Optimal Monetary Policy and the Timing of Asset Trade in Open Economies

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Abstract

This paper analyses the timing of asset trade and its implications for monetary policy and welfare in open economies. Optimal policy is shown to differ significantly depending on whether asset trade takes place before or after policy decisions are made.

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1. Introduction

A common practice in the recent open economy macroeconomic literature is to assume that international financial markets are complete. This is often imposed by assuming that a relationship of the following form holds in all states of the world

$$\frac{U_{C^*}}{U_C} = k\frac{SP^*}{P} \tag{1}$$

where U_C and U_{C^*} are the marginal utilities of consumption in the home and foreign countries, P and P^* are consumer price indices and S is the nominal exchange rate (defined as the domestic price of foreign currency). Having stated this equation many authors treat the coefficient k as fixed. Equation (1) is then imposed, along with other model equations, in the derivation of a general equilibrium solution. The resulting solution is then often used to analyse the positive and normative effects of policy. (See for instance Benigno and Benigno, 2003).

This paper shows that the assumption that k is constant and exogenous corresponds to an implicit assumption that trade in asset markets takes place *before* policy decisions are made. However, an alternative approach, which is rarely considered, is to reverse the timing of asset trade and policy decisions, so that asset trade takes place *after* policy is determined. Indeed, given the continuous nature of asset trade in real-world markets, it is arguable that this alternative approach may be more realistic. In this paper we analyse this alternative timing of asset trade and show that it can have important implications for the analysis of policy. In terms of equation (1) the main way in which the alternative approach differs from the usual practice is that k becomes an endogenous variable which depends on policy choices. (See Sutherland, 2004, for an example where this alternative timing of asset trade has been used.)

Whether or not the timing of asset trade matters depends on the purpose for which a model is being used. If the objective is to derive first-order accurate solutions for first moments or realised values then we show that the timing of asset trade has no critical implications. However, if the objective is to derive second-order accurate solutions to first moments, as would be the case in welfare calculations, then we show that the timing of asset trade proves to be critical.

2. The Model

These points are illustrated in a simple model which is typical of those commonly used for the analysis of monetary policy in open economies (following, for instance, Obstfeld and Rogoff (1995)). The model is for illustrative purposes only. The basic principle that the timing of asset trade matters for the welfare analysis of policy carries over to much more general models.

The world exists for a single period and consists of two equal-sized countries, which are referred to as home and foreign. There is a continuum of agents of unit mass in each country. Agents consume a basket of all home and foreign goods and each agent is a monopoly producer of a single differentiated product. The only source of stochastic shocks is the foreign money supply. Home monetary policy is modelled as a general feedback rule. All agents set prices (in their own currency) in advance of the realisation of shocks and the setting of monetary policy. The main equations of the model are shown in Table 1. Foreign real variables and foreign currency prices are indicated with an asterisk.

[insert Table 1 here]

All agents have utility functions of the form shown in row 1 of Table 1 where χ and K are positive constants, C is a consumption index defined across all goods, M denotes end-of-period nominal money holdings and $y(h)$ is the output of good h (which is produced by agent h). C is defined in row 2 of Table 1 where C_H and C_F are indices of home and foreign produced goods where the elasticity of substitution between individual goods is $\phi > 1$. The budget constraint of agent h is $M(h) = M_0 + p_H(h)y(h) - PC(h) - T + PR(h)$ where M_0 and $M(h)$ are initial and final money holdings, T is a lump-sum government transfer, $p_H(h)$ is the price of good h and *R(h)* is the income from a portfolio of state contingent assets (to be described below). P is defined in row 3 of Table 1 where P_H and P_F are the price indices for home and foreign goods respectively. The law of one price is assumed to hold, so $P_H = P_H^* S$, $P_F = P_F^* S$ and $P = SP^*$.

Aggregate home demands for home and foreign goods are shown in row 5 of Table 1 and foreign demands for home and foreign goods are shown in row 6. The total demand for home goods and the total demand for foreign goods are shown in row 7.

