How to make donuts and cut things in half

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In a short unpublished note, Gödel once remarked:

> [A]t least intuitively, if you divide a geometrical line at a point, you would expect that the two halves of the line would be mirror images of each other. Yet, this is not the case if the geometrical line is isomorphic to the real numbers. ([18, p. 3])

Because a division is *exhaustive* the 'center' point must fall either in the left half or in the right. And because a division is *exclusive* this point cannot be in both halves, leaving one half *open* in that it does not contain its boundary, and the other side *closed* in that it contains its boundary.

How strange. Which side is the lucky one? Which side gets to have its own boundary as a part? Any attempt to answer would surely be arbitrary. That is, we feel an intuitive pull toward a certain kind of symmetry: if there is no principled difference between two objects, then there is no principled difference between their boundaries, either. When one object is open and another is closed, there should be some reason as to why. What is strange is not merely that it is *possible* to divide a line in such an asymmetric way, but rather that it is *impossible* to do so symmetrically.

In this essay, I want to explore two possible solutions to this puzzle. The first is a paraconsistent one based on the metaphysics of identity advanced by Priest [17]. The second is a consistent solution that requires massive amounts of co-location. I apply the principles behind these solutions to a number of related topological puzzles, and evaluate their prospects.

1 *How to cut things in half*

To begin, it is worth noting that this puzzle is not merely a puzzle about geometrical lines, but may be generalized two and three (and higher) dimensional material objects. Casati and

Varzi [2] present a related puzzle about continuous material objects in three dimensions: imagining dissecting a solid sphere made of perfectly homogenous matter. Which of the two half-spheres will be (topologically) closed, containing the boundary as a part? Symmetry considerations make it appear arbitrary which way the boundary will go; but it must go one way or the other — it cannot belong to both. (Of course it is possible for part of the boundary to break one way and another part to break the other, leaving the half-spheres partly open and partly closed. But in this case, the symmetry problem simply re-appears at a smaller scale.)

Casati and Vazi [2] go on to suggest a novel solution to the puzzle.

> Which of the two half-spheres will be closed? This is an embarrassing question. But it arises, we submit, only on the basis of an incorrect model of what happens topologically when a process of cutting takes place. Topologically, the cutting of an object is no bloodstained process — there is no question of which severed halves keeps the boundary, leaving the other open and bleeding (as it were). Rather, topologically, the explanation is simply that the outer surface of the sphere is progressively deformed until the sphere separates into two halves [...] Eventually the right and left portions split, and we have two, *each with its own complete boundary*. A long continuous process results in an abrupt topological change. (p. 87)

This strategy recommends thinking of cutting a sphere in half rather as a smooth deformation of the boundary. In Figure 1, we imagine from step 1 to step 2 is a mere deformation of the boundary of the sphere inward toward its center. This process continues until step 3, where the two halves are connected by a single 'hinge' point. Casati and Varzi suggest that the real magic occurs at step 4, the moment when the two halves actually split into two closed entities. It is important to note that this picture of cutting is consistent with *classical mereotopology*, the standard formal theory of parts, wholes, and their boundaries.[1]

[1] By '*classical mereotopology*' I mean the system called *General Extensional Mereotopology with Closure* (GEMTC) by Casati and Varzi [2, p. 59].

