

PART I

Introduction and History



1

Composition as Identity

Framing the Debate

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Because, if a thing has parts, the whole thing must be the same as all the parts.

Theatetus 204a

Thus, totality is nothing else but plurality contemplated as unity.

Kant, *Critique of Pure Reason*, Categories of Understanding, III.7

A composite is nothing else than a collection or aggregatum of simple substances.

Leibniz, *The Monadology*, in *The Rationalists* (1960), 455

1 Groundwork: Motivations

Composition is the relation between a *whole* and its *parts*—the parts are said to compose the whole; the whole comprises the parts. But is a whole anything over and above its parts taken collectively?

It is natural to think ‘no’. Consider the following scenario.

Suppose a man owned some land which he divides into six parcels . . . He sells off the six parcels while retaining ownership of the whole. That way he gets some cash while hanging on to his land. Suppose the six buyers of the parcels argue that they jointly own the whole and the original owner now owns nothing. Their argument seems right. But it suggests that the whole was not a seventh thing. (Baxter 1988a, 85)

The land-buyers’ argument seems correct because the parts jointly make up the whole; the parts taken together and the whole are, in some sense, the *same*.

Some philosophers have expressed sympathy with the view of the land-buyers. Frege, in the *Foundations of Arithmetic*, claimed,

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If, in looking at the same external phenomenon, I can say with equal truth ‘This is a copse’ and ‘These are five trees’, or ‘Here are four companies’ and ‘Here are five hundred men’, then what changes here is neither the individual nor the whole, the aggregate, but rather my terminology. (Frege 1980, §46)

The five trees are parts of the copse. A whole battalion may have four companies as its parts; each company may have five platoons as parts; each platoon may consist of twenty-five soldiers. But the five hundred soldiers *just are* the battalion. Take away the soldiers and you have taken away the platoons, the companies, and the battalion. A copse is nothing but a group of trees. A battalion is nothing over and above a group of soldiers.

This idea has a long and complicated history. It was already a view under consideration among the ancients, making an appearance in Plato’s *Parmenides* and *Sophist*.¹ The view’s provenance and influence through the Middle Ages up to the Early Modern period is traced by Normore and Brown in Chapter 2 of this volume.

The intuitive notion that the whole is ‘nothing over and above’ its parts—that the whole is the *same* as its parts—may be clarified by claiming the whole is *identical*, in some sense or other, to its parts. This is the thesis of *composition as identity* (CAI).

But why should this *sameness* be considered *identity*? One intuitive line of thought comes from Armstrong (1978): consider two objects that have a part in common, say Hollywood Boulevard and Vine Street, where their common part is the famed intersection. It is natural to say that Hollywood is *partially* identical to Vine. But of course, we may consider further cases with larger areas of ‘overlap’, such as Seventh Avenue and Broadway. Since Seventh runs diagonally across Manhattan, its common part with Broadway is larger than usual, creating Times Square. Relatedly, large portions of the famous Route 66 were replaced by the I-40; the ‘partial identity’ of the two roads covers significant ground. And, of course, the limiting case of such overlap is just the case where two roads are *wholly* identical to each other. But the continuity between the cases indicates that the limit case is not different in kind. Similarly, each part of a whole is partially identical to the whole. But then shouldn’t we say that the parts taken together are identical to the whole as well?

Besides its intuitive appeal, there are other motivating concerns that lead one naturally to CAI. The first is that CAI can easily explain the particularly intimate relationship between parts and wholes. Sider (2007) notes various natural

¹ See Harte (2002, ch. 2) for details.

principles to which philosophers have been attracted that represent aspects of this intimacy. For example, take the following two:

Inheritance of Location: A whole is located where its parts are located.

Uniqueness of Composition: Any wholes having the same parts are identical.

For CAI theorists, the truth of these principles is no mystery. A whole shares its location with its parts because the whole is identical to its parts. If two wholes have the same parts, then because each whole is identical to those parts, the wholes are identical merely by the transitivity of identity. Perhaps other theories of composition can explain these and similar principles; indeed, Cameron's contribution in Chapter 5 of this volume attempts to do just that. But CAI does so plainly, taking the intimacy at face value.

A second motivation is that CAI satisfies an intuitively plausible *no double-counting* constraint on possible inventories of the world. Consider again an example from Baxter (1988a):

Someone with a six-pack of orange juice may reflect on how many items he has when entering a 'six items or less' line in a grocery store. He may think he has one item, or six, but he would be astonished if the cashier said 'Go to the next line please, you have seven items'. We ordinarily do not think of a six-pack as seven items, six parts plus one whole. (579)

Astonishment at the cashier is justified, we think, because she has counted *the same thing twice*. She has clearly violated the *no double-counting* policy. Of course, in this case (as in the case of the land-buyers above) there are a number of *practical* reasons why one should not count a six-pack as an additional thing over and above the six cans—presumably, one pays for such things all at once. But the general prohibition about double counting is not merely a practical constraint; it is thought to be an *ontological* constraint. And quite a few philosophers have endorsed it. Lewis claimed: 'If you draw up an inventory of Reality according to your scheme of things, it would be double counting to list the [parts] and then also list [the whole]' (Lewis 1991, 81). Likewise, Varzi (2000) argues that, while it is often useful to *countenance* wholes in addition to their parts, this should not be thought of as *counting* wholes in addition to their parts when drawing up our inventory of the world. This is particularly obvious, he claims, when the double counting involves wholes and their *undetached* parts. So, Varzi adopts the following policy:

Minimalist View: An inventory of the world is to include an entity *x* if and only if *x* does not overlap any other entity *y* that is itself included in that inventory.

Varzi then goes on to present various arguments and considerations in its favour.² Similar count policies are also endorsed by Cotnoir (2013) and Schaffer (2010).³

One advantage of CAI is that it makes satisfying such a policy easy. One way objects can overlap is if one is part of the other. But if wholes are *identical* to their parts, then wholes are never counted as distinct from their parts. And so overlapping objects of this sort are never counted as distinct. But there is another way objects can overlap, namely, by sharing a proper part. Suppose that one has managed to include two distinct overlapping wholes x and y in one's inventory of the world, such that neither is a part of the other, but they have some proper part z in common. But if wholes are identical to their parts, one's inventory of the world has included the parts of x as distinct from the parts of y . But the parts of x are not totally distinct from the parts of y , since both include z among them. And this violates the no double-counting rule.

