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UNIVERSALISM AND JUNK

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Those who accept the necessity of mereological universalism face what has come to be known as the 'junk argument' due to Bohn [2009], which proceeds from (i) the incompatibility of junk with universalism and (ii) the possibility of junk, to conclude that mereological universalism isn't metaphysically necessary. Most attention has focused on (ii); however, recent authors have cast doubt on (i). This paper undertakes a defence of premise (i) against three main objections. The first is a new objection to the effect that Bohn's defence of that premise presupposes far too much. I show that one can defend premise (i) from a much weaker set of assumptions. The second objection, due to Contessa [2012], is that those who accept unrestricted composition should only accept the existence of binary sums (which are compatible with junk) rather than infinitary fusions. I argue that this conception of unrestricted composition is problematic: it is in conflict with an intuitive remainder principle. The final objection is due to Spencer [2012]. His view is that there is no absolutely unrestricted plural universal quantifier; so any statement of the unrestricted fusion axiom will simply not rule out the existence of junky worlds. I argue that the failure of unrestricted quantification will not be enough by itself to establish the existence of junk. Furthermore, it is not clear whether this view counts as a form of mereological universalism. As a result, I suggest that if one wants to reject the junk argument, premise (ii) is the only viable option.

Keywords: junk, mereology, universalism, extensionality, unrestricted quantification

Universalism is the view that composition is unrestricted: that is, for any things whatsoever, there is a whole composed of them. A number of philosophers maintain that universalism is true, indeed that it is metaphysically necessary.[1] But universalism is one of the most controversial doctrines of classical mereology. A standard objection is that universalism commits us to too many composite objects, including gerrymandered objects composed of say, the Eiffel Tower and the tip of your nose. These are not objects recognised in our common-sense conception of the world.

A more recent objection, however, contends that universalism commits us to *too few* composite objects. It requires parthood chains to 'top out' at some largest object. But must there be such a largest object? Why couldn't it be that, for every composed whole, there is another that contains it? An argument has been put forward that undermines the view that universalism is metaphysically necessary; the argument appeals to the possibility of *junky*

[1] Lewis [1991], Rea [1998], Varzi [2000], Sider [2007], van Cleve [2008], and many more.

worlds. Junky worlds (introduced by Schaffer [2010]) are worlds where everything in that world is a proper part of something.[2] The junk argument, due to Bohn [2009a, 2009b, 2010], proceeds as follows:

(i) If universalism is necessarily true, then there can be no junky worlds.

(ii) Junky worlds are metaphysically possible.

(iii) Therefore, universalism is either metaphysically contingent or necessarily false.

Much attention has been devoted to premise (ii). Bohn [2009a: 29] contends that junky worlds satisfy three criteria for metaphysical possibility: *conceivability*, *advocacy*, and *consistency*:

> But if we can conceive of junky worlds, and several prominent philosophers have taken the idea seriously, and there are no logical contradictions lurking, then we are hard pressed to deny the mere possibility of the world being junky.

The case for conceivability involves thought experiments: one might imagine, for example, that a universe just like ours is contained as a particle in some much larger replica universe, which is itself merely a particle in another replica universe, and so on. Such a world would be junky. Leibniz [1765] and Whitehead [1919] both held that the actual world is junky.[3] Moreover, there are clearly formal models of junky mereologies, as we shall see below, and so the idea is logically consistent.

Schaffer [2010: 65] by contrast, thinks that junk is inconceivable: conceiving of a world as a *world* requires it to be viewed as a totality, a whole. And any world that contained junk would be an entity that isn't a proper part of anything, *per impossible*. Watson [2010] also takes up the case against conceivability by objecting to the specific thought experiments proposed. For the record, I do not think Bohn's three criteria are sufficient for (or even provide good evidence for) metaphysical possibility; but the matter is too complicated for discussion here.

In any case, I do not wish to challenge the case for premise (ii). More recently, premise (i) has come under fire. Contessa [2012] and Spencer [2012] have each provided independent reasons that 'universalists' should dispatch the junk argument by rejecting premise (i). In this paper, I undertake a defence of premise (i) against a variety of objections.

In §1 I put forward a new objection to Bohn's defence of premise (i)—that it is not available to proponents of non-classical mereologies. In §§2–3 I generalize the defence of premise (i) to hold in virtually any merelogy, and based on virtually any definition of mereological fusion. I address *unsupplemented* mereologies in §2 which explicitly reject Bohn's key assumption. In §3 I address *mutual parts* mereologists, who might accept counterfeit junk

[2] This is the *converse* of the notion of a *gunky* world in which everything in that world has something as a proper part.

[3] See Bohn [2009a: 28] and Simons [1987: 83] for relevant quotations.

that satisfies the definition in letter but not in spirit. A slight revision in the definition of junk re-establishes premise (i) (or a mild variant thereof). In §4 I defend premise (i) against Contessa's objection; and in §5 I defend it against Spencer's objection.

These defences of premise (i) show that it is particularly resilient and that it can be based on extremely minimal assumptions. The upshot, then, is that anyone who wants to reject the junk argument must do so by rejecting premise (ii) instead.

1. Extensionality and Junk

In his argument for premise (i), Bohn relies on the following mereological assumption, where $<$ is proper parthood, $\leq$ is parthood ('$x \leq y$' is defined as '$x < y \vee x = y$'), and $\circ$ is mereological overlap ('$x \circ y$' is defined as '$\exists z(z \leq x \wedge z \leq y)$').