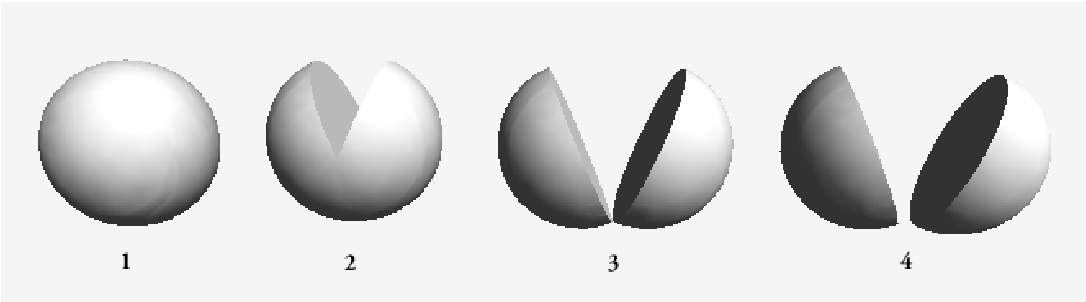

Figure 1: cutting a sphere

They admit that topology cannot deliver a complete explanation of the transition between 3 and 4: "Of course, there is something deeply problematic about the magic moment of separation. [...] [The puzzle] can only disappear on a more complete assessment, which mereotopology simply cannot deliver. (p. 88)" So, even if this were a correct description of the process of cutting, we still would not have a full explanation; in one sense the mystery still remains. But is this even a correct description of what happens?

To my mind, this description leaves out an important aspect of the process. There is not one but *two* important topological changes here. The first change occurs from step 2 to step 3, when the two halves of the object go from being topologically *continuous* ('one can go from one half to the other half without ever leaving the interior') to being merely *contiguous* ('one can get from one half to the other without going through the exterior').[2] The second change occurs from step 3 to step 4, when the halves go from being contiguous to being separated. Both changes, though, are equally mysterious.

The mystery is easily seen if we track two point-sized parts of the object: *a* and *b* on the top and bottom of the sphere (Figure 2). As we can see, at the change from continuity to contiguity (step 2 to step 3) the top surface and bottom surface of the sphere come into contact and hence *a* and *b* merge together to become one single point-sized part. (Alternatively, we might claim that one of *a* or *b* ceases to exist while the other survives. But of course there would be no

[2]These definitions are from Casati and Varzi [2, p. 80].

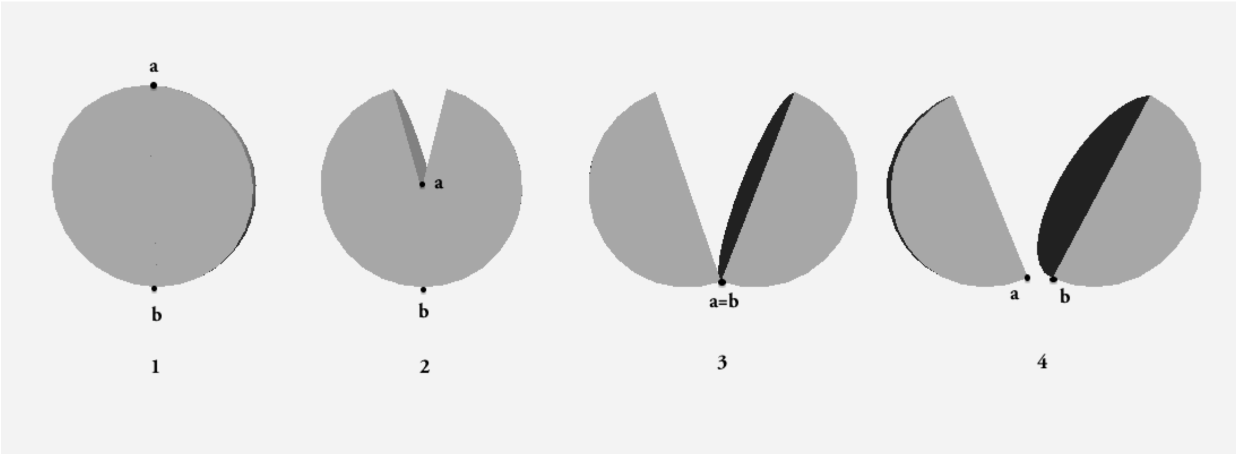

Figure 2: cutting a sphere (cross section)

principled metaphysical explanation as to which is which. It would also be strange for this non-existent point-sized object to pop back into existence again at step 4.) The challenge is to explain how such a change can happen.

