1^* = -\hat{\chi}^* - \hat{P}_{F,1}^*$ . Substituting into (27) yields the following expression for welfare

$$\Omega_D = -\frac{\psi (\sigma_K^2 + \mu^2 \sigma_\chi^2)}{\mu} \quad (73)$$

**2. Producer-price targeting:** From (32) and (33) it follows that producer-price targeting implies the following monetary rules

$$\hat{M} = \hat{\chi} - \frac{\hat{A} + \hat{K}}{\mu}, \quad \hat{M}^* = \hat{\chi}^* - \frac{\hat{A}^* + \hat{K}^*}{\mu} \quad (74)$$

These monetary rules imply

$$\hat{Y}_1 = -\frac{\hat{A} + \hat{K}}{\mu}, \quad \hat{Y}_2 = -\frac{\hat{A}^* + \hat{K}^*}{\mu} \quad (75)$$

When substituted into (27) the following expression for welfare is obtained

$$\Omega_D = -\frac{\psi\sigma_A^2}{\mu} \quad (76)$$

**3. Consumer-price targeting:** Targeting the consumer-price index in each country implies

$$\hat{P} = \nu\hat{P}_H + (1 - \nu) (\hat{P}_F + \hat{S}) = 0 \quad (77)$$

$$\hat{P}^* = \nu\hat{P}_F^* + (1 - \nu) (\hat{P}_H - \hat{S}) = 0 \quad (78)$$

Using (32) and (33) these two equations imply

$$\hat{M} = \hat{\chi} - \frac{2B - \gamma}{2B} \left( \frac{\hat{A} + \hat{K}}{\mu} \right) - \frac{\gamma}{2B} \left( \frac{\hat{A}^* + \hat{K}^*}{\mu} \right) \quad (79)$$

$$\hat{M}^* = \hat{\chi}^* - \frac{\gamma}{2B} \left( \frac{\hat{A} + \hat{K}}{\mu} \right) - \frac{2B - \gamma}{2B} \left( \frac{\hat{A}^* + \hat{K}^*}{\mu} \right) \quad (80)$$

where  $B = 1 - \psi(1 - \gamma)$ . When substituted into (27) the following expression for welfare is obtained

$$\Omega_D = -\frac{\psi [(1 - \zeta)\gamma^2\sigma_K^2 + [2B^2 - (2B - \gamma)(1 - \zeta)\gamma]\sigma_A^2]}{2\mu B^2} \quad (81)$$

**4. Nominal-income targeting:** Nominal-income targeting implies  $\hat{M} = \hat{\chi}$  and  $\hat{M}^* = \hat{\chi}^*$  so, from (28) and (29) it is apparent that  $\hat{Y}_1 = -\hat{P}_{H,1}$  and  $\hat{Y}_1^* = -\hat{P}_{F,1}^*$ . Substituting into (27) yields the following expression for welfare

$$\Omega_D = -\frac{\psi\sigma_K^2}{\mu} \quad (82)$$

**5. Symmetric fixed nominal exchange rate:** A fixed nominal exchange rate implies

$$\hat{M} - \hat{\chi} = \hat{M}^* - \hat{\chi}^* \quad (83)$$

From (32) and (33) it follows that targeting the aggregate world consumer price index implies

$$\frac{\hat{A} + \hat{K}}{\mu} + \hat{M} - \hat{\chi} + \frac{\hat{A}^* + \hat{K}^*}{\mu} + \hat{M}^* - \hat{\chi}^* = 0 \quad (84)$$

These two equations imply

$$\hat{M} = \hat{M}^* = \frac{1}{2} \left( \hat{\chi} + \hat{\chi}^* + \frac{\hat{A} + \hat{K}}{\mu} + \frac{\hat{A}^* + \hat{K}^*}{\mu} \right) \quad (85)$$

When substituted into (27) the following expression for welfare is obtained

$$\Omega_D = -\frac{\psi [(1 - \zeta)\sigma_K^2 + (1 + \zeta)\sigma_A^2]}{2\mu} \quad (86)$$

**6. Fixed exchange rate/foreign money targeting:** Foreign money targeting implies  $M^* = 0$  and a fixed nominal exchange rate implies  $\hat{M} = \hat{\chi} - \hat{\chi}^*$ . So  $\hat{Y}_1 = \hat{\chi}^* - \hat{P}_{H,1}$  and  $\hat{Y}_1^* = -\hat{\chi}^* - \hat{P}_{F,1}^*$ . Substituting into (27) yields the following expression for welfare

$$\Omega_D = -\psi (\sigma_K^2 + \mu^2 \sigma_\chi^2) \quad (87)$$

**7. Fixed exchange rate/foreign producer-price targeting:** Foreign producer-price targeting implies

$$M^* = \hat{\chi}^* - \frac{\hat{A}^* + \hat{K}^*}{\mu} \quad (88)$$

A fixed nominal exchange rate implies  $\hat{M} = \hat{\chi} - \hat{\chi}^* + M^*$ . So