Even though such a counting policy is natural, one might wonder whether it has any deep philosophical motivations. Though there are many possible reasons,⁴ I will only mention two that provide additional motivation for CAI. First is the avoidance of *colocation*. It is a commonplace metaphysical view that two distinct material objects cannot occupy the same region of space time. Wallace (2011a) argues that this thought extends to pluralities of material objects as well: two distinct pluralities of objects cannot occupy the same region of space time, and further one material object cannot occupy the same region as many distinct material objects. According to Wallace, colocation of the latter kind seems just as bad as the more usual kind. But CAI avoids the problem. Because they are *identical*, parts and wholes are colocated, but only in the trivial sense that everything is colocated with itself. Secondly and relatedly, CAI can handle cases of causal overdetermination. Merricks (2003) argues against the existence of wholes, since wholes would be in competition for the causal powers of their parts. But if CAI is true, wholes and their parts are not competitors. Parts collectively cause whatever wholes individually cause, because wholes and their parts are identical.⁵

² See also Berto and Carrara (2009) for objections.

³ In Cotnoir (2013), I suggested that a *count* should partition the universe. Schaffer's constraint, by contrast, applies only to fundamental, or basic, entities:

Tiling Constraint: The basic actual concrete objects collectively cover the cosmos without overlapping.

Schaffer then provides a number of arguments for why the fundamental entities in any ontology must satisfy it.

⁴ For a good start, see Varzi's (2000) and Schaffer's (2010) arguments.

⁵ For (much) more detail on this line of argument, see Wallace (2009), ch. 5.

Another possible philosophical motivation for CAI derives from considerations involving *supervenience*. Consider Armstrong (1997, 12):

The mereological whole supervenes upon its parts. But equally, the parts supervene upon the whole . . . This has the consequence that mereological wholes are *identical* with all their parts taken together. Symmetrical supervenience yields identity.

Armstrong here suggests that there is an important metaphysical interdependence between parts and wholes, and this is best explained as identity. Of course, the two-way supervenience between parts and wholes is controversial, but something the CAI theorist should accept.

Perhaps the major motivation for CAI is that it implies the ‘ontological innocence’ of classical mereology. Classical mereology (to be discussed in detail in §4) is the currently dominant formal theory of parts and wholes. It has been put to many applications, and served as a foundation for a great number of metaphysical theories. But classical mereology is ontologically extravagant; it has as an axiom that whenever there are some things there is a whole composed of them. This ‘universalist’ feature of classical mereology has been a source of much controversy. Proponents of restricted theories of composition often object that there is no such thing as an object composed of, say, the Eiffel Tower and some electron in the President’s nose. But if CAI is true, it would go some way toward alleviating these worries. Witness Lewis (1991, 81–2):

But given a prior commitment to cats, say, a commitment to cat-fusions is not a *further* commitment . . . Commit yourself to their existence all together or one at a time, it’s the same commitment either way . . . In general, if you are already committed to some things, you incur no further commitment when you affirm the existence of their fusion. The new commitment is redundant, given the old one.

And again Armstrong (1997, 12): ‘Mereological wholes are not ontologically additional to all their parts, nor are the parts ontologically additional to the whole that they compose.’

Insofar as one accepts the existence of the Eiffel Tower and that electron in the President’s nose, since the whole made up of them is *identical* to them, one accepts the existence of the object composed of them. Since everything is identical to itself, universalism should come as no surprise. And so, presumably, the CAI theorist can reap all the theoretical benefits of classical mereology without any additional ontological cost. Hawley’s contribution to this volume, Chapter 4, examines this possible motivation in detail. Chapter 3, by Varzi, also addresses this motivation by attempting to reconcile this thought about ontological innocence with a more standard Quinean approach to ontological commitment.

So much for motivations. In §2, I turn to the varieties of CAI that have been developed, and mention some options that have yet to be developed. These varieties are not without problems; in §3, I present some of the main objections to CAI in the contemporary debate. In §§4–5, I present some technical background that is often presupposed in the debate. Axioms and models for classical mereology are given in §4, highlighting relevant theorems along the way. In §5, I present an example of *plural* logic, and discuss some relevant issues involving plural identity and multigrade predicates. All this, I hope, will provide some groundwork and structural support for the excellent and intriguing essays in this volume.

2 Blueprints: Varieties of CAI

On the face of it, CAI seems to be a simple, straightforward thesis that reduces a difficult question, about the nature of composition, to a much easier question about the nature of identity. Most contemporary metaphysicians would agree with Lewis (2001, 192–3) when he writes,

Identity is utterly simple and unproblematic. Everything is identical to itself; nothing is ever identical to anything else except itself. There is never any problem about what makes something identical to itself; nothing can ever fail to be. And there is never any problem about what makes two things identical; two things never can be identical.

But this is quite a modern and philosophically loaded view. One can distinguish at least two different notions of identity: *numerical* identity and *qualitative* identity. Things are numerically identical when they are counted the same. Things are qualitatively identical whenever they have all their properties in common.

But philosophers have taken a stand and generated an orthodoxy: there is no such distinction. The ‘indiscernibility of identicals’ and ‘identity of indiscernibles’ jointly yield that things are numerically identical if and only if they are qualitatively identical.

$$\forall x\forall y(x = y \leftrightarrow (\varphi(x) \leftrightarrow \varphi(y)))$$

Call this biconditional ‘Leibniz’s Law’. Here φ is usually intended as schematic, ranging over any *extensional* predicate. That is, the principle must be restricted so as not to imply the indiscernibility of identicals with respect to *intensional* properties of objects. Leibniz’s Law is often taken to be definitive of identity; if a relation does not satisfy it, by definition it is not an identity relation. Obviously related is the standard elimination rule for identity, the ‘substitutivity of identicals’.

$$\frac{x = y \quad \varphi(x)}{\varphi(y)}$$

But then this rule, combined with the fact that identity is reflexive (e.g. everything is identical with itself), can be used to show that identity is unique; it is not possible for there to be two extensionally distinct relations satisfying Leibniz's Law.⁶ So, on the orthodox view, there appears to be only one way of developing the claim that composition *is* identity.