> **Weak Supplementation**. $x < y \rightarrow \exists z(z < y \wedge \neg z \circ x)$

Weak supplementation says that if something has a proper part, it must have another part disjoint from the first. Recall, a junky world is one in which everything is a proper part of something:

> **Junk**. $\forall x \exists y(x < y)$

Bohn's argument for premise (i) is as follows [Bohn 2010: 297n.1]:

> [A]ssume universalism is true in all possible worlds and that some possible world w is junky. Then universalism is true in w. Consider the plurality of everything there is in w, call it aa. By universal instantiation, $\exists y(aa \text{ compose } y)$; by existential instantiation, aa compose U. By definition [of junk], it is true in w that $\forall x \exists y(x < y)$. By universal instantiation, $\exists y(U < y)$. But U is composed of everything in w, and hence, by definition [of composition], everything in w is a part of it, including itself. But then nothing can have U as a proper part, because if so, by [weak] supplementation, there would be something that shares no part with U, and hence is not a part of U, which contradicts that everything is a part of U. So, if some possible world is junky, then universalism is not necessarily true. Q.E.D.

Now, the principle used in the argument is admittedly fairly plausible. Indeed, Bohn [2009a: 29] regards the weak supplementation principle as *analytic* of proper parthood:

> I assume at least some mereological principles without existential import are analytically true, and in virtue of that are necessarily true too. ... *Minimal Extensional Mereology* is a minimum of mereological necessary truths. This system includes (among other things) the asymmetry and transitivity of proper parthood, as well as a weak supplementation principle.

Plausible, but far from uncontroversial. I do not think weak supplementation is analytic, but I appreciate that more needs to be said.[4] In fact, as we

[4]For some considerations in this direction, see Cotnoir [2013].

will see below, there is a growing number of mereologists who reject weak supplementation. Some reject weak supplementation on its own terms, finding it independently objectionable.[5] Others reject weak supplementation because they reject[6] either *irreflexivity* or *asymmetry*,[7] which (in combination with transitivity) can be derived from weak supplementation.

Transitivity. $(x < y \wedge y < z) \rightarrow x < z$

Irreflexivity. $x \nless x$

Asymmetry. $x < y \rightarrow y \nless x$

Many of the reasons given for rejecting irreflexivity and asymmetry concern exotic counter-examples, of which one might be suspicious. But there is a more philosophically robust reason to generalize the junk argument: it turns out that these 'minimal' principles are sufficient, when combined with unrestricted composition, to yield perhaps the most controversial mereological principle of all—*extensionality*, which says that composite objects with the same proper parts are identical.[8] Extensionality forces us to identify objects, like perhaps the statue and the clay it is made from, which have all their parts in common. Given the *indiscernibility of identicals*, this entails that any two composed objects with all the same *proper parts* have all the same *properties*. And whether, for example, the statue and the clay are indiscernible is a matter of serious contention.

The argument that universalists who accept weak supplementation and transitivity are committed to extensionality is due to Varzi [2009]. Informally, imagine you had a counter-example to extensionality: suppose a statue s and its clay c are distinct objects with the same proper parts. By asymmetry, $s \nless c$, and $c \nless s$.[9] By unrestricted fusion, there must be a sum $s+c$. (Note: $s+c \neq s$, since $c \leq s+c$ but $c \nleq s$; likewise, $s+c \neq c$, since $s \leq s+c$ but $s \nleq c$.) Now, $s < s+c$; however, there is no proper part of $s+c$ that is disjoint from s, as any proper part of c is also a proper part of s by supposition. This violates weak supplementation. Hence, there can be no such counter-example to extensionality.

There are two ways to avoid Varzi's argument. As a result, there are two main approaches to mereology without extensionality: the *unsupplemented* view,[10] and the *mutual parts* view.[11] Both of these approaches are compatible with unrestricted composition; indeed, many proponents of these views

[5]Proponents include: Donnelly [2011], Forrest [2002], and Smith [2009]. Also see Caplan, Tillman, and Reeder [2010] who express sympathy for the view.
[6]Proponents of dropping irreflexivity include Kearns [2011], Kleinschmidt [2011], and Cotnoir and Bacon [2012].
[7]Proponents of dropping asymmetry (or antisymmetry for parthood) for extensionality-independent reasons include Sanford [1993], Kleinschmidt [2011], Cotnoir and Bacon [2012], and Tillman and Fowler [2012].
[8]See Varzi [2008] for a flavour of the debate.
[9]For suppose $c < s$. Asymmetry implies $s \nless c$. Hence, s and c are not collectively a counter-example to extensionality. *Mutatis mutandis* for the supposition that $s < c$.
[10]For extensionality-based reasons to drop weak supplementation, see Simons [1987] and Gilmore [forthcoming].
[11]For reasons to drop asymmetry to secure anti-extensionalism, see Thomson [1983, 1998] and Cotnoir [2010].

themselves accept it. Since each explicitly rejects Bohn's assumption of weak supplementation, Bohn's defence of premise (i) is not available to them. But there are defences of premise (i) that non-classical mereologists can (and should) accept. I provide such defences in the next two sections for virtually any mereology with virtually any conception of unrestricted fusion.

2. Weak Supplementation and Junk

The first main non-extensionalist option is the *unsupplemented* approach, which drops the weak supplementation axiom above but adds asymmetry. The unsupplemented approach has access to a well-developed mereology that includes unrestricted fusion. Here is a candidate list of axioms.

Asymmetry. $x < y \rightarrow y \nless x$

Transitivity. $(x < y \wedge y < z) \rightarrow x < z$

Unrestricted Composition. $\forall\, xx\, \exists\, y\, F(y, xx)$.

Fusions appearing in the unrestricted composition axiom are defined as follows:

Fusion. $F(t, xx)$ iff $xx \leq t \wedge \forall\, y(y \leq t \rightarrow y \circ xx)$(I write '$xx \leq t$' to mean that *every* x that is among xx is part of t, and '$y \circ xx$' to mean that y overlaps *some* x among the xx.)

