Non-Wellfounded Mereology

A binary relation $R$ is said to be wellfounded on a domain if and only if every non-empty subset of that domain has an $R$-minimal element; equivalently, iff every descending $R$ chain is finite. A proper parthood relation can be non-wellfounded in various ways; here are three: (i) gunk: for all $x$ there is a $y$ such that $y$ is a proper part of $x$; (ii) bounded infinite descent: there is an infinite chain of distinct proper parts $x_1 > x_2 > x_3 > \cdots$ which (after infinitely many steps) terminates in a smallest element $y$; (iii) loops: there are $x$ and $y$ such that $x$ is a proper part of $y$ and $y$ is a proper part of $x$.

Classical extensional mereology (CEM) is compatible with (i) and (ii), and hence models of CEM are not always wellfounded. A non-wellfounded model in sense (ii) is the set of non-empty subsets of the real interval from 0 to 1 with parthood interpreted as the subset relation. A non-wellfounded model in sense (i), due to Tarski 1956, is the set of non-empty regular open sets of the same real interval with the standard topology. However, there are no type (iii) models of CEM, since proper parthood in CEM is irreflexive and asymmetric.

There has been recent philosophical controversy over the irreflexivity and asymmetry of proper parthood and the corresponding reflexivity and antisymmetry of parthood. The controversy mainly involves a number of thought experiments that purport to describe metaphysical or conceptual possibilities in which these order-theoretic axioms fail (e.g. Sanford 1993; Cotnoir and Bacon 2012; Donnelly 2010; Effingham and Robson 2007; Gilmore 2007; Kearns 2011; Kleinschmidt 2011; Tillman and
Fowler 2012). But the controversy over antisymmetry is also connected to an earlier debate (Thomson 1983 and 1998) over the plausibility of mereological extensionality, the view that wholes with the same proper parts must be identical (Cotnoir 2010; Cotnoir forthcoming-a; Gilmore 2014; Rea 2010; Varzi 2008; Varzi 2009).

These debates have led to the development of mereologies that are non-wellfounded in sense (iii). A simple such mereology may be given as follows. We take mereological overlap \( \circ \) as primitive, with the following definitions.

\[
\text{Parthood} \quad x \leq y := \forall z (z \circ x \rightarrow z \circ y)
\]

\[
\text{Fusions} \quad Fu(y, X) := \forall z (z \circ y \leftrightarrow \exists x (X x \land z \circ x))
\]

Here \( X \) may be thought of as any non-empty collection of objects. A non-wellfounded mereology, then, is characterized by the following axioms:

\[
\text{Axiom 1} \quad x \circ y \leftrightarrow \exists z (z \leq x \land z \leq y)
\]

\[
\text{Axiom 2} \quad \forall X \exists y Fu(y, X)
\]

It follows by definition that parthood is reflexive and transitive, but not that it is antisymmetric. As we can see, antisymmetry would require the philosophically contentious assumption of extensionality – that whenever some \( z \) overlaps \( x \) iff \( z \) overlaps \( y \) then \( x \) and \( y \) are identical.

In the absence of the usual ordering properties of parthood, we can distinguish between two non-equivalent ways of defining ‘proper’ parthood.
Non-identical Parthood  \( x \preceq y := x \leq y \land y \neq x \)

Proper Parthood  \( x < y := x \leq y \land y \not\subseteq x \)

The different notions of correspond to different informal conceptions of ‘proper’ parts: the first being ‘parthood without identity’ and the second being ‘parthood with remainder’. Which definition is to be preferred is a matter of some contention, since proper parthood is compatible with Weak Supplementation, while non-identical parthood is not (Cotnoir 2010). Some philosophers (e.g. Varzi 2008, p. 110 and 2009; p. 60; Simons 1987, p. 116) contend that Weak Supplementation is analytic and so metaphysically necessary, whereas others (e.g. Caplan, Tillman, and Reader 2010; Cotnoir and Bacon 2012; Donnelly 2010 and 2011; Forrest 2002; Kleinschmidt 2011; Smith 2009) do not. Debate over the metaphysical and conceptual possibility of this ‘loopy’ type of non-wellfoundedness continues.

A related notion is that of a converse wellfounded or noetherian relation \( R \), which is such that every ascending \( R \) chain is finite. Might proper parthood be converse non-wellfounded? As above, bounded infinite ascent is compatible with CEM (Cotnoir 2013). However, a recently discussed example (e.g. Bohn 2009; Cotnoir forthcoming-b; Spencer 2012) regards junk: for all \( x \) there is a \( y \) such that \( x \) is a proper part of \( y \). In CEM, the doctrine of mereological universalism rules out junk.
Further Readings


[An argument against antisymmetry based on a proposition stating the existence of the universe.]

Kearns, S., 2011, ‘Can a thing be part of itself?’ *American Philosophical Quarterly*, 48 (1).

[A multitude of reasons for rejecting irreflexivity of proper parthood.]


[Some reasons for thinking that the non-extensionalist should reject anti-symmetry.]


[A systematic consideration of formal mereologies in which irreflexivity and asymmetry fail.]


[A range of thought experiments involving multilocation that question various axioms of mereology.]
Bibliographic Guide

Bohn 2009; Caplan, Tillman, Reeder 2010; Cotnoir 2010; Cotnoir forthcoming-a;
Cotnoir forthcoming-b; Cotnoir and Bacon 2012; Donnelly 2010; Donnelly 2011;
Eagle 2010; Effingham 2010; Effingham and Robson 2007; Forrest 2002; Gilmore
2007; Gilmore 2010; Gilmore 2014a; Gilmore 2014b; Kearns 2011; Kleinschmidt
2011; Rea 2010; Rossberg 2009; Sanford 1993; Simons 1987; Smith 2009; Spencer
2012; Tarski 1956; Thomson 1983; Thomson 1998; Tillman and Fowler 2012;
Uzquiano 2011; van Inwagen 1993; Varzi 2006a; Varzi 2008; Varzi 2009.

References

Bohn, E., 2009, ‘An Argument Against the Necessity of Unrestricted Composition’,


Cotnoir, A.J. 2010, ‘Anti-Symmetry and Non-Extensional Mereology’ The
Philosophical Quarterly, 60(239): 396–405.

Cotnoir, A. J., forthcoming-b, ‘Universalism and Junk’ Australasian Journal of
Philosophy.


Cotnoir, A. J. and A. Bacon, 2012, ‘Non-Wellfounded Mereology’ Review of


Donnelly, M., 2011, ‘Using Mereological Principles to Support Metaphysics’ The


Kearns, S., ‘Can a thing be part of itself?’ American Philosophical Quarterly, 48 (1).