This is a bit too quick, however, as CAI is a thesis about many things (some parts) being identical to one thing (a whole). As we will see concretely in §5, what is really needed is an identity predicate that takes not only singular terms, but also plural terms. One would also need to generalize Leibniz's Law accordingly. Notice that a whole is a *single* thing, while the parts are *many* things. It appears, then, that the whole and its parts are discernible, at least with respect to their *number*. So one must take some care in formulating the view. Moreover, the orthodox view also says nothing about the modal force of identity; for example, it might be argued that composition is *contingent* identity, to reflect the idea that wholes may survive changes to their parts.

As a result of these complications, there are a variety of ways to develop CAI.

Weak CAI: The relationship between the parts taken collectively and the whole is *analogous* to identity.

Moderate CAI: The relationship between the parts taken collectively and the whole is *non-numerical* identity.

Strong CAI: The relationship between the parts taken collectively and the whole is *numerical* identity.

Lewis (1991) concluded that the difficulties involving generalizing the identity relation and the initial troubles with Leibniz's Law showed that only weak CAI could be maintained. What is needed, then, is a theory of composition that preserves many of the relevant aspects of identity. Sider (2007), in a similar vein, attempts to construct a theory of composition that does just this.

Since moderate CAI takes composition to be non-numerical identity, there could feasibly be as many varieties of moderate CAI as there are variant theories of identity. The inspiration can be put by taking Lewis's words literally, 'Ordinary Identity is the special, limiting case of identity in the broadened sense' (Lewis 1991, 85). For example, Baxter (1988a; 1988b; 1999) sees the number of things as relative to what he calls 'counts'. A six-pack that is one thing in one count may

⁶ The proof, which presupposes classical logic, can be found in Williamson (2006) who cites Quine. But see Schecter (2011) for a theory of multiple identity relations satisfying the substitutivity of identicals based in a weakly classical logic.

be six things in another. More fundamental than the numerical identities within counts is the cross-count identity of what is counted variously. Composition, on this view, is a case of cross-count identity. Similarly, Cotnoir (2013) defends moderate CAI by taking composition to be a generalization of numerical identity; but one that is still an equivalence relation satisfying an appropriately generalized version of Leibniz's Law.

A more radical option is to fly in the face of orthodoxy by claiming that composition is purely qualitative identity, where qualitative identity does not imply numerical identity. Another option, inspired by considerations raised by Butler, and arguably developed in Baxter (1988a), would distinguish between identity in the 'strict and philosophical' sense (i.e. numerical identity) and identity in the 'loose and popular sense', and suggest that composition is the latter. Of course, there is an orthodox weak CAI variant of this view; it will include the thesis that loose identity is not a kind of *identity* at all, but merely one of its analogues. Or, one might have a version of moderate CAI holding that composition is *relative identity* (*à la* Geach 1962). This would reflect the idea that composed objects fall under different sortal predicates than their parts.

The view that has received the most attention is strong CAI. Although many have attributed strong CAI to Baxter (1988a, 1988b, 1999), this is incorrect since Baxter rejects the orthodox view of numerical identity, except within counts. In fact, as Yi (1999) rightly notes, Baxter argues against strong CAI insofar as he thinks that the whole and its parts are never to be included within the same count, and thus would never be numerically identical. Although Sider (2007) does not endorse strong CAI, his work certainly developed it in many ways. Bohn (2009) and Wallace (2011a, 2011b) appear to be the only adherents of strong CAI.

Finally, I have hinted at another important classification that cuts across the weak CAI, moderate CAI, and strong CAI taxonomy. This is the distinction between *count-based* views and *non-count-based* views. Recall that CAI was originally motivated by the idea that there are different ways of counting the *same external phenomena*: the six-pack vs. the six cans, the battalion vs. the five hundred soldiers. Some versions of CAI—the count-based ones—attempt to preserve this basic idea, and thus feature 'ways of counting' prominently. Other versions leave counts by the wayside and develop the view independently. It is notable that many of the defenders of CAI in the literature have endorsed *count-based* theories.⁷ However, many of the arguments against CAI have been levelled against *non-count-based* variants.

⁷ Baxter's (1988a, 1988b) counts are integral to understanding composition as cross-count identity, as they are in Cotnoir (2013). Wallace (2011a, 2011b) has counts feature in her notion of 'relative counting' in order to avoid certain objections.

As is clear, there are a number of rival versions of the thesis; no doubt there are others yet to be invented. Each variant will undoubtedly have strengths and weaknesses over others. It is to those weaknesses that I now turn.

3 Problems: Structural or Superficial?

CAI is not without criticism. Not only the various actual theories, but its very motivations have recently come under attack. This section will provide a brief summary of some of the most prominent objections, without pausing to supply any lines of response. I will start with objections to the motivations, proceed to objections based on linguistic considerations, and close with objections based on metaphysical considerations.

First, Sider (2007) argues that strong CAI does not explain the inheritance of location thesis. The inheritance of location thesis can be read two ways: (i) a whole is (wholly) located wherever its parts (taken collectively) are located; and (ii) a whole is (partly) located wherever its parts (taken individually) are located. Sider thinks that (ii) is the relevant fact to be explained, however, as this concerns a more fundamental relation between a thing and each of its parts. Strong CAI can explain only (i). And so strong CAI can only give an incomplete account of the intimacy of composition.

Second, some have suggested that it is unclear whether accepting CAI justifies a commitment to mereological universalism. Merricks (2005) gives a modal argument that CAI entails universalism; that is, he claims that, if composition is restricted, then CAI is false. The argument, which is too complicated to go into here, turns on the following premises: (i) if CAI is true, it is so necessarily; (ii) if two things are identical, they are so necessarily; and (iii) it is possible that some plurality of objects composes some singular object. Sider (2007) provides a variant of Merricks's modal argument. In addition, Sider gives the following line of argument. For any things, the *x*s, there are some *y*s identical to the *x*s (namely, the *x*s themselves). But the intuitive idea behind strong CAI is that speaking of the many *y*s is equivalent to speaking of them as a single *y*. Substituting yields that for any *x*s, there is some *y* identical to the *x*s, and hence some *y* composed of the *x*s, by strong CAI.