If *a* and *b* are a single point-sized part, to which half of the sphere does it belong? If it is part of the left half, then the right half is partly topologically open, and vice versa if it is part of the right half. Here we see the reintroduction of the same asymmetry with cutting a geometrical line. Perhaps the symmetrical thing to say is that it is part of *both* halves, and that the dissection up to this point has not (yet) partitioned the sphere into two disjoint parts.

But then the next step becomes more puzzling. In the change from contiguity to separation (step 2 to step 3), a single point-sized part can either split left or split right, in which case we have the same asymmetry as before. The only symmetric explanation of this separation requires a single point-size part to fission into two distinct point-sized parts, each of which partly comprised the boundary of one of the disjoint halves of the sphere. This fission also cries out for an explanation, and none seems forthcoming.

1.1 *A consistent solution*

I want to recommend an alternative model which gives more intuitive picture on which these mysterious transitions are avoided. A very natural thought is to think that, rather than identifying *a* with *b* at step 3, we ought to accept that *a* and *b* remain numerically distinct point-sized objects that become *co-located* with one another. So, the halves go from being continuous to being contiguous by virtue of parts of their boundaries coinciding — since there is no space separating them there are no interior parts of the sphere separating them, and thus one cannot traverse from one half to the other without leaving the interior. At step 4, of course, these two distinct point-sized parts cease to be co-located. So the halves change from being contiguous to separated by virtue of their boundaries failing to coincide; the space separating them is part of the exterior of the sum of the sphere's halves, and hence one cannot traverse from one half to the other without traversing the exterior. This alternative model offers a simple explanation of the both topological changes, without having to explain how two point-sized objects could become one or how one point-sized object could become two. Such metaphysical fusion and fission never occurs. We only require the possibility of the co-location of distinct point-sized objects. This conception of co-located boundaries has a robust history; it was defended by Brentano [1], revived and developed by Chisholm [3, 4] and formalized by Smith [19].

This model is not available in many standard mereotopologies. For example, it cannot be accepted in systems that define parthood in terms of topological connection thus: *x* is part of *y* iff every *z* connected to *x* is connected to *y*.[3] Since *a* and *b* are co-located they are connected to exactly the same things, and hence must be parts of each other. But then by the antisymmetry of parthood, *a* and *b* are identical. The model also cannot be accepted by anyone who accepts classical mereology together with the following principle of location: *x* is part of *y* iff the location of *x* is a subregion of the location of *y*.[4] On this principle the co-location

[3] These are the SMT systems of Casati and Varzi [2].

[4] This is the mereological harmony principle called 1ρ by Uzquiano [20, p. 204]. Oppenheim and Putnam [9] endorse the principle, as does Markosian [8].

of a and b entails their mutual parthood, which again by antisymmetry yields their identity.[5] This model also cannot be accepted in classical point-set topology which entails that any self-connected object (like the sphere) cannot be partitioned into two disjoint closed objects.[6] But there are well-behaved mereotopologies that accommodate such colocation, chief among them is the formal theory in Smith [19].

1.2 A paraconsistent solution

Paraconsistent solutions to the problems of boundaries have begun to see the light of day.[7] Mereotopology based in paraconsistent logic was first developed in Weber and Cotnoir [21]; the theory allows (*contra* classical mereotopology) for connected objects to be divided exclusively and exhaustively into two closed parts.

More recently, Priest [17] has developed a paraconsistent theory of identity and parthood that can be directly applied to the puzzle about how to cut a continuous object symmetrically in half. Priest is primarily concerned with the metaphysical explanation for the unity of the parts of a given whole. Take an object x, and its parts $y_1, \ldots, y_n$. Since x is genuinely counted as a single *whole* rather than a mere plurality of its parts, there must be something that unifies $y_1, \ldots, y_n$ to constitute x. A very natural answer here is that what unifies the parts to form a whole is a relation: the *composition* relation. Priest himself rejects this answer: as relations in general do not unify, we are owed some special explanation as to why this relation unifies and that seems not too far removed from the very thing we wanted to explain.