On this view, proper parthood is a strict partial order; the major change is that weak supplementation is explicitly rejected. Some unsupplemented mereologists simply drop the axiom and leave it at that.[12] Others (primarily Gilmore [forthcoming]) have suggested a replacement axiom to capture the intuition behind weak supplementation without risk of extensionality.[13] But, as Bohn's defence of premise (i) explicitly appeals to weak supplementation, this defence is not available to unsupplemented anti-extensionalists, many of whom do accept unrestricted composition.

There is, however, a straightforward argument from transitivity and asymmetry (in place of weak supplementation) to the conclusion that unrestricted composition entails the negation of junk. Let xx range over absolutely everything in a world. Since composition is unrestricted, there is some u such that $F(u, xx)$. As such, we have $xx \leq u$ (by the definition of fusion). Now, to satisfy the definition of junk, suppose $u < \hat{u}$. By asymmetry, $\hat{u} \nless u$, and thus $\hat{u} \nleq u$. But that contradicts the supposition that xx contains absolutely everything. So, there is no such $\hat{u}$, and hence the world is not junky.

[12]See the proponents listed in footnote 4 for the case for conceivability. It is also clear that the approach is consistent (the axioms are satisfied in any complete lattice; for quasi-supplementation, consider complete join semi-lattices with at least two distinct minimal elements). So, by Bohn's own criteria, the approach would appear to be metaphysically possible. It should at least be regarded as an available position in logical space.
[13]The relevant axiom is this:
Quasi-Supplementation. $x < y \rightarrow \exists\, w\, \exists\, z(z \leq y \wedge w \leq y \wedge \neg z \circ w)$
However, this addition is strictly optional; there is a defence of premise (i) that goes through without it.

None of this depends on any supplementation whatsoever. As a result, a version of the junk argument applies to unsupplemented anti-extensionalism.

Now, there is some debate over the correct notion of fusion for anti-extensionalists. In fact, some have held that the definition above is inadequate.[14] However, it is worth noting that the above argument only relies on the fact that the fusion of xx has each of the xx as a part. Surely, any adequate notion of mereological fusion must be such that it is an upper bound in this sense of the things it fuses (even if not a least upper bound). That is, one cannot think t is the fusion of the *cats* if some cat fails to be part of t. But then the above argument applies to unrestricted fusion of *any* sort.

3. Asymmetry and Junk

The second option for anti-extensionalists is the *mutual parts* view, which rejects asymmetry for proper parthood. This allows for two distinct objects, such as the statue and the clay, to be parts of each other. As it turns out, this is sufficient for avoiding extensionality principles of all sorts.[15] This approach has a growing number of advocates.[16]

The following axioms give a precise characterisation of the mutual parts approach.

Transitivity. $(x < y \wedge y < z) \to x < z$

Remainder. $y \not\leq x \to \exists z \forall w(w \leq z \leftrightarrow (w \leq y \wedge \neg w \circ x))$

Unrestricted Composition. $\forall xx \exists y\, F(y, xx)$.[17]

As above, weak supplementation entails asymmetry, and so the mutual parts view must reject weak supplementation. However, the mutual parts view has access to a variety of very strong supplementation principles. The strongest of these is the remainder principle, which says that if y fails to be part of x, then there is an object composed of all and only the non-x-overlapping parts of y—x's complement in y.[18] It is commonly thought that strong supplementation principles like the remainder principle entail weak supplementation; but the proof of this fact relies on asymmetry. So the mutual parts view can accept such strong principles without falling into extensionality.[19] This principle is again optional; however, we will return to this principle below (§5). But even in the presence of unrestricted fusion, this approach does not fall

[14]See for example the arguments in Varzi [2009] and Cotnoir [2014].
[15]See Cotnoir [2010] for an examination of the relation between antisymmetry and extensionality.
[16]Proponents of dropping antisymmetry (listed in footnote 5) have access to a well-developed mereology (which includes unrestricted fusion) with a class of well-defined models (see Cotnoir and Bacon [2012] for details). Thus, the approach is consistent. It appears the approach satisfies Bohn's own criteria for possibility. At least very least, then, dropping asymmetry should be worthy of serious consideration.
[17]A variant weaker definition of fusion is typically used here:

$$\mathrm{F}(t, xx) \text{ iff } xx \leq t \wedge \forall y(xx \leq y \to t \leq y)$$

However, the weakness of this definition is made up for by the corresponding strength of the remainder principle. (See Cotnoir and Bacon [2012: §3.2] for more details.)
[18]Simons [1987: 89] is responsible for the name. Varzi [2006] and Hovda [2009] call it *Complementation*.
[19]For more details, see Cotnoir [2010].

prey to the junk argument. There are models of this mereology that also satisfy Bohn's definition of junk, stating that everything is a proper part.

One such example is given by considering an a and b that are mutual parts such that everything else in the world is part of both. Such a world is consistent with unrestricted fusion, since a and b both count as fusions of the other. As is evident, a and b are distinct, so in fact there are *two* universal objects. Now, is this a junky world? In a sense, yes, since $a < b$, a has everything as a proper part but is itself a proper part of something (namely, of b). Likewise, since $b < a$, b is has everything as a part but is itself a proper part of something (namely, a). So, according to the letter of the current definition of junk, premise (i) is strictly speaking false.