But these arguments have recently come under fire. Cameron (2012) responds by pointing out that strong CAI only establishes that some parts compose a whole iff the parts are identical to a whole. But this does entail that, given some things, they in fact compose a whole; he thus rejects Sider's 'dodgy move' of replacing the *y*s with a single *y* in the argument. He also provides a more detailed response to the Merricks argument. Rather than responding directly to Sider and

Merricks, McDaniel (2009) provides a direct argument against the entailment. He shows that a mereological nihilist—one who accepts that composition *never* occurs—who accepts that extensionally equivalent properties are identical would be forced to accept strong CAI. But since nihilists reject universalism, strong CAI does not entail it. In Chapter 8 of this volume Bohn takes on these arguments and defends unrestricted strong CAI.

Third, some have argued that CAI does not imply the ontological innocence of mereology. The primary arguments are due to Yi (1999). His argument has two parts: (i) the only version of CAI that implies the innocence of mereology is strong CAI; and (ii) strong CAI is false. In favour of (i), Yi provides some level of detail in arguing that Lewis's version of weak CAI does not yield ontological innocence; but I take it he intends these criticisms to apply more generally to variants of moderate CAI as well. In favour of (ii), let Genie be the fusion of Tom and Jerry. Then strong CAI yields that Genie is identical to Tom and Jerry. Since Genie is one of Genie,⁸ we can substitute: Genie is one of Tom and Jerry. But this last claim is clearly false. Koslicki (2008) runs the following argument against the innocence of Lewis's weak CAI. She considers a world with only two objects: *a* and *b*. Universalism implies the existence of an object *c*, the fusion of *a* and *b*. She claims that because $c \neq a$ and $c \neq b$, *c* is a new (possibly objectionable) ontological commitment.

While the strengths of these arguments against the motivations for CAI vary, it is clear that the initial attractions of CAI are not without controversy.

Another class of objections to CAI aim at its commitments to various aspects in the philosophy of language. van Inwagen (1994) is an early example of this type of objection. He claims that CAI theorists cannot state their view grammatically in natural language.

There is the 'is' of (singular) identity. This word makes syntactical sense when it is flanked by singular terms and variables . . . There is the 'are' of (plural) identity. This word makes sense when it is flanked by plural terms and plural variables . . . But what kind of syntactical sense is there in taking either the 'is' or 'are' and putting a singular term or variable on one side of it and a plural term or variable on the other? (1994, 210–11)

Sider (2007) flags this concern as well: 'Grammatical revisionism was perhaps already in place right at the start' (2007, 57). Whether or not one takes this syntactic point seriously,⁹ the correctness of van Inwagen's and Sider's claims depends heavily on the results of the best theories of agreement in the syntax of English. The grammaticality of such sentences is an empirical question; and

⁸ See §5 for a discussion of the plural 'is one of' predicate. Ordinarily, 'is one of' takes a plural term on the right. Yi recognizes this, and suggests a fix.

⁹ van Inwagen seems to, Sider seems not to. See also Cameron (2012), n. 4.

Cotnoir (2013) argues that on at least one linguistic theory of plural agreement such claims are grammatical.

A similar sort of linguistic objection to CAI (particularly, strong CAI) is Sider's (2007) contention that it destroys the usefulness of plural quantification. I will defer discussion of these objections until after plurals are properly presented (§5).

More metaphysically minded objections can be found as well. The most obvious is the objection from Leibniz's Law. Consider Lewis (1991, 87):

[E]ven though the many and the one are the same portion of reality, and the character of that portion is given once and for all whether we take it as many or take it as one, still we do not really have a generalized principle of the indiscernibility of identicals. It does matter how you slice it—not to the character of what's described, of course, but to get the form of the description. What's true of the many is not exactly what is true of the one.

Lewis appears to be right. For example, consider a square divided diagonally into two right-angled triangles. The triangles compose the square, and so according to moderate CAI and strong CAI the triangles are identical to the square. But of course the triangles have the property of *being triangular* and lack the property of *being square*, whereas the square has the property of *being square* and lacks the property of *being triangular*. Yet, according to the orthodox view, if we do not have the indiscernibility of identicals, we do not really have identity. Compare Sider (2007, 57): 'Defenders of strong composition as identity must accept Leibniz's Law; to deny it would arouse suspicion that their use of "is identical to" does not really express identity.'

The objection seems most acute for strong CAI. It is perhaps less acute for more radical versions of moderate CAI, but only at the cost of having to reject (or at least generalize) the orthodox view of identity. Indeed, as is made clear in Chapter 13, Baxter does not generally accept the indiscernibility of identicals, and thinks there are principled reasons for rejecting it in the case of parts and wholes. Turner, in Chapter 12, formalizes these commitments in illuminating ways.

Merricks (1992) suggests a further metaphysical objection to CAI: some versions appear committed to the implausible view that wholes have their parts essentially. The argument, in effect, turns on the idea that, if parts are identical to a whole, and identity is necessary, then the whole is identical to those parts in every possible world. Of course the objection is a problem only insofar as mereological essentialism is; but one might wish to avoid the commitment if one can.¹⁰ One option is a form of CAI according to which composition is *contingent* identity. Another avenue of response involving *modal parts* is pursued by Wallace in Chapter 6.

¹⁰ See also Borghini (2005).

Another recent metaphysical objection to CAI is McDaniel's (2008) argument that strong CAI is incompatible with strongly emergent properties. McDaniel argues that any acceptable version of CAI ought to accept a *plural duplication principle*.

PDP: if the xs compose w , then z is a duplicate of w iff there are some ys that are plural duplicates of the xs and compose z .

But this principle is incompatible with strongly emergent properties, that is, natural properties of a whole that do not locally supervene on the natural properties of its (atomic) parts. Whether there are any such things as strongly emergent properties is controversial. But on the face of it, they do seem to go directly against the CAI theorist's contention that a whole is 'nothing over and above' its parts. But Sider, in Chapter 11 of this volume, shows how a strong CAI theorist might avoid this argument.

Again, there may be lots of ways to respond to these objections. Of course, different versions of CAI might be vulnerable to some objections and not others. I cannot canvass all the combinations and variations here. Some of the various options are explored in the chapters that follow.