So, to provide an explanation, Priest postulates the existence of a special class of objects that constitute this unity, which I will *unity particles* or u-parts for short.[8] A unity particle u

[5]Of course the rejection of the antisymmetry of parthood is available. See e.g. Cotnoir [5].

[6]See e.g. Hocking and Young [7, p. 14].

[7]Early suggestions along these lines can be found in Priest [13, ch. 11].

[8]Priest [17] following his earlier coinage [11] calls them 'gluons'. The name is unfortunate; 'gluon' is already the name for a fundamental particle of physics, the gauge boson for the strong force which responsible for holding matter together by binding quarks into protons and neutrons. Of course, Priest isn't literally talking about gluons in the physicist sense, since his gluons are responsible for the unity of all parts into any whole. Anyway, I avoid all talk of gluons in what follows.

for an object x cannot merely be another (distinct) object alongside its parts $y_1, \ldots, y_n$, since that would simply set off a vicious regress. After all, what makes $u, y_1, \ldots, y_n$ a true unity and not a mere plurality? Another unity particle? No, this would simply defer the explanation for unity indefinitely. Unity particles must be indistinct from the objects they unify.

Priest suggests, then, that a given unity particle is identical to each part of the object it unifies. That is, if u is the u-part for x, then $u = y_1$ and $u = y_2$ and so on. Does this mean that $y_1 = y_2$? Well, it would if identity were transitive; but Priest rejects transitivity of identity. To see why, we need a brief detour through paraconsistent logic.

In paraconsistent logics (like the variant Priest [17, §2.10] uses: second-order *LP*), interpretations of unary predicates include both an extension and an antiextension, where every object in the domain must be in either the extension or the antiextension but it could be in both. Then a sentence like '*Fa*' is true in an interpretation when the denotation of a is in the extension of F, false if the denotation of a is in the anti-extension of F, and both true and false whenever it is in both.

Now, Priest argues at length that identity should be defined in the usual Leibnizian way using the *LP* material conditional.[9]

Identity $x = y$ iff $\forall X(Xx \equiv Xy)$

Note that this is a metalinguistic definition of the identity relation rather than an a biconditional definition stated in the object language; you should read '$a = b$' simply as shorthand for the second-order *LP* formula '$\forall X(Xa \equiv Xb)$'.

The *LP* material biconditional is not transitive; $A \equiv B$ and $B \equiv C$ do not entail $A \equiv C$. (Take a model with A true, B is inconsistent, and C (just) false.) Similarly, identity fails to be transitive as $\forall X(Xa \equiv Xb)$ and $\forall X(Xb \equiv Xc)$ do not entail $\forall X(Xa \equiv Xc)$. (Take a model with exactly one unary property P comprising an extension containing just (the denotations of) a and b, and an antiextension containing just (the denotations of) b and c.)

[9]See §§2.6–2.7 of [17].

This theory of identity allows unity particles to unite the parts of a whole (by being identical to each of them), without forcing each part to be identical with any other part. Of course, if a u-part u is identical with some (consistent) part a, we will have that a and u share all their properties. And if u is identical to another (consistent) part b, they too will share all their properties. But it needn't follow that a and b share all their properties, as u itself may be inconsistent.

Every composite object, then, has a unity particle. Are unity particles unique? Priest [17, p. 20] argues as follows: suppose u and u' are both u-parts of x. Then u and u' are parts of x and so $u = u'$ and $u' = u$. One wonders, though, whether this is sufficient for uniqueness in a robust sense of each composite object having exactly one unity particle. Consider for example, an object a composed of b and c, and call its associated u-part u_a. But suppose further that b is itself composed further of d and e, and call its associated u-part u_b. Then u_b is part of b, and b is part of a. By the transitivity of parthood (accepted by Priest [17, p. 89], u_b is part of a. But then $u_b = u_a$; so the unity particle for and object may be identity to the unity particle for a distinct object, casting doubt on whether identity is sufficient for 'uniqueness'. (The only options here appear to be to reject that u-parts are themselves parts of the objects they unite, or reject the transitivity of parthood.)