However, the kind of 'junk' displayed above is not the sort of junk one has in mind when levelling the argument against opponents of unrestricted composition. Proponents of junk are arguing for the existence of certain metaphysical structures; they are not (at least not primarily) arguing for the existence of worlds that satisfy a particular formal definition. As is often the case when one moves between formal systems, sometimes definitions that previously captured the intended structures fail to do so in a certain setting.[20] Which definition captures the intended junky structures in the non-asymmetric setting? I contend that it is Junk*:

Junk*. $\forall x \exists y (x < y \wedge y \nleq x)$

So, a junky* world is a world in which everything is a proper part of something that is not also one of its own parts. In other words, junk* guarantees that everything is a *non-mutual* proper part (i.e. not a proper part of one of its proper parts, nor a proper part of itself). This sort of junk is incompatible with the above mereology. If u is the fusion of everything xx, then $xx \leq u$. Now, to satisfy the definition of junk*, suppose $u < \hat{u}$ and $\hat{u} \nleq u$. Contradiction.

This shows that premise (i) of the junk argument (on the intended reading, anyway) does not rely on any assumption of asymmetry.

4. Binary Sums and Junk

I have been using 'Universalism' in the usual way to mean that, whenever you have some things, there is a fusion of them. Contessa [2012] distinguishes between *strong* and *weak* versions of universalism.

Strong Universalism. For any plurality xx, there is an object that is their fusion.[21]

Weak Universalism. For any pair of objects x and y, there is an object that is their fusion.

[20] Compare the discussion regarding the correct definition of fusion (Varzi [2009] and Cotnoir [2014]) or the correct definition of proper parthood (Cotnoir [2010] and Rea [2010]) in non-extensional settings.

[21] It should be noted that Bohn states the thesis as follows: 'That is, assume that necessarily, any collection of things composes something' [2009a: 27]. Likewise, Contessa states the strong thesis as follows: 'for any collection of objects, there is always an object that is their mereological sum' [2012: 456]. But, given the connection between the notions of *collection* and *set*, we should be clear that it is only *non-empty* collections that have fusions; virtually no universalist thinks that the empty set has a fusion.

Contessa contends that those who accept universalism should accept only the weak version and reject the strong. So, for him, mereological universalists ought to accept only the existence of *binary sums* and to reject *infinitary fusions*.

To be clear about the context of this response, it is important to note that the junk argument was clearly originally intended as an objection to the strong version of the thesis. In fact, *no one* who calls herself a universalist accepts the weak thesis but not the strong. So, a natural reply springs to mind: The weak thesis isn't really a form of mereological universalism at all, is it? Of course it is compatible with universalism, as anyone who accepts the strong thesis would also accept the weak. But insofar as one accepts the weak thesis while rejecting the strong thesis, that view rejects the existence of certain kinds of fusions—it accepts that composition holds only in some cases (the finitary ones) and rejects that it holds under all cases (which would include the infinitary ones). For what it is worth, when Bohn [2009a] originally gives the junk argument he considers such a view, calling it explicitly a 'restricted view of composition'.

Contessa considers this thought and writes [2012: 456],

> However, according to weak mereological universalism, that should not be understood as a restriction on composition, for, according to weak mereological universalism, mereological composition would be restricted only if there were pairs of objects that, under some circumstances, did not have a mereological sum and, since junky worlds do not feature any such pair of objects, they do not constitute a counterexample to weak mereological universalism.

It is not clear from the passage precisely what Contessa has in mind here as a dialectically effective reply. The statement that 'composition would be restricted only if there were pairs of objects that . . . did not have a mereological sum' suggests the idea that composition is fundamentally a *binary* operation, not properly thought of as an operation on *collections*.[22] A related thought is that composition is *essentially* binary—the composition operation *by its very nature* takes two objects as inputs, and it outputs some further object. On that understanding of composition, weak universalists do think that composition is unrestricted. Infinitary cases of compositions are no counter-example to universalism because infinitary collections of objects aren't even '*composition-apt*'. Composition doesn't fail to obtain; it fails to apply.

There are a number of reasons one might find this strict version of weak universalism unsatisfying. Let me first consider a bad reason for rejecting it, before going on to present my own preferred reasons. Bohn [2009a] objects to this view by arguing that it is incompatible with the existence of *gunky* worlds—worlds in which everything has a proper part. He claims this [2009a: 30, emphasis mine]:

> That means that if the world is finite, it contains a universal object U, while if the world is infinite with mereological atoms, it is junky. The problem with this principle is that it is incompatible with the world being gunky. A gunky world is such that everything in it has a proper part. If everything in the world has a

[22]Contessa has confirmed in correspondence that this is in fact the view he had in mind.

> proper part, *then every fusion in the world is infinitely divisible, which means that every fusion in the world is of infinite cardinality, which again means that no fusion in the world is of finite cardinality.* Since our principle implies that necessarily all fusions are of finite cardinality, it must therefore be incompatible with the possibility of gunk.

There are two things to say about this argument. First, it is true that a gunky object a would be a case of infinitary fusion. Let aa be the plurality of all the proper parts of the gunky object a. Then aa is an infinite collection such that $F(a, aa)$. Must the weak universalist reject the existence of such an object? I think the answer is 'no'. A weak universalist might accept the existence of a so long as there exist some proper parts a_1 and a_2, such that their sum just is a. The problem with Bohn's argument is with the emphasised portion of the passage. Just because the world is infinite (and, indeed, every object is infinitely divisible), it does not follow that every fusion must be infinite. While gunky objects have infinitely *descending* proper parthood chains, the key issue is that of whether there are infinitely *ascending* chains leading up to the gunky object. The weak universalist might accept the possible existence of objects with infinitely many proper parts, so long as we can always fuse our way up to such an object in finitely-many steps.[23] So Bohn's objection to the weak thesis fails.

