Whether there can be as many angels in a place as there are alephs?  
(A defence of mereological universalism)  

A. J. Cotnoir

Supposing that angels can be located at a (zero-dimensional) point in space, and additionally that they can be co-located, the question naturally arises: how many such things can be co-located at a point in space? Where alephs are cardinals, here are two possible answers.

**Indefinite Extensibility** There could not be so many angels as to exceed each and every aleph, but for each ordinal $\alpha$ there could be at least $\aleph_\alpha$-many angels located at a point.

**Plenitude** The could be at least as many angels as there are alephs.

Hawthorne and Uzquiano [4] object to plenitude by showing that it is incompatible with mereological universalism — the view that for every plurality $xx$, there exists of fusion of $xx$. (They also show how the weaker answer of indefinite extensibility is incompatible with Lewis’s [6] modal realism and Williamson’s [10] necessitism.) Instead of their argument constituting a reductio against plenitude (or against indefinite extensibility for modal realists/necessitists), one might rather conceive of it as a reductio of mereological universalism.

1. If mereological universalism is true, then it is necessarily true.

2. If mereological universalism is necessarily true, then plenitude is false.

3. But, plenitude is true.

4. So, mereological universalism is false.

The options are: reject (1) a la Cameron [1]; reject (3) as apparently do Hawthorne and Uzquiano; or reject (2) and Hawthorne and Uzquiano’s case for it. In this article I argue against (2), that the mereological universalist who believes in co-location has a straightforward independently-motivated reply.

**Videtur Quod Non**

It would seem that there cannot be as many angels as there are alephs. For consider the following widely accepted principle connected to the philosophical conception of a set.

**Limitation of Size** A plurality forms a set iff they are not in one-one correspondence with the entire universe of all objects.
This principle entails that there is exactly one cardinality that is not characterised by the alephs, and that is the size of the universe. To see why, note that (on pain of the Burali-Forti paradox) the ordinals do not form a set. So, by LIMITATION OF SIZE the ordinals must be in a one-one correspondence with the entire universe. But since the aleph series can be put into a one-to-one correspondence with the ordinals, we know that the size of the alephs is the only size not characterised by them.

Say that a plurality \( xx \) is **disperse** iff every sub-plurality \( yy \) among the \( xx \) has a distinct mereological fusion. We also can show, by a generalization of Cantor’s diagonal argument, that any non-singular plurality it has more sub-pluralities than members.¹ What follows, then, is a particular **mereological result**: if a non-singular plurality is disperse, there are more fusions of its sub-pluralities that there are members of it.

A disperse plurality that is more numerous than one has more sub-pluralities than members. But given [universalism], every sub-plurality will have a fusion. By disperseness, different sub-pluralities will have different fusions, whence there are more fusions based on the initial plurality than there are members of it. [4, p. 10]

Notice that this mereological result rests crucially on the assumption of mereological universalism: once we assume that every sub-plurality of a disperse plurality has a fusion, then the plurality has more fusions than members.

At last, we come to the argument.

Our [mereological] result tells us that there are strictly more fusions of angels than there are angels. Limitation of size tells us that the size of the [angels] is at most the size of the alephs. […] Given Plenitude, we are forced to conclude that the size of the angels matches the size of the actual alephs. But now, by our mereological result, we must conclude that there are strictly more [fusions of angels] than there are alephs, which contradicts limitation of size. We conclude that Plenitude fails again. ([4, p. 14])

It would seem, then, that we cannot accept both PLENITUDE and universalism.

**Sed Contra**

To the contrary, Hawthorne and Uzquiano suggest the possibility of a non-standard mereology, when discussing their objection to Williamson’s **necessitism**:¹

¹Hawthorne and Uzquiano [4, p. 10].
Suppose we adopt a mereology that — as against classical extensional mereology — abandons the presumption that parthood is antisymmetric […] It now becomes possible to think of the possibly (but not actually) concrete objects, not as forming a disperse plurality, but as parts of each other, forming an entangled unity. When an object becomes concrete it breaks off from — that is, becomes mereologically discrete from those entangled entities, and when it ceases to be concrete, it returns to — that is, becomes mereologically reconciled to — those entities. While this is not perhaps a full vindication of Plotinus’ doctrine of a return to the One — carried into scholastic philosophy by the early Church Fathers — it is perhaps as close to a vindication as sober analytic metaphysics can provide. (21)

While Williamson’s necessitism does utilise the concrete/abstract distinction in crucial and controversial ways, he does nothing so far removed from sober analytic metaphysics as this. This sort of failure of antisymmetry seems wildly under-motivated.