4 Foundations: Mereology

We have been using 'part', 'whole', and 'composition' without providing any precise interpretation of these terms. There are a number of formal theories of parts, wholes, and the composition relation that holds between them; but *classical mereology* has been the most influential. It is probably the dominant view among contemporary metaphysicians; indeed, many of the chapters in this volume presuppose it. What follows is a brief introduction to classical mereology.¹¹ I start by presenting a standard axiom system, and proceed to discuss several important theoretical implications of classical extensional mereology that relate to CAI. In the background to mereology, let MAo be any axiom system sufficient for classical first-order logic with identity. In this axiom system, the parthood relation (symbolized by \leq) is the only primitive and must satisfy the axioms MA1–MA3.

MA1. *Reflexivity:*

$$\forall x(x \leq x)$$

¹¹ For a more complete introduction to mereology more generally, the reader should consult Varzi's excellent entry in the *Stanford Encyclopedia of Philosophy*. For more formal details, see Hovda (2009).

MA2. *Antisymmetry*:

$$\forall x \forall y (x \leq y \wedge y \leq x) \rightarrow x = y)$$

MA3. *Transitivity*:

$$\forall x \forall y \forall z ((x \leq y \wedge y \leq z) \rightarrow x \leq z)$$

MA1 says that everything is part of itself; in other words, identity is a limit case of parthood. MA2 says that things that are parts of each other are identical. MA3 says that if something is part of another thing which is part of a third thing, the first is part of the third. MA1–MA3 ensure that the parthood relation is a *partial order*.

We can now define several useful mereological notions:

MD1. *Proper Parthood*:

$$x < y := x \leq y \wedge x \neq y$$

MD2. *Overlap*:

$$x \circ y := \exists z (z \leq x \wedge z \leq y)$$

MD3. *Disjoint*:

$$x \wr y := \neg x \circ y$$

According to MD1, something is a proper part of a whole whenever it is a part distinct from the whole. MD2 says that two things overlap whenever they have a common part. MD3 tells us that two things are disjoint when they have no parts in common.

Given the notion of proper parthood, questions regarding the *decomposition* of objects may arise; for example: if an object has a *proper* part, shouldn't it have another? To guarantee this, one can add a *supplementation* axiom to MA1–MA3. Here is the standard candidate:¹²

MA4. *Strong Supplementation*:

$$\forall x \forall y (x \not\leq y \rightarrow \exists z (z \leq x \wedge z \wr y))$$

One special case of $x \not\leq y$ is when $y < x$; so MA4 tells us, in that case, that if y is a proper part of x , then there is some part of x disjoint from y —call it z . It is

¹² There are at least two other candidates, one weaker and one stronger:

MP1. Weak Supplementation: $\forall x \forall y (x < y \rightarrow \exists z (z \leq y \wedge z \wr x))$

MP2. Complementation: $\forall x \forall y (x \not\leq y \rightarrow \exists z \forall w (w \leq z \leftrightarrow (w \leq x \wedge w \wr y)))$

In the presence of MA1–MA3, MP2 implies MA4 which implies MP1, but none of the converse implications hold. Classical mereologists have favoured MA4 for reasons related to extensionality.

appropriate to think of z as the ‘remainder’ of x when y is removed. It may also be helpful to think of MA4 in its contraposed form: $\forall z(z \leq x \rightarrow z \circ y) \rightarrow x \leq y$. Thus, the axiom guarantees that, if every part of x overlaps y , then x is part of y .

But what is required for the *composition* of objects from others? Importantly, I haven’t yet specified when wholes exist. To do this, we need a definition of *fusion*.¹³

MD4. *Fusion*:

$$Fu(t, \varphi) := \forall y(y \circ t \leftrightarrow \exists x(\varphi \wedge y \circ x))$$

So, t is the fusion of the φ s when t overlaps exactly those things that overlap some φ . As already mentioned, in classical mereology fusions are unrestricted; we need to guarantee the existence of a fusion for every instance of φ with only x free.

MA5. *Unrestricted Fusion*:

$$\exists x \varphi \rightarrow \exists z Fu(z, \varphi)$$

Since we can substitute any suitably open sentence for φ , MA5 is an *axiom schema*; it has infinitely many instances since we have infinitely many suitably open sentences.¹⁴ This fusion axiom guarantees that for every (specifiable) subset of the domain objects, there is an object that overlaps anything that overlaps the members of that subset; that is, we always have a fusion of the members of that subset.

That’s it. MA0–MA5 is the standard axiomatization of classical mereology. To recap, we simply have classical logic (MA0), the partial order axioms for parthood MA1–MA3, a supplementation axiom (MA4), and a fusion axiom schema MA5.

It is worth pausing to notice that classical mereology itself yields some important—albeit controversial—connections between parthood, composition,

¹³ I use the term ‘fusion’ simply as the converse of the term ‘compose’. Wherever x is the fusion of the φ s the φ s compose x . This is in contrast to how some other authors use the term (e.g. van Inwagen (1990) and Varzi (2008), where composition is a relation that holds between non-overlapping objects and a whole).

¹⁴ Because MA5 is an axiom schema, like all first-order theories it will have unintended models. As an illustration, assume there are k -many atoms in our domain. Then a complete Boolean algebra will have size 2^k (subtracting the empty set: $2^k - 1$). If k is finite then the domain is finite, and if k is infinite then the domain is uncountable. In either case, we will not have a countably infinite domain. But by the Löwenheim–Skolem theorems, we know that, if any first-order theory has an infinite model, it will have a countably infinite model. In effect, the axiom schema MA5 fails to quantify over *all* subsets of the domain, but merely the first-order definable ones expressed by open sentences $\varphi(x)$. Formulating MA5 using plural logic avoids these issues.

and identity. In particular, there are several ‘extensionality principles’ that follow immediately from MA0–MA5.¹⁵

EO. *Extensionality of Overlap*:

$$\forall z(z \circ u \leftrightarrow z \circ v) \rightarrow u = v$$

EP. *Extensionality of Parthood*:

$$\exists z(z < u \vee z < v) \rightarrow (\forall w(w < u \leftrightarrow w < v) \rightarrow u = v)$$

UC. *Uniqueness of Composition*:

$$(Fu(u, \varphi) \wedge Fu(v, \varphi)) \rightarrow u = v$$

All three extensionality principles are theorems of classical mereology. In other words, EO states that if two things overlap all the same things, they *are* the same thing.¹⁶ EP states that if two composite objects have the same proper parts, then they are identical.¹⁷ The principle is restricted only to *composite* objects—objects that *have* proper parts—to allow for more than one uncomposed object, or atom.¹⁸ UC claims that if two things are fusions of the φ s, then they are the same thing.¹⁹

Some philosophers have rejected classical mereology on the grounds of extensionality principles.²⁰ But others have regarded extensionality principles as virtues. For example, Goodman (1951) endorses *hyperextensionality* for any type of ‘collection’ (e.g. sets, classes, fusions, etc.): objects built from the same atoms are identical.