The first-order conditions for price setting are shown in row 4 of Table 1, where E_P is the expectations operator conditional on information available at the time prices are set.

The home government's budget constraint is $M - M_0 + T = 0$. The first-order conditions for the choice of money holdings is given in row 8 of Table 1. The foreign money supply is assumed to be exogenous and subject to stochastic shocks such that $\log M^*$ is symmetrically distributed over the interval $[-\varepsilon, \varepsilon]$ with $E[\log M^*] = 0$ and $Var[\log M^*] = \sigma^2$. Home monetary policy is defined in terms of a general feedback rule for the home money supply of the following form

$$M = \bar{M} \left(M^* / \bar{M}^* \right)^{\delta} \qquad (2)$$

Below we consider equilibrium for a given value of δ and also the welfare maximising value of δ.

Figure 1 summarises the timing of events in the form of a time-line. The main events are: asset trade; monetary policy (i.e. decisions about the value of δ); price setting; realisation of the shock; and finally, production, trade and consumption. The main issue analysed by this paper is the ordering of the first two events, namely asset trade and monetary policy.

[insert Figure 1 here]

3. Financial Markets and the Timing of Asset Trade

It is assumed that sufficient financial instruments exist to allow efficient sharing of consumption risks. This can be achieved by allowing trade in two state-contingent assets, one which has a pay-off correlated with home aggregate real income and one with a pay-off correlated with foreign real income. Thus, a unit of the home asset pays $y = YP_H/(SP^*)$ and a unit of the foreign asset pays $y^* = Y^*P_F/P$. It is possible to show that the two asset prices are given by

$$q_H = E_A\left[\frac{y}{y+y^*}\right] \Big/ E_A\left[\frac{1}{y+y^*}\right], \qquad q_F = E_A\left[\frac{y^*}{y+y^*}\right] \Big/ E_A\left[\frac{1}{y+y^*}\right] \tag{3}$$

where q_H and q_F are the unit prices of the home and foreign assets and E_A is the expectations operator conditional on information available at the time asset trade takes place. Relative consumption levels in the two countries satisfy the following equation

$$\frac{C}{C^*} = \frac{q_H}{q_F}\frac{P^*S}{P} \tag{4}$$

This relationship holds true regardless of the timing of asset trade. The timing of asset trade comes into play in determining the information set relevant for the expectations operators in (3). If asset trade takes place after the home monetary authority chooses the value of δ (i.e. at point 2 in Figure 1) this information will be incorporated into the expectations formed at the time of asset trade and thus the value of δ will have an impact on asset prices, q_H and q_F. Therefore the choice of δ can have an effect on the ratio of home to foreign consumption. If, however, asset trade takes place before δ is chosen (i.e. at point 1 in Figure 1), asset prices will be fixed prior to the setting of policy. So the choice of δ will not have any effect on asset prices or the ratio of home to foreign consumption.

Clearly, in the context of the current model, equation (4) corresponds to equation (1) and the asset price ratio corresponds to the coefficient *k*. It is now clear how the timing of asset trade is embodied in assumptions about *k*. If asset market trade takes place before policy is determined, q_H and q_F are exogenous and thus *k* is also exogenous. But if asset market trade takes place after policy is determined then q_H and q_F depend on δ and thus *k* also depends on δ.

4. First-Order and Second-Order Solutions

Now consider the derivation of first-order accurate solutions for first moments and realised values. A first-order approximation of (3) is given by

$$\hat{q}_H = E_A[\hat{y}] + O(\varepsilon^2), \qquad \hat{q}_F = E_A[\hat{y}^*] + O(\varepsilon^2) \tag{5}$$

while (4) can be written exactly in log-deviation form as follows

$$\hat{C} - \hat{C}^* = \hat{q}_H - \hat{q}_F + \hat{P}^* + \hat{S} - \hat{P} \tag{6}$$

All log deviations of endogenous variables are mean-zero random variables so, up to first-order accuracy, (5) implies $\hat{q}_H = \hat{q}_F = 0$. And (6) simplifies to a linearised version of (1) with $k = 1$, i.e. the form of risk-sharing condition widely used in the current open-economy literature. It is therefore possible to solve the model, and obtain first-order accurate expressions for first moments and realised values, without having to consider the timing of asset trade. Note that it is possible to obtain second-order accurate solutions for second moments from first-order accurate expressions for realised values, so again in this case the timing of asset trade is irrelevant.