However, there is another argument against this particular essentialist version of weak universalism: namely, that it is in tension with an important mereological principle—the remainder principle above. Recall:

Remainder. $y \not\leq x \rightarrow \exists z \forall w (w \leq z \leftrightarrow (w \leq y \wedge \neg w \circ x))$

The principle entails that where y has a proper part x, there must be a 'remainder' or 'complement' of x in y. The remainder z has as parts all and only the non-x-overlapping parts of y. But the non-x-overlapping parts of y might well be infinite. And it turns out (by the definition of fusion) that z is the fusion of precisely these parts. To see this, fix some particular x and y, such that z is this remainder. Let ww be the plurality of objects satisfying the open sentence '$(w \leq y \wedge \neg w \circ x)$'. To establish that $F(z, ww)$ we need to show that $ww \leq z$ and $\forall w (w \leq z \rightarrow w \circ ww)$. The former we have by the right-to-left direction of the consequent of the remainder principle. For the latter, suppose $w \leq z$. Then, by the left-to-right direction of that consequent, we have it that w is such that $(w \leq y \wedge \neg w \circ x)$, which means by definition that it is identical to one of the ww, and hence overlaps one of them. Thus, the remainder principle entails that z is the fusion of ww.

The weak universalist cannot simply stipulate the remainder principle as an axiom without running the risk of thereby accepting infinitary fusions. Now, it may well be true in some cases (as in the case of gunk without infinite ascending $<$-chains above) that z can be constructed pair-wise from other

[23]Consider a non-junky but gunky model M; in particular, suppose M has no infinite *ascending* $<-$chains, but has infinite *descending* $<-$chains. It follows that, for every non-empty subset A of M, there is some finite subset F of A such that the least upper bound of F is the least upper bound of A. (For a proof of this fact, see Davey and Priestly [2002: 52], Theorem 2.41(i).) Since we can take all the members of F and fuse them pair-wise, it follows that every non-empty subset A has a fusion.

objects. But the weak universalist who rejects purely infinite fusions needs it to be the case that *every* remainder can be constructed in this way from finite sums. However, there is no general guarantee that this is the case; there is nothing in the axioms of her mereology that rules out structures for which remainders cannot be so constructed. Problematic cases involve worlds with infinite ascending proper parthood chains up to some composed whole—call them *non-Noetherian* worlds, using the standard mathematical term named after Emmy Noether.[24] Such worlds come in a number of varieties: worlds which are both junky and gunky (i.e. what Bohn [2009b] calls 'hunky') can be of this type, as are worlds where the proper parthood chain is *dense*, i.e. for every x and z such that $x < z$, there is some y such that $x < y < z$.

For a concrete example of the latter, suppose we have an object q that is the interval of rational numbers between 0 and 1 (inclusive). Now focus on the 'right half' r of q, namely the interval of numbers between and including $\frac{1}{2}$ and 1. Since $[0,1] \not\leq [\frac{1}{2},1]$, by the remainder principle there must be some object l (the 'left half') composed of all and only those parts of $[0,1]$ that do not overlap $[\frac{1}{2},1]$. So it must contain every number starting from 0 and up to (but *not* including) $\frac{1}{2}$, namely the interval $[0,\frac{1}{2})$. Notice that, because of the density of the rationals, there is no last number in this interval. So, if we are trying to generate it by composing it pairwise with other rational numbers, we will fail. For every object we sum up, there will always be infinitely many more. We would have a finite sum up to r if there were certain open intervals around, but these are precisely what we don't know to exist. And it's not clear there is any way of ensuring they exist, without also committing oneself to infinitary fusions.

There are a few avenues of reply: one might (a) reject the remainder principle, (b) reject the existence of non-Noetherian worlds (i.e. reject the possibility of hunky worlds or dense worlds), or (c) accept both and allow some cases of infinitary composition. Against (a), I would urge that the remainder principle is an important part of classical mereology; it is what makes mereology a *Boolean* algebra after all. The resulting mereology would be highly non-classical, in ways unrelated to the existence of junk.[25] Against (b), it would be very odd (even if not incoherent) if in defending the possibility of infinite ascent of one type—namely, junky worlds—one was forced into denying the possibility of infinite ascent of another type—namely, hunky or dense worlds. Why should it be the case that any world with proper parthood chains that continue infinitely *upward* cannot have any proper parthood chains that also continue infinitely *up to a whole*? Without some principled metaphysical reason, it is a dialectically weak position to find oneself defending, at any rate.[26] Moreover, given that many philosophers think that regions of space are *continuous* and hence dense, option (b) rules out a very common view about the structure of the actual world.

[24]Let M be a model of a non-Noetherian world. Since such a world has infinite ascending $<$-chains, we can no longer prove that every non-empty subset A of M has a corresponding finite subset F such that the least upper bound of F is the least upper bound of A. This may be (in the case of *junk*) because A has no upper bound at all in M, or it may be that even though A has a least upper bound, the only way to fuse to A is by taking the fusion of all of its members.

[25]For more discussion, see Hovda [2009], especially n.25.

[26]See also the arguments in Bohn [2009b: §2].

So I think the best option is (c). In fact, this is the view of the only mereologists (that I know of) who accept the weak thesis but not the strong thesis. For example, Bostock's [1979: ch. 2, §4] mereology was devised for the foundations of arithmetic in the rational numbers; as such, his mereology is quite robust. It allows for *gunk* and *junk* and *dense* proper parthood chains, and also satisfies the remainder principle. The main departure from classical mereology that Bostock proposes is a variant fusion axiom, which (in our notation) is:

Moderately Unrestricted Composition. $\forall xx(\exists y(xx \leq y) \rightarrow \exists z F(z, xx))$[27]

What this principle says is that if xx have an *upper bound*, then they must have a fusion. Of course, this entails that if some infinite collection is bounded from above, then it must have a fusion. This is compatible with the existence of junk, though, since not all infinite collections are required to be bounded from above. However, it is not compatible with the strict essentialist version of the weak universalist thesis, since it entails that some infinite collections have fusions.[28]

Moreover, 'moderately unrestricted' composition makes it plain that the view is not really a version of mereological universalism at all. The fusion axiom is *plainly* restricted by the antecedent—only collections that are bounded have fusions. And so I view this response as changing the terms of the debate. It may well be the most plausible version of a junky mereology, but it hardly constitutes a counter-instance to premise (i).