$$\hat{Y}_1 = \hat{Y}_2 = -\frac{\hat{A}^* + \hat{K}^*}{\mu} \quad (89)$$

Substituting into (27) yields the following expression for welfare

$$\Omega_D = -\frac{\psi [(1 - \zeta)\sigma_K^2 + \sigma_A^2]}{\mu} \quad (90)$$

**8. Fixed exchange rate/foreign consumer-price targeting:** Foreign consumer-price targeting implies

$$\hat{P}^* = \nu \hat{P}_F^* + (1 - \nu) (\hat{P}_H - \hat{S}) = 0 \quad (91)$$

and a fixed nominal exchange rate implies  $\hat{M} = \hat{\chi} - \hat{\chi}^* + M^*$ . Solving for output levels yields

$$\hat{Y}_1 = \hat{Y}_2 = -\frac{\gamma(\hat{A} + \hat{K}) - (2 - \gamma)(\hat{A}^* + \hat{K}^*)}{2\mu} \quad (92)$$

Substituting into (27) yields the following expression for welfare

$$\Omega_D = -\frac{\psi[(1 - \zeta)[2 - \gamma(2 - \gamma)]\sigma_K^2 + [2 - (1 - \zeta)\gamma(2 - \gamma)]\sigma_A^2}{2\mu} \quad (93)$$

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Figure 1: Effect of Psi on X1 and X2

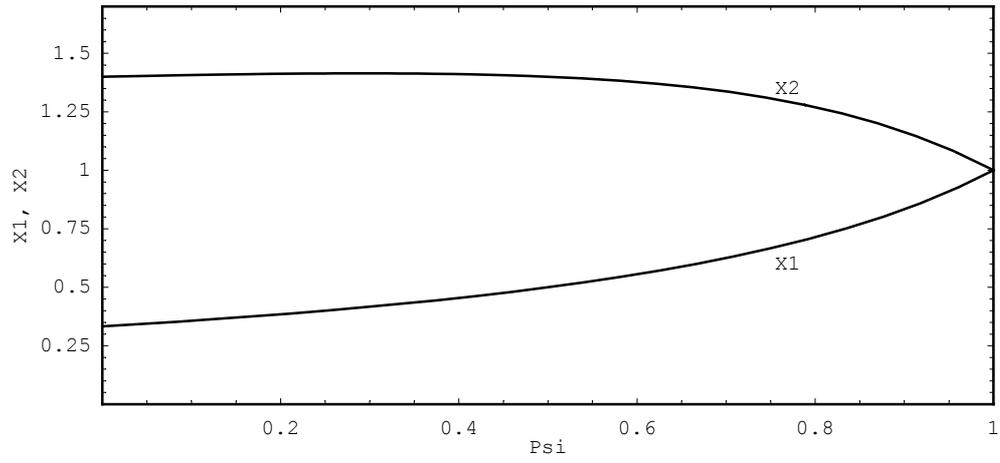


Figure 2: Effect of Gamma on X1 and X2

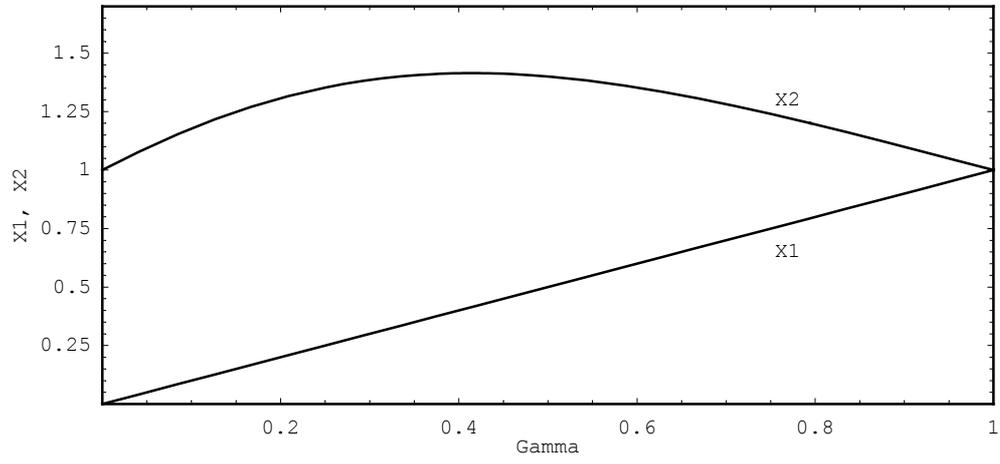


Figure 3: Effect of Zeta on X1 and X2