Now consider second-order accurate solutions for first moments. In particular assume we are interested in deriving first moments based on expectations at the time policy is set. This is the relevant case if one is deriving second-order accurate expressions for welfare. We use E_W to denote expectations formed conditional on information available at the time of policy decisions.

A second-order approximation of (3) is given by

$$\hat{q}_H = E_A[\hat{y}] + O(\varepsilon^3), \qquad \hat{q}_F = E_A[\hat{y}^*] + O(\varepsilon^3) \tag{7}$$

which may superficially appear to be identical to (5), but in fact has very different implications. This is because, when considering second-order accurate solutions, it is no longer true that $E_A[\hat{y}] = E_A[\hat{y}^*] = 0$, and, more importantly, q_H and q_F are endogenous in the case where asset trade takes place *after* policy decisions are made. To show this more clearly it is necessary to consider the two asset timing cases in detail.

First consider the case where asset trade takes place *before* policy (i.e. at point 1 in Figure 1). In this case asset prices are set before E_W-type expectations are formed. Thus, when solving for second-order accurate expressions for E_W-type first moments, q_H and q_F should be treated as exogenous in equation (6). When combined with second-order approximations for the other equations of the model this yields the solution for $E_W[\hat{C}]$ shown in the first column of Table 2.

[insert Table 2 here]

Now consider the case where asset trade takes place *after* policy is chosen (i.e. at point 2 in Figure 1). In this case asset prices are formed after E_W-type expectations are formed. Which implies that, in order to solve for E_W-type first moments it is necessary to solve for $E_W[\hat{q}_H]$ and $E_W[\hat{q}_F]$. However, by using the law of iterated expectations it follows that

$$E_W[\hat{q}_H] = E_W[E_A[\hat{y}]] = E_W[\hat{y}], \qquad E_W[\hat{q}_F] = E_W[E_A[\hat{y}^*]] = E_W[\hat{y}^*] \tag{8}$$

thus expected asset prices can be solved along with other E_W-type first moments. Notice in particular that the resulting expected asset prices will depend on the choice of policy rule. When equations (8) are combined with second-order approximations of the other equations in the model the expression for $E_W[\hat{C}]$ shown in the second column of Table 2 is derived.

It is immediately apparent from the expressions for $E_W[\hat{C}]$ shown in Table 2 that second-order solutions for the first moments differ depending on the timing of asset trade.

5. Welfare and Optimal Policy

One important reason for deriving second-order accurate solutions is to consider the welfare effects of policy. To illustrate the impact of asset market timing on welfare and optimal policy we present welfare results for our model. A second-order approximation of aggregate home utility (where the utility of real balances is assumed to be small enough to ignore) is given by

$$\tilde{\Omega} = E_W\left[\hat{C} - \Phi(\hat{Y} + \hat{Y}^2)\right] + O(\varepsilon^3) \qquad (9)$$

where $\tilde{\Omega}$ is the deviation of the level of welfare from the non-stochastic equilibrium and $\Phi = (\phi - 1)/\phi$. Expressions for welfare are shown in row 2 of Table 2. And expressions for the welfare maximising choice of δ are shown in row 3 of Table 2. It is immediately apparent that welfare depends on the timing of asset trade and thus the welfare maximising choice of δ also depends on asset market timing. A comparison of these expressions shows that δ is, in general, higher in the case where asset trade takes place before policy decisions are made than it is in the case where asset trade takes place after policy decisions are made. This implies policy is more active in the case where asset trade takes place before policy.