5. Unrestricted Quantification and Junk

Spencer [2012] has recently argued against premise (i) due to the fact that its defence relies on unrestricted plural quantification.[29] That is, the incompatibility of junk and unrestricted composition requires the supposition that there are some things xx—call them 'all things'—such that everything is amongst them [Spencer 2012: 71]:

> [T]his argument clearly relies on the premise that there are some things that all things are amongst. If there are no such things, then the argument is unsound. In fact, unrestricted composition and [the denial of 'all things'] together entail junk.

While this approach is the best attempt at avoiding premise (i), it is less than clear that the mere denial of 'all things' will deliver the compossibility of unrestricted composition and junky worlds in the required sense.

[27]The definition of fusion is different from above. For Bostock [1979: 118], fusion is defined thus: $F(t, xx)$ iff $\forall y(y \circ t \leftrightarrow y \circ xx)$, although in the presence of the remainder principle the two definitions are equivalent.

[28]For a sophisticated discussion of related issues, see Linnebo, Hellman, and Shapiro's [unpublished] mereological conception of the Aristotelian continuum. They openly admit that their view is not a universalist view (as such, none of the arguments in this paper are aimed at them), but they do wish to avoid actual infinities insofar as it is possible. The main difficulties involve securing the existence of differences (i.e. remainders), bisections, and biextensions, as well as proving the Archimedean principle. They propose a number of different fusion axioms to attempt to deal with the difficulties.

[29]This is related to Bohn's [2010: 297] explicit claim that '*Universalism* [is defined as] $\forall xx \exists y(xx \text{ compose } y)$, where the quantifiers are unrestricted and 'xx' is a plural variable taking one or more things as its value.'

Before delving into Spencer's arguments, we need to be clear on what it is that Spencer is rejecting. We need to distinguish between (a) absolutely unrestricted *singular* quantification and absolutely unrestricted *plural* quantification, and (b) two different readings of what it is for a plural quantifier to be absolutely unrestricted.

For (a), there is an important difference between absolutely unrestricted *singular* quantification and absolutely unrestricted *plural* quantification. The former is used in the definition of junk, while the latter is used in the fusion axiom. Spencer is clear that he is *not* denying that the singular universal quantifier is absolutely unrestricted. But one might deny this, and we should consider whether doing so might affect premise (i) of the junk argument. So, let us suppose that our singular universal quantifier fails to be unrestricted. Notice that junky worlds are themselves defined using the universal quantifier. So, if our universal quantifier fails to quantify over *everything*, it may well be that those objects not in its scope will fail to have a fusion. However, even if so, it will not follow that the world satisfies the definition of being junky, since it will not follow that $\forall x \exists y(x < y)$ is true. Why? Because those objects that fall outside the scope of the unrestricted fusion axiom will also fall outside the scope of the quantifier in the statement of junk. All that could be maintained in this case is that there might be worlds that are *inexpressibly* junky. But the metaphysical possibility of inexpressibly junky worlds has not been argued for. It needs an independent defence—a defence that goes well beyond the defence of premise (ii).

Returning to (b), we need to clarify what is at issue in claiming that a plural quantifier is unrestricted. There are two possible ways of thinking about this. First, we might think a plural quantifier is unrestricted if and only if it ranges over absolutely everything: that is, for every thing x, there is some plurality xx that x is among. On this reading, Spencer is definitely not arguing against unrestricted plural quantification.

However, there is a second way of thinking about what it is for a plural quantifier to be unrestricted: it is unrestricted if and only if it ranges over absolutely *all* pluralities, where pluralities are generated from all the instances of plural comprehension. On this reading, Spencer *is* arguing against unrestricted plural quantification, since he denies that there is a plurality of all things. To be precise, Spencer expresses the denial of 'all things' thus: $\neg \exists xx \forall yy(yy \text{ are among } xx)$. He has to deny plural comprehension since, were zz to contain all and only the objects satisfying the predicate $\exists x(z = x)$, it would consist of 'all things'.

Now we are in a position to consider Spencer's argument against premise (i).[30] His argument [2012: 72] purports to show that a world in which universalism is true and without 'all things' must be a junky world:

> Consider some arbitrary thing 'Chunk' and *call all of its parts 'Bits'. Now if [the existence of 'all things'] is false, there are some things that Chunk's parts are properly amongst*. Call those things 'Bits+Pieces'. Now, there are some things amongst Bits+Pieces that are not parts of Chunk ... [namely,] Pieces.

[30] I focus only on his second (more general) argument. What I say about the second applies equally to the first.

> According to unrestricted composition there is something composed of Chunk and Pieces, namely Big Chunk. Moreover since Big Chunk has parts that are not parts of Chunk, namely Pieces, it follows that Chunk is a proper part of Big Chunk. So, there is something that Chunk is a proper part of. But, since Chunk was arbitrarily chosen, it follows that for anything whatsoever, there is something that it is a proper part of. So Junk is true. Again, unrestricted composition and [the denial of 'all things'] entails Junk.

The argument has a number of moving parts; however, we do not have to read very far to find some potentially objectionable assumptions being made. The emphasised passage contains two such assumptions. The first is that, for any arbitrary thing, there is a plurality of all its parts (in this case, 'Bits'). The second assumption claims that, if there is no plurality of all things, then there must be a plurality that properly contains all of the parts of a given thing (in this case, 'Bits+Pieces'). But why think that either assumption is true? I will consider each in turn.