A class is different neither from the single individual that exactly contains its members, nor from any other class whose members exactly exhaust this same whole . . . the nominalist recognizes no distinction of entities without a distinction of content. (Goodman 1951, 26)

¹⁵ These principles are so-named due to the parallel extensionality principle of set theory: two sets are identical if and only if they have all the same members.

¹⁶ *Proof*: notice that $\forall z(z \circ u \rightarrow z \circ v) \rightarrow u \leq v$ is logically equivalent to Strong Supplementation (MA4). But then applying antisymmetry (MA2) to two converse instances of that implication suffices to prove EO.

¹⁷ *Proof*: from definitions MD1 and MD2, among non-atomic objects, if x and y have the same proper parts, then anything that overlaps x overlaps y and vice versa. Formally:

$$\exists z(z < u \vee z < v) \rightarrow (\forall w(w < u \leftrightarrow w < v) \rightarrow (w \circ u \leftrightarrow w \circ v))$$

But then, by EO and the transitivity of the logical implication, we have EP.

¹⁸ Suppose any two objects with the same proper parts were identical. Since atoms have no proper parts, any two atoms trivially have all the same proper parts. Thus every atom would be identical to every other atom.

¹⁹ *Proof*: assume that both u and v are fusions of the φ s. From MD5, we have $\forall y(y \circ u \leftrightarrow \exists x(\varphi \wedge y \circ x))$ and $\forall y(y \circ v \leftrightarrow \exists x(\varphi \wedge y \circ x))$. But, again by transitivity of implication, this implies that $\forall z(z \circ u \leftrightarrow z \circ v)$. Now, $u = v$ follows via EO.

²⁰ The literature is rife with objections to extensionality principles. For a start, see the discussion and references in Varzi (2008).

Of course, mereology is independent of these nominalist motivations. But CAI theorists are apparently committed to hyperextensionality for fusions, and indeed extensionality principles EP, EO, and UC. After all, if the whole is *identical* to its parts, then any two wholes composed of the very same parts must be identical to each other, by the transitivity of identity. In that case, then objections to extensionality principles would thereby be objections to CAI. McDaniel, in Chapter 7, explores various options for CAI theorists who reject UC.

5 Foundations: Plurals

Plural constructions are ubiquitous in natural language: ‘My children are loud’ contains the plural description ‘my children’; ‘Abe and Ian are playing with each other’ contains the plural term ‘Abe and Ian’ and the plural pronoun ‘each other’. Of course, these sentences are plural in form only; one could easily recast them so they contain only singular constructions: ‘My first child is loud, and my second child is loud’ or ‘Abe is playing with Ian and Ian is playing with Abe’.

But some sentences involving plurals cannot be recast in this way. Consider the sentence ‘The crowd is loud’ which contains a plural description ‘the crowd’. Attempting to recast would yield, ‘Crowd member 1 is loud and crowd member 2 is loud . . .’ But this is clearly not an adequate paraphrase; after all, a crowd of people may be loud, even if none of the members of the crowd is loud on her own. A more famous example is the Geach–Kaplan sentence: ‘Some critics admire only each other.’ There is no way to translate this sentence using singular quantification.²¹ These *irreducibly plural* constructions can be accommodated in various *plural logics*.

It would appear that merely stating the thesis of CAI necessarily involves a plural formulation. Recall Lewis: ‘The fusion is nothing over and above the [parts] that compose it. It just *is* them. They just *are* it’ (1991, 81). But claims like ‘they are it’ and ‘it is them’ are irreducibly plural. They cannot be reduced to the claim that each individual part is identical to the whole, as that is not what is meant.²² And CAI does not involve the claim that the *set* of parts is identical to the whole. After all, the set of parts is an abstract object, whereas the whole need not be; as Boolos (1984) notes, ‘I am eating the Cheerios’ does not involve my eating a set. This irreducibly plural character of the characteristic identity statements of CAI leads one to believe that CAI is closely bound up with the nature of the logic of plurals.

²¹ See Boolos (1984).

²² But see Baxter, Ch. 13 in this volume.

It will be useful, then, to give a formalization of an example of plural logic. Suppose we start with first-order classical logic with identity. In order to obtain a plural logic, add to our first-order language the following.

- *Plural Variables*: xx, yy, zz, \dots
- *Plural Constants*: aa, bb, cc, \dots
- *Plural Quantifiers*: $\dot{\forall}, \dot{\exists}, \dots$
- *Plural Predicates*: $\dot{F}, \dot{G}, \dot{H}, \dots$
- *Logical Predicate*: $<$

The set of formulas is defined in the usual way, except that ordinary non-logical first-order predicates (e.g. F, G, H, \dots) may only take singular terms and variables as arguments, while non-logical *plural* predicates (e.g. $\dot{F}, \dot{G}, \dot{H}, \dots$) may only take plural constants and variables. Likewise, singular variables must be bound by \forall or \exists ; and plural variables must be bound by $\dot{\forall}$ or $\dot{\exists}$. The set of sentences is merely the standard restriction to those formulas where all occurring variables (if any) are bound.