Returning now to the puzzle about cutting a sphere. According to Priest's theory of identity, the sphere by virtue of being a unitary object has an associated unity particle, call it u. And point-sized parts a and b of the sphere are each identical to u. When the sphere is transformed from a continuous whole (steps 1 and 2) to a contingous whole (step 3) it is connected by a single 'hinge' particle. But even at step 3, it is still a unified whole, and so u still exists. A natural thing to say is that it is this u itself which is the hinge particle. Indeed, because it already was the case (even at earlier stages) that $a = u = b$, there's no barrier to affirming this at step 3. Now, of course, what does change is whether $a = b$. At steps 1 and 2, $a \neq b$ (compatible with $a = u = b$), after all they are discernible with respect to their spatial location.[10] At

[10]Typically, only purely qualitative properties are thought to feature in Leibniz's Law. As locational properties

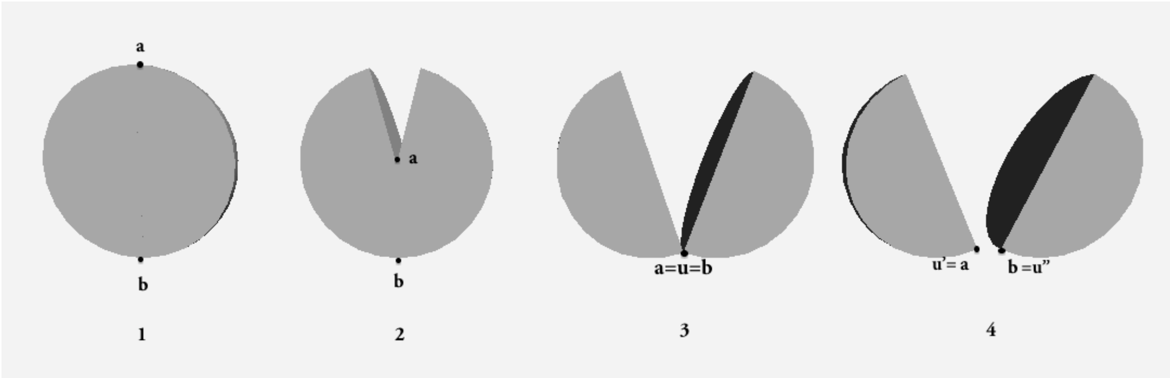

Figure 3: cutting a sphere 2 (cross section)

step 3, Priest can perfectly well affirm that $a = b$; after all, the only prior grounds we had for distinguishing them (their locations) now fail to do so.[11] Insofar as $\forall X(Xa \equiv Xb)$, then the transition from continuity to contiguity (step 2 to step 3) has an explanation.

What about the transition from the sphere going from a contiguous object to two separated objects (step 3 to step 4). The answer must be that because there is no longer a single unified whole, the unity particle u goes out of existence. And so the single hinge particle connecting the two halves of the sphere ceases to connect them because it ceases to be.

Hang on, we might think, what about a and b? Do they still exist? They must, after all both halves of the sphere remain completely bounded — there's no point-sized whole in the boundary of either. But how can u go out of existence when things that were identical to it stick around? I'm not sure I have much to say by way of explanation on this front. But it is not as if this is an unintended consequence of Priest's view of unity — it *is* his view of unity. A u-part unifies some objects by virtue of being identical to them. They cease to be unified when the u-part ceases to be identical to them by virtue of the u-part ceasing to exist. How exactly one *expresses* this ceasing to exist is another question. After all, for any composite object (any object having more than one part) its u-part will be non-self-identical.[12] After all,

aren't purely qualitative, they wouldn't typically be allowed to play a distinguishing role. Priest, however, rejects this restriction ([17, p. 23]) allowing locational differences to play a distinguishing role. (See also [17, §2.7].)

[11] Priest [17, p. 26] accepts that identities are temporary.

