Non-Wellfounded Mereology

A binary relation R is said to be *wellfounded* on a domain if and only if every non-empty subset of that domain has an R-minimal element; equivalently, iff every descending R chain is finite. A proper parthood relation can be non-wellfounded in various ways; here are three: (i) *gunk*: for all x there is a y such that y is a proper part of x; (ii) *bounded infinite descent*: there is an infinite chain of distinct proper parts $x_1 > x2 > x_3 > \cdots$ which (after infinitely many steps) terminates in a smallest element y; (iii) *loops*: there are x and y such that x is a proper part of y and y is a proper part of x.

Classical extensional mereology (CEM) is compatible with (i) and (ii), and hence models of CEM are not always wellfounded. A non-wellfounded model in sense (ii) is the set of non-empty subsets of the real interval from 0 to 1 with parthood interpreted as the subset relation. A non-wellfounded model in sense (i), due to Tarski 1956, is the set of non-empty regular open sets of the same real interval with the standard topology. However, there are no type (iii) models of CEM, since proper parthood in CEM is irreflexive and asymmetric.

There has been recent philosophical controversy over the irreflexivity and asymmetry of proper parthood and the corresponding reflexivity and antisymmetry of parthood. The controversy mainly involves a number of thought experiments that purport to describe metaphysical or conceptual possibilities in which these order-theoretic axioms fail (e.g. Sanford 1993; Cotnoir and Bacon 2012; Donnelly 2010; Effingham and Robson 2007; Gilmore 2007; Kearns 2011; Kleinschmidt 2011; Tillman and

Fowler 2012). But the controversy over antisymmetry is also connected to an earlier debate (Thomson 1983 and 1998) over the plausibility of mereological *extensionality*, the view that wholes with the same proper parts must be identical (Cotnoir 2010; Cotnoir forthcoming-a; Gilmore 2014; Rea 2010; Varzi 2008; Varzi 2009).

These debates have led to the development of mereologies that are non-wellfounded in sense (iii). A simple such mereology may be given as follows. We take mereological overlap ∘ as primitive, with the following definitions.

Parthood $\quad x \leq y := \forall z(z \circ x \rightarrow z \circ y)$

Fusions $\quad Fu(y, X) := \forall z(z \circ y \leftrightarrow \exists x(Xx \land z \circ x))$

Here *X* may be thought of as any non-empty collection of objects. A non-wellfounded mereology, then, is characterized by the following axioms:

Axiom 1 $\quad x \circ y \leftrightarrow \exists z(z \leq x \land z \leq y)$

Axiom 2 $\quad \forall X \exists y\, Fu(y, X)$

It follows by definition that parthood is reflexive and transitive, but not that it is antisymmetric. As we can see, antisymmetry would require the philosophically contentious assumption of *extensionality* – that whenever some *z* overlaps *x* iff *z* overlaps *y* then *x* and *y* are identical.

In the absence of the usual ordering properties of parthood, we can distinguish between two non-equivalent ways of defining 'proper' parthood.

Non-identical Parthood $x \lessgtr y := x \leq y \wedge y \neq x$

Proper Parthood $x < y := x \leq y \wedge y \nleq x$

The different notions of correspond to different informal conceptions of 'proper' parts: the first being 'parthood without identity' and the second being 'parthood with remainder'. Which definition is to be preferred is a matter of some contention, since *proper parthood* is compatible with Weak Supplementation, while *non-identical parthood* is not (Cotnoir 2010). Some philosophers (e.g. Varzi 2008, p. 110 and 2009; p. 60; Simons 1987, p. 116) contend that Weak Supplementation is analytic and so metaphysically necessary, whereas others (e.g. Caplan, Tillman, and Reader 2010; Cotnoir and Bacon 2012; Donnelly 2010 and 2011; Forrest 2002; Kleinschmidt 2011; Smith 2009) do not. Debate over the metaphysical and conceptual possibility of this 'loopy' type of non-wellfoundedness continues.

A related notion is that of a *converse wellfounded* or *noetherian* relation R, which is such that every ascending R chain is finite. Might proper parthood be converse non-wellfounded? As above, *bounded infinite ascent* is compatible with CEM (Cotnoir 2013). However, a recently discussed example (e.g. Bohn 2009; Cotnoir forthcoming-b; Spencer 2012) regards *junk*: for all x there is a y such that x is a proper part of y. In CEM, the doctrine of mereological universalism rules out junk.

Further Readings

Tillman, C. and G. Fowler, 2012, **'**Propositions and Parthood: The Universe and Antisymmetry**'**, *Australasian Journal of Philosophy* 90: 525–539.

[An argument against antisymmetry based on a proposition stating the existence of the universe.]

Kearns, S., 2011, 'Can a thing be part of itself?' *American Philosophical Quarterly*, 48 (1).

[A multitude of reasons for rejecting irreflexivity of proper parthood.]

Cotnoir, A. J., 2010, **'**Anti-Symmetry and Non-Extensional Mereology**'** *The Philosophical Quarterly*, 60(239): 396–405.

[Some reasons for thinking that the non-extensionalist should reject anti-symmetry.]

Cotnoir, A. J. and A. Bacon, 2012, 'Non-Wellfounded Mereology**'** *Review of Symbolic Logic*, 5(2): 187–204.

[A systematic consideration of formal mereologies in which irreflexivity and asymmetry fail.]

Kleinschmidt, S., 2011, **'**Multilocation and Mereology**'** *Philosophical Perspectives*, 25: 253–276.

[A range of thought experiments involving multilocation that question various axioms of mereology.]

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