While I will not provide a full semantics for this language, the main idea is that while singular terms (i.e. singular constants and variables) denote single objects from the first-order domain, plural terms (i.e. plural constants and variables) denote ‘pluralities’ of objects from the first-order domain. A plurality of objects is, intuitively, just some objects. Just as singular predicates are usually interpreted as sets of objects, plural predicates are interpreted as sets of pluralities.²³

The primitive predicate $<$ is meant to represent the ‘is one of’ relation: $a < bb$ is true if the thing denoted by a is *one of* the things denoted by bb . So, $<$ relates singular terms variables to plural terms (e.g. $\dot{\exists}xx\dot{\forall}y(y < xx)$ is well formed). Plural identity $\dot{=}$ is a generalization of standard first-order identity, and may be defined as follows: $xx \dot{=} yy := \forall z(z < xx \leftrightarrow z < yy)$. So $\dot{=}$ takes plural terms in both argument places.

There are some key additional principles governing the standard logic of plurals that one may wish to be satisfied.²⁴

PA1. *Comprehension*:

$$\dot{\exists}y\varphi(y) \rightarrow \dot{\exists}xx\dot{\forall}y(y < xx \leftrightarrow \varphi(y))$$

²³ Giving a full semantics would take us too far afield. The typical semantics, modelling pluralities via sets, is sometimes considered to have an objectionable ontology. But the ontological innocence of plural quantification has been highly controversial. See Boolos (1984), Resnik (1988), and Linnebo (2003) for a start.

²⁴ See Rayo (2007) for motivations and arguments involving these principles.

PA2. *Extensionality*:

$$\dot{\forall} xx \dot{\forall} yy (\forall z (z < xx \leftrightarrow z < yy)) \leftrightarrow (\dot{\varphi}(xx) \leftrightarrow \dot{\varphi}(yy))$$

The plural comprehension principle PA1 states that every satisfiable predicate φ has a corresponding plurality of things that satisfy it. PA1 is restricted to satisfiable predicates since it is typically thought that there is no such thing as the ‘empty’ plurality. PA2 says that any two pluralities having exactly the same things among them have all and only the same plural predicates true of them. Another way of seeing PA2: pluralities that are $<$ -indiscernible are indiscernible *tout court*. Taking seriously the identity of indiscernibles, PA2 implies plural identity of any such xx and yy .

An important fact about the plural logic shown: plural predicates and singular predicates are *distinct*. I noted that some plural predications are irreducibly plural, while others are not. That is, some plural predication is *distributive*: $\dot{F}(aa)$ implies $F(a)$ for each $a < aa$. Some plural predication is *collective*: $\dot{G}(bb)$ is true while $G(b)$ may be false (for some $b < bb$).

On this approach to plural logic, plural predicates like *is loud* are ambiguous: there are two distinct predicates, one of which is *plural* (\dot{L}) and the other *singular* (L). This approach proliferates homonymous predicates. Moreover, since I have not drawn any semantic connections between \dot{L} and L , there is nothing that could validate (or invalidate) the inference from $\dot{L}(aa)$ to $L(a)$. In other words, there is no way to draw the collective/distributive distinction.

One option is to forget about plural predicates like $\dot{F}, \dot{G}, \dot{H}, \dots$ and simply allow our first-order predicates F, G, H, \dots to take either plural or singular terms as arguments. On this approach both $F(a)$ and $F(aa)$ are well formed. The primitive predicate, $<$, could likewise be extended to relate either singular or plural terms to plural terms. As such, it would represent both the ‘is one of’ relation and the ‘are among’ relations. Such predicates are called *multigrade*. Allowing multigrade predicates into plural logic opens up a variety of new issues, too many to explore here.²⁵

Note that the identity claims characteristic of CAI are naturally thought to be multigrade. If the parts (plural) *just are* the whole (singular) we would need an identity relation \approx that could be flanked by a plural term on the left and a singular term on the right ($aa \approx b$). Likewise, if the whole just is the parts, we require \approx to be flanked by plural terms on the right and singular terms on the left ($b \approx aa$). As such, \approx must be multigrade in both argument places in order

²⁵ For an excellent exploration of the history and philosophy of multigrade predicates, see Oliver and Smiley (2004). For the first major contribution to the study of their logic, see Morton (1975).

to express the relevant *many-one* and *one-many* identities.²⁶ Presumably, CAI theorists would want *one-one* identities expressed by $(a \approx b)$ to coincide with the singular identity $=$ of first-order logic. Likewise, *many-many* identities expressed by $(aa \approx bb)$ ought to coincide with the plural identity \doteq of standard plural logic.

These considerations were partly responsible for Lewis's retreat to weak CAI.

I know of no way to generalize the definition of ordinary one-one identity in terms of plural quantification. We know that x and y are identical iff, whenever there are some things, x is one of them iff y is one of them. But if y is the fusion of the xx , then there are some things such that each of the xx is one of them and y is not; and there are some things such that y is one of them but none of the xx is. (1991, 87)

Indeed, whether and how CAI should be formulated in plural logic is an interesting and open question addressed by some of the chapters in this volume.

Philosophers who write on mereology—indeed, even those who endorse CAI—often use *plural* quantification to give the fusion definition and axiom, rather than relying on an axiom schema. For instance:

MD4'. *Plural Fusion*:

$$Fu(t, xx): = \forall y(y \circ t \leftrightarrow \exists x(x < xx \wedge y \circ x))$$

MA5'. *Plural Unrestricted Fusion*:

$$\forall xx \exists z Fu(z, xx)$$

Note that because there is no 'empty' plurality, the fusion axiom may be simplified from MA5, by eliminating its antecedent.²⁷

It is worth highlighting, however, that combining mereology and plural quantification proves to be expressively very powerful. Lewis (1991, 1993) called this combination 'megethology' and shows how it allows one to express hypotheses about the size of the universe, how it (combined with a theory of singleton functions) has the expressive resources of *ZFC* and (with Hazen and Burgess) how to simulate quantification over relations.

But there is some reason to think that if some versions of CAI are true, megethology has nowhere near this sort of expressive power. Indeed, certain varieties of CAI have consequences for plural logic. Yi (1999) suggests that considerations from plural logic rule out CAI as a possible view, and in Chapter 9 in this volume Yi develops and adds to these arguments. Sider (2007) argues that strong CAI has numerous bad consequences for plural logic and otherwise wreaks havoc

²⁶ The concept of many-one and one-many identity is due to Baxter (1988b).