[12] See Fact 1, [17, p. 27].

the parts must be distinct by being discernible via some property. But then the u-part must be discernible from itself, and hence non self-identical. But the underlying *LP* logic yields that non-self-identical things do not exist; i.e. $u \neq u$ entails that $\neg\exists x(x = u)$.[13] So, in a sense, u-parts never exist, and as a result it is hard (impossible?) to express the situation when a u-part did exist, when $\exists(x = u)$ goes from being both true and false to being (just) false.

Expressive limitations aside, the view does have something to say about the puzzle, and in particular offers a solution to the problem that does not require point-particles to be co-located or leave their fusion and fission completely unexplained. The solution comes at a fairly heavy cost: accepting inconsistent objects, non-existent objects, non-transitive identity, widespread failures of the substitutivity of identicals, temporary and contingent identity, failures of modus ponens, and more. These costs are completely independent of one another. And Priest has provided further motivations in other work.[14]

2 *How to make a donut*

I want to turn now to a related puzzle involving a different sort of topological change: turning a continuous sphere made of homogenous material into a donut. In keeping with the previous

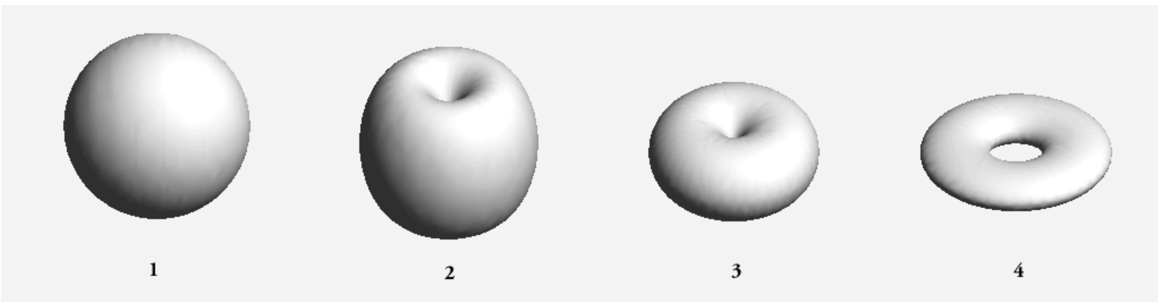

Figure 4: making a donut

example, we can imagine smoothly deforming the surface of the sphere (step 1 to step 2), until

[13]See Fact 2, [17, p. 27].

[14]See e.g. Priest [10, 11, 12, 13, 14, 15, 16].

the top surface and bottom surface meet at a point in the center, forming a 'horn torus' (step 3). This point of connection then 'breaks' as it were (much like the hinge point between the two halves of the sphere) leaving a hole in the middle, forming a 'ring torus' (step 4).

How exactly to make sense of such a change in keeping with the solutions to the previous puzzles is not immediately obvious. The mystery becomes more puzzling when we track points *a* and *b* as before. The old mystery about how *a* and *b* could become identical (at step 3) remains. But the second transition (step 3 to step 4) is made even more puzzling. Here *a* and *b* must split into distinct point parts. But in this case it is not just two boundary points (as before), but an (uncountable) infinite number of them that serve as a boundary around the circumference of the hole. How exactly are we supposed to explain such a radical fission?[15]