Regarding the first assumption, here is a possible scenario. Suppose we can quantify over every macro-level object but we cannot quantify over all their parts. It may well be that the world gets 'too numerous' as we move to more and more finely grained levels of reality. We may be simply unable to quantify over all things because we cannot quantify over all the *parts* of things.

If we can quantify over objects at some macro-level specification, by unrestricted fusion (of any sort), we will have it that every parthood chain has an upper bound. Hence, we know that the world has at least one $<$-maximal element (an element with nothing containing it as a proper part).[31] But a maximal element entails the *failure* of junk. For a concrete example, imagine the domain of all subsets of the $[0, 1]$ interval of the real numbers. Imagine that pluralities can take at most countably many subsets of that interval. Let *yy* be the plurality containing two intervals: $[0, \frac{1}{2}]$ and $[\frac{1}{2}, 1]$. The fusion of *yy* is the whole unit, even though there is no plurality of all subsets of the unit.

It is worth noting that Spencer recognises that his argument relies on this assumption, and suggests that its failure would be an 'interesting metaphysical result' [2012: 72]. Still, such a possibility shows that a universalist who rejects unrestricted plural quantification is not committed to junk. What is required, then, is an argument for thinking that our quantifier fails to quantify over absolutely everything going *upwards* along parthood chains, rather than the failure occurring as we go *downwards*. What we need, in short, is an argument in favour of premise (ii) of the junk argument.

There is a second assumption in Spencer's argument: namely, that the denial of all things entails that, for any plurality, there must be another plurality that properly contains it. Presumably, what Spencer is thinking is this:

[31]This requires an application of Zorn's Lemma, which is equivalent to the Axiom of Choice:

> **Zorn's Lemma**. Let M be any non-empty partially ordered set. If every chain has an upper bound, then M has a maximal element.

Suppose that M here is a model of the mereological structure of the world, and that the partial order on M is just the parthood relation. A *chain* is defined to be a totally ordered subset of M; so, a subset such that for every x and y, either $x \leq y$ or $y \leq x$.

Suppose *aa* are some things and that there is no plurality that properly contains *aa*. Then *aa* would be a plurality of everything, which we are supposing does not exist. But this does not follow, as *aa* might just be the largest plurality over which we can quantify. As such, the assumption is entirely unwarranted in the context.

To make the matter concrete, let us give a metaphysical example. Suppose there are too many mereological simples over which to quantify. In that case, we couldn't quantify over all things because we couldn't quantify over all the *smallest* things. If there are too many simples to form a plurality, those simples might not be parts of anything, due to the corresponding expressive weakness of the unrestricted composition axiom. Still, each individual simple will be an upper bound of the (trivial) chain containing only itself. Every other chain will have an upper bound, by unrestricted fusion. Hence, we will have a maximal element of the world—in this case, perhaps lots![32] But the existence of such maximal elements means that the world cannot be a junky world. Note well: this is all perfectly compatible with the failure of the universal quantifier to be absolutely general.

Again, what is not given, and what is required for the argument, is a reason to think that, for any plurality, there is always a more expansive plurality containing it, i.e. $\forall xx \exists yy(xx$ are properly among $yy)$. This is strikingly close to the assumption of junk, i.e. $\forall x \exists y(x < y)$.[33] The best way forward, I think, would motivate the failure of unrestricted plural quantification via *indefinite extensibility*. That is, whenever we have 'all things' at one level, we can construct some new thing which has all previous things as proper parts. So, at each level α (where α is an ordinal) in the construction, there is some further thing at level $\alpha + 1$ which failed to be quantified over at level α. This *typed* approach is seen clearly by indexing our quantifiers. Thus, a kind of 'junk' becomes expressible:

Typed Junk. $\forall_\alpha x \exists_{\alpha+1} y(x < y)$

Likewise, the corresponding version of 'unrestricted' composition becomes this:

Typed Composition. $\forall_\alpha xx \exists_{\alpha+1} y \, F(y, xx)$

On this view, junk is compatible with typed composition. However, one should immediately begin to wonder whether this typed version of composition is really *unrestricted* after all. How are we supposed to admit that our quantifiers *fail* to be unrestricted, and yet to accept that composition *is* unrestricted if they exhibit the very same phenomena? We might say that junk is still incompatible with *absolutely* unrestricted composition; and we might reword premise (i) accordingly.

[32]Again, this follows by an application of Zorn's Lemma.
[33]Where $\preccurlyeq$ is the symbol for the plural 'are among' relation, the denial of 'all things' is logically equivalent to $\forall xx \exists yy(yy \not\preccurlyeq \mathrm{xx})$. This is structurally analogous to something entailed by the definition of junk, but which does not itself entail junk: namely, $\forall x \exists y(y \not\leq x)$.

Of course, the reply will come: 'But there is no such thing as absolutely unrestricted composition—some variant of our typed composition is the best anyone can do!' I am not sure how to adjudicate this debate over whether or not something counts as 'unrestricted'. The issues are too complex to delve into here. But I'll just note that, if the reply is correct, then it depends on the arguments for the metaphysical possibility of indefinite extensibility, and on the corresponding sort of junk generated from that possibility. And so it still seems to me to boil down to a case of requiring a robust defence of premise (ii).

6. Concluding Remarks

I have argued that the junk argument can be modified to apply to virtually any mereology—extensional or not—that accepts unrestricted composition of any sort. I've shown that the retreat to the weak version of mereological universalism is not a promising move. Either it requires more widescale revisions to mereological structure, or it is explicitly a form of restricted composition. Moreover, I have argued that rejecting absolutely general quantification does not (at least by itself) ensure the possibility of junky worlds. One might allow for inexpressible gunk, but this option goes well beyond anything that premise (ii) provides. Or if one expresses junk and composition via type restrictions, then it's unclear that the view really does represent a form of unrestricted mereological composition. I conclude that premise (i) of the junk argument is undefeated. The case against universalism rests entirely on premise (ii).[34]

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REFERENCES

Bohn, Einar D. 2009a. An Argument Against the Necessity of Unrestricted Composition, *Analysis* 69/1: 27–31.