²⁷ Moreover, mereology with MA5' does not suffer the same problem with non-standard models that was noted with MA5.

on the usefulness of plural logic. Primarily, it eliminates the possibility of *distributive* plural predicates, it requires a rejection of plural comprehension (PA₁), and it forces a *collapse* in that ‘is one of’ has to behave exactly like ‘is part of’. The formal and philosophical consequences of these results are explored in Sider’s Chapter 11. Others have suggested that plural logic can come to the CAI theorist’s aid. Cotnoir (2013) argues that the moderate CAI theorist can co-opt considerations from plural logic to provide independently motivated responses to objections. Hovda’s Chapter 10 develops a number of plural languages that are friendly to the CAI theorist. The expressive power of mereology, plurals, and any of the varieties of CAI is an area that is just beginning to be discovered in full detail.²⁸

References

- Armstrong, D. M. 1978. *Universals and Scientific Reasoning*. Cambridge: Cambridge University Press, ii.
- 1997. *A World of States of Affairs*. Cambridge: Cambridge University Press.
- Baxter, D. L. M. 1988a. “Identity in the Loose and Popular Sense.” *Mind*, 97: 576–82.
- 1988b. “Many–One Identity.” *Philosophical Papers*, 17: 193–216.
- 1999. “The Discernibility of Identicals.” *Journal of Philosophical Research*, 24: 37–55.
- Berto, F., and M. Carrara. 2009. “To Exist and to Count: A Note on the Minimalist View.” *Dialectica*, 63(3): 343–56.
- Bohn, E. D. 2009. “Composition as Identity.” Ph.D. thesis, University of Massachusetts, Amherst: <http://scholarworks.umass.edu/open_access_dissertations/92>.
- Boolos, G. 1984. “To Be is to Be the Value of a Variable (or to Be Some Values of Some Variables).” *Journal of Philosophy*, 81: 430–50.
- Borghini, A. 2005. “Counterpart Theory Vindicated: A Reply to Merricks.” *Dialectica*, 59: 67–73.
- Cameron, R. P. 2012. “Composition as Identity doesn’t Settle the Special Composition Question.” *Philosophy and Phenomenological Research*, 84(3): 531–54.
- Cotnoir, A. J. 2013. “Composition as General Identity.” In D. Zimmerman and K. Bennett (eds), *Oxford Studies in Metaphysics*. Oxford: Oxford University Press, viii. 295–322.
- Frege, G. 1980. *The Foundations of Arithmetic: A Logico-Mathematical Enquiry into the Concept of Number*, ed. J. L. Austin. 2nd edn. Evanston, IL: Northwestern University Press.
- Geach, P. T. 1962. *Reference and Generality*. Ithaca, NY: Cornell University Press.
- Goodman, N. 1951. *The Structure of Appearance*. Cambridge, MA: Harvard University Press.
- Harte, V. 2002. *Plato on Parts and Wholes: The Metaphysics of Structure*. Oxford: Oxford University Press.

²⁸ Thanks to Don Baxter, Doug Edwards, Elisabetta Lalumera, Federico Luzzi, Aidan McGlynn, and Giacomo Melis.

- Hovda, P. 2009. "What is Classical Mereology?" *Journal of Philosophical Logic*, 38(1): 55–82.
- Inwagen, P. van. 1990. *Material Beings*. Ithaca, NY, and London: Cornell University Press.
- 1994. "Composition as Identity." In J. E. Tomberlin (ed.), *Philosophical Perspectives*. Atascadero, CA: Ridgeview, viii: 207–20.
- Koslicki, K. 2008. *The Structure of Objects*. Oxford: Oxford University Press.
- Lewis, D. 1991. *Parts of Classes*. Oxford: Basil Blackwell.
- 1993. "Mathematics is Megethology." *Philosophia Mathematica*, 1: 3–23.
- 2001. *On the Plurality of Worlds*. Oxford: Blackwell.
- Linnebo, Ø. 2003. "Plural Quantification Exposed." *Noûs*, 37: 71–92.
- McDaniel, K. 2008. "Against Composition as Identity." *Analysis*, 68(2): 128–33.
- 2010. "Composition as Identity does Not Entail Universalism." *Erkenntnis*, 73(1): 97–100.
- Merricks, T. 1992. "Composition as Identity, Mereological Essentialism, and Counterpart Theory." *Australasian Journal of Philosophy*, 77(2) (June): 192–5.
- 2003. *Objects and Persons*. Oxford: Oxford University Press.
- 2005. "Composition and Vagueness." *Mind*, 114: 615–37.
- Morton, A. 1975. "Complex Individuals and Multigrade Relations." *Noûs*, 9: 309–18.
- Oliver, A., and T. Smiley. 2004. "Multigrade Predicates." *Mind*, 113: 609–81.
- Rayo, A. 2007. "Plurals." *Philosophy Compass*, 2: 411–27.
- Resnik, M. 1988. "Second-Order Logic Still Wild." *Journal of Philosophy*, 85: 75–87.
- Schaffer, J. 2010. "Monism: the Priority of the Whole." *Philosophical Review*, 119(1): 31–76.
- Schecter, J. 2011. "Weakly Classical Theories of Identity." *Review of Symbolic Logic*, 4(4): 607–44.
- Sider, T. 2007. "Parthood." *Philosophical Review*, 116: 51–91.
- Varzi, A. 2000. "Mereological Commitments." *Dialectica*, 54: 283–305.
- 2006. "Mereology." In Edward N. Zalta (ed.), *The Stanford Encyclopedia of Philosophy*: <http://plato.stanford.edu/archives/win2006/entries/mereology>.
- 2008. "The Extensionality of Parthood and Composition." *Philosophical Quarterly*, 58: 108–33.
- Wallace, M. 2009. "Composition as Identity." Ph.D. thesis, University of North Carolina, Chapel Hill.
- 2011a. "Composition as Identity: Part I." *Philosophy Compass*, 6(11): 804–16.
- 2011b. "Composition as Identity: Part II." *Philosophy Compass*, 6(11): 817–27.
- Williamson, T. 2006. "Absolute Identity and Absolute Generality." In A. Rayo and G. Uzquiano (eds), *Absolute Generality*. Oxford: Oxford University Press, 369–90.
- Yi, B. 1999. "Is Mereology Ontologically Innocent?" *Philosophical Studies*, 93: 141–60.