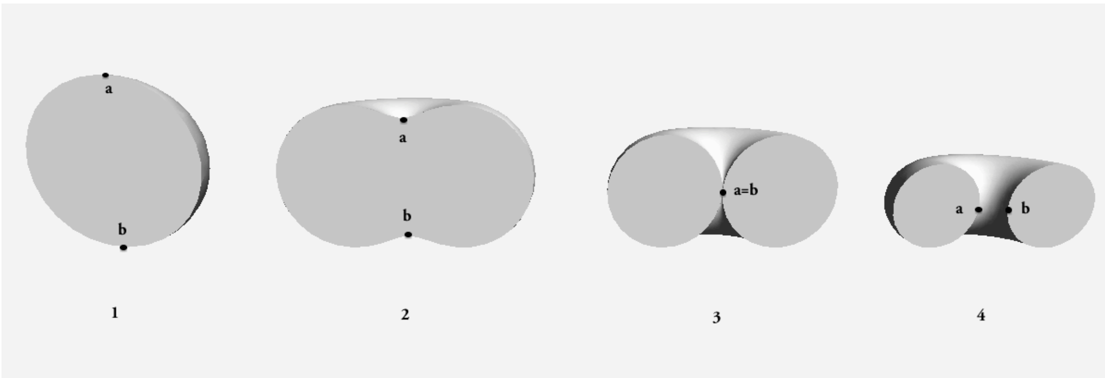

Figure 5: making a donut (cross section)

2.1 *A paraconsistent solution?*

To what extent does the paraconsistent solution generalize? The explanation for the first key transition between steps 2 and 3 ports over nicely. Priest's theory of identity allowed us to explain the fusion between *a* and *b* since at step 3 there is no longer anything to distinguish

[15]Of course there are other ways of making a donut. We could insist that the correct model involves simply punching out a portion of matter from the middle of the sphere, leaving a donut and its remainder (similar to the lumps of dough sometimes sold as 'donut holes'); this method would not be all that different from the model of cutting the sphere in half. The puzzle isn't that there's no way of making a donut, but that there's seemingly no explanation for a very natural way of doing so.

them.

But the explanation for the second key transition between steps 3 and 4 does not transfer very well at all. First, let's try to explain why *a* and *b* 'break' and a hole appears. In the previous example, the explanation for the 'break' was that the sphere's unity particle went out of existence. But as the donut is still a singular object, it would seem that the *u*-part for it still exists. So the explanation for the topological change cannot be that the u-part goes out of existence.

On the other hand, it's possible that the unity particle for the sphere *does* go out of existence, if one takes seriously the thought that the sphere itself goes out of existence *qua* sphere. That is, the unitary object undergoes a substantial change from sphere (at step 1) to donut (at step 4), and so somewhere along the way the unity particle for the sphere is replaced by a numerically distinct unity particle for the donut. The question then arises: when? After all, at step 2 we arguably no longer have a (true) sphere, but something spheroid (maybe it is a deformed sphere?). At step 3 we already have something that can truly be called a torus, but maybe not a donut. One wonders then, what exactly are the identity conditions on unity particles? We are told they come into existence when an unified object does, and go out of existence when the object does. But what sorts of changes can they survive?

Even so, we still do not have a full explanation of how *a* and *b* fuse and then fission (going from non-identical to identical and back). And we also need an explanation of how infinitely (uncountably) many points split from *a*/*b* in order to form a closed loop boundary. This is particularly puzzling, since it would appear the donut gains a lot of new point-sized parts *ex nihilo*. Of course, Priest is not committed to mereological essentialism — objects (and their u-parts) can survive the addition or subtraction of their material parts. The problem is rather a question of where the boundary points come from.

One way around the conclusion that the donut gains a bunch of new parts from nowhere is to simply insist that the new parts are not 'new', but that they are all simply identical to *a* (and *b* for that matter), even at step 4. This would require postulating that the inner boundary of the hole is composed of a single multi-located particle. I suggest this no because it is particularly plausible, but only because it seems to be a natural consequence of Priest's [17, §5.7] account of fission.

> Suppose we have an amoeba. Call it a. At some time t, it divides down the middle to form two new amoebas, b and c. After t, where is a? If it exists at all, it must be either b or c; it has not transmigrated elsewhere. But by symmetry, if it is b it is also c and vice versa. Hence either it has gone out of existence, or it is both b and c. Now it is difficult to suppose it has gone out of existence. It has not, after all, died. So it is both b and c. [...] After t, 'a' denotes b and c, even though these are distinct. Hence, $a = b$ and $a = c$. (For the same reason, $a = a$ is false — as well as true — after t.) Notice that we cannot apply the substitutivity of identity to infer that $b = c$ because, as we saw [...] this principlce of inference breaks down in the case of multiple denotation. [10, p. 369-70]