6. Conclusions

This paper demonstrates that the timing of asset market trade has important implications for the analysis of welfare and optimal policy in open economies. The current literature has tended to focus on cases where asset trade takes place before policy. It is at least arguable that the alternative assumption, that asset trade takes place after policy, is empirically more relevant. The analysis of this paper suggests that this alternative case will produce results which differ from those reported in the recent literature and is thus worthy of further investigation.

References

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Table 1: Model equations.

	Home Country Equations	Foreign Country Equations
1	$U(h)=\log C(h)+\chi\log\frac{M(h)}{P}-\frac{K}{2}y^{2}(h)$	$U^{*}(f)=\log C^{*}(f)+\chi\log\frac{M^{*}(f)}{P^{*}}-\frac{K}{2}y^{*2}(f)$
2	$C=\left[\left(\frac{1}{2}\right)^{\frac{1}{\theta}}C_{H}^{\frac{\theta-1}{\theta}}+\left(\frac{1}{2}\right)^{\frac{1}{\theta}}C_{F}^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}$	$C^{*}=\left[\left(\frac{1}{2}\right)^{\frac{1}{\theta}}C_{H}^{*\frac{\theta-1}{\theta}}+\left(\frac{1}{2}\right)^{\frac{1}{\theta}}C_{F}^{*\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}$
3	$P=\left[\frac{1}{2}P_{H}^{1-\theta}+\frac{1}{2}P_{F}^{1-\theta}\right]^{\frac{1}{1-\theta}}$	$P^{*}=\left[\frac{1}{2}P_{H}^{*1-\theta}+\frac{1}{2}P_{F}^{*1-\theta}\right]^{\frac{1}{1-\theta}}$
4	$P_{H}=\frac{\phi}{\phi-1}\frac{KE_{P}\left[Y^{2}\right]}{E_{P}\left[Y/(PC)\right]}$	$P_{F}^{*}=\frac{\phi}{\phi-1}\frac{KE_{P}\left[Y^{*2}\right]}{E_{P}\left[Y^{*}/(P^{*}C^{*})\right]}$
5	$C_{H}=\frac{1}{2}C\left(\frac{P_{H}}{P}\right)^{-\theta}$	$C_{F}=\frac{1}{2}C\left(\frac{SP_{F}^{*}}{P}\right)^{-\theta}$
6	$C_{H}^{*}=\frac{1}{2}C^{*}\left(\frac{P_{H}}{SP^{*}}\right)^{-\theta}$	$C_{F}^{*}=\frac{1}{2}C^{*}\left(\frac{P_{F}^{*}}{P^{*}}\right)^{-\theta}$
7	$Y=C_{H}+C_{H}^{*}$	$Y^{*}=C_{F}+C_{F}^{*}$
8	$\frac{M}{P}=\chi C$	$\frac{M^{*}}{P^{*}}=\chi C^{*}$

Table 2: Model solutions and asset market timing.

	Asset trade before policy	Asset trade after policy
$E_w[\hat{C}]$	$\frac{-2-A-2\delta(2-A)-\delta^2(2+A)}{8}$	$\frac{-6+3\theta-\theta^2-2\delta(2-A)+\delta^2(2-5\theta-\theta^2)}{8}$
$\tilde{\Omega}$	$\frac{-2-A-2\delta(2-A)-\delta^2(2+A)}{8}+\frac{(\delta^2-1)(\theta-1)\theta}{2(\theta+1)}\Phi$	$\frac{-6+3\theta-\theta^2-2\delta(2-A)+\delta^2(2-5\theta-\theta^2)}{8}$
δ_{opt}	$\frac{(\theta+1)(A-2)}{2+\theta^3+\theta^2(2-4\Phi)+\theta(3+4\Phi)}$	$\frac{A-2}{\theta^2+5\theta-2}$

where $A=\theta(\theta+1)$

Figure 1: The timing of events