Bohn, Einar D. 2009b. Must There Be a Top Level? *The Philosophical Quarterly* 59/235: 193–201.

Bohn, Einar D. 2010. The Necessity of Universalism vs. The Possibility of Junky Worlds: A Rejoinder, *Analysis* 70/2: 296–8.

Bostock, David. 1979. *Logic and Arithmetic: Rational and Irrational Numbers, Vol. 2*, Oxford: Clarendon Press.

Caplan, Ben, Chris Tillman, and Pat Reeder. 2010. Parts of Singletons, *The Journal of Philosophy* 107/10: 501–33.

Contessa, Gabriel 2012. The Junk Argument: Safe Disposal Guidelines for Mereological Universalists, *Analysis* 72/3: 455–7.

Cotnoir, A.J. 2010. Anti-Symmetry and Non-Extensional Mereology, *The Philosophical Quarterly* 60/239: 396–405.

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Cotnoir, A.J. 2013. Strange Parts: The Metaphysics of Non-Classical Mereologies, *Philosophy Compass* 8/9: 834–45.
Cotnoir, A.J. 2014 (forthcoming). Does Universalism Entail Extensionalism? *Noûs*.
Cotnoir, A.J. and Andrew Bacon 2012. Non-Wellfounded Mereology, *Review of Symbolic Logic* 5/2: 187–204.
Davey, Brian A. and Hilary A. Priestly 2002. *Introduction to Lattices and Order*, 2nd edn, Cambridge: Cambridge University Press.
Donnelly, Maureen 2011. Using Mereological Principles to Support Metaphysics, *The Philosophical Quarterly* 61/243: 225–46.
Forrest, Peter 2002. Non-Classical Mereology and Its Application to Sets, *Notre Dame Journal of Formal Logic* 43/2: 79–94.
Gilmore, Cody forthcoming. Quasi-Supplementation, Plenitudinous Coincidentalism, and Gunk, in *Substance: New Essays*, ed. Robert K. Garcia, Munich: Philosophia Verlag.
Hovda, Paul. 2009. What is Classical Mereology? *Journal of Philosophical Logic* 38/1: 55–82.
Kearns, Stephen 2011. Can a Thing be Part of Itself? *American Philosophical Quarterly* 48/1: 87–102.
Kleinschmidt, Shieva 2011. Multilocation and Mereology, *Philosophical Perspectives* 25/1: 253–76.
Leibniz, G. W. 1996 [1765]. *New Essays on Human Understanding*, ed. Peter Remnant and Jonathan Bennett, Cambridge: Cambridge University Press.
Lewis, David 1991. *Parts of Classes*, Oxford: Basil Blackwell.
Linnebo, Øystein, Geoffrey Hellman, and Stewart Shapiro (unpublished manuscript). *An Aristotelian Continuum*.
Rea, Michael C. 1998. In Defense of Mereological Universalism, *Philosophy and Phenomenological Research* 58/2: 347–60.
Rea, Michael C. 2010. Universalism and Extensionalism: A Reply to Varzi, *Analysis* 70/3: 490–6.
Sanford, David H. 1993. The Problem of the Many, Many Composition Questions, and Naïve Mereology, *Noûs* 27/2: 219–28.
Schaffer, Jonathan. 2010. Monism: The Priority of the Whole. *ThePhilosophical Review* 119: 31–76.
Sider, Theodore 2007. Parthood, *The Philosophical Review*116/1: 51–91.
Simons, Peter M. 1987. *Parts: A Study in Ontology*, Oxford: Clarendon Press.
Smith, Donald 2009. Mereology without Weak Supplementation, *Australasian Journal of Philosophy* 87/3: 505–11.
Spencer, Joshua. 2012. All Things Must Pass Away, in *Oxford Studies in Metaphysics: Volume 7*, ed. Karen Bennett and Dean W. Zimmerman, Oxford: Oxford University Press: 67–92.
Thomson, Judith Jarvis 1983. Parthood and Identity Across Time, *TheJournal of Philosophy* 80/4: 201–20.
Thomson, Judith Jarvis 1998. The Statue and the Clay, *Noûs* 32/2: 149–73.
Tillman, Chris and Gregory Fowler 2012. Propositions and Parthood: The Universe and Antisymmetry, *Australasian Journal of Philosophy* 90/3: 525–39.
van Cleve, James 2008. The Moon and the Sixpence: In Defense of Mereological Universalism, in *Contemporary Debates in Metaphysics*, ed. Theodore Sider, John Hawthorne, and Dean W. Zimmerman, Oxford: Blackwell: 221–40.
Varzi, Achille C. 2000. Mereological Commitments, *Dialectica* 54/4: 283–305.
Varzi, Achille C. 2006. Mereology, in *The Stanford Encyclopedia of Philosophy*, ed. Edward N. Zalta. URL=<http://plato.stanford.edu/archives/win2006/entries/mereology/>.
Varzi, Achille C. 2008. The Extensionality of Parthood and Composition, *The Philosophical Quarterly* 58/230: 108–33.
Varzi, Achille C. 2009. Universalism Entails Extensionalism, *Analysis* 69/4: 599–604.
Watson, Duncan 2010. An Argument Against an Argument Against the Necessity of Universal Mereological Composition, *Analysis* 70/1: 78–82.
Whitehead, Alfred N. 1919. *An Enquiry Concerning The Principles Of Natural Knowledge*, Cambridge: Cambridge University Press.