Because Priest gives up the transitivity of identity (and similar instances of the substitutivity of identicals), he can endorse that each new particle is identical to a after the split. And the transitivity of identity "fails to be truth preserving when the medial object, a in this case, has contradictory properties." [17, p. 67]

I suspect this is the sort of explanation Priest would want to endorse in this case. It fits best with his overall theory of identity and unity. It does raise some questions, however. First, if this is the account of fission that takes place in the puzzle of the donut, why isn't it also what happens between a and b in the original puzzle of the sphere? That is, are a and b identical post-split there too? If so, then contra appearances, the two halves of the sphere have a part in common. Is there ever a way of dividing an object into two non-overlapping parts? If not, then what explains the difference between halving a sphere and making a donut?

2.2 *A consistent solution?*

The consistent solution outlined above can be generalized to the current puzzle. As before, they key idea involves coincident objects. The transition between step 2 and 3 is explained as before: *a* and *b* are two distinct point-sized parts that become co-located. The tricky part is explaining the transition between step 3 and 4. It would appear that we require that there are an uncountable infinitude of co-located boundary points stacked up at *a*/*b*. What reason could there be for that?

The Brentano-Chisholm theory of boundaries supplies us just such a reason.[16] Spatial locations have what Brentano called the *plerosis* or 'fullness' of the point in space. How 'full' a spatial location is depends on how many boundary parts are located there. And how many are located there depends on how many such parts there need to be in order to ensure that everything has a boundary. For example, the spatial point bisecting a line segment could serve as a boundary in two directions, and so there are two *plerotic* parts co-located there. A spatial point on the surface of a sphere (such as the location of *a* in step 1) is potentially a location for a boundary in infinitely many directions. So there are infinitely many point-sized plerotic parts stacked up there. Similarly the location of centerpoint of the horn torus (step 3) could serve as a boundary in infinitely (uncountably) many directions. And so there are also uncountably many plerotic parts co-located at exactly that point. What happens at the transition (step 3 to step 4) — when the donut 'breaks' and a hole appears — is that all of these pre-existing boundary parts spread out into their respective directions, continuing to serve as the boundary particles of their respective objects.

This view raises symmetry considerations anew: what explains which point-part goes where after the split? There are number of options. One could say there's no fact of the matter; it's metaphysically indeterminate which is which. Another option is that it is a brute fact about identity and distinctness. Another option, endorsed by the Brentano-Chisholm view, is that

[16] The following presentation of these ideas owes much to Smith [19].

the underlying asymmetry is explained by the ontological dependence of boundary parts on their wholes. Boundary parts cannot exist except as proper parts of the objects they serve as boundaries for. This is not a brute asymmetry, since the boundary of an object is *defined in terms* of the object and essentially dependent on it. In any case, it's not clear that this kind of 'directionality' is problematic. Or at the very least, it is not the *same* problem as the asymmetries with which we began.

As usual, there are no perfect solutions in philosophy, but there are many good ones. And not all good solutions are equally good. They must be adjudicated on the usual criteria for theory choice. One key criteria: to what extent does the theory require widespread revisions to theoretical commitments in distal parts of philosophy? On this score, at least, it would appear that the paraconsistent view as developed by Priest fares worse than co-location view. After all, the co-location view is consistent, requires no non-existent objects, with a transitive, necessary, atemporal identity relation that satisfies the substitutivity of identicals, and a detachable conditional. Of course, there are many other criteria, and so many other potential applications that a full evaluation cannot be carried out here.

We have seen that there is reason to agree with Dummett [6, p. 505] when he writes,

> The classical model is to be rejected because it fails to provide any explanation of why what appears to intuition to be impossible should be impossible. It allows as possibilities what reason rules out.

How exactly to fix the classical model is up for debate. It seems to me that if we want to cut continuous things exactly in half, we need to allow that point-sized objects can be co-located.

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