Respondeo Dicendum

I reply saying that one does not have to appeal to any strange doctrines like the ‘return to the One’ to defend mereological universalism. Indeed, the failure of antisymmetry is much better motivated in the context of mereological universalism with the possibility of co-location.

We begin with a single binary mereological primitive: overlap $\circ$.²

**Parthood** $x \leq y := \forall z(z \circ x \to z \circ y)$

**Proper Parthood** $x < y := x \leq y \wedge y \not\leq x$

**Non-identical Parthood** $x \preceq y := x \leq y \wedge x \not= y$

**Fusions** $F(y, xx) := \forall z(z \circ y \leftrightarrow z \circ xx)$

In the definition of fusion, we use ‘$z \circ xx$’ to mean that $z$ overlaps some $x$ among the $xx$.³

The definition of parthood allows us to prove reflexivity and transitivity of $\leq$; but we do not allow anti-symmetry of $\leq$ which is equivalent to $\forall z((z \circ x \leftrightarrow z \circ y) \to x = y)$, an extensionality principle of overlap. Thus, we might have distinct objects $a \not= b$ such that $a \leq b$ and $b \leq a$. As a consequence, we need to be clear about the relevant notion of proper part that we are using. On one conception, $x$ is a proper part of $y$ whenever $x$ is non-mutual part of $y$. By the definition

---

²We could have chosen disjointness $\sqcap$, since they are interdefinable as the negations of each other.

³These are fusions in the style of Leonard and Goodman [5] and Simons [8], rather than the fusions used by Lewis [7] and Tarski [9]. Although this mereology could have been axiomatized using those definitions, too (see Cotnoir [3]).
of parthood in terms of overlap, we thus have that if \( x < y \) then there’s a \( z \) which overlaps \( x \) but doesn’t overlap \( y \). This is a form of \emph{weak supplementation}. By contrast, there is another conception of proper part — that of a \emph{non-identical} part, expressed by ‘\( \preceq \)’ — according to which we are not guaranteed supplementation.

In any case, we have two axioms of our mereology:

\[
A_1 \quad x \circ y \leftrightarrow \exists z (z \preceq x \land z \preceq y)
\]

\[
A_2 \quad \forall x \exists y F(y, xx)
\]

Indeed, this mereology is very well-suited to allowing co-location. Indeed, if one accepts certain assumptions about locations, this mereology would even be required:⁴

\textbf{Harmony} if \( x \) is weakly located at every region in which \( y \) is weakly located, then anything that overlaps \( x \) overlaps \( y \).

On this view, co-located objects \( x \) and \( y \) are mutual parts: \( x \preceq y \land y \preceq x \). By the definition of \( \preceq \), this is equivalent to: \( \forall z (z \circ x \leftrightarrow z \circ y) \) i.e. two objects are co-located whenever they overlap the same objects.

\textit{Ad Primum Ergo Dicendum}

Therefore to the first objection based on the \textit{limitation of size}, we can see that for any co-located \( xx \) and any \( x_i \) among the \( xx \), we have it that \( F(x_i, xx) \); after all, for co-located \( xx \), \( \forall z (z \circ x_i \leftrightarrow z \circ xx) \). In the case at hand, each individual angel counts as a fusion of the the angels. Hence according to our co-location mereology, the angels are not a disperse plurality. So the \textit{limitation of size} argument fails.

We do need some assurance that the \textit{only} fusions of \( xx \) that exist are identical to some \( x_i \) among the \( xx \). In other words, we want to be sure that the angels together with their fusions do not force there to be strictly more things than there are alephs.

There is, however, a straightforward way of showing why the number of fusions of angels is just the same as the number of angels themselves. We merely need appeal to a single plausible principle about locations of fusions:

\textbf{Inheritance} If \( y \) is the fusion of \( xx \), then for all regions \( r \), \( y \) is weakly located at \( r \) iff some \( x \) among \( xx \) is weakly located at \( r \).

⁴Cotnoir [2] considers the following: the location of \( x \) is a subregion of the location of \( y \) iff \( x \preceq y \).
This principle entails that if none of the $xx$ are located in a region, then neither is their fusion. That is, a whole inherits the locations of its parts. We can then argue as follows. Let the $xx$ be the plurality of all things exactly located at a point $r$. By limitation of size, the $xx$ can be at most as many as the alephs. Now, let $F(y, xx)$. Since fusions are upper bounds of the things fused, $xx$ are all parts of $y$. Hence, by inheritance, $y$ is weakly located at $r$ and only $r$; i.e. $y$ is exactly located at $r$. Hence $y$ is one of the $xx$. As a result we can rest assured that the cardinality of angels does not exceed the cardinality of the alephs.  

References


⁵Thanks to [XXXX].